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16. Abstract

In this report, we study information propagation via inter-vehicle communication along two parallel roads. By identifying an inherent Bernoulli process, we are able to derive the mean and variance of propagation distance. A road separation distance of $\frac{\sqrt{3}}{2}$ times the transmission range distinguishes two cases for approximating the success probability in the Bernoulli process. In addition, our results take the single road as a special case. The numerical test shows that the developed formulas are highly accurate. We also explore the idea of approximating the probability distribution of propagation distance with the Gamma distribution.

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CHARACTERIZING INFORMATION PROPAGATION THROUGH INTER-VEHICLE COMMUNICATION ON A SIMPLE NETWORK OF TWO PARALLEL ROADS

by

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Research Report SWUTC/10/161005-1

October 2010

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ABSTRACT

In this report, the information propagation via inter-vehicle communication is studied along two parallel roads. By identifying an inherent Bernoulli process, it is able to derive the mean and variance of propagation distance. A road separation distance of $\frac{\sqrt{3}}{2}$ times the transmission range distinguishes two cases for approximating the success probability in the Bernoulli process. In addition, the results take the single road as a special case. The numerical test shows that the developed formulas are highly accurate. In addition, this study also explores the idea of approximating the probability distribution of propagation distance with the Gamma distribution.

EXECUTIVE SUMMARY

An elementary concept in the initiative of IntelliDrive™ under the federal Department of Transportation Intelligent Transportation Systems (ITS) program, the Inter-Vehicle Communication (IVC) aims to integrate fast mobile computing and advanced wireless communication technologies with vehicles for the sake of traffic safety, mobility, efficiency and high quality of life. The IVC system connects vehicles with each other (via on-board equipment) and with roadside infrastructure (via roadside stations) so that vehicles can disseminate and share such information as traffic condition, safety related warning, and other matters of importance. A unique cross-disciplinary area between transportation engineering and wireless communication has emerged.

Routing algorithms and communication protocol, critical to this system, both depend on the estimated volume of information potentially transmitted between vehicles. The volume of information is largely dependent on the size of ad hoc mobile vehicular networks. The network size is a function of vehicular connectivity. This connectivity can also be equivalently characterized through information propagation. In this report, we particularly studied the process of information propagation on two parallel roads.

An IVC system depends on various parameters such as (1) communication range, (2) bandwidth of the device, and (3) the number of hops of communication. To simplify this study, the following assumptions are made:

(1) The information propagation distance is measured based on instantaneous connectivity.
(2) The communication range is deterministic and no channel interference is considered.

Therefore, the information propagation in this report is represented by a distance of the last receiving vehicle connected to the information originating vehicle through a transmission range. In other words, it is a connected distance of vehicles. Unlike the previous work that studies the same process on a single road, this work expands onto a network of two parallel roads. The process of instantaneous propagation on two parallel roads is significantly more complex on one road. The complexity is due to interactions of vehicles between the two roads. Information can take a ‘detour’ through the other road when it comes to a vehicle gap larger than the transmission range on one road. Modeling of this process was challenging.

An approximate model was developed, which determines a similar process of information propagation on two parallel roads. The general framework was to develop a Bernoulli process. When information propagation succeeded, the information moved forward by one vehicle gap, called a successful Bernoulli trial. In this process, a Bernoulli region is defined, which was determined by a vehicle gap on one road. A Bernoulli region covers sections on both roads. The Bernoulli region was used to calculate the probability of a successful trial in the Bernoulli process. With this Bernoulli process, it is able to characterize the propagation process by its mean, variance, and probability distribution.

In addition, extensive numerical tests are conducted on the proposed model. This study especially examined the cases when the vehicle headway followed a Gamma distribution, truncated normal distribution, and when the two parallel roads were zigzag. The results showed that the approximation method generally worked well in an overwhelming majority of the cases tested.

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CHAPTER 1:
INTRODUCTION

Recently, a fascinating technology that allows vehicles to exchange traffic information has emerged in a new traffic system called Inter-Vehicle Communication (IVC), also referred to as IntelliDrive in the new literature. It aims to enhance significantly safety, mobility, and quality of life of the public (IntelliDrive website, Dion *et al.* 2010). By integrating fast mobile computing and advanced wireless communication technologies, the IVC system can connect vehicles with each other (via on-board equipment) and with roadside infrastructure (via roadside equipment). Drivers could be warned of potential hazards through in-vehicle display by receiving information, such as speed, acceleration and deceleration rate, road conditions, etc., from neighboring vehicles and roadside infrastructure. The transportation monitors could also use the traffic information transmitted from vehicles on the road networks to adjust transportation system operations. Figure 1 illustrates this scheme of communication. Compared with the traditional systems that distribute information from a centered control station, this system provides more flexibility to individual vehicles and has the capability of strengthening a driver's awareness of surrounding traffic. It is anticipated that this system will remarkably improve the efficiency of the transportation operations. In order to design and deploy the IVC system, one of the important issues is to know the property of information propagating in a vehicular network. In this report, we explore this issue.

