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A Methodology for Design of a Linear Referencing System for Surface Transportation

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Abstract

The transportation community has recently placed significant emphasis on development of data models, procedural standards, and policies for management of linearly-referenced data. There is an Intelligent Transportation Systems initiative underway to create a spatial datum for location referencing in one, two, and three dimensions. Most recently, a call was made for development of a unified linear referencing system to support public, private, and military surface transportation needs.

Before a unified linear referencing system can be produced, it must be designed. Before it can be designed, a design methodology must be developed. The linear referencing systems in use today were never designed. They merely evolved. An appropriate design methodology must provide supportable assurances that the linear referencing system will meet the accuracy requirements of users.

Such a methodology for design of the linear referencing system was developed from geodetic engineering principles and techniques used for designing geodetic control networks. The method is founded upon the law of propagation of random error and the statistical analysis of systems of redundant measurements, used to produce best estimates for unknown parameters. A complete mathematical development is provided. Example adjustments of linear distance measurement

systems are included. The classical orders of design are discussed with regard to the linear referencing system. A simple design example is provided. A linear referencing system designed and analyzed with this method will not only be assured of meeting the accuracy requirements of users, it will have the potential for supporting delivery of error estimates along with the results of spatial analytical queries.

Modeling considerations, alternative measurement methods, implementation strategies, maintenance issues, and further research needs are discussed. Recommendations are made for further advancement of the unified linear referencing system concept.

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Executive Summary

The significance of linearly-referenced information to the operations of transportation agencies is being increasingly recognized. A number of agencies have recently begun to establish internal policies and procedures targeted at ensuring integrity and providing consistency in linearly-referenced information. Furthermore, there is an emerging need to provide linkages between linearly-referenced data and data referenced in higher dimensions. At the same time, there is a national-level initiative to develop a spatial datum for ITS that supports location referencing in one, two, and three dimensions. This initiative provides a significant opportunity for transportation agencies at state and local levels to develop linear referencing system components and integrate them with the ITS datum. Ultimately, it should be possible to create a unified linear referencing system for public, private, and military purposes. A unified system requires a consistent conceptual model and a rigorous design methodology.

Recent advances in linear referencing system data modeling are leading to a developing consensus on the need for a linear datum, very similar to the ITS datum, which supports multiple networks, multiple cartographic representations, and multiple linear referencing methods. Many transportation agencies have spatial databases at different scales that cannot be integrated and many different methods for determining linear locations that cannot be integrated. The linear datum provides the sought-after integration and transformation mechanism.

The linear referencing systems in place today were never designed. They merely evolved. No statistically-supportable statements can be made about their abilities to support the accuracy requirements of their users. This unfortunate circumstance does not hold for geodetic referencing systems and mapping. Geodetic referencing systems are designed with methods that ensure their positional integrities. Classification standards for accuracy and specifications for measurement procedures provide assurances of reliability to users of these systems. Similarly, there are well-established testing standards, for the positional accuracies of maps, that serve as a basis for product development. Similar methods and standards must be developed for the linear referencing system if it is to become a reality.

The design methodology, developed herein for a linear referencing system is based upon the mathematical and statistical principles of geodetic referencing system design and analysis. The methodology is driven by user requirements for accuracy in event locations, expressed as a variance-covariance matrix in linear referencing system parameters. This matrix is a by-product of a least squares adjustment of a system of measurements related to the unknown parameters. In the design process, the matrix is specified *a priori* and used in reverse fashion to determine an optimum configuration for the referencing system and the necessary accuracies of the measurements to be made. This information can then be used to develop specifications for selecting datum and reference objects and for making measurements in the field.

A linear referencing system designed and analyzed in this manner is assured, at a statistically-based level of confidence, of meeting the accuracy requirements of users. The design method is statistically rigorous both globally and locally. The method produces not only system-wide measures of reliability but also a means for determining the maximum size of an undetectable gross error in each individual measurement. Uncertainties in system parameters and measurements can be propagated through functions of linearly-referenced data using mathematical expressions. In this way, it should be possible to report estimates for error along with the results of spatial queries against linearly-referenced data.

Of particular interest is the third-order design problem which addresses the development of linkages between systems. Linkages between components under different jurisdictions must be designed if the linear referencing system is to be unified. Third-order design also addresses the incorporation of new information in, and deletion of old information from, an existing system. When an alignment changes, decisions must be made concerning which new datum and reference objects to create and what new measurements to make, while upholding the global and local integrities of the system.

An incremental approach to linear referencing system development could be most cost effective. A comprehensive initial design would yield the ultimate system configuration and measurement needs. Advantage could then be taken of on-going data collection efforts, that, with some refinement, could be used to make many of the necessary measurements. “As-built” information provides a potential resource for measurement data, but its consistency, currency, and accessibility must be assessed. Emerging technologies such as high-resolution satellite imagery, synthetic aperture radar, and feature extraction techniques could potentially provide low-cost, high-accuracy spatial data over large extents. They should be monitored for applicability to the linear referencing system problem.

The following recommendations address further advancement of the unified linear referencing system concept:

Technical Recommendations

1) Resources should be devoted to development of a design tool. Such a tool should allow the designer to interact with a spatial representation of the transportation network, testing various configurations of datum and reference objects, various systems of measurements among those objects, and various accuracies of the measurements against a specified variance-covariance matrix for the linear referencing system parameters.
2) The design method should be tested, preferably after development of the design tool. The test should include national, state, and local levels of the linear referencing system hierarchy. It should be performed over a limited extent, perhaps at the district level, incorporating components of the ITS datum, state highways, and local roads. At least one municipality should be included.
3) The test design should be partially implemented, with a subset of the datum and reference objects actually selected. The designed measurements among this subset of objects should be made and analyzed. The results should be compared to the design criteria.
4) The model for linear distance error, $\sigma_l = (k^2 + ppm^2)^{1/2}$, should be calibrated and validated for various measurement methods and equipment, including calibrated odometers and GPS.
5) The status of “as-built” information at the state and local levels should be assessed. Its potential for use is very high if it is accurate, consistent, current, and accessible.
6) The long-needed study that links spatial data accuracy requirements to risk in decision making should be undertaken. The assumption that the accuracy of linearly-referenced

data should be compatible with that of two-dimensional spatial databases might not be valid. Perhaps there are aspects of linear spatial analysis that require different accuracies.

Institutional Recommendations

1) The federal government should take a proactive role in development of the unified linear referencing system. The ITS datum initiative is at the national level. It does not directly account for linear referencing needs at the state and local levels. A coordinating, standard-setting, and facilitating role should be fulfilled at the federal level. The National Geodetic Survey has traditional federal-level responsibility for civilian location referencing systems. NGS should be encouraged to participate in continuing development of the linear referencing system.
2) State and local agencies should be encouraged to take advantage of the opportunity afforded by the ITS datum initiative. Economies of scale exist when components of the linear referencing system within a jurisdiction will be developed by another authority. Existing partnerships should be strengthened and new partnerships should be pursued. Forums on the integration of ITS and GIS-T should be encouraged.
3) Linear data issues should be incorporated in the on-going standards activities of FGDC. Linear referencing should be included in positional accuracy classification systems. Standards for linear distance measurement should be developed.

1. INTRODUCTION

1.1 Background

The National Cooperative Highway Research Program (NCHRP) 20-27(2) linear referencing system data model was developed in response to a growing awareness for the need to integrate increasing amounts of linearly-referenced data used by the transportation community (Vonderohe, *et al*, 1995). Data integration problems are pervasive, having both internal and external manifestations. They arise from bottom-up, application-specific approaches to data collection and representation — and from lack of consensus on a model for integration. The intent of the location referencing workshop, convened in August 1994 to address this need, was development of a generic linear referencing system data model that would support as many application areas as possible.

The historical approach of bottom-up application development, based on individual linear referencing methods, has resulted in multiple stand-alone databases supporting limited numbers of functions. Yet, the business functions of an organization (*e.g.*, planning, engineering, and operations) or several organizations (*e.g.*, DOT's, transit authorities, and emergency management agencies) have common data needs. These data needs include the locations, conditions, and states of transportation facilities and events that occur along them.

Early link/node network data models supported various network analytical functions (*e.g.*, trip generation and assignment, shortest path, optimum route, location/allocation). Later work defined the need for modeling of linearly-referenced data to support infrastructure management (Fletcher (1987); Dueker (1987); Nyerges (1990)). Ries (1993) and Deighton and Blake (1993) drew upon the concept of Baker and Blessing (1974) that a linear referencing system is a set of office and field procedures that includes multiple linear referencing methods and support for transformations among them. The data model proposed by Ries (1993) went further by including a generic topologic object (link/site) to which were linked not only multiple linear referencing methods but also multiple cartographic representations. This aspect of the link/site model addressed the notion that the same linear facility might be represented on many different maps at many different scales and levels of resolution.

The NCHRP workshop participants drew upon this earlier work and others and derived a linear referencing system data model which supports multiple linear referencing methods, multiple cartographic representations, and multiple network representations. Support for multiple networks considerably extends the range of potential applications (*e.g.*, the optimum network model for transit applications might not be optimum for emergency management).

The NCHRP model supports integration of data through transformations among methods, networks, and cartographic representations by association with a central object referred to as a "linear datum". The linear datum consists of a connected set of anchor sections (each having a single "distance" attribute). The anchor sections have anchor points (each having a "location description" attribute) at their junctions and termini. Anchor sections and anchor points are similar to links and sites, respectively, in the Ries (1993) model. Since publication of the NCHRP linear referencing system data model, it has been incorporated in the GIS-T

Pooled Fund Study Phase B architecture (Fletcher, 1995) and is being considered for adoption by several state departments of transportation.

A number of transportation agencies, through recognition of the significance of spatially-referenced information (especially, linearly-referenced information) to their operations, have recently developed internal policies and standards with regard to the management of location referencing. Among them are the Departments of Transportation of the States of Colorado (Allen and Joy, 1995), Minnesota (MinnDOT, 1992, 1994), Utah (Deighton and Blake, 1993), Washington (Cihon, 1996), and Wisconsin (Ries, 1993; WisDOT, 1996).

In parallel with the data modeling work being done by infrastructure managers, the Intelligent Transportation Systems (ITS) community was also developing models for location referencing and data integration (*e.g.*, Okunieff, *et al*, 1995; Goodwin, *et al*, 1995). More recent ITS work produced a location reference message specification (Goodwin, *et al*, 1996) and a proposed ITS datum for location referencing (Siegel, *et al*, 1996).

Recognizing the commonalities among the linear referencing systems models for infrastructure management and ITS, and drawing upon expertise from the military transportation community, the most recent published work calls for development of a unified linear referencing system with a common linear datum to support the transportation and navigational data needs of civilian government, the military, and the private sector (Fletcher, *et al*, 1996).

Before the linear datum can be implemented it must be designed. Before it can be designed a methodology for its design must be developed. In order for the datum to be usable by all parties, it must be non-proprietary even though many of the applications it will support will be proprietary. The datum must have the precision, accuracy, and resolution necessary to support the needs of the most stringent users, both current and future. In this way, the unified linear referencing system will support not only intra- and inter-agency data needs, but also the products of multiple vendors and the needs of users of multiple mapping systems, ITS or otherwise.

1.2 Objectives

The primary objective of this research is development of a methodology for design of the linear datum, based upon the accuracy requirements of users. A datum design consists of specifications for:

1) Locations of anchor points and anchor sections, and
2) Measuring the distances of anchor sections.

A design methodology consists of rules for developing these specifications. Of particular concern are the required spatial densities of anchor points and anchor sections and the required accuracies of the distances measures.

Implementation of the linear datum also requires the development of guidelines for selecting appropriate anchor points in the field and procedures for preparing location descriptions of anchor points. Of particular concern are the identifiability and recoverability (persistence) of anchor points. Finally, if linear referencing systems are to be linked with systems of higher dimensionality, a means for doing so must be devised. All specifications, guidelines, and procedures must be applicable to initial development of the datum and to its maintenance over time as changes are made to infrastructure.

As the research progressed, it became apparent that design of the linear datum and design of at least the most stringent linear referencing method are inextricably related. Thus, the scope of the research expanded to include simultaneous design of the linear datum and referencing methods, that is, design of the overall linear referencing system.

1.3 Research Approach

The foundation for a linear referencing system design methodology lies in existing theory for design of geodetic referencing systems (vertical, horizontal, and three-dimensional). These three kinds of geodetic referencing systems were characterized, then compared to and contrasted with the linear referencing system.

A model for propagation of error through the linear referencing system was derived. Application of basic geodetic engineering principles to the linear referencing problem led to the conclusion that design of the linear datum is intertwined with design of linear referencing methods. A mathematical model for determination of linear referencing system parameters from systems of redundant measurements was developed. This mathematical model provides the basis for deriving the required accuracy of datum and reference objects from user specifications for the accuracies of event locations.

The orders of geodetic referencing system design were reviewed and related to the linear referencing system design problem. These, and other factors (including the capabilities of measurement technologies), establish the method for deriving optimum anchor section distances and, thereby, optimum anchor point densities.

Desirable characteristics of anchor points are identified through a review of current practice and through development of their roles in both the linear referencing system data model and the real world.

1.4 Organization of this Report

Chapter 2 provides a review of the NCHRP linear referencing system data model, including clarification of one aspect and correction of two others. Additional modeling issues and some implementation issues are discussed. Chapter 3 compares the linear referencing system with geodetic referencing systems in one, two, and three dimensions. Commonalities and differences are identified and described.

Chapter 4 describes the model for propagation of error through functions of linearly-referenced data. Concepts of relative and absolute accuracy are discussed within the context of relating user requirements at the event level to accuracies at the method and datum levels. Chapter 5 provides the mathematical model for derivation of linear referencing system parameters and their accuracies from systems of measurements. It is an extension of a general linear model with specific treatments of distance measurements between reference and datum objects. A number of examples are provided.

Chapter 6 presents the design methodology by reference to the orders of geodetic referencing system design. The discussion includes linkages to referencing systems of higher dimension and models of error in distance measurements. A simple example of linear referencing system design by trial and error is provided.

Chapter 7 discusses a number of design considerations, including implications of the ITS datum initiative, determining user accuracy requirements, guidelines for selection of datum objects, alternative measurement methods, possible implementation strategies, and issues related to maintenance. Chapter 8 includes a summary of the work and recommendations for action.

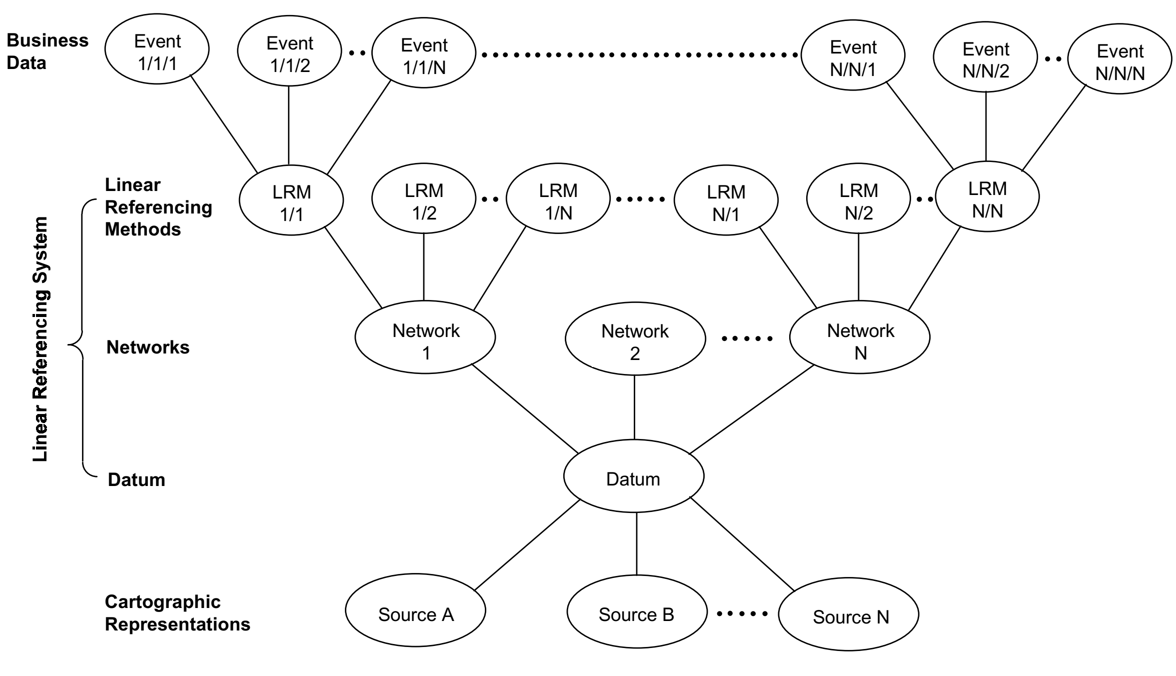
2. THE NCHRP LINEAR REFERENCING SYSTEM DATA MODEL REVISITED

It is apparent, through both a review of the literature and a re-evaluation of the data model by the authors, that a certain issue needs to be clarified and two minor revisions need to be made to the model. This chapter presents a review of the linear referencing system data model as published, provides clarification on the nature of the linear datum, describes an association that must be added to the model to eliminate potential spatial ambiguities, and discusses a necessary revision to the cardinality of an existing association. Treatment of odd-ordered intersections, options concerning bi-directional and multi-lane facilities, and certain implementation issues are also discussed.

