



IDEA

**Innovations Deserving
Exploratory Analysis Programs**

Intelligent Transportation Systems Program

USE OF GPS ATTITUDE DETERMINATION TO CALIBRATE AN ARRAY OF INEXPENSIVE ACCELEROMETERS

Final Report for ITS-IDEA Project 77

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Use of GPS Attitude Determination to Calibrate an Array of Inexpensive Accelerometers

Abstract

The next generation of driver assistance/vehicle safety and control systems (AVCS, AICC, CWS, CAS, AHS, etc.) require inexpensive (much less than \$50) and accurate (approximately 50 micro G in acceleration and less than 0.1 deg/hr in rotation rate) inertial and location (centimeter-level position and 3-D velocities) sensors at high update rates (25-100 Hz) to gauge 3-D vehicle dynamics. Conventional mechanical and advanced fiber optic and ring laser gyroscopes have not thus far been able, and may not ever be able, to provide sufficiently accurate inertial data inexpensively. Under the proper conditions, GPS can provide data of sufficient location accuracy (2-3 centimeters has been demonstrated in a highway environment), but is limited by update rate and the occlusion of satellite signals by natural and manmade features.

This project takes advantage of a novel "virtual gyroscope" concept under development at PATH/UCB that employs multiple accelerometers in a 3-D grid framework, combined with 3-D GPS interferometry attitude determination techniques developed by SRI for calibrating the inertial array in a geodetic coordinate system.

SRI has provided GPS attitude data and instrumentation to the PATH/UCB team, and early simulation data indicates the combination of the virtual gyroscope and GPS interferometry improves upon the state of the art of an inexpensive inertial navigation system.

The project makes use of current and expected advancements in micro-machining, microprocessor, and GPS technologies. The project has the potential to provide inexpensive inertial sensors of sufficient accuracy and other characteristics (update rate, size, and operating conditions) to provide advanced vehicle control and driver assistance and safety systems in all price ranges of vehicles.

Acronyms

ACC	active/adaptive cruise control
AHS	automated highway systems
AICC	advanced interactive cruise control
AVCS	advanced vehicle control systems
CAS	collision avoidance system
CWS	collision warning system
deg	degrees
G	acceleration due to gravity
IDEA	Innovations Deserving Exploratory Analysis
INS	inertial navigation system
ITS	intelligent transportation systems
LIDAR	light detection and ranging
OEM	original equipment manufacturer
PATH	Partners for Advanced Transit and Highways
SRI	Stanford Research Institute
TRB	Transportation Research Board
UCB	University of California, Berkeley

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I INTRODUCTION

A. CONCEPT EXPLORATION PROJECT DESCRIPTION

SRI International and California Partners for Advanced Transit and Highways (PATH)/University of California at Berkeley (UCB) are performing separate but related ITS IDEA concept exploration projects that are composed of two major technical innovations:

- (1) Use of an array of multiple accelerometers to create a "virtual gyroscope" (being developed by UCB)
- (2) Use of GPS interferometry/attitude determination to calibrate the accelerometer array (being developed by SRI).

The motivation for integrating these two innovations is to provide an inexpensive and accurate inertial navigation system (INS) to be used in advanced driver-assistance systems (ADAS), advanced vehicle control systems (AVCS), advanced intelligent/interactive cruise control (AICC) systems, collision warning and avoidance systems (CWS and CAS), and automated highway systems (AHS) for passenger, commercial, and transit vehicles. The inexpensive accelerometer array negates the need for expensive mechanical, fiber optic, or laser-based gyroscopes, and provides data at rates sufficient for vehicle safety and control applications. GPS-based attitude determination provides an accurate and convenient method of aligning the accelerometer array in three dimensions.

B. POTENTIAL IMPACT AND PAYOFF IN PRACTICE

An inexpensive and accurate INS with three-dimensional (3-D) attitude has the potential to dramatically affect the capabilities of advanced driver-assistance systems. Current conventional inertial navigation systems are either too expensive for widespread use in automobile vehicle safety and control systems, or they do not provide the accuracy needed to achieve the potential benefits of the next generation of these systems.