BACKGROUND

In the 1970s, a project that enabled vehicles to communicate by radio was initiated in Japan (Kawashima 1990) in order to reduce road congestion and accidents. Although the technology at that time could not provide high quality communication, it explored the potential benefits of sharing information among individual vehicles. In these days, the development of technologies in wireless local area network (WLAN) enables automobile industries to develop and equip reliable wireless devices on vehicles at low cost (Papadimitratos 2009). In 2009, the U.S. Department of Transportation (USDOT) and its public and private partners developed a strategic plan for the IntelliDrive. This USDOT five-year (2010 – 2014) plan mainly focuses on safety, mobility, and environmental applications and research (Website of IntelliDrive). In this system, vehicles are equipped with wireless communication devices, enabling IVC, Vehicle-Infrastructure Integration (VII), and connectivity among infrastructure, vehicles, and other wireless stations. Similar programs, for example, eSafety program in Europe and Advanced Safety Vehicles (ASV) program in Japan, are also under way.

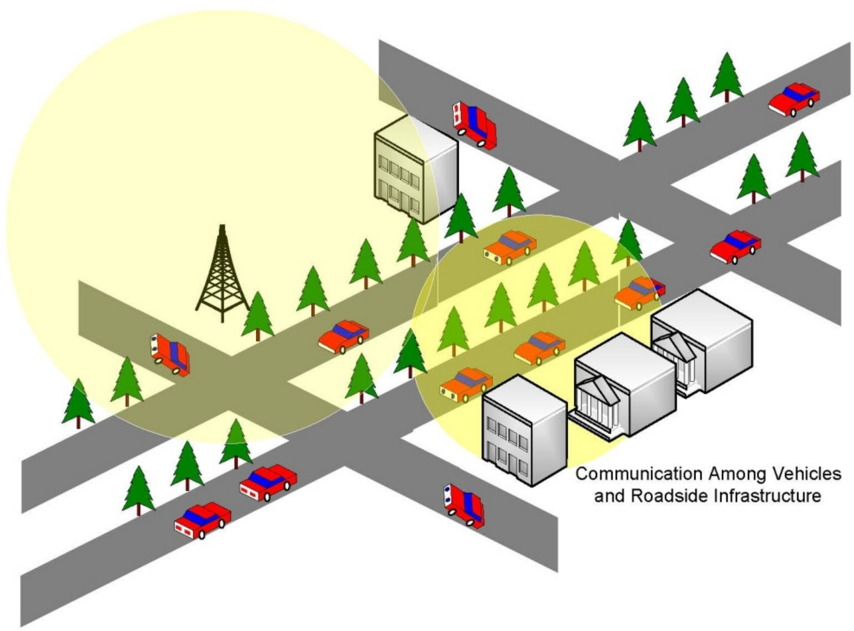

Figure 1: Vehicles Communicate with Neighboring Vehicles and Roadside Infrastructure.

There are two main objectives of the IntelliDrive system. The first objective is to improve safety. IVC and VII techniques can monitor traffic situations and alert drivers as soon as potential hazards take place, and even automatically navigate the vehicles, by exchanging driving information such as acceleration and deceleration. It also can potentially reduce a large number of accidents caused by careless driving or drivers' misperception. For instance, the application of blind spot warning could alert drivers who tend to change lanes frequently when another vehicle is in the blind spot (IntelliDrive Website). Moreover, the relative analysis shows that the estimated total safety benefit equals to 41.8 billion dollars in accident cost savings (VII

Initiative Benefit-cost Analysis 2008). The second objective is to increase mobility. It is recognized that ineffective traffic control and inefficient use of road system capacity are due to limited traffic information available to planners and performance limitations of drivers. The intent of the IVC system is to provide information about vehicles in the traffic network, such as the location and route of each vehicle, so that traffic managers are able to adjust traffic signals to improve overall traffic flow. An application such as adaptive cruise control can also reduce the headway between two vehicles, resulting in an increase in the road capacity (Shladover *et al.* 2009). Another benefit of lessening traffic and congestion is an estimated savings of 1.2 million gallons of fuel a year (VII Initiative Benefit-cost Analysis, 2008).