2.1 Review

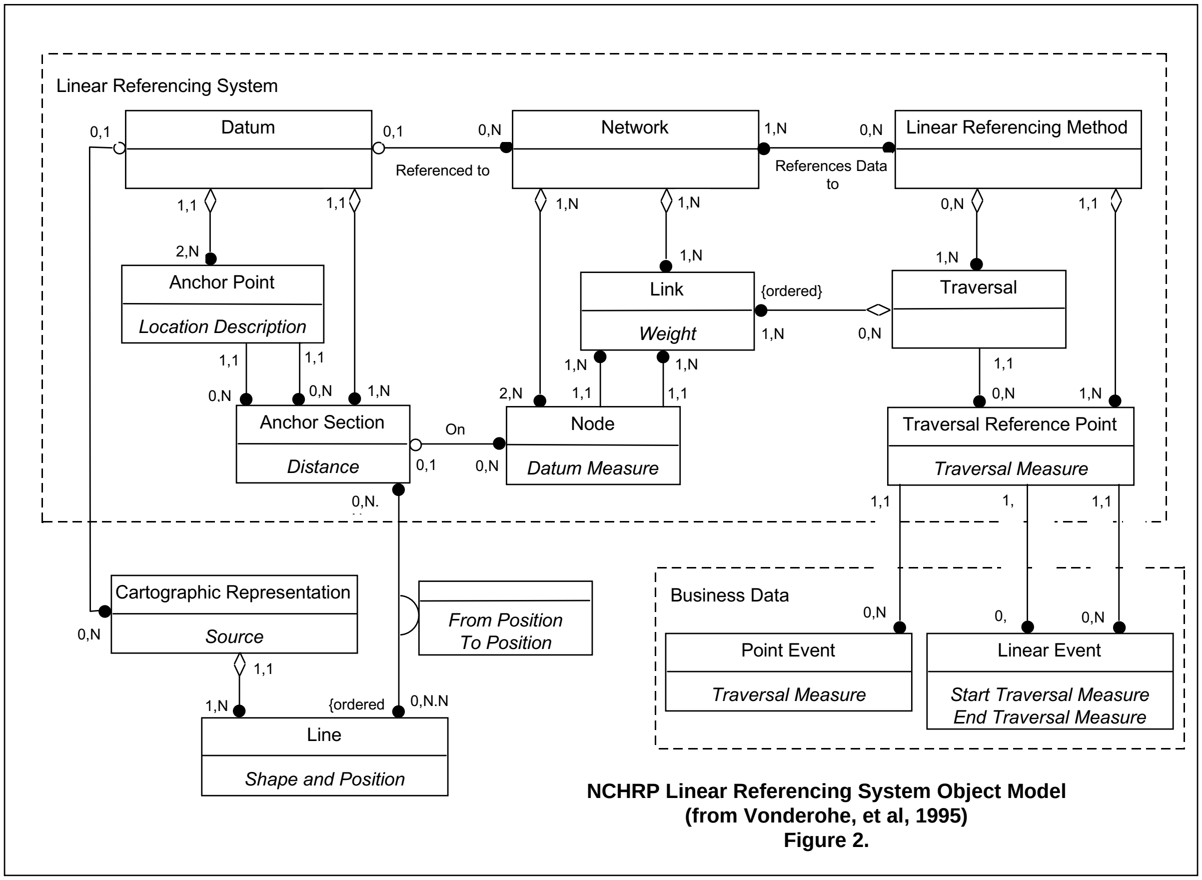
This section is primarily composed of excerpts from the NCHRP report as presented at the 1995 AASHTO GIS-T Symposium (Vonderohe, *et al*, 1995).

Figure 1, a conceptual overview, illustrates the central role of the linear datum, providing a linkage and generic transformation space among multiple cartographic and network representations. Each network, in turn, supports multiple linear referencing methods, with each method serving to locate multiple events.



Conceptual Overview of NCHRP Linear Referencing System Data Model (from Vonderohe, et al, 1995)
Figure 1.

Figure 2 illustrates the object model, using the notation of Rumbaugh *et al* (1991), including object classes and associations with cardinality and optionality.



NCHRP Linear Referencing System Object Model
(from Vonderohe, et al, 1995)
Figure 2.

Definitions of the object classes follow.

Linear Datum

The collection of objects which serve as the basis for locating the linear referencing system in the real world. The datum relates the database representation to the real world and provides the domain for transformations among cartographic representations. The datum consists of a connected set of anchor sections that have anchor points at their junctions and termini. No attributes are assigned to the datum.

Anchor Point

A zero-dimensional location that can be uniquely identified in the real world in such a way that its position can be determined and recovered in the field. Each anchor point has a "location description" attribute which provides the information necessary for determining and recovering the anchor point's position in the field. Forms of location descriptions can vary and be quantitative or descriptive or both.

Anchor Section

A continuous, directed, non-branching linear feature, connecting two anchor points, whose real-world length (in distance metrics), can be determined in the field. Anchor sections are directed by specifying a "from" anchor point and a "to" anchor point. Anchor sections have a "distance" attribute which is the length of the anchor section measured on the ground. Values are expressed in units of linear distance measure (*e.g.*, kilometers).

Cartographic Representation

A set of lines that can be mapped to a linear datum. The set of lines can be either fully or partially connected. That is, the set can consist of groups that are externally unconnected but internally connected.

Cartographic representations have a "source" attribute that denotes the source (scale and lineage) of the object. Scale values are expressed as ratios or as equations that relate distances measured on the source form of the cartographic representation to distances measured on the ground.

Cartographic representations provide coordinate references; the basis for to-scale visualization of other components of the linear referencing system model; and linkages to extended topological, vector-based GIS data models.

Line

"A generic term for a one-dimensional object" (SDTS definition (USGS, 1992)). The Spatial Data Transfer Standard (SDTS) goes on to define five specific kinds of lines: 1) line segment, 2) string, 3) arc, 4) link, 5) chain. A line, as defined herein, can be any of these except a link. This is because lines, as defined herein, have a "shape and position" attribute.

Network

A graph without two-dimensional objects or chains. If projected onto a two-dimensional surface, a network can have either more than one node at a point and (or) intersecting links without corresponding nodes. Note: This is a modification of the definition provided by SDTS. Modification is necessary to exclude chains. Within the context of the linear referencing system data model, a network is an aggregate of nodes and links and is, thus, a purely topological object. The network component of the model provides the basis for analytical operations such as path finding and flow. No attributes are assigned to networks.

Node

A zero-dimensional object that is a topological junction of two or more links, or an end point of a link. Note: This is a modification of the definition provided by SDTS. Modification is necessary to remove reference to chains. In this data model, nodes do not have coordinates. They are located geometrically by reference to the datum.

Each node has a "datum measure" attribute which is used to locate it on an anchor section. "Datum measure" is an offset measured from the "from" anchor point of the anchor section. "Datum measure" is expressed as a distance measure in the same units as the "distance" attribute of the associated anchor section.

Link

A topological connection between two ordered nodes. Note: This is a modification of the definition provided by SDTS. Modification is necessary to require directionality. Each link has a “weight” attribute that is a linear measure of impedance associated with travel along the link. Weights are often expressed in distance measure, but they could be in other linear metrics such as travel time or cost.

Linear Referencing Method

A mechanism for finding and stating the location of an unknown point along a network by referencing it to a known point. Note: This is a modification of the definition provided by Deighton and Blake (1993). All linear referencing methods consist of traversals and associated traversal reference points, that together provide a set of known points, a metric, and direction for referencing the locations of unknown points. No attributes are assigned to linear referencing methods.

Traversal

An ordered and directed, but necessarily connected, set of whole links. Coding conventions are required for establishing traversal directionality (in contrast to link directionality) and for specifying non-connected traversals. No attributes are assigned to traversals.

Traversal Reference Point

A zero-dimensional location along a traversal that is used to reference events along the traversal. Each traversal reference point has a “traversal measure” attribute that is used to locate it along the traversal. “Traversal measure” is an offset measured from the initial node in the traversal to the traversal reference point. It is in the same units as the “weight” attribute of the links in the traversal.

Point Event

A zero-dimensional phenomenon, that occurs along a traversal and is described in terms of its attributes in the extended database. Each point event has a “traversal measure” attribute. “Traversal measure” is an offset measured from the referenced traversal reference point to the point event. Point event traversal measures are in the same units as the traversal measures of the traversal reference points that they reference. A positive point event traversal measure expresses measurement in the direction of the traversal. A negative point event traversal measure expresses measurement against the direction of traversal.

Linear Event

A one-dimensional phenomenon that occurs along a traversal and is described in terms of its attributes in the extended database. Each linear event has “start traversal measure” and “end traversal measure” attributes that locate the linear event along the traversal. The traversal measures are offsets measured from the traversal reference points that they individually reference. Linear event traversal measures are in the same units as the traversal measures of the traversal reference points that they reference. Rules for direction of measurement are identical to those of point event traversal measures.

2.2 Clarification and Necessary Revisions

2.2.1 Clarification of the Datum Object Class

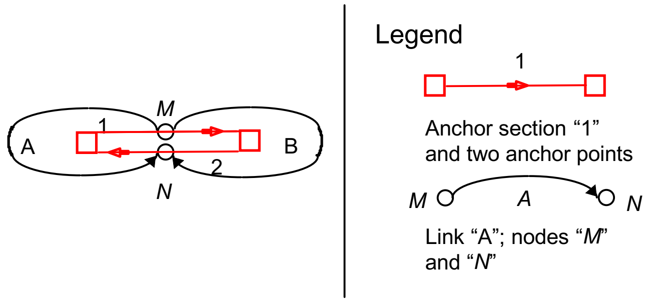
There is some confusion in the GIS-T community concerning the nature of the linear datum, possibly due to inclusion of a "datum" object class in the model (Scarponcini, 1995). At any point in time there is only one datum. It was included in the model as an object class more to indicate its nature as an aggregate of anchor points and anchor sections than to suggest that there are multiple aggregate objects called "datums". A possible utility for including "datum" as an object class is in version tracking. Retrieval of earlier versions of the datum is important for some applications. Temporal datum objects could hold lists of pointers to appropriate anchor points and anchor sections.

The linear datum is a singular construct, representing one "real world." Multiple administrative levels may have jurisdiction over separate components, but there is only one datum underlying the physical transportation facility. Separate authorities might be responsible for distinct components of the datum and associated information. Users might work with the entire datum, or a subset. For example, the national highway system and a city's street system are represented by separate subsets of the datum except where they overlap. There, they are both represented by the same subset.

Cartographic representations exist in higher dimensions than the datum. Multiple cartographic representations may show the same component of the datum in different forms, but it is the same component.

2.2.2 Link - Anchor Section Association

The current data model relates the links of a network to the datum only through their end nodes, which have offsets along anchor sections. As shown in Figure 3, an ambiguity is created when anchor sections form loops. The path between the "from" node (*M*) and the "to" node (*N*) is not unique. Links A and B are both "from *M* to *N*". A many-to-many optional association between object class "link" and object class "anchor section", including specification of link direction in terms of anchor section direction, resolves the ambiguity. As can be seen in Figure 3, the ambiguity remains if only the order of the anchor sections is specified. Links A and B are both on anchor section 1, then anchor section 2 as they go from *M* to *N*. However, only link A is against the direction of anchor section 1, then against the direction of anchor section 2 and only link B is with the direction of anchor section 1, then with the direction of anchor section 2. The overall association is optional because a network can exist without being tied to the datum. However, data referenced to such a network cannot be integrated with data referenced to other networks.

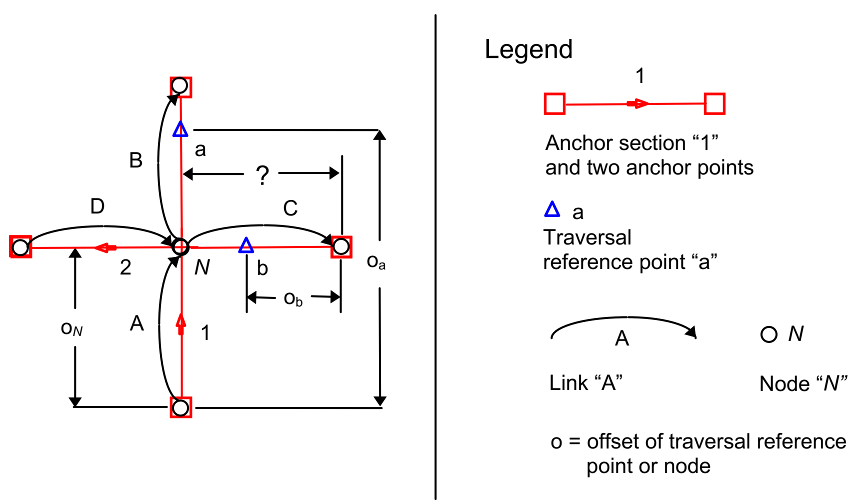


Link - Anchor Section Ambiguity
Figure 3.

2.2.3 Node - Anchor Section Cardinality

The current cardinality / optionality between nodes and anchor sections (a node may be associated with zero or one anchor section) must be changed to zero or many. With reference to Figure 4, anchor sections 1 and 2 cross without a common anchor point. In the network representation, node *N* appears at the junctions of four transport links. Datum addresses of node *N*, traversal reference point a, and traversal reference point b are o_N and o_a on anchor section 1 and o_b on anchor section 2, respectively.

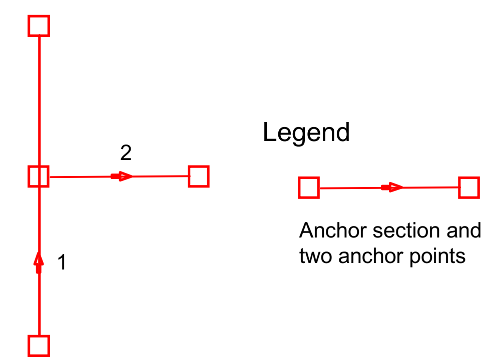
The spatial lengths of links C and D are indeterminate. If a traversal is specified to contain links C and B, the distance between traversal reference points b and a cannot be determined. If the cardinality restriction between nodes and anchor sections is relaxed, an offset for node *N* along anchor section 2 could be included in the database. Although this modification appears to add complexity to the database maintenance problem, it reduces the number of datum objects that must be developed and managed.



Node - Anchor Section Association
Figure 4.

2.3 Odd-Ordered Intersections

Each odd-ordered intersection must include an anchor point. However, as illustrated in Figure 5, all but one anchor section can pass through such an intersection without including the anchor point. The “from” anchor point of anchor section 2 is at the intersection, but it does not “intersect” anchor section 1. This characteristic further reduces the number of datum objects to be developed and managed. It also favorably impacts the database maintenance problem. If the “from” anchor point of anchor section 2 is disturbed, anchor section 1 does not have to be updated.

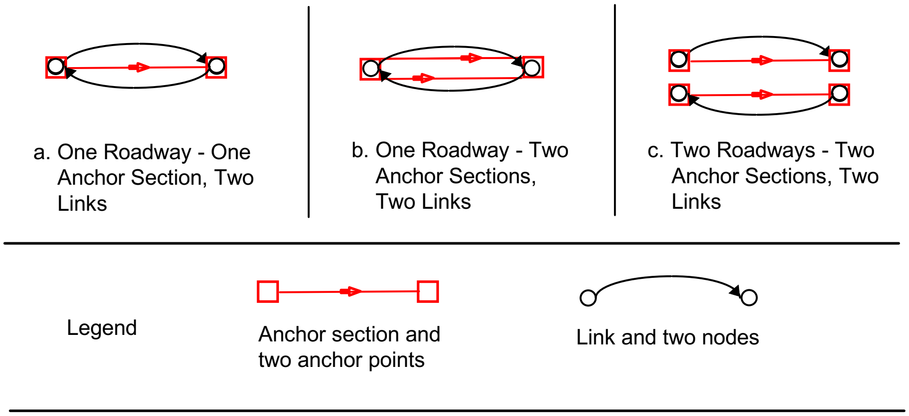


Anchor Point on One Anchor Section at Odd-Ordered Intersection
Figure 5.

This characteristic also places a lower bound on the number of anchor points required in the datum. The minimum number is the sum of the number of termini and the number of odd-ordered intersections. As discussed in Section 6.1.2., other design considerations can cause the number of anchor points to increase beyond this theoretical minimum.

2.4 Bi-Directional and Multi-Lane Facilities

Figure 6 illustrates three choices for representing bi-directional facilities. The objects in Figure 6a could represent a single roadway with traveled ways in both directions. There is an anchor section for the roadway and a link for each traveled way. This choice results in a minimum number of datum objects and has the further advantage that any individual object in the field (real world), serving as a reference point for traversals in both directions, has a single datum location. An example of such an object is the intersection of a roadway centerline with an edge of a bridge abutment. With this choice, spatial distinctions among events must be maintained at the link or traversal level. Otherwise, events of the same kind, which should be associated with opposite directions of travel, risk “collision” in the single datum object.



Choices for Modeling Bi-Directional Facilities
Figure 6.

The objects in Figure 6b could represent a single roadway with traveled ways in both directions. There is an anchor section and a link for each traveled way. This choice has the advantage that spatial distinctions among events are maintained at the datum level. Disadvantages include doubling the number of anchor sections and developing and maintaining two datum addresses for any single object in the field that serves as a reference point for traversals in both directions.

Two anchor sections are necessary in the unlikely occurrence of two traveled ways with lengths differing by an amount large enough to require their resolution. A similar statement can be made for representation of multiple lanes in a single direction. Information on events at the lane level is useful for construction and emergency vehicle routing, signal timing, congestion management, and other operational functions and ITS applications. Fohl, *et al*, (1996) propose representing roadway centerlines as continuous linear spatial objects with start and stop points for lanes located by offsets along them. They argue that theoretical and technological limitations on the accuracy of absolute positioning obviate the need for explicit spatial representation of lanes. In fact, the potential accuracy of relative measures, such as lane length, might be great enough to make metric distinctions among lanes, but the need for doing so from an application perspective probably does not exist.

The objects in Figure 6c represent a divided highway with separate roadways for each direction of travel. Here, each of the roadways has an anchor section. The anchor sections terminate at distinct anchor points and often have different values for the distance attribute. Distinct objects in the field serve as reference points for traversals along the individual roadways.

2.5 Scalability

Scalability is traditionally an issue of cartographic representation and how spatial abstractions are affected by display at different scales. Much research has been done on map generalization and methods for transforming maps from larger to smaller scales. The linear referencing system has a certain level of abstraction, but it has no single cartographic scale. In fact, it has as many cartographic scales as there are cartographic representations with distinct source scales linked to it. This aspect of scalability of the linear referencing system is manifested in the one-to-many association between datum and cartographic representation.

A divided highway, represented by two anchor sections, might appear as a single line on a small scale map and as two lines on a large scale map. The anchor sections would both be associated with the single line for display at small scale. Each of them would be associated with an individual line for display at large scale. Two point events, each with a datum address in a separate anchor section, would appear to be co-linear at small scale and in different lines at large scale. The Wisconsin Department of Transportation has an effective demonstration of this characteristic of the Ries (1993) link/site model.

Zero-dimensional objects in the linear referencing system data model (*e.g.*, anchor points and traversal reference points) are zero dimensional objects on the ground (*e.g.*, intersections of centerlines). Distances in the data model are distances as measured on the ground. Spatial abstraction takes place at the one-dimensional level. Roadways, pavements, and many of the things that happen along them (events) are two- and three-dimensional, yet they are represented as having linear locations. The second and third dimensions are sometimes represented as attributes (*e.g.*, pavement width and thickness). In this manner, the linear referencing system is similar to a surface model that represents elevation as an attribute of horizontal location. This aspect of scalability of the linear referencing system is manifested in the choices discussed in Section 2.4.