Inexpensive but very inaccurate inertial sensors are currently used in skid-control systems that are available on selected luxury automobiles. Radar and LIDAR-based collision warning and AICC systems currently available on expensive passenger vehicles and commercial trucks and buses will become more available as components become more affordable and capable.

Currently, both Cadillac and Mercedes Benz offer advanced skid-control systems as expensive options for top-of-the-line luxury vehicles. These systems calculate the intended path of the vehicle by sensing the direction of the steering wheel, then compare this direction to the actual path of the vehicle through use of a solid state micro-machined set of accelerometers. If the intended and actual paths differ, selective braking is applied to individual wheels to alter the direction of the vehicle to better approximate the intended path. The systems are rudimentary in that the position of the vehicle relative to the roadway or possible obstructions is unknown, and the systems are ineffective if the difference between the intended and actual paths are too great (i.e. greater than 45 deg.).

For several years, Eaton VORAD has offered a radar-based CWS for commercial trucks and buses. The system senses the presence of potential obstacles and warns the driver when a

collision is calculated to occur within a certain settable time period. Because the system does not have knowledge of the lane location, “non-obstacles” outside of the vehicle lane, especially in curves, and frequently result in false alarms of impending collisions.

The Mitsubishi Diamante sold in Japan has an option for an AICC that employs a LIDAR sensor. LIDAR and radar-based AICCs are currently available on a limited number of luxury Jaguars in Europe, and will soon be offered on several other passenger vehicles with prices for the options expected to range from \$3000 to \$5000.

All of these systems, and future systems, could benefit from accurate, low-cost INS sensors that can increase their capabilities (lanekeeping, for example) at lower cost, thereby enabling their widespread use in the future.

II INVESTIGATIVE APPROACH

A. OVERVIEW

A persistent decrease in the price of GPS receivers and solid-state inertial sensors can be reliably predicted, due to the continual improvements in semiconductor and micro-machining technology, and economies of scale. Each drop in price increases the range of feasible applications, including the navigation of automobiles, trucks, boats, and agricultural equipment, for which the use of integrated navigation systems become commercially viable.

The traditional approach to designing integrated navigation systems is based on the combination of (1) determining absolute navigation parameters (coordinates and velocity), which are obtained from the GPS, and (2) determining relative navigation parameters (relative changes in coordinates and velocity), which are obtained from the inertial navigation system (INS). The INS approach investigated in these two projects provides the required update rate of the navigation parameters between the GPS fixes and also during loss of a GPS signal. However, the INS errors in determination of linear and angular parameters of motion grow continuously with time. The correction of linear (coordinates and velocity) errors is based on the GPS data. For precise determination of angular parameters, accurate gyroscopes are used. Use of gyros in INSs leads to: (1) high system costs (\$10,000 - \$80,000), and (2) inclusion of an additional operation mode (gyroscope initialization), which takes additional time. Therefore, it is more economical to replace gyroscopes with an array of relatively inexpensive semiconductor accelerometers. In this approach, determination of the linear and angular parameters of motion is based on the weighted averaging of data from the accelerometers. However, substituting data from an array of (inexpensive) accelerometers in place of (expensive) gyroscope readings leads to a superlinear increase in attitude errors. This, in turn, causes a rapid increase in errors of linear parameters of motion (coordinates and velocity) between GPS fixes. Therefore, in a gyro-free system, abrupt degradation of navigation system accuracy can be observed.

It is important to mention that a single-antenna GPS, which is traditionally used in integrated navigation systems, regards the vehicle as a single point with no angular orientation. Our use of a multiple-antenna GPS will allow determination of the vehicle attitude; although this adds to system cost, GPS receiver prices have been falling rapidly while gyro prices have fallen slowly. Thus, the critical point in designing gyro-free navigation systems is incorporation of an algorithm for the initial attitude determination and the correction of attitude errors on the basis of GPS data. The idea of a "virtual gyro," consisting of distributed accelerometers combined with a multiple-antenna, GPS-based, attitude-determination system, is an economically attractive solution.

B. TECHNICAL ISSUES

1. INS Errors

The predominant sources of errors for the INS under development can be categorized as either angular or linear in nature. Accelerometer noise contributes to errors in both the angular and linear domains. Linear errors are also caused by misalignments between the accelerometer

array reference frame and a geodetic (earth referenced) frame, thus leading to errors, plus incorrect compensation of gravity due to the same misalignment errors.