Networks such as IntelliDrive systems are called Vehicular ad hoc Networks (VANETs) (Willke *et al.* 2009), referring to self-organizing wireless communication networks among vehicles without the aid of established infrastructure. In these systems, the information of one vehicle and/or its surroundings (for instance, speed and relative distance) is coded into a piece of so-called *beacon message* for wireless transmission. For each wireless device, the power of a signal attenuates as the distance from this device increases. When the power is below a certain value, the message cannot be decoded. That means two vehicles equipped with the same kind of wireless communication device can exchange information only when their distance headway is less than a certain distance, so-called *communication range*. If two vehicles are not within the communication range, their communication depends on *multi-hop* transmission. Since vehicles constantly enter and exit a segment of road, the communication connectivity among vehicles may vary constantly, which leads to concerns about performance and challenges in designing VANETs.

It is recognized that the *information propagation distance*, defined from the source of a vehicle delivering one piece of message to the furthest receiver, is a fundamental measure for the performance of the IVC system, as it can help estimate the connectivity in a vehicular network and thereby guide the design of wireless communication devices and communication protocols. However, previous studies on this issue have been limited to the case of one (lane) straight road. Interactions among vehicles from different roads on a discrete traffic network have significant impact on effective information propagation along traffic streams. A basic situation is parallel roads, which might be found as two divided highway lanes or an elevated road and its frontage road. The rationale for studying this case was based on the question: "If one cannot handle this case, how can one expect to address the case of discrete networks?" This research began with a study of the statistic property of information propagation on two parallel roads.

PROBLEM STATEMENT

Information propagation in VANETs depends on various parameters such as communication range, bandwidth of the device, and number of hops. To avoid the complexity that prohibits development of useful results, in this study, the following three major assumptions are made:

- Message is transmitted and delivered instantaneously with respect to vehicle movement.
- The information propagation distance is measured based on instantaneous connectivity.
- The communication range is deterministic and no channel interference is considered.

The first assumption gives reasonable representation of the current wireless communication technologies, according to which the data transmission interval is less than 100 ms (Briesemeister *et al.* 2000, Ohyama *et al.* 2000). This was also true for most applications of vehicle-to-vehicle communications (Chen *et al.* 2010). During such a short period, the effect of vehicle movement can be omitted. Admittedly, this requirement could only be realized in practice with a large communication bandwidth device and short messages. The second assumption disregards the connectivity through vehicle mobility. If two neighboring vehicles, which travel in opposite directions, have distance headway larger than the communication range of a wireless device, the message could not be delivered between them instantaneously. Although such a message could possibly be sent to one vehicle in the opposite direction after a period during which the transmitter holds it, this kind of information propagation could not be effective when applications relate to, for example, urgent safety issues. Hence, this study only considered the vehicles with instantaneous connectivity. The third assumption states that the fading effect of the wireless device is not considered. Otherwise, it would lead to the highly complex expression results without better understanding of the basic network property. In fact, it is not difficult to extend the proposed method to the fading case.

Consider two parallel straight roadways denoted by R_1 and R_2, respectively. R_1 and R_2 are separated by a distance d. The traffic density on each road is λ_1 and λ_2, respectively. According to the assumptions, the direction of movement of traffic is not considered explicitly. Since usually the communication range L is significantly larger than the width of lanes, each road of multi-lanes can be considered a single lane. As illustrated in Figure 2, starting from a vehicle A on road R_2, information is propagated forward in one direction of interest, say, rightward. Vehicles on both roads within the communication range of A are able to receive and instantly further transmit the information forward. If we only consider the information transmission on road R_2, and if there is no vehicle present on road R_2 within the communication range L, vehicle A is not able to reach directly vehicle C. However, with help from vehicle G' on road R_1 within range L, information is able to propagate to vehicle C. Clearly, compared with the case of one road, information propagation is enhanced by vehicles on the second road (Wang 2007).