Should single roadways with travel in both directions be modeled by one anchor section and two anchor points, two anchor sections and two anchor points, or two anchor sections and four anchor points? The answer lies in what we are truly trying to represent and in what level of the model we select for maintaining spatial distinctions. The former must be addressed from an application perspective. The latter must be addressed from a data management perspective.

2.6 Implementation Issues

Sutton and Bespalko (1995) identify five linear referencing "pathologies" that arise from route definition problems: discontinuous routes, dog-leg routes, split roads, cul-de-sacs, ramps. These are, perhaps, more appropriately characterized as implementation issues rather than modeling issues. This remark is not intended to diminish the significance of these problems. No model is useful unless it can be implemented.

Discontinuous Routes (Traversals)

The authors state that routes may start and stop for various reasons, creating gaps and thus requiring decisions on whether to reference the route as if the "missing" sections were in place or to re-start offset measurements at the start of each new section. They go on to state that the latter approach is usually adopted. It is this latter approach that is included in the model described above. It is important to note that route or traversal designations may change for a number of reasons, but changes in datum addresses of traversal reference points and events occur only when the locations of those objects change or when changes in alignment cause changes in the datum.

Dog-Leg Routes (Traversals)

The authors state that signed routes share common sections of highways. This is modeled by the many-to-many association between links and traversals in Figure 2, above. The authors point out that decisions must be made concerning the assignment of attributes (events) to individual routes along the common sections. With the NCHRP model, event locations can be transformed among traversals at will. Event locations can be stored as datum addresses and transformed into whatever traversal offset (method) is required by the application at hand. Of course, some attributes, such as administrative designation, should be traversal-specific.

Split Roads

The authors suggest that divided highways can have two roadways of unequal length and that this poses a dilemma. This situation should be managed as illustrated in Figure 6c and described in Section 2.4. Each of the two roadways should have separate datum objects, links, and traversal designations. The authors also suggest that another link length problem arises in hilly areas where length is difficult to represent with a planar graph map. The NCHRP model is not based on a planar graph map. The distance attribute of an anchor section is the traveled distance between anchor points.

Cul-de-Sacs

The authors point out that cul-de-sacs can be linearly referenced either clockwise or counter-clockwise and that a convention should be adopted to prevent offsets from being non-uniquely identified. This is true. A number of coding standards must be in place before sharable databases can be developed, but the robustness of the underlying linear referencing system model is not diminished by this need.

Ramps

The authors point out that ramps, which are transitions between routes, must be dealt with in special ways. A number of transportation agencies have adopted ramp coding schemes that uniquely identify ramps and associate them with routes (see, for example, Allen and Joy (1995) or Cihon (1996)). When applying the NCHRP model, a ramp is best represented at the datum level by an individual anchor section, due to the odd-ordered intersections at each end.

These implementation issues described by Sutton and Bespalko (1995), and other issues as yet unidentified, are certain to arise as linear referencing systems are further developed and the need to share data continues to grow. The ITS datum, being proposed at the national level, is both a cause and an opportunity for transportation agencies to plan for and begin designing improvements in, and linkages among, heretofore independent linear referencing systems.

So many implementation issues seem to be based upon the capabilities of existing software. Hopefully, the vendor community will respond to recently-expressed needs with products that overcome some of these problems.

3. LOCATION REFERENCING SYSTEMS COMPARED

There are established referencing systems and datums for locating phenomena of interest in one, two, and three dimensions in support of public, private, and military applications. Table 1 identifies a number of aspects of these systems that serve as a basis for comparison with the linear referencing system.

Name	Dimension	Datum Object	Reference Object	Location Specification	Transformation	Authority
WGS84	3 D	3D Cartesian Axes	GPS Satellite	X,Y,Z	X,Y,Z ⇔ ϕ,λ,η	Dept. of Defense
NAD83	2 D (Horizontal)	Ellipsoid	Horizontal Control Station	ϕ,λ	ϕ,λ ⇔ x,y ϕ,λ ⇔ N,E	National Geodetic Survey / States
PLSS	2 D (Non-Mathematical)	Section Corner	Cadastral Survey Monument	Township, Range, Section, Aliquot Part	None	Bureau of Land Management / Counties
NAVD88	1 D (Vertical)	Geoid	Benchmark	Elevation	None	National Geodetic Survey / States
LRS	1 D (Linear)	Anchor Point / Anchor Section	Traversal Reference Point	Offset along Anchor Section	LRM_i ⇔ LRM_j	

Table 1. Characteristics of Location Referencing Systems

WGS84 is a three-dimensional referencing system, for establishing locations in space, developed by the Department of Defense. NAD83 is a two-dimensional referencing system, for establishing horizontal locations, developed by the National Geodetic Survey and now being managed in cooperation with the states. The Public Land Survey System (PLSS) is a two-dimensional, non-mathematical referencing system, for establishing the locations of real properties, developed by the Bureau of Land Management and now being maintained by counties. NAVD88 is a one-dimensional referencing system, for establishing elevations, developed by the National Geodetic Survey and now being managed in cooperation with the states. The linear referencing system (LRS) is a one-dimensional system for establishing locations along linear facilities. The LRS has no lead authority. However, Fletcher, *et al*, (1996) proposed that the Department of Transportation work in cooperation with the states to develop a unified linear referencing system for the nation. Siegel, *et al*, (1996) proposed and are developing a prototype location referencing datum for ITS.

Each of the location referencing systems in Table 1 have datum objects, reference objects, and location specifications. All but two of them support transformations among various location referencing methods. All of them are similar beyond the characteristics identified in Table 1 in that their designs are, or should be, based upon closed systems of redundant measurements that ensure their spatial integrities to the level required by the user community. These aspects of the systems identified in Table 1 are discussed in more detail below.

3.1 Datum Objects

A datum object provides the basis for location referencing. It is the object to which everything else in a given location referencing system is tied, either directly or indirectly. Datum objects link the referencing system to the real world. They are sometimes defined as abstractions of real objects. In other cases, they are, themselves, physical objects.

The Cartesian coordinate axes of WGS84 and the ellipsoid of NAD83 are mathematical constructs. Both of these are linked to the real world by determining the locations of their origins with respect to the Earth's center of mass and the rotations of their axes with respect to the spin axis of the Earth.

The geoid, that serves as the datum object for NAVD88, is the real gravitational equipotential surface that would be the level of the sea if the Earth's oceans were permitted to flow unrestricted within the continents and be unaffected by tides (Bomford, 1980, pp. 94–95). Within the NAVD88 referencing system, the location of the geoid is estimated by measurements.

A section corner, the datum object of the Public Land Survey System, is a zero-dimensional location at the place of the centroid of the first monument set by the original surveyor. The original monument might be long since destroyed.

Anchor sections and anchor points, the datum objects of the linear referencing system, are low-level abstractions of a transportation facility. They represent the centerline of the traveled way and points along that centerline, respectively. The anchor section representation is solely through identification of "from" and "to" anchor points and the distance as measured along the centerline of the traveled way. The linear datum is linked to the real world through the location description attribute of anchor points.

Datum objects are not dedicated to a singular coordinate system or referencing method, but allow multiple coordinate systems to be imposed upon them. A datum may be designed for a particular type of coordinate system, such as Cartesian axes, or angular coordinates, but there are different possibilities for origin, orientation and scale.

3.2 Reference Objects

Reference objects are those things to which measurements are made such that coordinates for a point of interest may be computed. Reference objects have physically identifiable locations and known coordinate values referenced to the datum. They are the linear referencing system objects that are most familiar to the user community. In fact, the density and physical character of reference objects in a well-designed location referencing system should be derived from the needs of those who will be using the system in the field. The design accuracies of the coordinates of reference objects, and therefore the design accuracy of the datum, should be derived from the needs of those who will be basing decisions upon the analysis of data located by measurements to reference objects.

In three-dimensional satellite navigation systems the reference objects are the satellites themselves. To locate an unknown point, distances are measured to satellites whose coordinates are known with respect to the datum object (WGS84 Cartesian axes). In differential

GPS applications, at least one marked location on the ground also serves as a reference object. Satellite orbital parameters are determined through a redundant ground-based network of fixed tracking stations.

The reference objects for NAD83 and NAVD88 are the monumented stations of the horizontal and vertical control networks, respectively. Within each of these control networks, locations of individual reference objects are computed from an interconnected and over-determined system of measurements.

Within the PLSS, cadastral survey monuments witness the locations of section corners. Surveyors make measurements to the monuments and then use these measurements and other evidence (*e.g.*, the physical character of the monument versus that described in the record) to decide which, if any, monuments should be accepted as representing section corners. A surveyor might form an expert opinion as to the location of a section corner, but its ultimate location is determined in a court of law (Brown, *et al*, 1986, pp. 296-299). A similarity between the PLSS and the linear referencing system is that users measure distances to traversal reference points (reference objects) and not to anchor points (datum objects). A difference is that unlike section corners, whose actual locations are subject to legal decisions, both anchor points and traversal reference points are recoverable in the field. The PLSS includes a redundant system of measurements which link adjacent section corners and which were used to provide initial estimates for the sizes, shapes, and relative locations of sections of land. This original system of measurements serves as the ultimate reference data set for restoring the locations of section corners if all higher forms of evidence fail (BLM, 1974).

3.3 Location Specification

Each location referencing system specifies location in a particular way, with a particular metric. WGS84 uses distance offsets from each of the X,Y,Z axes. NAD83 uses angular measures of latitude (ϕ) and longitude (λ); the latitude of a point being the angle between the ellipsoid normal at that point and the equatorial plane, as measured in the meridional plane containing the point; and the longitude of a point being the angle between the meridional plane containing the point and the meridional plane of zero longitude (Greenwich). NAVD88 uses elevations, which are distances above or below the datum. The PLSS uses a naming convention (Section, Township, Range, Principal Meridian) to distinguish locations. The linear referencing system uses distance offsets along anchor sections.

For all of the location referencing systems except the PLSS, the location of an unknown point can be computed from measurements of direction and distance from a known point; all that differs is the dimensionality of the direction. For WGS84, direction can be specified by direction cosines along the three-dimensional coordinate axes. For NAD83, direction is specified by a two-dimensional azimuth. For NAVD88, direction is either “up” or “down”. For the linear referencing system, direction is either “with” or “against” the direction of a traversal.

3.4 Transformation

Location specifications are such that there is a unique description for each location in a datum. In many cases, the location referencing datum then provides a common basis for additional coordinate systems or location referencing methods, thereby enabling transformations among them based upon calculations through a set of mathematical functions. For example, point locations in WGS84 are often published as both X,Y,Z and ϕ,λ,h (on an ellipsoid nearly identical with that of NAD83). Here, h is height above or below the ellipsoid, measured along the normal, yielding three-dimensional location when given with ϕ and λ. The transformation functions are:

$$\rho = (X^2 + Y^2)^{1/2} \qquad (1)$$

$$\theta = \mathrm{Tan}^{-1}(aZ / b\rho) \qquad (2)$$

$$\phi = \mathrm{Tan}^{-1} ((z + (a^2e^2 / b)\sin^3\theta) / (\rho - a^2e^2\cos^3\theta)) \qquad (3)$$

$$\lambda = \mathrm{Tan}^{-1} (Y/X) \qquad (4)$$

$$h = r / \cos \phi \qquad (5)$$

where a is the ellipsoid's semi-major axis,
b is the ellipsoid's semi-minor axis, and
e is the ellipsoid's eccentricity $((a^2 - b^2)^{1/2} / a)$.

(Hofmann-Wellenhof, *et al*, 1994, pp. 255-257)

With NAD83, ϕ,λ are often transformed into rectangular coordinates (x,y or N,E) on Lambert or Mercator map projections. Such map projections provide the basis for institutionalized coordinate systems such as Universal Transverse Mercator and State Plane Coordinates, the most common coordinate systems used in GIS spatial databases. Transformations between map projections usually involve two steps: 1) x,y on the first projection to ϕ,λ on the datum, then 2) ϕ,λ on the datum to x,y on the second projection. The transformation function requires datum parameters (a,b), map projection parameters (*e.g.*, longitude and scale factor on central meridian), and starting coordinates. The transformed coordinates constitute the output.

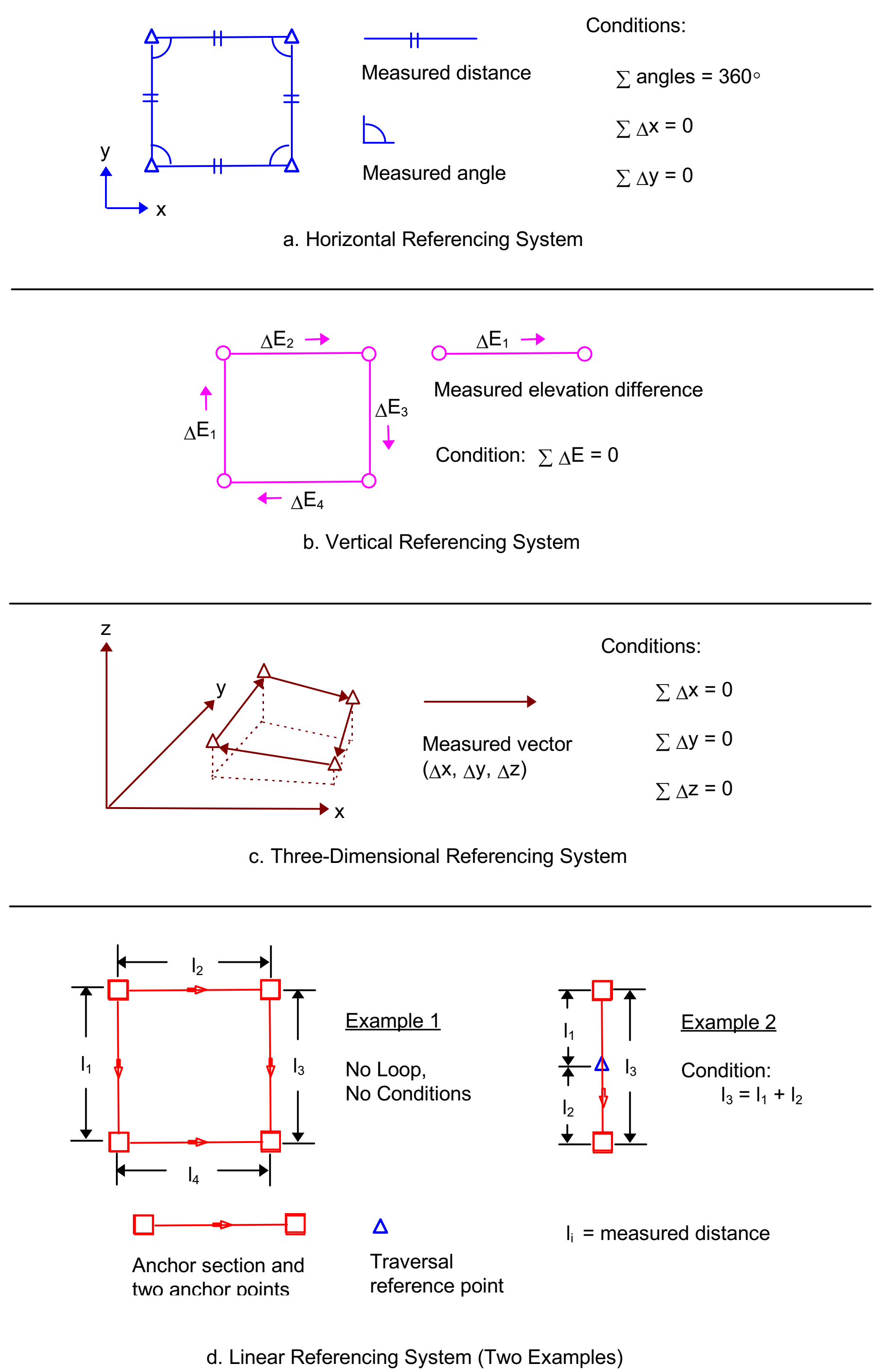
With the linear referencing system, transformations between linear referencing methods, such as reference post and milepoint, take place in a similar way. An offset from a reference post is transformed into an offset along an anchor section which is then, in turn, transformed into a milepoint. The transformation function requires datum parameters (anchor section distances), linear referencing method parameters (offsets of traversal reference points along anchor sections) and a starting coordinate. The transformed coordinate constitutes the output.

3.5 Redundant Measurements and Enabled Conditions

All of the location referencing systems in Table 1 include collections of redundant measurements that establish the datum objects and determine the locations of the reference objects. Redundancies in the measurements allow the imposition of geometric conditions that enable quality checking and provide a mathematical basis for determination of best estimates for datum and reference object parameters (see Figure 7). The same underlying mathematics provide the foundation for methodologies for *a priori* design of the measurement systems.

In horizontal referencing systems, such as NAD83 and the PLSS, measured angles and distances form loops that impose conditions on the sums of the interior angles and the sums of the coordinate differences around the perimeters (Figure 7a). In vertical referencing systems, such as NAVD88, measured elevation differences form loops that must ultimately sum to zero after adjustment of the observed values (Figure 7b). In three-dimensional referencing systems, such as WGS84, measured space vectors form loops whose sums of coordinate differences must be zero after adjustment of the observed values (Figure 7c). In the linear referencing system, measured anchor section distances that apparently form loops, as in Figure 7d, example 1, provide no redundancy and no conditions. This is because of the independence of each anchor section. However, measurements that span anchor sections or tie traversal reference points to one another or to anchor sections, as in Figure 7d, example 2, can provide redundancy and allow the imposition of conditions. In Figure 7d, example 2, the anchor section distance must be equal to the sum of its two parts after adjustment of the observed values.

The conditions imposed within any spatial measurement system can be used to place maximum tolerances on misclosures (the amounts that the measurements fail to meet the conditions). These tolerances are derived from the accuracy requirements of the system. It should be noted, however, that misclosures are neither the only, nor the best, indicators of accuracy within a measurement system. It is possible, and appropriate, to use other indicators of both global and local accuracy. The accuracies and positional interdependencies of individual point events can be described using the mathematics of redundant measurements.



Closed Figures with Redundant Measurements in Various Location Referencing Systems
Figure 7.