In practice, it is possible for the misalignment angles to grow unbounded and render the INS output useless. In such cases, the error of INS grows dramatically. Therefore, it is necessary to provide accurate attitude determination to the INS. As mentioned above, the navigation parameters of a moving vehicle can be determined in an earth-referenced coordinate frame through the use of GPS.

A discussion of the gyro-free INS developed and tested by PATH/UCB is included in Appendix A.

2. GPS Errors

GPS error sources can be grouped into four categories: satellite clock and ephemeris; ionospheric and tropospheric; multipath; and selective availability (SA).

- Satellite ephemeris errors result from the fact that the satellite's position is not known exactly and, therefore, cannot be perfectly reflected in the broadcast ephemeris parameters. The satellite clock error occurs because the clock does not keep perfect time and its drift rate is not absolutely constant. Typically the clock and ephemeris errors are about 3 m each.
- The radio signals from the satellites travel at different speeds in the ionosphere and troposphere than in free-space. The speed in each of these media can be estimated from the parameters of the medium--such as electron density (commonly called total electron content [TEC]) of the ionosphere and the temperature and humidity of the troposphere. However, these estimates are never exact; consequently, a residual, unmodeled delay of a few meters is not accounted for. The delay through the ionosphere is a function of frequency and, therefore, can be measured directly if more than one carrier frequency is available.

GPS satellite transmits on two separate frequencies specifically to allow measurement of the ionospheric delay. However, very few commercial receivers are currently able to directly measure and correct for errors induced by the ionosphere because the U.S. military encrypts the coded signals modulated on the two frequencies required to calculate the delay. Errors induced by the ionosphere and troposphere are more prevalent when receivers are tracking satellites below 20 deg above the horizon, because the signal path from the GPS satellites to the GPS receiver passes through a longer path in the ionosphere and troposphere than do the signals from satellites higher above the horizon.

- Multipath occurs when a receiver antenna, located in the vicinity of reflecting objects such as buildings, trees, vehicles, or the Earth's surface, receives (in addition to or instead of the direct signal from the satellite) energy that has bounced off one or more of the nearby surfaces. When multipath occurs, both the modulation and phase of the signal will be distorted, and the timing measurements are likely to exhibit increased errors. Multipath errors are more prevalent when receivers are tracking satellites below 20 deg above the horizon because signals along low grazing angles are more likely to encounter a surface that will reflect the signal. Stationary GPS receivers and antennas are more likely to experience multipath than those that are moving.
- Selective availability (SA) is a satellite clock error intentionally introduced by the DoD to reduce the accuracy available to non-authorized users, and thereby reduce the probability that adversaries can use GPS to their military advantage. SA location errors are typically 40-60 m and can range as high as 100 m. SA-induced velocity errors are typically 0.5 m/s.

Of these four types of error sources, all but multipath can be substantially reduced by employing differential GPS, a technique described later in this enclosure.

III. ADVANCED GPS TECHNIQUES

As originally conceived, GPS was intended to provide approximately 100-m accuracy to the "unauthorized," non-military user. Standard differential GPS techniques can provide location accuracies of 1-10 m. However, several techniques (discussed in the following paragraphs) are available to commercial users that, in combination, yield position accuracies of a few centimeters and velocity accuracies of a few centimeters per second.

A. CARRIER PHASE TRACKING

In addition to determining range, most of the newer GPS receivers track the phase of the carrier frequency. "Carrier phase tracking" means that the receiver monitors the total number of whole and partial cycles that elapse between successive measurements. Carrier phase information is useful because phase can be measured to about 1/200 of the 19-cm carrier wavelength--about two orders of magnitude better than the accuracy of code measurements.

The number of elapsed cycles is measured for several reasons. First, it provides Doppler information that can be used to accurately determine vehicle velocity. Second, the phase data provide an independent measure of how far the receiver has traveled since the preceding measurement, and therefore can be used to smooth the noisier "code" measurements from which the receiver's position is initially computed. This smoothing process reduces the error-inducing effects of multipath and receiver noise.