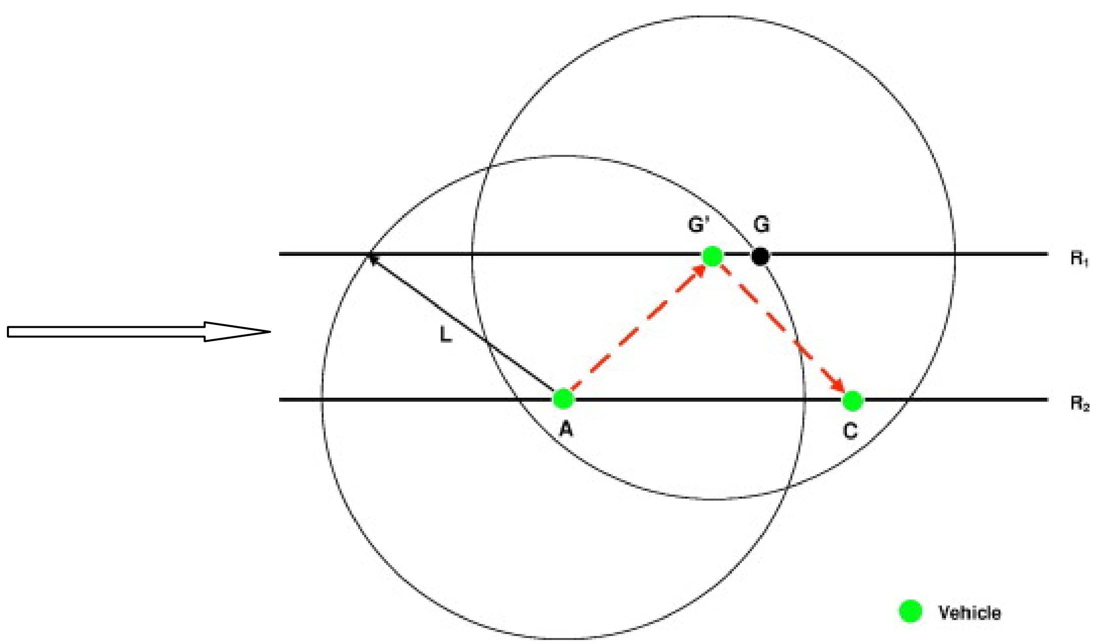

Figure 2: An Illustrative Process of Information Propagation.

The objective of this study was to find out the probabilistic property of the information propagation distance in terms of its expectation, variance, and probability distribution. The propagation distance measures from the initiating vehicle to the last receiving one on the same road, as illustrated in Figure 3. According to such definition and assumptions, one can also treat the information propagation distance as the diameter of a random graph (Penrose 2003). At the beginning of the next chapter, a mathematical description of the issue is provided.

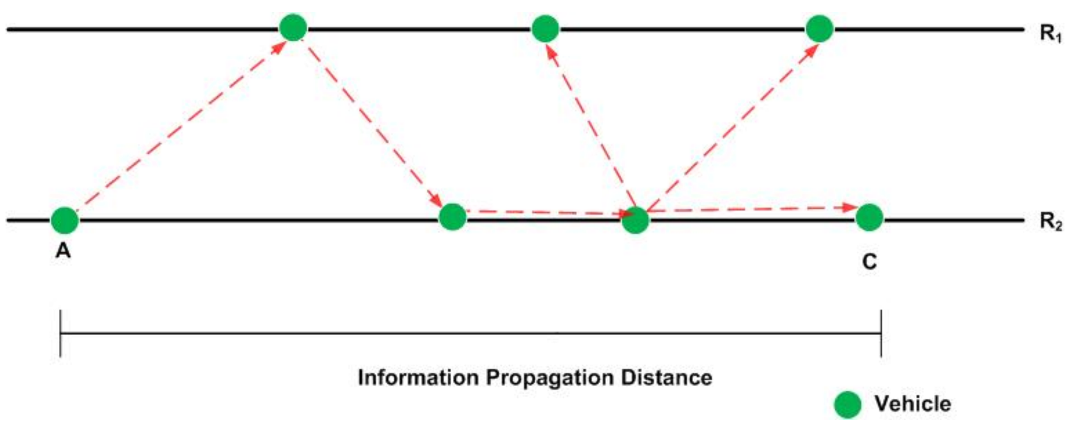

Figure 3: An Illustration of Information Propagation Distance.

OVERVIEW OF THE PROPOSED METHODOLOGY

To deal with the complexity of information propagation on two parallel roads, we developed an approximate method. By identifying the conditions that the information fails to propagate further, the propagation process was approximated by a Bernoulli process. This method can also be applied to the case of one road (Wang 2007). For this model, the distance headway can be generalized to any distribution. The approximation generally works well but suffers significant errors in some cases.

Considering the particular vehicular network studied here, the information process can be viewed as an instance of a stochastic geometry process. Due to the inherent important relationship between geometry process and wireless communication network, the proposed analytical method has a potential to be generalized for other research on ad hoc networks (Kendall and Molchanov 2010).

RESEARCH OBJECTIVE

The objective of this research was to model the information propagation process along two parallel roads. The focus was to develop models to estimate the expectation, the variance, and the probability distribution of information propagation along two parallel roads under certain distributions of vehicle headway. In details, this study will do as follows.

- Review and assessment of the current research on IVC systems.
- Develop an approximate model to address explicitly the information propagation distance by assuming a general distribution for the distance headway.
- Implement numerical tests of the proposed models, including numerical calculation and simulation.

CHAPTER 2:
LITERATURE REVIEW

An appropriate model of wireless ad hoc networks, particularly a probabilistic model simplifying physical world without losing basic characteristics, can give some insights into the performance and help design IVC systems. Many groundbreaking analytical results are obtained ranging from connectivity issues to network capacity. On the other hand, simulation is necessary to explore the performance, because very often the complexity of IVC systems exceeds the capability of analytical methods due to a large number of factors, such as signal fading and interference, and because field tests are too costly to conduct (Dion *et al.* 2010). This chapter focuses on the simulation and two basic topics: network capacity and connectivity.

