Design and Evaluation
of Toll Plaza Systems

by

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Project Summary

This mini-project supported the effort for some preliminary studies of several design issues in the toll plazas of highway systems. Of particular attention is paid to the toll plaza on Garden State Parkway (GST), one of the two major toll highways in New Jersey.

With the increasing population density and industrial base in New Jersey, highways such as GST or NJ Turnpike experience severe traffic jams at toll plazas during the rush hours. Good designs of these systems can impact significantly to the effective use of the infrastructure, and contribute to increasing standard of living of its residents. We analyze several designs of toll plazas and use it to search for the optimal layout when the number of each type of toll collection booths is fixed. We also consider the optimal number of each type of toll booths for given traffic intensities of each type of traffic. Our ultimate goal is to provide a decision tool to dynamically control tollbooth designs during the different time of the day and the week to best serve vehicle drivers. We also discuss future research topics, including some theoretical studies in the delay analysis of the toll plaza queueing problem, and simulation software development with animation capabilities that demonstrate different strategies and their effect on the traffic congestion.

1. Introduction

Nobody in New Jersey can avoid the traffic congestion on, for example, the Garden State Parkway, during the rush hours. As a matter of fact, it is common to experience long delays at a toll plaza even during non-rush hours. Recently, debate is going on in NJDOT with regard to whether we should remove some toll plazas and increase the toll fees at the remaining ones. Such a change will certainly relieve part of the congestion problem, but it will undoubtedly create unfairness among the travelers, since those travelers who drive short distance but have to pass a toll plaza will end up paying more. Worse, some transportation professionals worry that it may deregulate the traffic process because some travelers may skip the toll plazas by switching a portion of the route to local road, resulting in other problems.

The design of a toll plaza is clearly an important problem in any toll highway. Typically, there are four types of toll collection methods, they are

(a) E-Z (electronic) pass,

(b) toll only,

(c) exactly change or tolls, and

(d) full service.

The current practice in Garden State Parkway is that these four types of lanes are mixed. The advantage of such a design is that every vehicle has access and adjust to each type of toll booths. It has been argued, and experiences proved, that with such a design each traveler has to search for the particular lane that he/she wants to (and can) take, resulting in slowed traffic and vehicle intervention. This phenomenon can worsen the already congested traffic during rush hours.

There are a number of important research issues in the toll plaza design problem. Some of these are

1. how many of each type of booth should we have
2. how should we layout and what should be the relative positions of the different types of toll payment methods
3. what is the traffic delay as a function of the numbers of different types of toll booths and layouts,
4. what other traffic control methods should be used to ease the traffic congestion.

The importance of the first two problems is obvious. The third issue comes from the fact that, at different period of the day or different days of the week there are different traffic intensities, hence a good toll plaza design and personnel scheduling should be dependent on the delay analysis for different parameters of the traffic patterns/intensities. Finally, the last issue should incorporate traveler behavior and human factors into the design to make the system most convenient for travelers.

2. Analysis

To the best of our knowledge there has been no analysis of tollbooths with different types of traffic and different types of toll collection methods. The most classical work in toll design is the work of Leslie Edie (1954), who considers traffic delays at tollbooths of homogenous booths and homogenous vehicles. When there are different types of traffic the problems becomes considerably harder to analyze. Operations research methods are usually used for the study of design and optimization of transportation systems (e.g., Belenky (1998), and Erlander (1991)). Typically queueing theory is used in analyzing toll booth problems, see for example, Hall (1990).

In this study we first assume that the number of each type of tollbooths is fixed. For instance, let the number of full service, toll or exactly change only, tolls only, and E-Z pass be A, B, C and D respectively. For fixed A, B, C and D we analyze the optimal layout of the tollbooths for the traveler delays to be minimized. We then consider the performance analysis problem as functions of A, B, C and D, which is of importance for optimally determining these parameters.

(a) AVERAGE VEHICLE DELAY

Our first result is that, the average delay, i.e., mean waiting time in the toll plaza for all travelers, is not affected by the relevant positions, i.e., the layout, of the different toll collection booths.

During the off peak hour the design issue of toll plaza is not critical. Drivers long before arriving at the toll plaza start to search for their appropriate booth and, because the traffic is light, they can easily adjust themselves and make sure that they get to the tollbooth they want to pay tolls, and experience very minimal delays. Typically the wait in such situations is only one or two cars before paying toll.