4. ACCURACY AND ERROR PROPAGATION IN THE LINEAR REFERENCING SYSTEM

4.1 Accuracy and Reference System Parameters

The accuracy of the location of a point event, determined by measurement to a reference object, is subject to many factors, including the accuracies of reference system parameters and any random, systematic, and gross errors that might be present in the measurement itself. Gross errors cannot be managed in any way except to detect and eliminate them through refined analysis and measurement techniques. Systematic errors are typically modeled and removed from measurements before final statements regarding location accuracies are made. These final statements, therefore, usually express location accuracies in terms of variances (σ^2) that result from combinations of purely random errors in measurements and reference system parameters. Random errors in reference system parameters, expressed as a variance-covariance matrix, arise, in turn, from an additional underlying system of measurements used to compute the parameters. Knowledge of maximum tolerable variances in event locations can lead to specification of an appropriate variance-covariance matrix for reference system parameters, thus yielding the required accuracies of the underlying measurements that determine the parameters.

Linear referencing system parameters are lengths (distances) of anchor sections and offsets along anchor sections of traversal reference points and nodes. From the standpoint of measurement system design, there is no distinction between traversal reference points and nodes. They all have locations that are ultimately described by offsets along anchor sections. These locations are all determined through the same system of measurements.

4.2 Absolute Accuracy and Relative Accuracy

In the linear referencing system, absolute accuracy relates to uncertainty in the location of a point event or linear event with respect to datum objects. This uncertainty is expressed as the variance in an offset along an anchor section.

Relative accuracy relates to uncertainty in the location of a point event with respect to another point event or to uncertainty in the length of a linear event. These uncertainties are expressed as variances in distances that are linear functions of measurements and reference system parameters. Relative accuracy is sometimes denoted by a ratio of standard deviation in distance to the distance itself (*e.g.*, 1:10,000).

Many of the accuracy requirements expressed by users pertain to relative accuracies. A safety engineer who wants to know the location of the start of a guard rail to “within 2 feet” probably needs to relate the location of the guard rail to that of a bridge abutment or a point of change in curvature of an alignment. A pavement management analyst who wants to know location to “within 0.01 mile” probably needs that level accuracy within a unit of analysis (a section or linear event).

If linear referencing system design methods are to be founded upon stochastic properties of measurements, it is necessary to express user requirements, whether they be relative or absolute accuracies, in probabilistic terms. Must we know location to within 0.01 mile all of the time, 90% of the time, 68% of the time, or some other percentage of the time? Probabilistic expressions can be transformed into variances in absolute and relative locations, thereby providing a basis for linear referencing system design.

4.3 Error Propagation

Relative and absolute accuracy requirements can be related through the principles of error propagation, thus providing further insight into the nature of the linear referencing system design problem. The General Law of Propagation of Random Error (Mikhail and Gracie, 1982, pp. 152-153) states that if y is a linear function of random variables x_i, thus

$$y = a_1x_1 + a_2x_2 + \ldots\ldots + a_nx_n + c \tag{6}$$

where c is a constant, then

$$\sigma_y^2 = \sum_{i=1}^{n} a_i^2\sigma_{x_i}^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} a_ia_j\sigma_{x_ix_j} \quad i \neq j \tag{7}$$

where σ_y^2 is the variance in y,

$\sigma_{x_i}^2$ is the variance in x_i, and

$\sigma_{x_ix_j}$ is the covariance between x_i and x_j.

Measurements are typically uncorrelated (zero covariances). However, reference system parameters are typically correlated (non-zero covariances). Locations, computed from linear combinations of measurements and parameters, have variances that can be determined using equation (7).

The absolute accuracy of the point event, z, in Figure 8a can be expressed as the variance ($\sigma_{o_z}^2$) or the standard deviation (σ_{o_z}) in its offset (o_z) along the anchor section. The offset is computed from

$$o_z = o_a + l_{az} \tag{8}$$

where o_a is the offset of the traversal reference point and

l_{az} is the measurement.

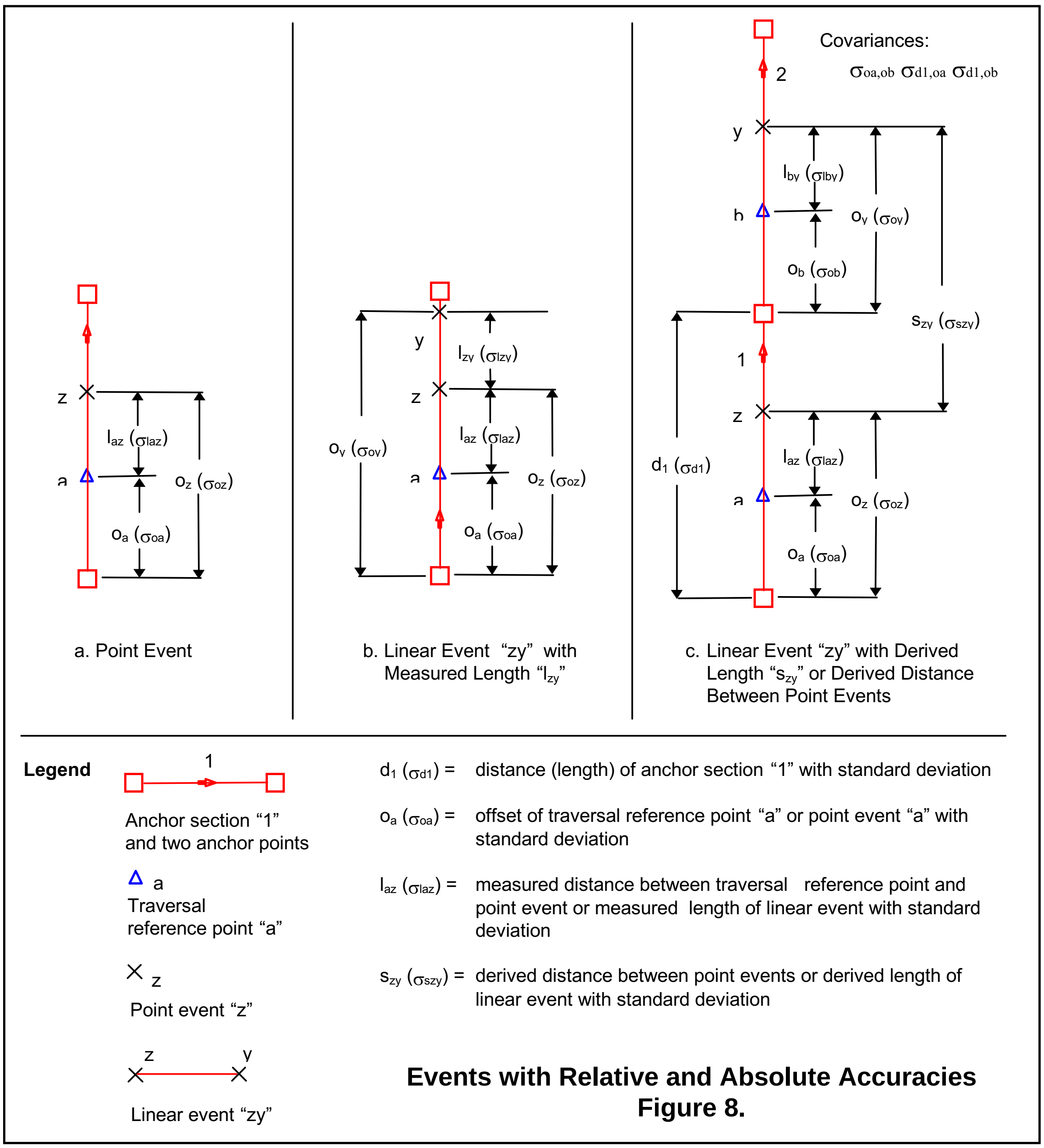
Applying equation (7),

$$\sigma_{o_z}^2 = \sigma_{o_a}^2 + \sigma_{l_{az}}^2 . \tag{9}$$

The linear event, zy, in Figure 8b has both absolute and relative accuracies. Its absolute accuracy is expressed as the variances or standard deviations in the offsets (o_z and o_y) of its start and end points. These are given by equation (9) and

$$\sigma^2_{o_y} = \sigma^2_{o_a} + \sigma^2_{l_{az}} + \sigma^2_{l_{zy}} . \qquad (10)$$

Its relative accuracy is expressed as the variance ($\sigma^2_{l_{zy}}$) or standard deviation in the direct measurement of its length (l_{zy}) or as the ratio 1: (l_{zy} / $\sigma_{l_{zy}}$).



Events with Relative and Absolute Accuracies
Figure 8.

Figure 8c illustrates a linear event, zy, whose length, s_{zy}, is derived from the offsets of its end points. Alternatively, s_{zy} might represent a distance between two point events, computed for analysis purposes. To illustrate a general case, s_{zy} spans parts of two anchor sections. It is computed by

$$s_{zy} = d_1 - o_z + o_y$$

$$= d_1 - o_a - l_{az} + o_b + l_{by} \qquad (11)$$

where d_1 is the length (distance) of anchor section 1,

o_a and o_b are traversal reference point offsets, and

l_{az} and l_{by} are measurements.

The reference system parameters, d1, oa, and ob, are correlated. Applying equation (7),

$$\sigma^2_{s_{zy}} = \sigma^2_{d_1} + \sigma^2_{o_a} + \sigma^2_{o_b} + \sigma^2_{l_{az}} + \sigma^2_{l_{by}} + 2\sigma_{d_1o_b} - 2\sigma_{d_1o_a} - 2\sigma_{o_ao_b} \,. \qquad (12)$$

Equations (9), (10), and (12) indicate that if the linear referencing system is to be designed according to the business data accuracy requirements of users, then the reference objects (traversal reference points), datum objects (anchor points and anchor sections), and measurements that will be used to locate events must be considered simultaneously.

Given that it is impossible to anticipate all possible types of measurement methods and standards that will be used to locate events throughout the lifetime of the linear referencing system, it might be appropriate to design the datum and reference objects to be 2-3 times more accurate than the event locations that will be referenced to them. Precedent for this approach lies within the accuracy requirements of control points used for making and checking topographic maps. Wolf (1983, p. 396) states that horizontal control points used for mapping should contain errors no greater than one-half the horizontal accuracy tolerance of the map. The American Society for Photogrammetry and Remote Sensing’s accuracy standard for large-scale maps requires standard deviations in the coordinates in the reference data set to be equal to or less than one-third of the limiting root-mean-square error selected for the map being tested (ASPRS, 1990). If we specify the maximum allowable standard deviation in the derived distance s_{zy} in Figure 8c to be ±0.01 mile and attribute two-thirds of this to the measurements, l_{az} and l_{by}, then the total error budget for the datum and reference objects is

$$\pm 0.0033 \text{ mile} = \left[\sigma^2_{d_1} + \sigma^2_{o_a} + \sigma^2_{o_b} + 2\sigma_{d_1o_b} - 2\sigma_{d_1o_a} - 2\sigma_{o_ao_b}\right]^{\frac{1}{2}} . \qquad (13)$$

It is tempting to suggest that reference objects be treated in a similar manner, and that all that need be designed are datum objects, making them, perhaps, 3-4 times more accurate than the event locations that will be referenced to them. This is an alternative. However, it is based upon assumptions concerning reference objects that need not be made if they are included in the design. For example, measurement technologies for locating traversal reference points can be identified, and measurement procedures can be controlled. Locating traversal reference points is far less *ad hoc* than locating events.

Ideally, all linear referencing methods should be accounted for in a simultaneous design, but, practically, this might not be feasible. It might be appropriate to include only traversal reference points of one or two methods having the most stringent accuracy requirements, along with datum objects, in the design. Traversal reference points of remaining methods, having lower accuracy requirements, can then be made to fit the designed framework.

5. MATHEMATICAL MODEL FOR DERIVATION OF LINEAR REFERENCING SYSTEM PARAMETERS AND THEIR ACCURACIES FROM MEASUREMENTS

5.1 The General Linear Model

In Figure 7d, example 2, the unknown system parameters are the traversal reference point offset (observed value = l1) and the anchor section distance (observed value = l3). Assume that l1, l2, and l3 are 1.010, 1.020, and 2.045 miles, respectively. The misclosure is

$$mc = l_3 - l_1 - l_2 = +0.015 \text{ miles} \qquad (14)$$

Assuming equal reliability in the measurements, one-third of the misclosure should be attributed to each, yielding estimates of 1.015 miles for the traversal reference point offset and 2.040 miles for the anchor section distance. Estimates for the errors in the measurements, referred to as "residuals", are -0.005, -0.005, and +0.005 miles for l_1, l_2, and l_3, respectively.

In a complex system, with many anchor points, many traversal reference points, and many measurements, the full set of conditions (containing overlapping sets of measurements) is difficult to identify. An alternative approach is to build a system of "observation equations" each of which expresses a functional relationship between a single measurement and a set of system parameters (Mikhail and Gracie, 1981, p. 72). In a linear system, the general form of an observation equation is

$$l_i - \varepsilon_i + c_i = a_{i1} x_1 + a_{i2} x_2 + \dots\dots\dots + a_{in} x_n \qquad (15)$$

where l_i is the ith measurement,
ε_i is the error in the ith measurement,
c_i is a constant,
the x's are the unknown parameters, and
the a's are coefficients of the unknown parameters.

A system of m observation equations may be written as

$$L - E + C = AX \qquad (16)$$

where L is an mX1 matrix of measurements,
E is an mX1 matrix of errors,
C is an mX1 matrix of constants,
A is an mXn matrix of coefficients, and
X is an nX1 matrix of unknown parameters.

If $m>n$, the parameters are over-determined with $m-n$ degrees of freedom. If the errors are normally distributed with means of zero (*i.e.*, there are no systematic or gross errors present), then the solution to equation (16) that yields the best estimates for the unknown parameters is

$$\bar{X} = (A^T PA)^{-1} A^T P(L + C) \tag{17}$$

where $$P = Q_{ll}^{-1}, \tag{18}$$

$$Q_{ll} = \frac{1}{\sigma_0^2} \Sigma_{ll}, \tag{19}$$

σ_0^2 is the reference variance, and

Σ_{ll} is the mXm variance-covariance matrix of the measurements.

σ_0^2 is used for scaling purposes and is often assigned a value of one. For independent measurements, Σ_{ll} and, therefore, Q_{ll} are diagonal. The elements of Σ_{ll} may be based upon experience or derived from repetitions of individual measurements. Subsequent to the solution for $\bar{X}$, the residuals (V) can be computed from

$$V = L + C - A\bar{X}. \tag{20}$$

An *a posteriori* estimate for the reference variance can then be computed as

$$\hat{\sigma}_0^2 = (V^T PV) \ / (m - n). \tag{21}$$

$\hat{\sigma}_0^2$, σ_0^2, and the Chi square statistic can be used to perform a global test of the overall quality of the weights of the measurements and the measurements themselves. The variance-covariance matrix of the parameters can be obtained from

$$\Sigma_{xx} = \hat{\sigma}_0^2 (A^T PA)^{-1}. \tag{22}$$

The variance-covariance matrix of the residuals is given by

$$\Sigma_{vv} = \hat{\sigma}_0^2 (Q_{ll} - A(A^T PA)^{-1} A^T). \tag{23}$$

(Mikhail and Gracie, 1981, p. 169)

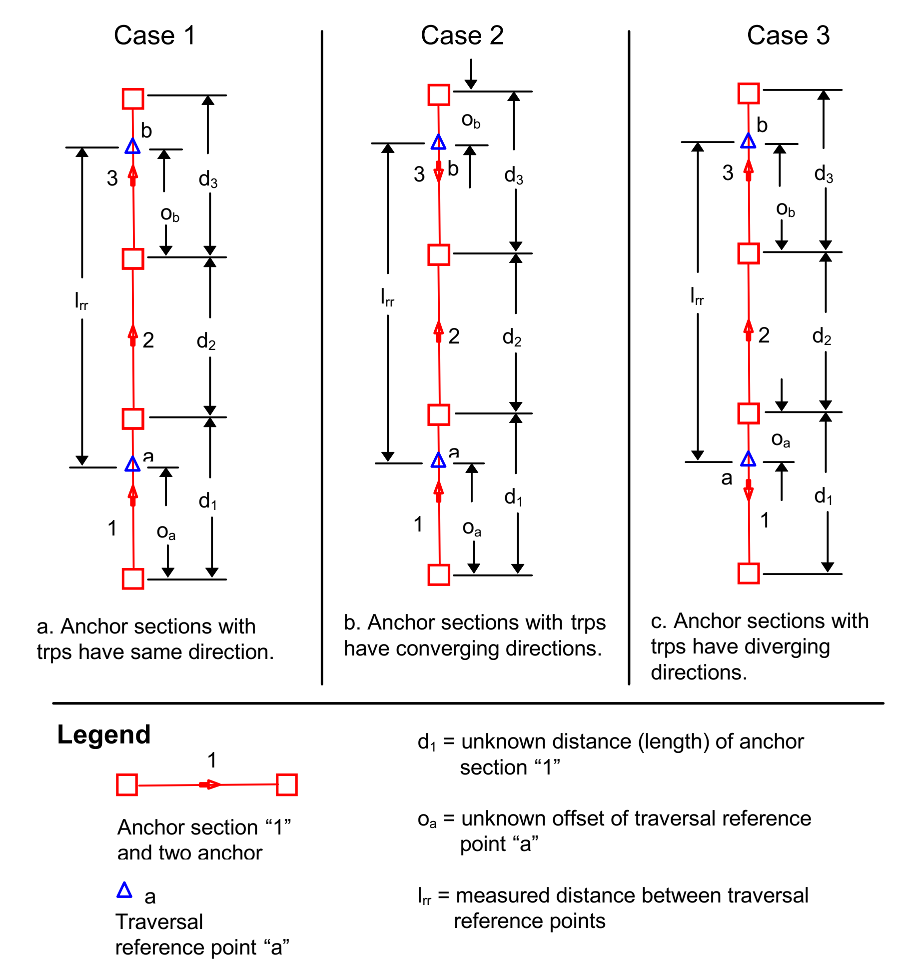
Σ_{vv} can be used to examine the local redundancies in the measurements, detect gross errors, determine the minimum size of a detectable gross error (internal system reliability), and determine the effects of undetected gross errors on the parameters (external system reliability) (Kuang, 1996, pp. 169-173).

5.2 Linear Referencing System Observation Equations

All measurements within the linear referencing system are distances. Anchor sections have directions, but these are assigned, not observed. There are three kinds of distances, categorized with regard to the objects they connect: 1) connecting two traversal reference points, 2) connecting a traversal reference point and an anchor point, and 3) connecting two anchor points.