B. CARRIER CYCLE AMBIGUITY RESOLUTION

Comparing the relative phase of two receivers (such as a stationary reference receiver and a receiver on a moving vehicle) offers the potential of determining, to within a few centimeters or less, the position of the moving receiver relative to the fixed receiver. To reap the benefits of this measurement, however, the "total" cycle and phase difference between the two receivers must be determined. The phase measurement provides only the single-cycle fraction (0 to 360 deg) of the difference, and the number of whole cycles must be determined by another process, sometimes referred to as "carrier phase ambiguity resolution" or, more correctly, "carrier cycle ambiguity resolution."

Until recently, ambiguity resolution has been largely a trial-and-error effort, in which various whole wavelength combinations (of the signals from all the tracked satellites) are tried until a solution is found that "fits" and continues to fit over time as the vehicle and the satellites move. This requires setting up an initial search volume that has a high degree of certainty of containing the correct location. Moreover, the ambiguities often cannot be resolved instantaneously. A time interval varying from several seconds to several minutes is often needed to completely determine the carrier integer values.

Ambiguity resolution can be performed more quickly if the receiver tracks both the L_1 and L_2 frequencies, rather than just L_1 . If phase information on both carriers is known, the effect is mathematically equivalent to having a single carrier at the wavelength of the difference frequency or, in this case, 86 cm. This technique is referred to as "wide-laning." GPS receivers are currently available that, in low-multipath environments, have differential code measurement

accuracies of less than 1 m. Since the wavelength of the wide-laning difference frequency is close to a meter, the carrier phase ambiguities can be nearly determined by examining the code measurement, thus simplifying considerably the ambiguity resolution process because there are fewer combinations within the search volume.

On-the-fly (OTF) carrier cycle ambiguity resolution techniques that do not require lengthy static initialization routines are seen as very advantageous to the operation of advanced highway and construction vehicles. Techniques developed for surveying and aviation applications hold great potential, and future advances and techniques optimized for the highway environment are expected.

C. GPS ATTITUDE DETERMINATION

Using carrier phase measurements and carrier cycle resolution techniques GPS satellite signals can be used to determine attitude. For example, with two antennas placed a known distance apart and at a known orientation relative to the longitudinal axis of a vehicle, the heading and pitch of that vehicle can be determined because the relative horizontal and vertical positions of the antennas can be determined by a common receiver. For each tracked satellite, the phase difference between the two antennas is measured and converted to the angle the antennas make with respect to the satellite. Attitude accuracies of 0.1-0.25 deg are currently available for antenna separations of 1 m. It should be noted that GPS attitude determination through interferometry is not dependent on high-accuracy carrier phase GPS differential corrections.

IV PROJECT PERFORMANCE

This project was performed in two stages: Stage 1--Integration of Existing "Virtual Gyro" hardware and software (developed by PATH/UCB) with existing GPS Attitude-Determination hardware and software (developed by SRI), and Stage 2-- Testing of the Integrated System. Due to the differences in the start and end dates of the separate contracts to SRI and UCB, it was necessary for SRI to support PATH/UCB in two ways during Stage 1: 1. Provide a time ordered playback file of GPS generated attitude data; and 2. Build an instrumentation set that performed GPS attitude determination and then provide this instrumentation set to UCB. PATH/UCB is currently integrating the different hardware and software components of the system, and will be testing the system to analyze performance. PATH/UCB and SRI are both currently negotiating with a major automaker on development of an advanced driver-assistance system, and other automakers and OEM suppliers have also expressed interest. The results of this project will provide the basis for further development of a product that has the potential to be an integral part of advanced driver-assistance systems. PATH/UCB and SRI will continue development of the resulting product, for both the AVCS research community and the worldwide auto market. Research results from this project will be made available to the AVCS community.