During the rush hours long delays build up at the tollbooth. During rush hours it almost never happens that some of the booths are idling, while the other long line waiting, even though it might be the case that some drivers took significant effort to finally arrive at the particular type of booth he/she is seeking. This shows that the aggregated departure process from the tollbooth, i.e., the vehicles that left the booth after paying the toll, is probabilistically the same regardless of the relative orders of the different types of toll collecting booths. This shows that the total number of vehicles waiting at the toll plaza is probabilistically the same, and therefore by Little's formula (Little (1961)), which states that the average number of vehicles at the toll plaza equals the average vehicle arrival rate at the plaza multiplied by the average delay of a vehicle at the plaza, the average delay of a vehicle is not affected by the design of the layout of the different toll collection booths.

(b) VARIANCE OF VEHICLE WAITING TIME

We show in this section that, the variance of waiting time for all travelers, or in plain English, the fairness, is the determining factor for a good design of a tollbooth, i.e.,

a better design of toll plaza gives smaller variance of waiting times and more fair to vehicle drivers.

The result shown in (a) does not imply that the delays experienced by the vehicles at the toll plaza are identical. That the mean delay is the same simply implies that the delay of some of the vehicles are longer than that in an alternative design, while some other vehicles actually experience shorter delays. The reasoning behind that fact is the following. If the different methods of toll collecting booths are relatively spread out, the different classes of vehicles have to watch very carefully to figure out which lanes to join. It happens very regularly that some vehicles have to move to the left, and some others move to the right. As a result different classes of vehicles have to slow down and give ways to some other vehicles, yielding longer delay. On the other hand, such confusions would also create relatively less traffic intense territories which would allow portion of the "opportunistic" vehicles to get to his/her desired booths faster. Furthermore, the disorders created by the vehicles gives opportunities to the aggressive drivers, who can find ways to go through the toll plaza in shorter amount of time, yielding unfairness to an average traveler.

Since the average delay of the waiting time at the tollbooth is the same, the difference lies in the variance of waiting times. A better design gives smaller variance, and a not-so-good design gives rise to larger variance. The arguments show that a better design of the tollbooth is more fair than other designs, because the amount of delay experienced by the different vehicles are close to each other. And a bad design leads to a situation that some vehicles experience longer delay and some others shorter delays.

(c) THE OPTIMAL TOLLBOOTH LAYOUT

The next question is, for a fixed number of each type of toll collection booths, how should they be arranged in order to minimize the variance of the vehicle delay?

To answer this question, we first analyze the driving behavior as the vehicles approach the toll plaza. Tollbooths, as well as vehicles, are divided into four different classes. Some vehicles have E-Z, and they will typically only go through the E-Z lanes, which we call class 1. The second class is toll only. Only vehicles with tolls are allowed to use these lanes. The third class is toll or exact change. Clearly, both vehicles with exactly change and those with tolls can use these lanes, but vehicles with exact change but with no toll cannot use the second class of tollbooths. Finally, the fourth class of tollbooths is full service, where customers of class 2, 3, as well customers without exact change can use this lane. The processing times of vehicles at different lanes may have different distributions.

The current practice of toll plaza is that different classes of tollbooths are mixed in order to provide easy access for each vehicle. The problem associated with such a design is the intensive interaction between vehicles approaching the tollbooths. This interaction not only increases the contention among vehicles which consumes drivers' energies, making drivers tired, giving increasing feelings of a bad design, but it also increases delays for the drivers. For this reason, we propose to segment the four types of tollbooths into four sections, and within each section there is only one type of booths. For instance, if we arrange the four sections, from left to right, as types 1, 2, 3 and 4, then different types of traffic would seek their own section long before reaching the toll plaza. The reduction in vehicle interactions not only reduces the effort of the driver in reaching the tollbooths, and it also reduces the chance for having vehicle accident, making the toll plaza a safer place to use.

To reach the best result for having section design various methods should be used in designing the layout for toll plazas to avoid a driver ending up in a wrong lane. For instance, if a driver realizes to be in the wrong section until very close to the tollbooth it may create tremendous amount of effort to correct the problem. This may happen, for example, to some drivers new to the area. Such confusion can be removed by putting in multiple signaling mechanisms before the plaza (500 feet, 1000 feet, etc., before the plaza), warning the drivers long before they reach the toll plaza, that informs the driver which sections should he/she go (e.g., E-Z pass keep left). It is interesting to note that such a design has not been used in Garden State Parkway. The reason behind it contributes to the lack of understanding of its performance and how does it compare to the other designs.

d. PERFORMANCE ANALYSIS

The difficult problem left is to evaluate the delays of different types of vehicles that approach the toll plaza. At any given period of time, e.g., 7:30am – 8:30pm, the traffic intensity of each type of vehicles are somehow known and constant. The traffic intensities of the different vehicle classes should be given as system parameters, and the control variable, or the decision variable, is the number of each type of tollbooths we should have at the plaza.