5.2.1 Distance Connecting Two Traversal Reference Points

There are three cases illustrated in Figure 9.



Distances Connecting Traversal Reference Points
Figure 9.

5.2.1.1 Case 1 — In the same direction

In Figure 9a, the traversal reference points are on anchor sections that have the same direction. The observation equation is

$$l_{rr} + \varepsilon_{rr} = o_b - o_a + d_1 + d_2 \,. \tag{24}$$

The general form of equation (24) is

$$l_{rr} + \varepsilon_{rr} = o_b - o_a + \sum_{i=1}^{n-1} d_i \,. \tag{25}$$

where the first anchor section contains traversal reference point a and the nth anchor section contains traversal reference point b. If the two traversal reference points are on the same anchor section, there is no summation of anchor section distances.

5.2.1.2 Case 2 — In converging directions

In Figure 9b, the anchor sections containing the two traversal reference points have converging directions. The observation equation is

$$l_{rr} + \varepsilon_{rr} = - o_b - o_a + d_1 + d_2 + d_3 \,. \tag{26}$$

The general form of equation (26) is

$$l_{rr} + \varepsilon_{rr} = - o_b - o_a + \sum_{i=1}^{n} d_i \,. \tag{27}$$

5.2.1.3 Case 3 — In diverging directions

In Figure 9c, the anchor sections containing the two traversal reference points have diverging directions. The observation equation is

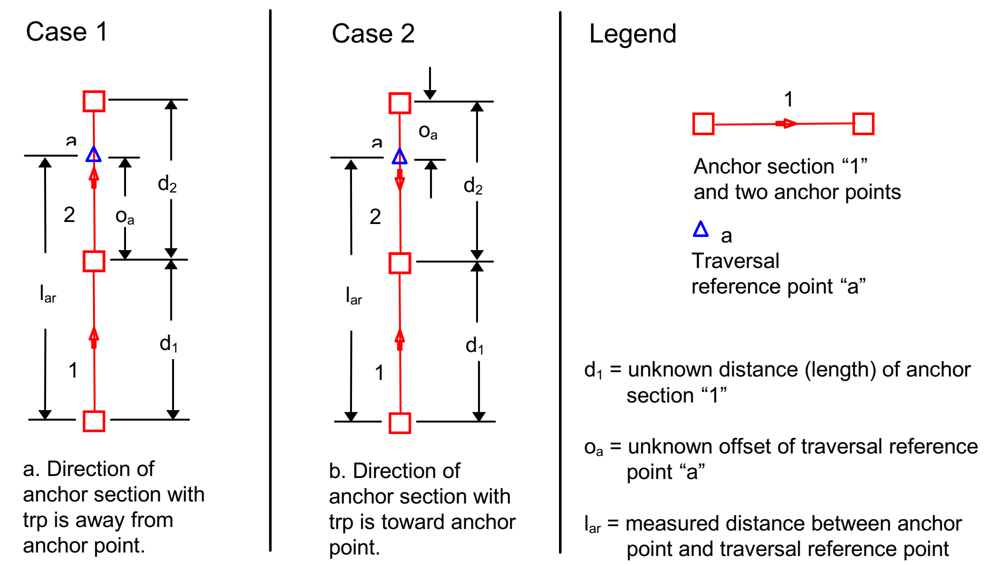
$$l_{rr} + \varepsilon_{rr} = o_b + o_a + d_{2.} \tag{28}$$

The general form of equation (28) is

$$l_{rr} + \varepsilon_{rr} = o_b + o_a + \sum_{i=2}^{n-1} d_i \,. \tag{29}$$

5.2.2 Distance Connecting a Traversal Reference Point and an Anchor Point

There are two cases illustrated in Figure 10.



Distances Connecting Anchor Points and Traversal Reference Points
Figure 10.

5.2.2.1 Case 1 — Direction away from anchor point

In Figure 10a, the direction of the anchor section containing the traversal reference point is away from the anchor point included in the measurement. The observation equation is

$$l_{ar} + \varepsilon_{ar} = o_a + d_1 \,. \tag{30}$$

The general form of equation (30) is

$$l_{ar} + \varepsilon_{ar} = o_a + \sum_{i=1}^{n-1} d_i \,. \tag{31}$$

Where the first anchor section contains the anchor point and the nth anchor section contains the traversal reference point. If the traversal reference point is on the first anchor section, there is no summation.

5.2.2.2 Case 2 — Direction toward anchor point

In Figure 10b, the direction of the anchor section containing the traversal reference point is toward the anchor point included in the measurement. The observation equation is

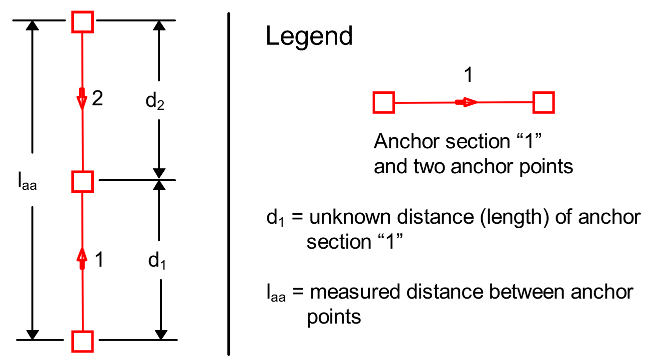
$$l_{ar} + \varepsilon_{ar} = -\,o_a + d_1 + d_2 \,. \tag{32}$$

The general form of equation (32) is

$$l_{ar} + \varepsilon_{ar} = -\,o_a + \sum_{i=1}^{n} d_i \,. \tag{33}$$

5.2.3 Distance Connecting Two Anchor Points

Figure 11 illustrates a measured distance connecting two anchor points and spanning two anchor sections.



Distance Connecting Two Anchor Points
Figure 11.

The observation equation is

$$l_{aa} + \varepsilon_{aa} = d_1 + d_{2.} \tag{34}$$

The general form of equation (34) is

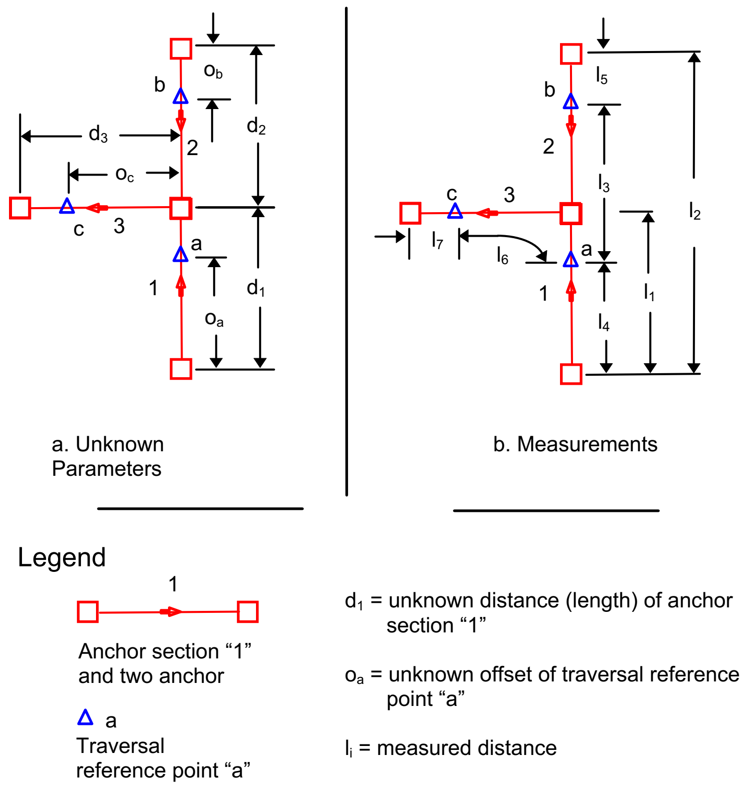
$$l_{aa} + \varepsilon_{aa} = \sum_{i=1}^{n} d_i \; . \tag{35}$$

5.3 Examples

5.3.1 Example 1a

Figure 12a illustrates a configuration of three anchor sections and three traversal reference points with unknown parameters

$$X = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ o_a \\ o_b \\ o_c \end{bmatrix} . \tag{36}$$



Unknown Parameters and Measurements for Example 1a
Figure 12.

Figure 12b illustrates a set of measurements among the traversal reference points and anchor points. If the measured values are 5.25, 10.55, 6.41, 2.23, 1.94, 4.72, and 2.33 miles for the successive l_i's, then the observation equations are

$$\underset{L}{\begin{bmatrix} 5.25 \\ 10.55 \\ 6.41 \\ 2.23 \\ 1.94 \\ 4.72 \\ 2.33 \end{bmatrix}} - \underset{E}{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ o_a \\ o_b \\ o_c \end{bmatrix}} . \tag{37}$$

If the measurements are independent, l1 and l2 have standard deviations of ±0.005 miles, the remaining measurements have standard deviations of ±0.010 miles, and σ_0^2 is assigned a value of one, then the weight matrix is

$$P = \begin{bmatrix} 40{,}000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 40{,}000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10{,}000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10{,}000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10{,}000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10{,}000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10{,}000 \end{bmatrix}. \qquad (38)$$

The solution for the parameter estimates is

$$\bar{X} = (A^TPA)^{-1}A^TPL = \begin{bmatrix} 5.25 \\ 5.30 \\ 4.02 \\ 2.22 \\ 1.93 \\ 1.69 \end{bmatrix}. \qquad (39)$$

The residuals are 0.000, 0.002, -0.009, -0.009, -0.009, 0.000, and 0.000 miles, respectively. There is one degree of freedom in the measurement system. There is no local redundancy in the measurements l_1, l_6, and l_7. The *a posteriori* estimate for the reference variance is 2.77. The standard deviations (miles) in the parameter estimates are

$$\begin{bmatrix} \sigma_{d_1} \\ \sigma_{d_2} \\ \sigma_{d_3} \\ \sigma_{o_a} \\ \sigma_{o_b} \\ \sigma_{o_c} \end{bmatrix} = \begin{bmatrix} \pm 0.008 \\ \pm 0.011 \\ \pm 0.028 \\ \pm 0.014 \\ \pm 0.014 \\ \pm 0.023 \end{bmatrix}. \qquad (40)$$

The variance-covariance matrix for the parameter estimates is

$$\Sigma_{xx} = \begin{bmatrix} \sigma^2_{d_1} & \sigma_{d_1d_2} & \sigma_{d_1d_3} & \sigma_{d_1o_a} & \sigma_{d_1o_b} & \sigma_{d_1o_c} \\ \sigma_{d_2d_1} & \sigma^2_{d_2} & \sigma_{d_2d_3} & \sigma_{d_2o_a} & \sigma_{d_2o_b} & \sigma_{d_2o_c} \\ \sigma_{d_3d_1} & \sigma_{d_3d_2} & \sigma^2_{d_3} & \sigma_{d_3o_a} & \sigma_{d_3o_b} & \sigma_{d_3o_c} \\ \sigma_{o_ad_1} & \sigma_{o_ad_2} & \sigma_{o_ad_3} & \sigma^2_{o_a} & \sigma_{o_ao_b} & \sigma_{o_ao_c} \\ \sigma_{o_bd_1} & \sigma_{o_bd_2} & \sigma_{o_bd_3} & \sigma_{o_bo_a} & \sigma^2_{o_b} & \sigma_{o_bo_c} \\ \sigma_{o_cd_1} & \sigma_{o_cd_2} & \sigma_{o_cd_3} & \sigma_{o_co_a} & \sigma_{o_co_b} & \sigma^2_{o_c} \end{bmatrix}$$

$$= \begin{bmatrix} 0.693 & -0.693 & -0.693 & 0.000 & 0.000 & -0.693 \\ -0.693 & 1.330 & 0.914 & 0.222 & 0.222 & 0.914 \\ -0.693 & 0.914 & 8.144 & 1.911 & -0.859 & 5.374 \\ 0.000 & 0.222 & 1.911 & 1.911 & -0.859 & 1.911 \\ 0.000 & 0.222 & -0.859 & -0.859 & 1.911 & -0.859 \\ -0.693 & 0.914 & 5.374 & 1.911 & -0.859 & 5.374 \end{bmatrix} \text{X } 10^{-4} \quad (41)$$

.

5.3.2 Example 1b

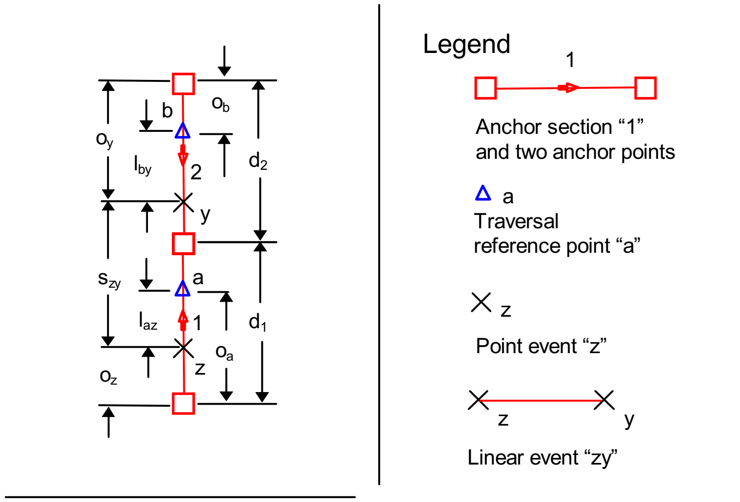
Given the linear referencing system in Figure 12 and the result in equation (41), the example is now extended to include analysis of the accuracies of point and linear events. Let point event z be located on anchor section 1 by measurement (l_{az} = 0.50 mile) from traversal reference point a and point event y be located on anchor section 2 by measurement (l_{by} = 0.75 mile) from traversal reference point b, as shown in Figure 13. If

$$\sigma_{l_{az}} = \pm 0.005 \text{ mile and} \quad (42)$$

$$\sigma_{l_{by}} = \pm 0.010 \text{ mile, then} \quad (43)$$

the absolute accuracy of the location of point event z is given by

$$\begin{aligned} \sigma_{o_z} &= \left[\ \sigma^2_{o_a} + \sigma^2_{l_{az}}\ \right]^{\frac{1}{2}} \\ &= \left[\ 1.911 + 0.250\ \right]^{\frac{1}{2}} 10^{-2} \\ &= \pm 0.015 \text{ mile.} \end{aligned} \quad (44)$$



d_1 = distance (length) of anchor section "1"

o_a = offset of traversal reference point "a" or point event "a"

l_{az} = measured distance between traversal reference point "a" and point event "z"

s_{zy} = derived distance between point events or derived length of linear event

Events on Anchor Sections 1 and 2 for Example 1b
Figure 13.

The absolute accuracy of the location of point event y is given by

$$
\begin{aligned}
\sigma_{o_y} &= \left[\sigma^2_{o_b} + \sigma^2_{l_{by}} \right]^{\frac{1}{2}} \\
&= \left[1.911 + 1.000 \right]^{\frac{1}{2}} 10^{-2} \\
&= \pm 0.017 \text{ mile.}
\end{aligned}
\qquad (45)
$$

The derived distance, s_{zy}, is given by

$$
\begin{aligned}
s_{zy} &= d_1 + d_2 - o_a + l_{az} - o_b - l_{by} \\
&= 5.25 + 5.30 - 2.22 + 0.50 - 1.93 - 0.75 \\
&= 6.15 \text{ miles.}
\end{aligned}
\qquad (46)
$$

The relative accuracy of the locations of z and y with respect to one another or the accuracy of the length of the linear event zy is given by

$$\sigma_{s_{zy}} = \left[\sigma^2_{d_1} + \sigma^2_{d_2} + \sigma^2_{o_a} + \sigma^2_{o_b} + \sigma^2_{l_{az}} + \sigma^2_{l_{by}} + 2\sigma_{d_1d_2} - 2\sigma_{d_1o_b} - 2\sigma_{d_1o_a} - 2\sigma_{d_2o_a} - 2\sigma_{d_2o_b} + 2\sigma_{o_ao_b} \right]^{\frac{1}{2}}$$

$$= \left[0.693 + 1.330 + 1.911 + 1.991 + 0.250 + 1.000 + 2(-0.693) - 2(0.000) - 2(0.000) - 2(0.222) - 2(0.222) + 2(-0.859) \right]^{\frac{1}{2}} 10^{-2}$$

$$= \pm 0.018 \text{ miles.} \qquad (47)$$

Expressed as a ratio, the relative accuracy of zy is 1 : (6.15 / 0.018) or 1 : 340.