Appendix A

Gyro Free INS

PATH/UCB has developed and tested a gyro-free INS system. An analysis of the growth of errors in the determination of angular and linear parameters of motion shows several sources of error:

- (1) Accelerometer noise is the major source of errors in angular parameters (Error Component 1). The errors show continuous and rather regular growth characterized by a constant parameter.
- (2) The sources of errors in linear parameters (errors in the determination of coordinates and velocity) are the following.
 - (a) Accelerometer noise--The character of growth of this component (Error Component 2a) approximately coincides with the character of attitude error growth.
 - (b) A mismatch between angular characteristics of body frame and an earth-referenced frame leading to an incorrect determination of the vehicle's direction of motion--This component (Error Component 2b) is characterized by continuous growth. Here, with the increase in magnitude of angle mismatch, an increase in the parameter which determines error growth rate can be observed.
 - (c) An incorrect compensation of gravity due to an angular mismatch between the body frame and the earth-referenced frame--The growth of this component (Error Component 2c) is more rapid than that of the previous component and has an abrupt character. This phenomenon can be explained by the fact that for Component 2b, the accelerations have, in general, a sign-changing character. For Component 2c, however, the direction of gravity is always the same in the earth-referenced frame.

An analysis of the results of the simulation studies shows the possibility of classification of the error components. First, the errors can be categorized as either stochastic or deterministic: Components 1 and 2a have a stochastic nature; Components 2b and 2c have a deterministic nature and are determined by the misalignment angles. In contrast, errors can be sorted according to their magnitude:

- In cases when the correction of angular parameters is performed at the frequency equal to the frequency of updates of the linear parameters (or inversely proportional to the time between the two successive GPS fixes), Components 1, 2a, 2b, and 2c are commensurable. Here, for error correction, it is necessary to perform joint correction of deterministic and stochastic error components.
- In cases of initial angular alignment (with initial angular uncertainty), and after the loss of GPS signal (for example, while in a tunnel or while moving in the vicinity of skyscrapers, etc.), Components 2b and 2c grow rapidly with time and their magnitudes are significantly larger than magnitudes of Components 1 and 2a.

Based on the conducted research and analysis, the development of an attitude-correction system and algorithm will be performed in two successive stages:

- Stage 1: Development of an algorithm for correction of deterministic components of total error--as the basic part of the algorithm. This algorithm will be based on measurements from GPS receivers with multiple antennas.
- Stage 2: A combination of an algorithm for deterministic error correction with methods

of stochastic error filtering (particularly on the basis of adaptive Kalman filtering)--as a modernization of the algorithm.

Accelerations, measured by accelerometers, are determined in the moving frame or the body frame. Spatial orientation of the body frame is determined by the spatial orientation of INS accelerometers' sensitive axes. Suppose that the angle between the body frame axes and the earth-referenced frame axes is equal to alpha (for a two-dimensional case).

Determination of the center of the mass acceleration \vec{a} and angular velocity derivative $\dot{\vec{\omega}}$ in the body frame of the vehicle is carried out based on the following relationships.

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \frac{1}{\rho} S \\ T \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} - TC' \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \rho \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_y \omega_z \\ \omega_z \omega_x \\ \omega_x \omega_y \end{bmatrix}$$

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{bmatrix} \quad T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Where:

- ρ = half of the cube side length, and
- C = matrix determining the orientation of body frame, with respect to earth-referenced frame.

However, as noted above, accelerometer noise leads to the accumulation of errors in the determination of angle alpha. Therefore, the INS determines the coordinates in some fictitious coordinate frame, rather than in an earth-referenced frame. The angle between the earth-referenced frame axes and the fictitious frame axes is equal to α_0 . Thus, at the initial moment of time (after initial alignment), the fictitious frame and the earth-referenced frame coincide with each other. Thereafter, with accumulation of error in alpha determination, the fictitious frame rotates relative to the earth-referenced frame and the angle between these coordinate frames becomes equal to α_0 .

Thus, the attitude-correction algorithm must periodically determine the values of angular mismatch (for the two-dimensional case, it is α_0 ; for the three-dimensional case, there are three mismatch angles: ψ, ϕ, θ), and provide the alignment of the fictitious frame to make it collinear to the earth-referenced frame. Taking into consideration a gyro-free INS structure, an attitude-correction algorithm must periodically define the correction to the direct cosine matrix C_{CORR} , which depends on misalignment of angles values. Multiplication of the current value of the direct cosine matrix by C_{CORR} must be equivalent to the alignment of the fictitious frame with the earth

During the time between two successive GPS fixes the change in vehicle coordinates is characterized by the vector of relative displacement r . The coordinates of this vector are known in: (a) earth-referenced frame, based on GPS data; and (b) fictitious frame, based on INS data. When the coordinate frames coincide with each other (for example, after initial alignment), the components of the relative displacement vector will be equal in both coordinate systems.