The first step is to characterize the traffic intensities. Once the traffic intensity is specified we have a queueing problem. Simulation of these queueing problems is straightforward, but displaying these results on the computer systems with user-friendly interface may take tremendous effort and time, and it will be left as future research work.

The performance evaluation problem of the tollbooth design turns out to be one of the hardest problem in queueing theory, and even the simplest special case is well known to be analytically intractable. In the following we present the queueing model for some of these problems, and demonstrate how to solve them.

To make the model general, we have J classes of servers, and J classes of customers. For j=1, 2, …, J, class j customers can be served by any of the servers $k \leq j$ but cannot be served by class $k>j$ servers. In general the service time distributions at the different servers are different, and the service time distribution at server j is F_j (t). Class j customers arrive according to a Poisson process with rate λ_j The service discipline at each server is First Come First Served.

For convenience we first assume $F_1=F_2 = \dots = F_J$. With this assumption when a class j customer arrives he intends to minimize his waiting time and chooses to join the shortest queue in front of the servers 1, 2, …, j. We note that when J=2 and there is only one class of customer, class 2, then the queueing problem is reduced to the shortest queue problem with two servers. Even this simplest case does not have a closed form solution and we have to rely on approximations. But simulation can be conducted immediately without any difficulties since, upon arrival the customer observes the queue lengths at the two queues and joins the shorter queue.

To demonstrate, consider the case of two tollbooths, one for type 1 and one for type 2. Type 2 vehicles have to use tollbooth 2 while type 1 vehicles can use either type.

Let X_1 and X_2 be the queue lengths at the time of an arrival. If the arrival is of class 1 then its waiting time can be written as

$$X=\min\{X_1, X_2\}$$

$W=S_1+S_2+\ldots+S_X$

where S_j is the processing time of vehicle j, that can be simulated using $Sj=F^{-1}(U)$, where F^{-1} is the inverse of the processing time distribution and U is a uniform random variable on [0,1].

On the other hand, if the arrival is a class 2 vehicle then his waiting time is equal to

$W= S_1+S_2+\ldots+S_{X1}$

The analysis above shows that, even though searching for closed form solution to evaluating the tollbooth systems is not possible, simulation is an easily implementable tool to apply in evaluating the performances of different designs of a toll plaza system.

3. Conclusion and Future Research

In this project we studied several design problems in toll plaza systems. This problem is important to the overall transportation infrastructure to the nation in general, and to the state of New Jersey in particular. Clearly, there are many interesting research work remain to be done. The first one is a simulation software that incorporates user friendly interfaces with animation capabilities. The success of such a simulation software relies heavily on the understanding of the behavior of drivers. The usefulness of such a software is clear, it will enable use to test different designs on the computer screen, and convince the transportation officials to adopt these designs.

Another research issue that needs to be addressed is, as we have already showed, a good design depends on our understanding of vehicle delays for each type. Even though we can calculate the vehicle delays by simulation, and qualitative understanding is not easy to obtain only through simulation, even if the simulation is not expensive to run at all. Thus, some analytical results for the delays for each type of vehicles in terms of traffic intensities are desired. Such analytical results will be extremely important and will play significant roles in identifying a good design. On the other hand, as we explained earlier, even in the simplest case the resulting queueing problem is not analytically tractable. The only remaining resort is to develop approximation formulas for the delays. Once these approximations results are obtained, they can be used in determining the number of each type of tollbooths as well as at different times of the day, of the week, and of the month. The design should be given in the form of a chart, which instructs the specific design at any given period of time of the day, e.g., 7:30am-8:30am, Monday, or 10:00am – noon, Saturday, etc.

4. References:

1. Belenky, A.S. (1998). *Operations Research in Transportation Systems - Ideas and Schemes of Optimization Methods*, Kluwer Academic Publisher.

2. Edie, A. C. (1954). Traffic delays at toll booths, *Journal of Operations Research Society of America*, 2, 107-138.

3. Erlander, S. (1991). *Optimal Spatial Interaction and the Gravity Model*, Springer-Verlag.

4. Hall, R. (1991). *Queueing Methods for Service and Manufacturing*, Prentice-Hall, New Jersey.

5. Little, J. D.C. (1961). A proof for the queueing formula $L=\lambda W$. *Operations Research*, 9, 383-387.