SIMULATION ISSUES

Computer simulations serve as a basic tool to evaluate the functionality of IVC systems. An appropriate simulator must be capable of modeling both wireless network communication and traffic flow. Unfortunately, currently no integrated simulator exists with a full range of components, though there are many advanced simulators in each individual area and a number of efforts in progress (Piorkowski *et al.* 2008, Balcioglu *et al.* 2009). From the perspective of wireless communication, one challenge of simulation comes from simulating radio signal propagation with various environment factors. Besides the path loss effect, i.e., the power reduction as signal propagates in space, the radio wave signal may be diffracted and scatted by the environment, and the vehicle mobility may also cause a time-variant distortion to the signal strength, known as shadowing and fading, respectively (Killat 2009). The multi-path loss effect also plays a significant role in communication. It arises when the same messages transmitted through different internodes arrive at a receiver simultaneously. In vehicular networks, information propagation usually takes place in a multi-hop manner. The percentage of equipped vehicles, known as *market penetration rate*, may affect the multi-hop delivery since this value can vary significantly in different places in a large-scale network. To deal with some of these issues in simulation, currently, there are several wireless communication simulators, such as Qualnet, NS-2, OPTNET, and NCTUns.

A traffic simulator is needed to describe the movement of vehicles according to some constrains such as road topology and speed limit. From the perspective of traffic engineering, a qualified simulator should employ at least microscopic and macroscopic mobility models. The former tracks individual vehicles and mimics individual driver behaviors with response to surrounding traffic, such as car following and lane changing. The latter deals with the vehicular traffic flow at an aggregate level, describing the phenomena such as trip generation (Hoogendoom and Bovy 2001, Harri *et al.* 2006). Current studies tend to integrate the traffic and wireless communication simulators, for examples, combing NS-2 and microscopic simulator VISSIM (Park and Lee 2009), and to explore the performance of vehicular networks, topics including transverse message delivery (Kesting *et al.* 2010), packet-routing protocols (Wang *et al.* 2009), packet collision rate (Jarupan *et al.* 2008), and safety improvement (Killat 2009).

There are many benefits for establishing a realistic simulation framework. It is crucial to test current theoretical models and design of next generation networks (Harri *et al.* 2006). Although it is argued that field tests provide much valuable information, field tests are often restricted to addressing small-scale issues. This is because large-scale deployment of IVC systems would be too expensive (Shladover 2009). Hence, evaluation of IntelliDrive systems in simulation will remain crucial for a long time.

CAPACITY OF VEHICULAR NETWORKS

It is important to measure the "amount of information flow" in a wireless ad hoc network. This issue is addressed as the *capacity* of networks from different perspectives. In a pioneered work, Gupta and Kumar (2000) study the transport capacity that measures the distance-rate throughput that is transported per second. They investigate a model of fixed ad hoc networks where the source and destination are randomly located with a fixed communication range. The main result shows that the capacity reduces linearly as $\Theta(\frac{1}{\sqrt{n}})$ with the decreasing number of nodes n.

Along this strategy, the work in (Xie and Kumar 2006) shows the expected transport capacity can be upper-bounded by the multiple of total transmission powers of nodes in some cases. Franceschetti *et al.* (2007) apply percolation theory to find a lower bound on the achievable bit rate in a wireless network. Although there may be some interference in the communication, their results show that in some cases the total amount of information sent by all sources can be transmitted by the nodes along the end-to-end connected paths. For the case of independent fading channel between any pair of nodes, the exact expression of per-node capacity can also be found if multi-hop connection only allows data forwarding (Nebat *et al.* 2009).

The transmission capacity, defined as the throughput of successful transmitting nodes in the network per unit area, is first introduced in Weber *et al.* (2005) with an outage constraint. Most interestingly, the transmission capacity can be tightly bounded in many situations. Currently, the transmission capacity has been studied from the perspectives of design and performance analysis, addressing the issues such as interference cancelation (Weber *et al.* 2007a), power control (Weber *et al.* 2007b), and the relationship with outage probability and data rate (Haenggi 2009). However, most of these studies focus on a one-hop wireless ad hoc network. For the VANETs, several works show the mobility of nodes increase the capacity of the networks (Grossglauser and Tse 2002, Diggavi *et al.* 2005). The results show that the end-to-end throughput does not significantly decrease with the growth of number of nodes in the entire network. From the transportation engineering perspective, Du *et al.* (2009) consider the VANETs with a broadcast transmission protocol and study the broadcast capacity measured by the maximum number of successful concurrent transmissions. Due to the particular characteristics of traffic flow, various effects such as traffic density and vehicular distribution are investigated by two integer programming models.