However, in the presence of angular mismatching, the components of the relative displacement vector will be different in different coordinate systems. This can be explained by the rotation of relative displacement vector (due to the rotation of the fictitious frame) and incorrect compensation of gravity. Thus, it is necessary to rotate the fictitious frame back for providing the coincidence of the relative displacement vector components in both coordinate systems. When this is achieved, the components of the displacement vector will be equal in both coordinate systems, and the errors due to incorrect compensation of gravity and incorrect attitude determination can be minimized.

Appendix B

GPS PRINCIPLES OF OPERATION

1. MULTILATERATION USING SATELLITE CODE SIGNALS

GPS is a navigation system that uses a constellation of 24 satellites whose orbits are arranged in such a way that in populated areas of the Earth, six or more satellites are almost always visible above the local horizon. The satellites continuously transmit coded signals that, when tracked and decoded by a GPS receiver, can be used to accurately determine the location and velocity of the receiver. The receiver is "passive" in the sense that it does not transmit any signals back to the satellite. Therefore, any number of receivers may operate in a given area without interfering with one another.

GPS is based on the principle that, if the distance from three or more known points is known, the location relative to those points can be determined by triangulation. In GPS, the satellites serve as the known points because, at any given time, the position of each satellite in its orbit can be calculated. The location of the satellites is calculated by the GPS receiver using as inputs to the orbital equations certain parameters called ephemeris data. Ephemeris data are broadcast from the satellites to the receiver by means of a 50-bps data message encoded on each satellite's signal. The receiver can measure the distance to the satellites because each satellite signal contains information that indicates the time the signal left the satellite; the receiver also contains a clock that allows it to know the time when the signal arrives. In practice, the GPS receiver must be tracking four (or more) GPS satellites to arrive at a 3-D location because of the need to calculate time in addition to latitude, longitude, and elevation.

2. C/A CODE

The distance from a GPS satellite to a GPS receiver is determined indirectly by measuring the elapsed time between transmission and reception. The satellite signal is modulated in such a way that its arrival time at the receiver can be measured to an accuracy of about 10 ns. The transmission time can be deduced from information contained in the satellite data messages, which are also modulated onto the transmitted signal. For commercial receivers, the timing modulation is referred to as coarse/acquisition (C/A) code, which is transmitted at the rate of 1 MHz, and repeated once per second.

The satellites are equipped with expensive atomic clocks; hence, the transmission time is accurate to within ~10 ns (neglecting SA). The GPS receiver, however, to minimize size and cost, is equipped with an inexpensive crystal oscillator, thus making the estimate of receive time somewhat erroneous. The satellite-to-receiver distance, as measured by the receiver, is therefore referred to as a "pseudorange," indicating the uncertainty resulting from the local clock error. As mentioned in the previous section, range measurements from four satellites are needed to generate a 3-D solution for the position of the receiver. Data from the fourth satellite are required to calculate the receiver clock error.

3. P-CODE

Because GPS was developed as a military system, several additional features are available to "authorized" users. For example, each satellite actually broadcasts three separate ranging signals. The first, called L_1 C/A-code, is transmitted at 1575.42 MHz and is available to all users. However, the accuracy of this signal is intentionally degraded by SA to prevent an enemy from using it to guide weapons. (This intentional degradation is one of the error sources that is removed by a technique called "differential GPS," which is discussed in the next subsection.)

In addition to the C/A code, the satellite also transmits a precision (P)-code signal on the L_1 frequency, which is modulated 90 deg out of phase of the C/A code. The P-code is repeated once per second, as is the C/A code, but is transmitted at a 10-MHz rate, a bandwidth ten times that of C/A code. The P-code signal is also transmitted on a second frequency, known as L_2 , at 1227.6 MHz. The P-code signals are encrypted (then called Y-code) in a process called anti-spoofing (A-S) and can be interpreted only by receivers that are equipped with special decoding circuitry and an encryption key. The P-code signals offer two advantages, in addition to not being intentionally degraded by SA. First, the bandwidth is larger, thereby making more precise ranging measurements possible. Second, because the delay through the ionosphere is a function of frequency, a dual-frequency, P-code receiver can directly measure the code delay rather than having to rely on an approximate mathematical model, as do C/A-code receivers. This is somewhat less important in differential applications, because the ionospheric delay is one of the error sources removed in the differential process.