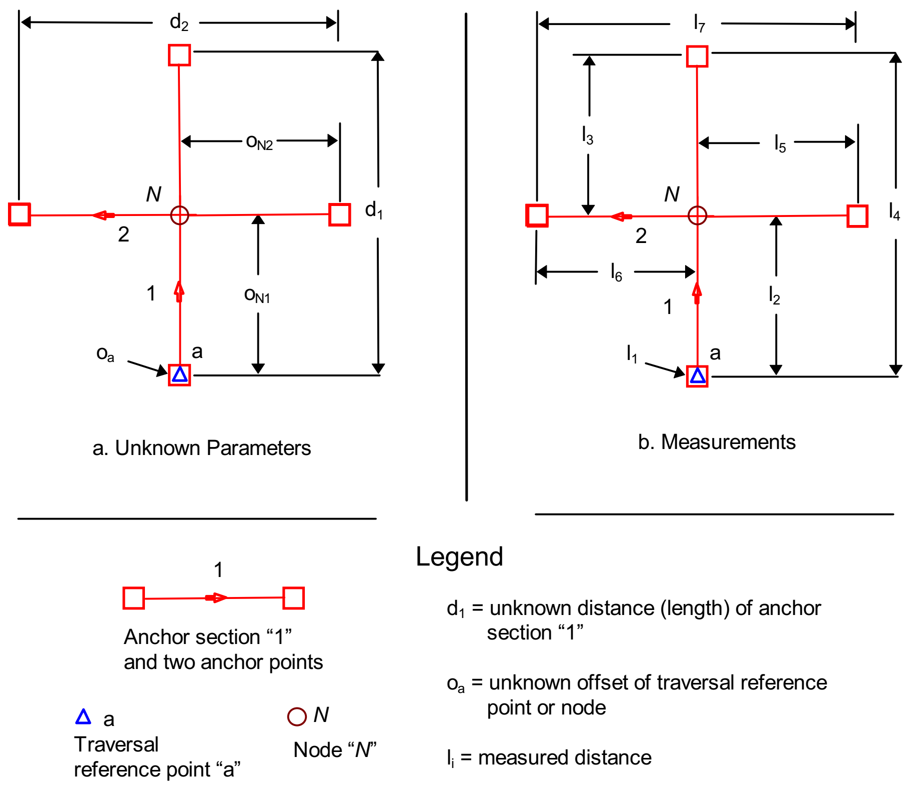
5.3.3 Example 2

Figure 14a illustrates a configuration of four anchor points, two anchor sections, one traversal reference point, and one node for a second example. The anchor sections cross, but do not have a common anchor point. The node appears at their intersection. The traversal reference point is coincident with the "from" anchor point of anchor section 1. Measurements among the traversal reference point, node, and anchor points are shown in Figure 14b. This example illustrates 1) the association of a node with more than one anchor section and 2) the ability to enforce constraints on the system through weighting of the measurements. The unknown parameters are

$$X = \begin{bmatrix} d_1 \\ d_2 \\ o_{N1} \\ o_{N2} \\ o_a \end{bmatrix} . \qquad (48)$$

The node has offsets (o_{N1} and o_{N2}) on both anchor sections. If the measured values are 0.00, 1.25, 1.36, 2.67, 1.73, 1.54, and 3.24 miles for the successive l_i' s, then the observation equations are

$$\underset{L}{\begin{bmatrix} 0.00 \\ 1.25 \\ 1.36 \\ 2.67 \\ 1.73 \\ 1.54 \\ 3.24 \end{bmatrix}} - \underset{E}{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}} \underset{X}{\begin{bmatrix} d_1 \\ d_2 \\ o_{N1} \\ o_{N2} \\ o_a \end{bmatrix}} . \qquad (49)$$



Unknown Parameters and Measurements for Example 2
Figure 14.

The traversal reference point is forced to coincide with the anchor point by providing an artificial measurement of 0.00 mile with an artificially high weight (low standard deviation (±0.001 miles)). If the reference variance is assigned a value of one and the measurements are independent with standard deviations ±0.001, ±0.010, ±0.010, ±0.010, ±0.010, ±0.010, and ±0.005 miles, respectively, then the weight matrix is diagonal with elements 1,000,000, 10,000, 10,000, 10,000, 10,000, 10,000, and 40,000.

The solution for the parameter estimates is

$$\bar{X} = (A^T P A)^{-1} A^T P L = \begin{bmatrix} 2.65 \\ 3.24 \\ 1.27 \\ 1.72 \\ 0.00 \end{bmatrix}. \qquad (50)$$

The residuals are 0.000, 0.020, 0.020, -0.020, -0.013, -0.013, and 0.003 miles, respectively. The *a posteriori* estimate for the reference variance is 7.95. There are two degrees of freedom in the measurement system. The standard deviations (miles) in the parameter estimates are

$$\begin{bmatrix} \sigma_{d_1} \\ \sigma_{d_2} \\ \sigma_{o_{N1}} \\ \sigma_{o_{N2}} \\ \sigma_{o_a} \end{bmatrix} = \begin{bmatrix} \pm 0.023 \\ \pm 0.013 \\ \pm 0.023 \\ \pm 0.021 \\ \pm 0.002 \end{bmatrix} . \tag{51}$$

The variance-covariance matrix for the parameter estimates is

$$\Sigma_{xx} = \begin{matrix} \\ (d_1) \\ (d_2) \\ (o_{N1}) \\ (o_{N2}) \\ (o_a) \end{matrix} \begin{bmatrix} (d_1) & (d_2) & (o_{N1}) & (o_{N2}) & (o_a) \\ 5.326 & 0.000 & 2.703 & 0.000 & 0.080 \\ 0.000 & 1.749 & 0.000 & 0.875 & 0.000 \\ 2.703 & 0.000 & 5.326 & 0.000 & 0.000 \\ 0.000 & 0.875 & 0.000 & 4.452 & 0.000 \\ 0.080 & 0.000 & 0.000 & 0.000 & 0.080 \end{bmatrix} \text{X } 10^{-4} . \tag{52}$$

The covariances $\sigma_{d_1d_2}$, $\sigma_{d_1o_{N2}}$, $\sigma_{d_2o_{N1}}$, $\sigma_{d_2o_a}$, $\sigma_{o_{N1}o_{N2}}$, and $\sigma_{o_{N2}o_a}$ are zero, reflecting the independence of the two anchor sections. The zero covariance $\sigma_{o_{N1}o_a}$ results from the high weight assigned to l_1. The traversal reference point remains coincident with the anchor point.

6. LINEAR REFERENCING SYSTEM DESIGN METHODOLOGY

If the reference variance is assigned a value of one, the variance-covariance matrix of the parameters of any location referencing system is given by

$$\Sigma_{xx} = (A^T PA)^{-1} . \tag{53}$$

If the *a posteriori* estimate for the reference variance is sufficiently close to one, the variance-covariance matrix of the residuals following an adjustment of measurements is given by

$$\Sigma_{vv} = \Sigma_{ll} - A(A^T PA)^{-1} A^T . \tag{54}$$

As stated in Chapter 5, Σ_{vv} can be used to examine the local redundancies in the measurements, detect gross errors, determine the minimum size of a detectable gross error (internal system reliability), and determine the effects of undetected gross errors on the parameters (external system reliability) (Kuang, 1996, pp. 169-173).

In location referencing system design problems, Σ_{xx} is known *a priori* . It is an expression of the accuracy requirements of users. The measurements to be made, as expressed in the A matrix by their interactions with the parameters, and their necessary accuracies, as expressed by P^{-1}, are what must be determined. None of Σ_{ll}, A, or P, depend upon actual observed values (L). Therefore, equations (53) and (54) can be used in design to ensure a location referencing system's internal reliability, external reliability, and ability to meet the accuracy requirements of the user community before any actual measurements are made. Of course, many possible configurations and measurement schemes might provide these assurances. An optimal design provides them at least cost.

6.1 Orders of Location Referencing System Design

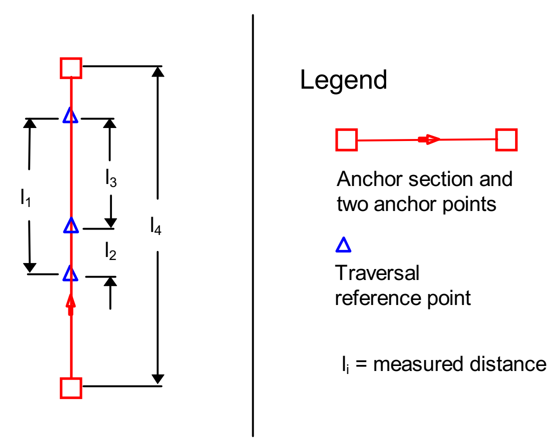
The geodetic community recognizes four orders (zero, first, second, and third) of location referencing system design problems (Grafarend and Sanso, 1985, p. 7).

6.1.1 Zero-Order Design

Zero-order design, sometimes referred to as "the datum fixation problem" can be characterized as selecting an optimum set of minimum constraints to be imposed upon the parameters (X) in order to ensure that the inverse $(A^T PA)^{-1}$ exists. Constraints on X affect A because A describes the relationship between the measurements (L) and the parameters (X).

For example, a fully observed vertical network of differential leveling cannot be used to determine elevations unless the elevation of at least one benchmark in the network is provided. Also, a fully observed horizontal network of triangulation cannot be used to determine coordinates of control points unless the coordinates of one point, a distance (for scale), and an azimuth (for orientation) are provided. In these kinds of location referencing systems, the unknown parameters are not observed directly. The measurements are all indirectly related to the parameters. A network with more than enough measurements to determine the coordinates or elevation of each point uniquely must still be provided with constraints before the computation can be done.

In the linear referencing system, the unknown parameters (anchor section lengths (distances)) and traversal reference point offsets, can be observed directly. A fully observed system of measurements does not have to be constrained. This does not mean that $(A^TPA)^{-1}$ will exist for any collection of measurements. But, if it does not exist, then the system of measurements is not fully observed. For example, a sequence of traversal reference points along an anchor section could have distances measured between each successive pair, as in Figure 15. The length (distance) of the anchor section could also be measured, as could any other distance connecting two traversal reference points. In fact, the number of measurements in Figure 15 is equal to the number of unknowns, suggesting a unique solution. But, if there is no measurement connecting a traversal reference point and an anchor point, no traversal reference point offsets can be computed. This problem, assuring that the measurement system is appropriate, is actually more closely related to first-order design.

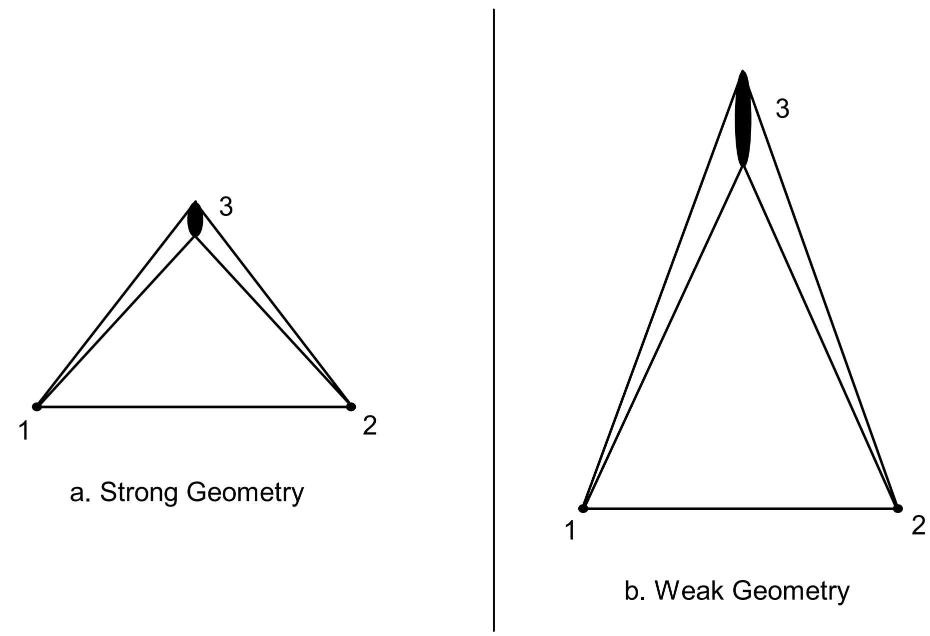


Under-Observed Measurement System
Figure 15.

6.1.2 First-Order Design

First-order design refers to choosing the optimum configuration for the location referencing system, that is selecting the numbers and locations of datum and reference objects and the measurements that will be made. It amounts to determining an optimum matrix A from Σ_{xx} and P, the former being the expression of user requirements and the latter being known for a selected measurement method.

First-order design of two- and three-dimensional reference systems must account for the geometry of reference objects and measurements. Figure 16 illustrates the principle that, for a given measurement accuracy, the geometry of the reference system affects the accuracies of the reference objects. The accuracy of point 3, relative to points 1 and 2, is greater for the geometry in Figure 16a than for the geometry in Figure 16b, even though the accuracy of direction measurements from points 1 and 2 to point 3 is the same in both cases. This consideration is referred to as the classical "strength of figure" problem. One of its manifestations is the PDOP (position dilution of precision) of the Global Positioning System. PDOP is a parameter, expressing the impact of satellite geometry on the accuracy of ground positions (Hofmann-Wellenhof, *et al*, 1994). It is critical during planning of GPS surveys because satellite geometry is continually changing. For any given latitude, longitude, and elevation, PDOP can be computed from the satellites' ephemerides.



Strength of Figure in Two- and Three-Dimensional Reference Systems
Figure 16.

In one-dimensional reference systems (vertical and linear), there are no geometric intersections of measurements to affect accuracy. In this case, first-order design is concerned only with the spatial density and distribution of datum objects and reference objects and the system of measurements that connect them.

The density of traversal reference points, for any given linear referencing method, depends primarily upon the needs of field personnel who make measurements to locate events. These needs vary with the application. For example, reference points used for highway inventory probably have a different optimal spacing than timing points for transit routes. The Wisconsin Department of Transportation places route reference points at approximate 1-mile intervals, with no adjacent pair being more than 2 miles apart (WisDOT, 1996). It is feasible, although not likely, that measurement accuracies and user accuracy requirements for reference system parameters could combine to force a more dense spacing of traversal reference points than that needed for convenience in the field.

The appropriate density of anchor points and, therefore, anchor sections is not as straightforward. As stated in Section 2.4, the minimum number of anchor points needed to define the linear datum is equal to the number of termini plus the number of odd-ordered intersections. Densities of anchor points beyond the theoretical minimum are driven by two factors 1) their utility as linkages to systems of higher dimensionality and 2) the accuracies of measurement methods.

6.1.2.1 Higher Dimensionality

In order to support as many applications as possible, the linear referencing system should be linked to location referencing systems of higher dimensionality. This can be done by providing three-dimensional coordinates for anchor points as proposed by Siegel, *et al*, (1996) for the ITS datum.

The primary utility of these coordinates as a linkage is in associating map databases with the linear referencing system. The NCHRP model provides this association without requiring coordinates at anchor points. As indicated in Figure 2, any number of lines in any number of cartographic representations can be associated with any number of anchor sections through "from position" and "to position" attributes. With the NCHRP model, coordinates for anchor points could be derived from each cartographic representation. At any given anchor point, the derived coordinates would vary among the cartographic representations.

The advantage in providing independent coordinates for anchor points lies in matching all cartographic representations at each set of independent coordinates. Anchor points with coordinates could serve not only as registration points but also as control points for computing parameters for global transformations of cartographic representations in two and three dimensions. Of course, the cartographic representations would still vary at all locations other than anchor points.

Whether or not coordinates must be provided for each anchor point, or for a selected subset, for this purpose is an open question, to be decided according to the desires of the designer for frequency of map registration. Perhaps some rules of thumb could be developed from those for distribution of control points for coordinate transformations. For example, when applying an affine transformation to a digitized map, a minimum of three control points are required and at least five are desirable. An appropriate distribution of five control points places one near each map sheet corner and the fifth near the center. For the 7.5´, 1:24,000 quadrangle series, this suggests an approximate control point spacing of 3–4 miles. This spacing is compatible with that of local geodetic control at the tertiary level being considered by some states (Wisconsin GPS Standards Work Group, 1995, p. 9).

The prototype ITS datum has a datum node (analogous to an anchor point) with latitude, longitude, and elevation at each intersection in the National Highway Planning Network, resulting in approximately 50,000 nodes distributed across the United States with varying densities (Siegel, *et al*, 1996). Final ITS datum nodes density requirements are still under investigation (see Section 7.1).

Anchor point coordinates must have a greater accuracy (by 2–3 times) than the most accurate cartographic representation tied to them, no matter their distribution. Otherwise, the quality of the cartographic representation is at risk when transforming it to fit the anchor point coordinates.

6.1.2.2 Measurement Accuracy

The typical precision of a distance measurement is given by:

$$\sigma_l = (k^2 + ppm^2)^{\frac{1}{2}} \tag{55}$$

where k is a constant expressing the error in the measuring device and in positioning it with regard to the object being measured, and

ppm (parts per million) is an error proportional to the distance being measured.

For any particular measurement method, the ratio $\sigma_l : l$ decreases with l, indicating that if relative accuracies are of importance, then longer distances are desirable. This argues for sparse datum objects. However, there might also be an upper bound on the tolerable amount of overall error in an anchor section distance (length), no matter the value of the distance. This, in conjunction with equation (55), would put an upper bound on anchor section distances.

6.1.3 Second-Order Design

Second-order design involves determination of the optimum accuracy of the measurements. It amounts to determining an optimum P matrix from A and Σ_{xx}. P depends upon the choice of measurement technologies and the detail of measurement procedures. Second-order design results in selection of equipment and specifications for its use. Estimates for measurement accuracies can be based upon experience, analysis of repeated measurements, or empirical formulas such as equation (55).

Clearly, first-order and second-order design are closely related. We cannot select A using P unless we know P. Neither can we select P using A unless we know A. Both A and P must be determined in an overall design. Therefore, first-order and second-order design are usually coupled in an iterative process.

6.1.4 Third-Order Design

Third-order design is a hybrid of first-order and second-order design applied to the problem of improving or adding to an existing location referencing system. In third-order design, new parameters are to be added with resulting additions to Σ_{xx} and changes to A and P. An appropriate design finds optimum changes (*i.e.*, what new measurements to make and how well to make them so that new datum and reference objects are best integrated with those already in place). Third-order design is clearly applicable to the problem of how to best add state and local components to a linear referencing system established at the national level, such as the proposed ITS datum.

Two approaches can be used when adding components to an existing location referencing system:

1) Treat existing parameters as fixed and include them as absolute constraints during computation for the new parameters. This risks forcing any distortions in the existing system into the new components. The risk is minimized if the components are developed in a hierarchical fashion, with the most accurate components developed first over the widest extents. Less accurate, regional and local components can then be added in succession.

2) Treat existing parameters as observations and use the inverse of their variance-covariance matrix as a weight matrix during computation for the new parameters. This risks having two sets of values for the existing parameters (*i.e.*, the existing values will change during the new computation).