CONNECTIVITY OF VEHICULAR NETWORKS

Estimating connectivity of wireless ad hoc networks like IVC systems is important for understanding the effectiveness of the system and designing routing protocols. Bettstetter and Eberspacher (2003) investigate the probability distribution of the minimum number of hops, given fixed number of uniformly distributed nodes on a rectangular area. Although closed form expressions can be derived for the cases that the nodes are one or two hops connected, the cases of two hops or more are only studied by simulation. Mullen (2003) presents two approximate models for the distance distribution of connected nodes within a rectangular region, assuming that the static nodes are independently distributed in each dimension. The work by Orris and Barton (2003) and its correction by Zanella *et al.* (2009) provide the distribution of the number of nodes within one-hop area with log-normal fading effect. Although this work does not allow multi-hop connection, it provides some guideline and inspires further research. Mukherjee and Avidor (2008) extend the study to the multi-hop network with homogeneous Poisson distributed nodes in a plane, considering the fading channel and power consumption with battery-operated nodes.

Although there are many studies on the connectivity of ad hoc wireless networks, the work on IVC systems from a connectivity perspective is rare. The IVC systems distinguish themselves from other ad hoc networks in several ways. In contrast to the previous studies where nodes are randomly distributed within a circle or a rectangular plane and are randomly assigned with a speed, groups of vehicles travel in a segment of road along a fixed direction whereby the traffic density, volume, and speed follow certain relationship. That means the mobility of nodes is predictable with a constrained road topology. Since vehicles can provide enough power to the wireless devices, power consumption is not an issue. Additionally, most previous work focuses on one-hop broadcast communication, but IVC systems adopt multi-hop strategy in real application. Inspired by these particular features, many researchers from transportation and electronic engineering areas make great effort. Wang (2007) studies the information propagation along one road. This work defines a relay process to obtain a closed formula for information propagation distance when the traffic follows homogeneous Poisson distribution. Wu *et al.* (2009) considers the mobility effect on the information propagation. The vehicles are assumed to have the same communication range and a constant delay between receiving and transmitting at the same vehicle. In light of this strategy, the study in Kesting *et al.* (2010) considers the effects of two-direction traffic flow, assuming the messages to be delivered within a tolerable delay. When no vehicle locates within the communication range, a vehicle can broadcast the information until it reaches another one in the opposite direction. The investigation of this kind of connectivity is conducted by simulation. Jin and Recker (2010) extend study to the inhomogeneous Poisson traffic where the positions of vehicles are known through simulation or field observation.

CHAPTER 3:
BERNOULLI APPROXIMATION

INTRODUCTION

This chapter discusses the process of instantaneous information propagation along two parallel roads of traffic, as stated in Chapter 1. In the context of instantaneous propagation, the propagation distance is equivalent to the distance of the furthest connected vehicle on the same road. This problem can be also described by the random geometric graph (Penrose 2003) discussed in the following section.

RANDOM GEOMETRIC GRAPHS DESCRIPTION

The assumptions in Chapter 1 enable to use random geometric graphs to model the vehicle ad hoc networks. It is assumed that the network nodes were randomly placed on two parallel lines, and a communication link connects two nodes if the distance between them was not greater than the communication range. A mathematical model for this case is as follows. Let $\|\cdot\|$ be the Euclidean norm, and R_1 and R_2 be two parallel lines on $\mathbb{R}^2$. The distance of these two lines is $d > 0$. Let $f_1(\cdot)$ and $f_2(\cdot)$ be some two probability density function (p.d.f.) on R_1 and R_2, respectively. Note that $f_1(\cdot)$ and $f_2(\cdot)$ are defined on $\mathbb{R}^1$. Also, let $X_1, X_2, X_3, \ldots\ldots$ be independent and identically distributed (i.i.d.) random variables with the density $f_1(\cdot)$, where X_i denotes he random location of node i on the line R_1. Similarly, let $Y_1, Y_2, Y_3, \ldots$ be i.i.d. random variables with the density $f_2(\cdot)$ on the line R_2. The ensemble of graphs with undirected links connecting all those pairs $\{z_i, z_j\}$ (where $z_i \in \{X_i\}$ or $\{Y_i\}$) with $\|z_i - z_j\| \le L, L > 0,$ is called *random geometric graph* (Penrose 2003), which is denoted by $G(\chi, L)$.