4. CARRIER SMOOTHING

In addition to measuring the arrival time of the GPS signal using the C/A code modulation, many of the GPS receivers currently on the market also measure the phase of the carrier frequency and the total phase change (both whole and partial cycles) since the previous measurement. This parameter, referred to as "integrated carrier phase" or "accumulated delta range," is statistically independent of the code measurement and is about two orders of magnitude less noisy. The code and carrier measurements can be jointly processed to yield a range estimate that is substantially more accurate than that available from the code measurement alone. The process, known as "carrier smoothing," typically yields pseudorange measurements with noise errors ± 1 m.

5. DIFFERENTIAL GPS

Differential GPS is based on the principle that two receivers in the same general region of the Earth's surface will make approximately the same errors in measuring satellite signals. The reason for this is that the major sources of error are common: uncertainties in the satellite's orbital position, errors through the Earth's ionosphere and troposphere, errors in the satellite's clock, and intentional degradation of GPS accuracy SA. Therefore, GPS accuracy can be improved significantly by placing a "reference" receiver at a fixed, surveyed location and measuring the aggregate effect of these errors. The errors are calculated by comparing the known location of the reference receiver to the location computed from the satellite measurements. Similarly, because the reference receiver is fixed, velocity errors experienced by the navigating receiver can be calculated. When this information is provided to another receiver, that receiver can correct its own location and velocity accordingly. SA, satellite clock errors, and ephemeris errors are common to any C/A code receiver seeing a particular satellite. Because of a decoupling of ionospheric and tropospheric errors with increasing distance between the reference receiver and the navigating receiver, the ionospheric and tropospheric corrections become less valid at increasing distance from the reference receiver. This is especially true for tropospheric corrections when the reference receiver and the navigating receiver are at increasingly different

altitudes. At constant height differences, all of the errors (with the exception of SA) are very slow changing. How often differential corrections need to be calculated and applied, and the separation distance between the reference receiver and the navigating receiver, is dependent upon the desired accuracies. An authorized receiver, which does not experience SA, requires differential correction updates at a much slower rate, and the corrections can be from a reference receiver at a larger separation than an unauthorized receiver would require to maintain the same accuracy. Experiments at SRI indicate that for authorized dual-frequency receivers (not experiencing SA and capable of autonomously calculating ionospheric corrections), differential corrections supplied at 30-min intervals from a reference receiver up to 1000 miles away are sufficient to maintain 2-3 m x,y location accuracies. When corrections to individual satellite measurements are employed in the form of pseudorange corrections (PRCs) and pseudorange rate corrections, the calculations are said to be performed in navigation space. When corrections are applied to a location, such as x, y, and z (and the commensurate velocities) offsets from the location calculated by the navigating receiver, solution space is being operated on. Navigation space corrections are generally preferred because if a reference receiver provides corrections for measurements from eight satellites, a navigating receiver seeing those eight or a subset of those eight can apply the corrections individually. In solution space, both the reference receiver and the navigating receiver would have to be seeing the exact same set of satellites for the correction to be valid, or the reference receiver would have to supply different corrections for all of the different combinations of the subsets of the eight satellites.

6. RELATIVE GPS

Relative GPS is closely related to differential GPS. However, instead of using a reference receiver at a surveyed stationary location, relative GPS removes the common errors between two GPS receivers, one or both of which may be moving. Relative GPS corrections have historically been operating in solution space, and the technique is used when two identical GPS receivers are tracking a common set of satellites. The reason for using two identical receivers is to ensure that the internal operations of the receivers, both hardware and software, produce data that are comparable. GPS receiver manufacturers often use very different hardware and software designs that could exhibit different characteristics. Kalman filter designs, ionospheric and tropospheric models, means of correcting for clock biases and drifts, and measurement accuracies are frequently different, causing different models to produce different data even when co-located and experiencing the same dynamics.