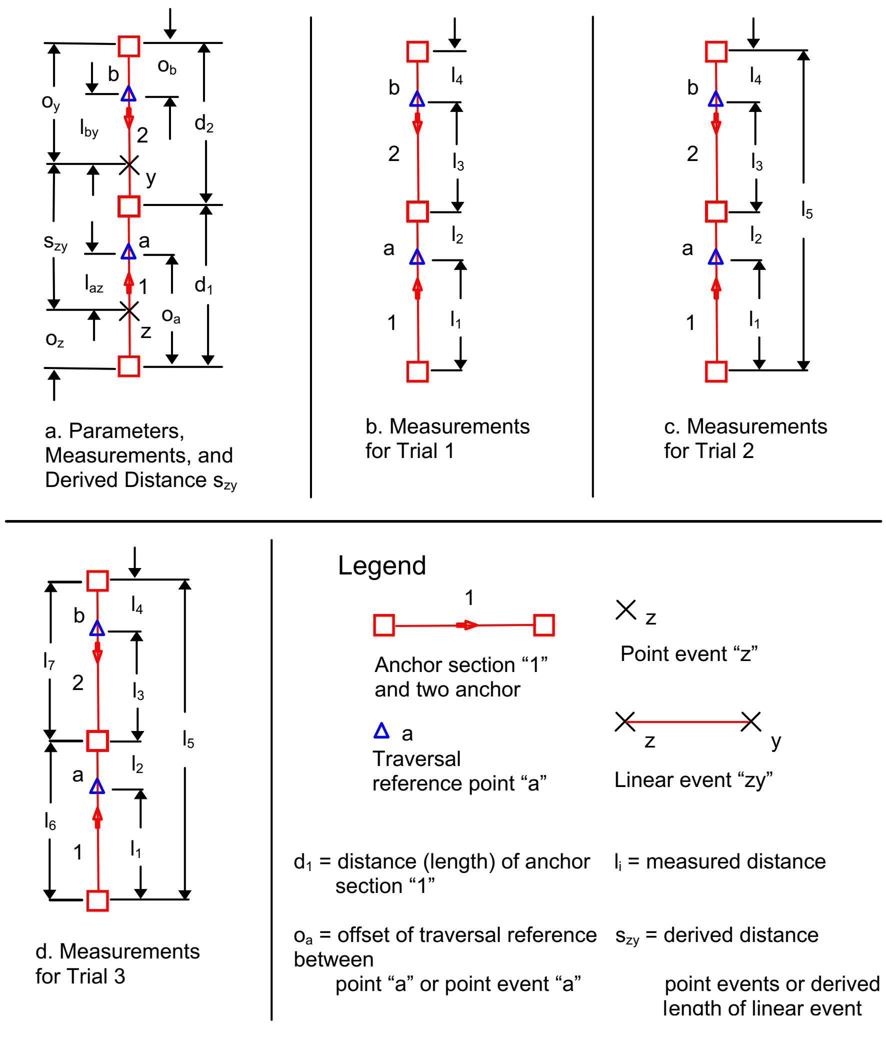
6.2 Example Design by Trial and Error

Consider the linear referencing system in Figure 17a with parameters and derived distance, s_{zy}, between point events z and y as shown. Derived distances are to have standard deviations not exceeding ±0.015 miles. The point events are located by measurements, l_{az} and l_{by} to traversal reference points a and b, respectively. Measurements locating point events have standard deviations of ±0.010 miles. Measurements made to establish referencing system parameters are made with better care and technology. They have standard deviations of ±0.005 miles. First-order design by trial and error will now be used to determine which measurements to make.

The standard deviation of s_{zy} is given by error propagation as

$$\sigma_{s_{zy}} = \Big[\sigma^2_{d_1} + \sigma^2_{d_2} + \sigma^2_{o_a} + \sigma^2_{o_b} + \sigma^2_{l_{az}} + \sigma^2_{l_{by}} + 2\sigma_{d_1 d_2} - 2\sigma_{d_1 o_b} - 2\sigma_{d_1 o_a} - 2\sigma_{d_2 o_a} - 2\sigma_{d_2 o_b} + 2\sigma_{o_a o_b} \Big]^{\frac{1}{2}} \tag{47}$$

The challenge is to find a set of measurements that will generate variances and covariances in the parameters that, when introduced in equation (47) along with the variances in l_{az} and l_{by}, will result in a standard deviation less than or equal to ±0.015 miles.



Parameters and Measurements for Design Example
Figure 17.

6.2.1 Trial 1

Consider the measurements in Figure 17b. There is no redundancy. Each parameter is determined uniquely. The A matrix is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{56}$$

The variance-covariance matrix of the parameters is

$$\Sigma_{xx} = (A^TPA)^{-1} = \begin{bmatrix} 0.500 & 0.000 & 0.250 & 0.000 \\ 0.000 & 0.500 & 0.000 & 0.250 \\ 0.250 & 0.000 & 0.250 & 0.000 \\ 0.000 & 0.250 & 0.000 & 0.250 \end{bmatrix} X\ 10^{-4} \qquad (57)$$

This results in

$$\sigma_{s_{zy}} = \left[0.500 + 0.500 + 0.250 + 0.250 + 1.000 + 1.000 - 2(0.250) - 2(0.25) \right]^{\frac{1}{2}} 10^{-2}$$

$$= \pm 0.0158 \text{ miles}, \qquad (58)$$

which is too large.

6.2.2 Trial 2

An overall measurement spanning the two anchor sections is added in Figure 17c. There is now one degree of freedom. The new A matrix is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}. \qquad (59)$$

The variance-covariance matrix of the parameters is

$$\Sigma_{xx} = \begin{bmatrix} 0.300 & -0.200 & 0.150 & -0.100 \\ -0.200 & 0.300 & -0.100 & 0.150 \\ 0.150 & -0.100 & 0.200 & -0.050 \\ -0.100 & 0.150 & -0.050 & 0.200 \end{bmatrix} X\ 10^{-4}. \qquad (60)$$

This results in

$$\sigma_{s_{zy}} = \pm 0.0152 \text{ miles}, \qquad (61)$$

which meets the design criterion when rounded off. The effect of adding two more measurements is examined in Trial 3.

6.2.3 Trial 3

Measurements of each individual anchor section distance are added in Figure 17d. There are now three degrees of freedom. The A matrix is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}. \tag{62}$$

The variance-covariance matrix of the parameters is

$$\Sigma_{xx} = \begin{bmatrix} 0.120 & -0.050 & 0.060 & -0.020 \\ -0.050 & 0.120 & -0.020 & 0.060 \\ 0.060 & -0.020 & 0.150 & -0.010 \\ -0.020 & 0.060 & -0.010 & 0.150 \end{bmatrix} \text{X } 10^{-4}. \tag{63}$$

This results in

$$\sigma_{s_{zy}} = \pm 0.0150 \text{ miles.} \tag{64}$$

For larger systems, design can be facilitated by a computer tool, allowing the user to interact with a spatial representation of the system and modify elements of A and P at will to develop alternative designs. Design alternatives can be developed using not only Σ_{xx} but also Σ_{vv}, thereby ensuring acceptable internal and external reliabilities in the linear referencing system in addition to the variances and covariances in its parameters.

The trial and error approach, although suitable for first-third order designs, does not guarantee an optimum design from the standpoint of cost. Grafarend and Sanso (1985) and Kuang (1996) present analytical methods that solve directly for A and P and can include optimization for cost.

Location referencing systems, designed by the methods in this chapter and adjusted and analyzed by the methods in Chapter 5, allow managers to make statistically defensible statements about the quality of their data, allow for computation of error in functions that operate on the data, and provide assurances to users that the spatial integrity of the data meets their needs.

7. OTHER DESIGN CONSIDERATIONS

7.1 The ITS Datum

The proposed ITS datum has two components (Siegel, *et al*, 1996):

1) WGS84 as the geodetic datum for latitude, longitude, and ellipsoid height and recognition that a geoid model is required in order to obtain orthometric heights (elevations); and

2) The ITS Datum Node Set, consisting of a nationwide set of nodes which are accurately geo-referenced and whose inter-node distances are accurately measured.

The roles of the ITS Datum Node Set and the linear datum are quite similar, but there are differences in their emphases. ITS datum design places emphasis on two- and three-dimensional location referencing and provides support for linear referencing, whereas linear datum design places emphasis on linear referencing and provides linkages to two and three dimensions.

The ITS datum will include latitude, longitude, and elevation at each "datum node". Required densities of datum nodes and required accuracies of their coordinates and inter-node distances are still under study (Siegel, *et al*, 1996). A prototype of the ITS datum, derived from the National Highway Planning Network (1:100,000 map database) became available during late 1996. It includes a datum node at each intersection with coordinates accurate to about 80 meters. Inter-node distances in the prototype have accuracies approaching 1 percent of the distance. Development of ITS datum node coordinates, with accuracies in the 3–5 meter range, using differential GPS is under consideration by the U.S. Department of Transportation.

The ITS datum requirement for coordinates at each datum node becomes problematic at complex intersections. An "intersection object model" has been proposed to address this issue (Siegel, *et al*, 1996). The object model includes a reference node, placed off any traveled way, for which precise coordinates are determined. All nodes in the intersection (a node at each gore point) are located by coordinate offsets relative to the reference node.

The primary application driving the design of the ITS datum is vehicle navigation. Much of the location referencing for vehicle navigation is expected to be two- and three-dimensional. ITS location references must be transmitted through limited bandwidths (Goodwin, *et al*, 1996). These considerations, and others that are independent of linear referencing, are leading to the requirement for coordinates at each datum node, the accuracy requirements for those coordinates, and requirements for datum node densities. For example, transmission bandwidth is one of the factors that could determine node density. Location references to be transmitted include coordinate offsets from datum nodes. Bandwidth restrictions set an upper bound on the number of digits that can be transmitted for an offset, thereby establishing a minimum node density such that no offsets have more digits than the upper bound.

Clearly, the requirements of the infrastructure management activities of many transportation agencies differ at least somewhat from those of ITS. Such activities most likely do not require anchor points at every intersection and, as discussed in Section 6.1.2., might not require coordinates for every anchor point. They certainly do not require location references to be constrained by telecommunication bandwidths. This does not imply that there is a need for two different location referencing systems for surface transportation. On the contrary, if ITS requirements are more stringent than infrastructure management requirements, and the ITS datum will inevitably be constructed and maintained, all transportation agencies can take advantage by linking their linear referencing systems to the ITS datum using third-order design methods.

7.2 Accuracy Requirements

The required accuracies of reference system parameters, as expressed in the Σxx matrix, constitute the most critical factor in linear referencing system design. They determine what measurements to make and how well to make them. They can also affect the density of datum objects.

Statements of required accuracies vary widely with function. At the program planning level, concern is placed more on topological correctness than on positional accuracy. Program planners want highways to cross rivers at bridges but they are not terribly concerned about the absolute accuracy of coordinates of any of those features or even about the relative accuracy of coordinates of two highway intersections with respect to one another[1]. On the other hand, highway safety analysts have said that they must know the location of the start of a guard rail to "within two feet"[2]. This is almost certainly an expression of a relative accuracy requirement. The location of the start of the guard rail must be known relative to the location of a bridge abutment, a change in alignment, the beginning of a skid mark, or some other features of importance to highway safety. Bossler and Toth (1995) report a need for determining coordinates, with sub-meter absolute accuracies, of features, such as tracks and switches, along the Burlington Northern Railroad right-of-way.

Figure 18, after Vonderohe, *et al*, (1993), indicates various map scales and spatial database accuracies associated with ranges of infrastructure management activities. The data are based upon the source scales and accuracies of GIS databases supporting various applications within transportation agencies. The accuracies in Figure 18, derived from the National Map Accuracy Standard, indicate upper bounds on horizontal positional errors, to be exceeded by no more than 10% of tested, well-defined features on any given map. These allowable positional errors are relative to the ground coordinate system of the map, as located on the ground by reference to monumented, horizontal control points and are, therefore, expressions of absolute accuracy requirements. The National Map Accuracy Standard does not address accuracies of features relative to one another.

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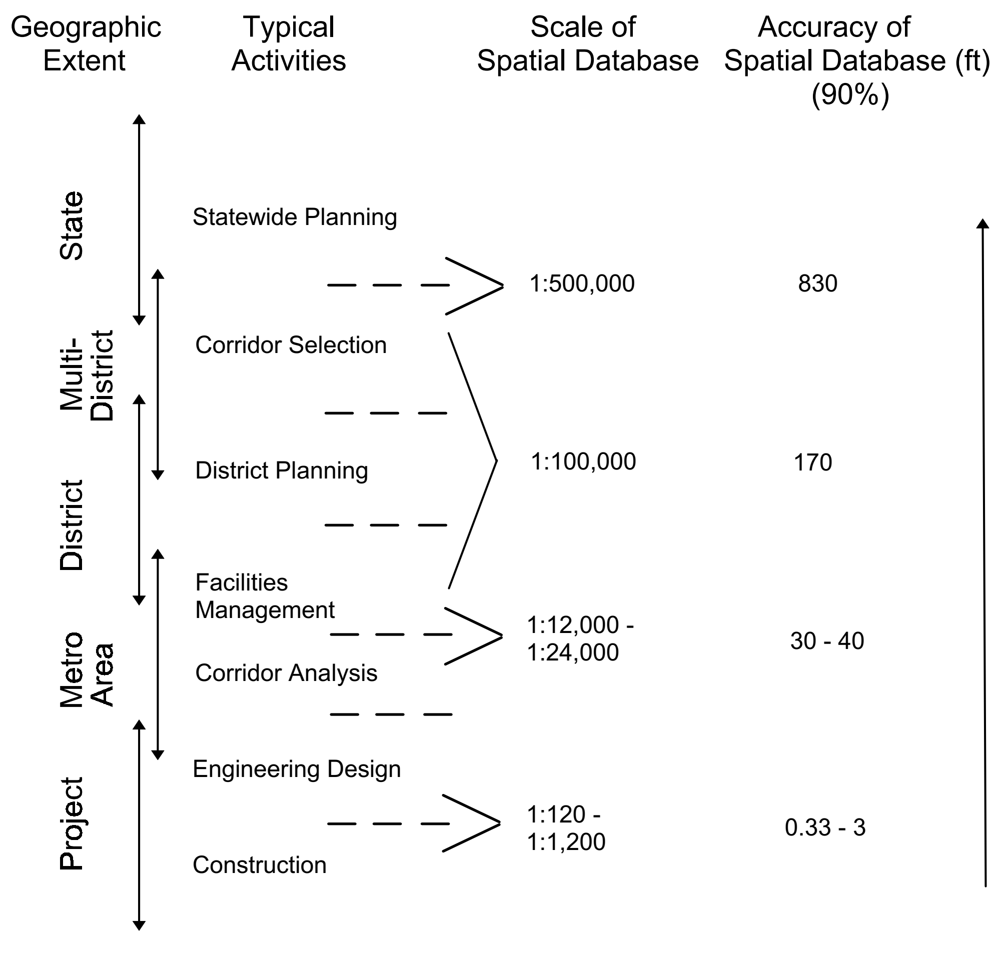
[1] Interview with Diann Danielsen, Geographic Information System Specialist, Wisconsin Department of Transportation, June 25, 1996.

[2] Interview with Ron Cihon, Geographic Information Systems Technical Expert, Washington State Department of Transportation, July 14, 1996.



Relationships among Geographic Extent, Typical Activities, and Scale and Accuracy of the Associated Spatial Database (after Vonderohe, et al, 1993)
Figure 18.

Activities, ranging from planning through preliminary design and operations, that establish requirements for the linear referencing system, are supported by source scales of 1:12,000 and smaller, with 1:24,000 being typical. The linear referencing system need not be designed for accuracies required by engineering design and construction. Rather, engineering activities produce data that might be used for development of the linear referencing system.

If we assume that well-defined, linearly-referenced point events are to be compatible with spatial databases derived from 1:24,000 scale maps, an approximate upper bound on the diagonal elements of Σ_{xx} can be readily calculated. The 90% accuracy of 40 feet, associated with 1:24,000 scale maps, translates to about 24 ft (approximately 0.005 mile) at the one standard deviation level. If locations of reference objects are to be 2–3 times as accurate as the events tied to them, then standard deviations in linear referencing system parameters should be in the range of 8–12 feet. Furthermore, if we assume that accuracies of two- and three-dimensional coordinates for anchor points should be compatible with accuracies of linear parameters, then the 8–12 foot requirement is on the higher-accuracy side of the 3–5 meter coordinate accuracy being considered for ITS datum nodes.

Anchor section distances and traversal reference point offsets collected and maintained at a resolution of 0.01 miles are not adequate for referencing events that must be compatible with 1:24,000 mapping. In fact, 0.01-mile resolution in linear referencing system parameters is barely adequate for events that must be compatible with 1:100,000 mapping.

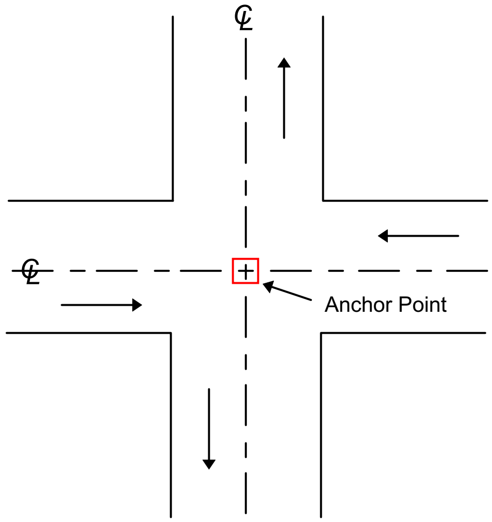
Statements of required accuracies are often based upon the capabilities of technology (*e.g.*, GPS) or the availability of data (*e.g.*, 1:24,000 scale maps, 0.01-mile odometer readings), rather than upon actual needs for decision making. Moreover, although it might be reasonable to assume that the accuracies of linearly-referenced data should be compatible with those of two- and three-dimensional data, a definitive study of error propagation through typical spatial analytical operations on linearly-referenced data, and its impact on risk in decision making, has yet to be done.

7.3 Selecting Anchor Points

Choices for modeling bi-directional and multi-lane facilities are discussed in Section 2.4. This section describes desirable characteristics of anchor points and suggests methods for selecting them in the field after modeling choices have been made.

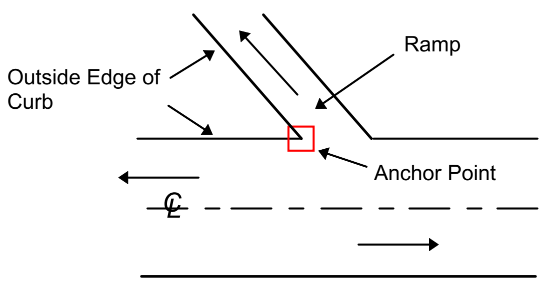
Anchor points must be readily and unambiguously identifiable in the field in such a way that measurements might be made to them from moving vehicles. Anchor points should be physically stable over time. That is, they should be in locations that are not likely to be grossly disturbed. Slight changes in position (millimeters or centimeters), that are intolerable for horizontal control points and benchmarks, are not of concern for anchor points. Anchor points should be on the facility represented by anchor sections, preferably at intersections of centerlines or intersections of sharply-defined edges.

Figure 19, after WisDOT (1996), illustrates a typical at-grade intersection of two roadways. The indicated anchor point location is appropriate for either single or dual anchor section representation of each roadway. The anchor point could be common to as few as two anchor sections (*e.g.*, one each to the east and west) or as many as eight (two each to the north, south, east, and west).