In this model, when saying 'one node can send the information to another one,' it means there exists a link connecting these two nodes. The problem of *information propagation* can be described as follows. Let W be any connected component of $G(\chi, L)$ and $\{Y_0, Y_1, ..., Y_N\} \subset W$, where N is a random integer valued variable and it depends on W. The node Y_i should locate between Y_{i-1} and Y_{i+1} on R_2. Define the *information propagation distance* D_2 on R_2 in this chapter as follows:

$$D_2 \triangleq \sup\{\|y_i - y_j\| : i, j \in \{0,1,...,N\}\}.$$

In this definition, one can see that $D_2 = \|y_0 - y_N\|$. Let $H_i = \|y_i - y_{i-1}\|$. Then one can express D_2 as

$$D_2 = \sum_{i=1}^{N} H_i. \qquad [1]$$

It is primarily concerned with the expectation $E[D_2]$ and variance $V(D_2)$ in this chapter.

IDEAS ABOUT THE SOLUTION

In this chapter, the ideas about the solution of Equation 1 are straightforward: to find an approximation that all H_i are independent. In details, the steps of this study are:

- to find some geometric region (called Bernoulli region) related to each pair $\{y_i, y_{i-1}\}$ (they may not necessarily connect with each other), where the length of one edge of this region is equal to H_i;
- to find the probability of H_i by conditioning on the same connected component;
- to treat approximately all Bernoulli regions to be independent under some situations; and
- to consider the probability for each Bernoulli region occurring the same, so that the information propagation can be approximated by Bernoulli process.

In the next sections, we will first identify two cases in terms of the distance d between R_1 and R_2, where in the first case the proposed approximation performs well.

With the help of the Bernoulli region, we derived the mean and variance of the propagation distance. Then, numerical simulations were conducted to assess accuracy of the results from using the Bernoulli process, and to show how distance between the two parallel roads impacts the propagation distance.

APPROXIMATION WITH A BERNOULLI MODEL

This study considered the information relayed along one road rightward, say R_2. Vehicle A is the transmitting vehicle on R_2, as shown in Figure 4. Suppose G and H are the furthest points directly reachable by vehicle A on both roads. $ABGH$ is the according parallelogram with $BG = AH = L$. Two cases are identified. Left side of Figure 4 shows that point B within the communication range of vehicle A, representing Case I. In Case I, it is clear that vehicles left of the parallelogram $ABGH$ do not matter in further propagating information rightward from vehicle A: any vehicle left of point B on road R_1 would have to resort to vehicles within BG for further propagation. By the language of random graph, it means that any node in BG can connect with A and only these nodes can connect the nodes (if any) at the right of points G and H. We further define $r = 2(L - \sqrt{L^2 - d^2})$ in Case I, representing two times the horizontal *shift* of the two horizontal sides of each parallelogram.

If point B of the parallelogram is outside vehicle A's transmission region, one gets Case II as shows on the right of Figure 4. In Case II, it is not possible to have a parallelogram $ABGH$ within the communication range of A, and point B will be out of reach by the vehicle at point A as shown in Figure 4. In the later part, we define for Case II the point B as the leftmost reachable point by A, in which case $ABGH$ will not be a parallelogram.

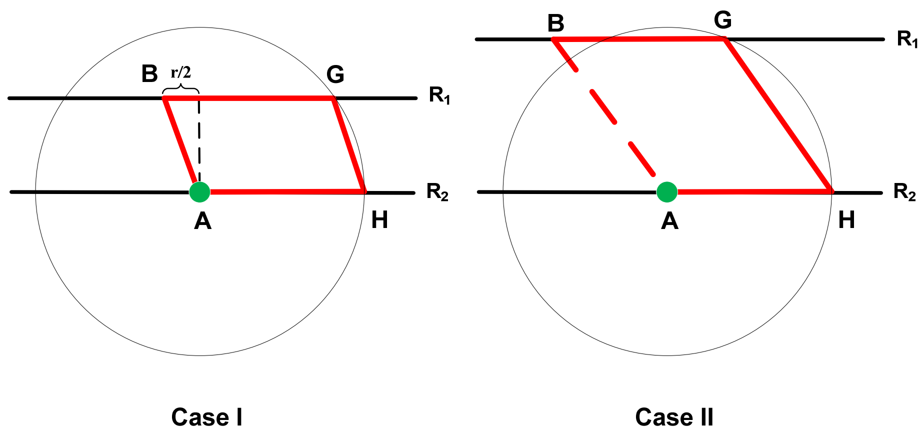

Figure 4: Two Cases of Point B on Road R_1.

Case I has the following relationship: $d \le \frac{\sqrt{3}}{2}L$. Similarly, Case II corresponds to the situation in which $d > \frac{\sqrt{3}}{2}L$. Here $d = \frac{\sqrt{3}}{2}L$ is the critical point separating two cases, at which the parallelogram $ABGH$ has exactly three points BGH on the circle of radius L about point A. The increase of the separation distance d makes point B fall outside the circle, implying Case II, while the decrease draws point B to within the circle, implying Case I. We first discuss Case I.