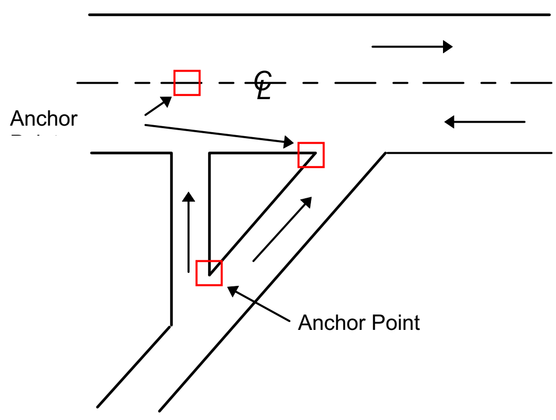


Anchor Point at Intersection of Centerlines (after WisDOT, 1996) Figure 19.

Centerline intersections at ramped interchanges can be difficult to identify in the field. In this case, anchor points are appropriately selected at gore points where the outside edges of curbs intersect, as illustrated in Figure 20 (after WisDOT, 1996). The anchor point in Figure 20 could be common to as few as one anchor section (the ramp) or as many as five (the ramp and two each to the east and west). Where there is no curb, the edges of paved shoulders can be used. If there is no curb and the edge of shoulder is difficult to identify, the edges of grassy medians can be used. Figure 21, after WisDOT (1996), illustrates appropriate choices for anchor points if ramps with islands are modeled.



Anchor Point at Intersection of Outside Edges of Curbs at Gore (after WisDOT, 1996) Figure 20.



Anchor Points at Ramp with Island
(after WisDOT, 1996)
Figure 21.

Circumstances might require selection of anchor points at locations other than intersections. It is feasible, although not likely, for design criteria to drive anchor point density beyond intersection density. In this case, anchor points should be selected at the intersections of roadway centerlines with permanent features on the facility such as the edge of a bridge abutment or the centerline of a pair of railroad tracks.

As stated in Section 2.4., an anchor point must appear at each terminus in the transportation facility. The requirement is for physical termini, not jurisdictional or administrative termini. If there is to be a unified linear referencing system, there is no need to require datum components to match jurisdictional or administrative boundaries. Such features do not constitute good anchor points in any case because they are often difficult to identify in the field. On the other hand, the physical termini of a facility should be both readily identifiable and relatively stable.

Selection or establishment of appropriate features for traversal reference points for any particular linear referencing method are germane to the method and are not addressed here. Of course, they must be addressed as the particulars of any given method are developed.

7.4 Measurement Methods

There are at least three methods for determining linear distances along transportation facilities: 1) calibrated odometers, 2) in-vehicle GPS, and 3) derivations from "as-built" information. In addition, emerging technologies for high-resolution imaging and detailed terrain modeling over large extents have the potential for providing accurate measurements of linear distances. Deriving linear distances from digitized maps at 1:24,000 and smaller source scales is not considered here because the necessary accuracies cannot be achieved in this way.

7.4.1 Calibrated Odometers

Calibrated vehicle odometers are likely to be one of the most frequent technologies employed in linear distance measurement. These devices count wheel revolutions and assume a constant tire diameter to convert revolutions to distance. Sensing devices include a fixed detector along with as many as three magnets mounted on the rotor of a wheel's brake drum, yielding a resolution of one-third of the wheel's diameter[3]. Alternatively, an electronic detector, capable of counting up to 20,000 pulses per mile, can be mounted on a vehicle's transaxle.

Converting revolutions to distances based on a constant tire diameter introduces error that must be tightly controlled. Tire diameter changes with temperature and pressure, and tire temperature increases (asymptotically approaching a given limit) as a function of distance traveled at highway speeds. The Wisconsin Department of Transportation employs a field procedure with its photolog van that requires tires to be over-inflated (80 psi) and that vehicle operators check and maintain a "steady" tire pressure. The odometer is initially calibrated along a four-mile baseline on a straight alignment over flat terrain. Experience has shown that not following these guidelines can produce errors on the order of ½ mile in 40-50 mile measurements, whereas strict adherence reportedly produces repeatabilities of ±0.01 mile.

An advantage of this technology is that it has a developed history with a substantial amount of gathered data, so that the accuracy of measurements can be analyzed. Such analysis should lead to calibration and validation of the error model in Equation (55) for various manufacturer's devices used under various conditions.

7.4.2 In-Vehicle GPS

GPS is another likely candidate for anchor section distance measurements. A number of transportation agencies have converted, or are converting, their photolog and inventory vehicles to include GPS technology.

A mobile mapping system that includes GPS, inertial navigation technology, calibrated odometers, and digital photogrammetric technology, has been used to locate features along a railroad right-of-way to a planimetric accuracy of 50 centimeters, root mean square (Bossler and Toth, 1995). Use of GPS alone to determine linear distances, requires the summation of multiple chords along the roadway. Using epochs of one-sixth second and a speed of 60 miles per hour, each chord would be approximately 14.7 feet in length. Errors in chord lengths combine with errors in positioning the measuring center of the device at each end of a driven distance, and with errors in driving along a fixed path, to produce an overall error in a measured distance. Although there has been considerable research on the ability of mobile mapping systems to determine absolute coordinates of features, the accuracies of distances measured in this way have not been investigated in depth. Equation (55) might serve as an appropriate error model, but its parameters must be determined.

The ITS concept includes GPS-based in-vehicle navigation systems for hundreds of thousands of vehicles on the nation's highways every day. Conceptually, each vehicle could "measure" the length of the path it travels. If these data could somehow be captured and re-

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[3] Interview with Bill O'Mara, Geodetic Engineer, Wisconsin Department of Transportation, May 30, 1996.

lated to the components of the linear referencing system, the task of establishing system parameters would become one of data management instead of measurement.

7.4.3 Derivation from "As-Built" Information

Extracting information from "as-built" drawings provides an attractive method for establishing anchor section distances. Engineering stationing along centerlines of single roadways could provide some anchor section distances and traversal reference point offsets directly. In other cases, alignment and pavement width information, along with coordinate geometry calculations, could be used to derive some anchor section distances and traversal reference point offsets. In still other cases, distances between traversal reference points might be derivable.

Although there is no current store of "as-built" information at the federal level[4], many state transportation agencies attempt to collect and compile such information. Unfortunately, there are questions concerning its completeness, timeliness, and accessibility. Quite often, procedures for collecting and compiling "as-built" information vary with project engineers and contractors. The degree to which they reflect the true situation in the field also varies. Delivery of "as-built" information from the project level, through district headquarters, to state headquarters is sometimes delayed so long that, to keep GIS databases current, they are updated with design information, before "as-built" information arrives.

The potential benefit in using high-quality "as-built" information for development of a linear referencing system is so great that a more complete study of its status should be performed. The study should address accuracy, completeness, currency, and accessibility of "as-built" information and should make recommendations for improving these characteristics, if necessary. The errors in "as-built" information should be characterized so that their contribution to the error budget of the linear referencing system can be accounted for during its design.

7.4.4 Emerging Technologies

The use of a vehicle to measure distance, whether it be by calibrated odometer, GPS, inertial navigation, or a combination of them, requires alignment of a measuring mark on the vehicle with anchor points or traversal reference points on the ground. Simple, low-cost electronic beacons imbedded in the anchor points and traversal reference points, combined with a sensor in the measuring vehicle, would minimize this problem. As navigating and location referencing become more electronic, the role of these beacons might evolve from marking the locations of datum and reference objects to actually becoming the datum and reference objects.

Within the next few years, a new generation of civilian imaging technology will be launched into orbit aboard a number of satellites. Some of these imaging systems will have 1-meter ground resolution, an increase of a hundredfold in the amount of data per unit area over currently available satellite images. Some will also have stereo-imaging capability. At the same time, recently-declassified synthetic aperture radar, with the potential for rapidly generating very dense, highly-accurate, digital elevation models over large areas, will become

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[4] Interview with Tom Pettite, Federal Highway Administration, June, 1996.

available. These developments, in conjunction with improved feature-extraction techniques, have the potential for making dramatic changes in mapping, spatial database development and maintenance, and the delivery and use of geographic information in general. We will still have linearly-referenced data and we will still need to do analysis in the linear domain, but the need to provide linkages to higher dimensions, and to integrate with and extract from data referenced in those higher dimensions will be even greater than it is today.

7.5 Implementation Strategies and Needs

Most transportation agencies support a number of linear referencing methods. Very few of them have an operational linear referencing system. However, linearly-referenced data are collected on a routine basis. Many state DOTs drive every mile of their highways every year, all the while making linear distance measurements for photolog and inventory purposes. With some refinement, these on-going data collection efforts could be used to gather some of the measurements necessary for development of the linear referencing system.

An incremental approach, that takes advantage of these on-going data collection activities and other initiatives, such as the ITS datum, could be most cost effective. Data collection must be preceded by a comprehensive design that identifies the datum and reference objects and the necessary system of measurements among them. In this way, necessary refinements to existing procedures and necessary new procedures will be well understood.

An incremental strategy should perhaps target priority areas for more complete early development, rather than homogenous distribution of measurement effort over time and space. Preliminary estimates for linear referencing system parameters could be obtained from adjustments of subsystems of measurements. These preliminary estimates could be used in priority areas until the complete system of measurements is available for analysis.

A unified linear referencing system should be possible if each responsible party fulfills its role. Authorities at the national, state, and local levels must be responsible for design and development of components under their jurisdictions. All authorities must cooperate on design and development of linkages among the components. In this way, a hierarchical linear referencing system can be developed that meets needs at all levels of use and geographic extent. Such a hierarchy is analogous to geodetic referencing systems that have sparse, highly-accurate control supplemented by successive levels of denser, less-accurate control. The sparse geodetic control is developed at the national level and the densest geodetic control is developed at the local level.

Consistent geodetic control frameworks and consistent mapping are possible because of rigorous standards and procedures for data collection, classification, analysis, and quality testing. There are orders of accuracy for geodetic control and specifications for measurement methods to support those orders of accuracy. There is a set of standards for testing the positional accuracy of maps. No such uniform standards, specifications, and testing procedures exist for the linear referencing system and data referenced to it. If a unified linear referencing system is to become reality, uniform quality measures and standard procedures must be developed and agreed upon.

7.6 Maintenance

A well-designed linear referencing system will require a minimum of maintenance. Maintenance is necessary when unacceptable errors are detected in reference system parameters or when datum or reference objects undergo physical changes.

The Wisconsin Department of Transportation's location control management policy includes procedures for updating link distances when a field measurement disagrees with a value in the database by more than a specified tolerance (WisDOT, 1996). This is certainly good practice, given the evolution of WisDOT's linear referencing system, but it should be unnecessary once a statistically designed and analyzed system is in place. Such a system should have sufficient local redundancy in the measurements to ensure that no system parameters are unacceptably affected by undetectable gross errors.

Alignment changes cause changes in datum and reference objects that lead to requirements for maintenance. The determination of what new datum and reference objects to create after an alignment change, and what new measurements are needed to integrate them with the existing system, while maintaining the accuracy required by users, is a classical problem in third-order design.

If datum and reference objects are sparse, the number of them affected by a change in alignment will be small. If they are dense, the number will be greater. However, there is a tradeoff. For a given density of events, the number of datum addresses that need to be modified might actually be higher for sparse datum objects. For example, if the average density of events is 500 per mile and the length of a single anchor section that must be modified, due to an alignment change over a short distance, is ten miles, then 5,000 datum addresses must be updated. If the same alignment change affected only an anchor section whose length was five miles, 2,500 datum addresses would need to be updated.

As described in Section 2.3, having the anchor point at odd-ordered intersections fall on one anchor section only favorably impacts the maintenance problem if that anchor point is disturbed. None of the other anchor sections will be affected.

It is possible for datum objects to undergo minor physical changes that do not require maintenance operations on the linear referencing system. A minor change in alignment might cause a small change in the location of a centerline intersection, represented by an anchor point. Hypotheses can be formulated and tested, using elements of Σ_{xx}, to determine whether or not the change is significant at any desired level of confidence.

Traversal addresses are reported as offsets from the initial node of the traversal. This would seem to imply that all traversal addresses downstream of an alignment change must be updated. However, if traversal reference points and events are located by datum addresses, only those bounded by affected anchor sections need updating. Correct downstream traversal addresses can be generated "on-the-fly".

From a database management perspective, changes in the linear referencing system require updates to more than merely the datum and reference objects. Nearly all objects, from cartographic representations, through networks and events are affected by direct or indirect association with the datum. Datum and reference objects that are obliterated in the field should be archived in the database so that historical views and analyses of the data are supported.

Necessary fieldwork for updating the linear referencing system should become a matter of routine practice. The Wisconsin Department of Transportation includes measurement of new link distances in the activities of projects that affect alignments (WisDOT, 1996).

8. SUMMARY AND RECOMMENDATIONS

8.1 Summary

The significance of linearly-referenced information to the operations of transportation agencies is being increasingly recognized. A number of agencies have recently begun to establish internal policies and procedures targeted at ensuring integrity and providing consistency in linearly-referenced information. Furthermore, there is an emerging need to provide linkages between linearly-referenced data and data referenced in higher dimensions. At the same time, there is a national-level initiative to develop a spatial datum for ITS that supports location referencing in one, two, and three dimensions. This initiative provides a significant opportunity for transportation agencies at state and local levels to develop linear referencing system components and integrate them with the ITS datum. Ultimately, it should be possible to create a unified linear referencing system for public, private, and military purposes. A unified system requires a consistent conceptual model and a rigorous design methodology.

Recent advances in linear referencing system data modeling are leading to a developing consensus on the need for a linear datum, very similar to the ITS datum, which supports multiple networks, multiple cartographic representations, and multiple linear referencing methods. Many transportation agencies have spatial databases at different scales that cannot be integrated and many different methods for determining linear locations that cannot be integrated. The linear datum provides the sought-after integration and transformation mechanism.

The linear referencing systems in place today were never designed. They merely evolved. No statistically-supportable statements can be made about their abilities to support the accuracy requirements of their users. This unfortunate circumstance does not hold for geodetic referencing systems and mapping. Geodetic referencing systems are designed with methods that ensure their positional integrities. Classification standards for accuracy and specifications for measurement procedures provide assurances of reliability to users of these systems. Similarly, there are well-established testing standards, for the positional accuracies of maps, that serve as a basis for product development. Similar methods and standards must be developed for the linear referencing system if it is to become a reality.

The design methodology, developed herein for a linear referencing system is based upon the mathematical and statistical principles of geodetic referencing system design and analysis. The methodology is driven by user requirements for accuracy in event locations, expressed as a variance-covariance matrix in linear referencing system parameters. This matrix is a by-product of a least squares adjustment of a system of measurements related to the unknown parameters. In the design process, the matrix is specified *a priori* and used in reverse fashion to determine an optimum configuration for the referencing system and the necessary accuracies of the measurements to be made. This information can then be used to develop specifications for selecting datum and reference objects and for making measurements in the field.

A linear referencing system designed and analyzed in this manner is assured, at a statistically-based level of confidence, of meeting the accuracy requirements of users. The design method is statistically rigorous both globally and locally. The method produces not only system-wide measures of reliability but also a means for determining the maximum size of an undetectable gross error in each individual measurement. Uncertainties in system parameters and measurements can be propagated through functions of linearly-referenced data using mathematical expressions. In this way, it should be possible to report estimates for error along with the results of spatial queries against linearly-referenced data.

Of particular interest is the third-order design problem which addresses the development of linkages between systems. Linkages between components under different jurisdictions must be designed if the linear referencing system is to be unified. Third-order design also addresses the incorporation of new information in, and deletion of old information from, an existing system. When an alignment changes, decisions must be made concerning which new datum and reference objects to create and what new measurements to make, while upholding the global and local integrities of the system.

An incremental approach to linear referencing system development could be most cost effective. A comprehensive initial design would yield the ultimate system configuration and measurement needs. Advantage could then be taken of on-going data collection efforts, that, with some refinement, could be used to make many of the necessary measurements. "As-built" information provides a potential resource for measurement data, but its consistency, currency, and accessibility must be assessed. Emerging technologies such as high-resolution satellite imagery, synthetic aperture radar, and feature extraction techniques could potentially provide low-cost, high-accuracy spatial data over large extents. They should be monitored for applicability to the linear referencing system problem.

8.2 Recommendations

The following recommendations address further advancement of the unified linear referencing system concept:

Technical Recommendations

1) Resources should be devoted to development of a design tool. Such a tool should allow the designer to interact with a spatial representation of the transportation network, testing various configurations of datum and reference objects, various systems of measurements among those objects, and various accuracies of the measurements against a specified variance-covariance matrix for the linear referencing system parameters.

2) The design method should be tested, preferably after development of the design tool. The test should include national, state, and local levels of the linear referencing system hierarchy. It should be performed over a limited extent, perhaps at the district level, incorporating components of the ITS datum, state highways, and local roads. At least one municipality should be included.

3) The test design should be partially implemented, with a subset of the datum and reference objects actually selected. The designed measurements among this subset of objects should be made and analyzed. The results should be compared to the design criteria.

4) The model for linear distance error, expressed in equation (55), should be calibrated and validated for various measurement methods and equipment, including calibrated odometers and GPS.

5) The status of “as-built” information at the state and local levels should be assessed. Its potential for use is very high if it is accurate, consistent, current, and accessible.

6) The long-needed study that links spatial data accuracy requirements to risk in decision making should be undertaken. The assumption that the accuracy of linearly-referenced data should be compatible with that of two-dimensional spatial databases might not be valid. Perhaps there are aspects of linear spatial analysis that require different accuracies.

Institutional Recommendations

1) The federal government should take a proactive role in development of the unified linear referencing system. The ITS datum initiative is at the national level. It does not directly account for linear referencing needs at the state and local levels. A coordinating, standard-setting, and facilitating role should be fulfilled at the federal level. The National Geodetic Survey has traditional federal-level responsibility for civilian location referencing systems. NGS should be encouraged to participate in continuing development of the linear referencing system.

2) State and local agencies should be encouraged to take advantage of the opportunity afforded by the ITS datum initiative. Economies of scale exist when components of the linear referencing system within a jurisdiction will be developed by another authority. Existing partnerships should be strengthened and new partnerships should be pursued. Forums on the integration of ITS and GIS-T should be encouraged.

3) Linear data issues should be incorporated in the on-going standards activities of FGDC. Linear referencing should be included in positional accuracy classification systems. Standards for linear distance measurement should be developed.

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