CASE I

Suppose vehicle A is the transmitting vehicle on R_2 and C is the immediate next vehicle to A on R_2. That is, there is no vehicle in the segment AC. As shown in Figure 5, $ABGH$ is the according parallelogram defined earlier. We further assumed that E is the leftmost point on road R_1 that can reach vehicle C on road R_2. Let $ED = EC = FC = L$, and $CDEF$ is the according parallelogram. The region $ABDC$, according to a vehicle gap AC on road R_2, is called the *Bernoulli region* of vehicle A in Case I. It is worth noting that BD has the length $AC + r$. In addition, one has to note that Figure 5 just illustrates an example for the Bernoulli region. The distance headway of vehicles A and C can be any value. The two parallelograms $ABGH$ and $CDEF$ could have some overlap. Hence, any distance headway on R_2 is associated with a Bernoulli region. Or equivalently, *any* vehicle (as A in Figure 5) on road R_2 has a Bernoulli region with according points B and D defined above. Such region is critical to information propagation. In summary, we have the next proposition.

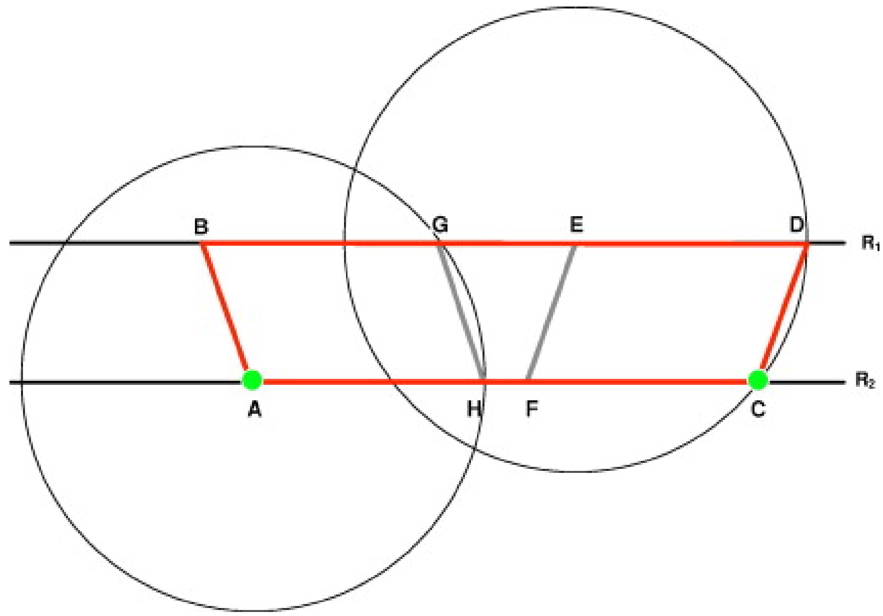

Figure 5: A Bernoulli Region *ABDC* during Propagation.

Proposition 1. Information propagation along road R_2 is gapped out at a vehicle location A if, and only if, the associated Bernoulli region has a vehicle gap larger than L on both roads.

By the language of random graph, when we say 'the information propagation is gapped out,' we mean that two neighboring vehicles on R_2 are not in the same connected component. The Proposition 1 is obvious: if there is no link to connect A and C, the only situation for them in the same connected component is that some node within one connected component in BD connects with A and C, as shown in Figure 5.

It is clear that information propagation gaps out if, and only if, the process fails to get through a Bernoulli region. Each vehicle on road R_2 has an according Bernoulli region with a probability of success. Since the vehicle gaps on each road are *i.i.d.*, one may roughly treat each Bernoulli region as having an *identical and independent* probability of success. Propagating through vehicles on road R_2 can, therefore, be considered as a Bernoulli process whose "trials" are the Bernoulli regions. The success means successful propagation through the Bernoulli region and failure means otherwise. The gap-out probability associated with a Bernoulli region is denoted by p_r. In this report, we assess the value of p_r by assuming independent Bernoulli regions. This assessment of p_r could be fairly accurate but not exact, as explained below. This is why we call the Bernoulli process here an *approximate* Bernoulli process.

This treatment using the Bernoulli process is not accurate because the probability of gap-out at a "trial" could be slightly correlated to the probability of no gap out at its proceeding trial. The Bernoulli regions associated with two consecutive vehicles on road R_2; i.e., $ABDC$ and $CGEF$, as in Figure 6, have an overlap of length r on road R_1. When two consecutive gaps larger than L are present on road R_2, not gapping out in the first Bernoulli region might indicate, to a certain degree, vehicle presence in the overlap section on road R_1. From such perspective, r is an important parameter in this study as it indicates the extent to which vehicular interaction takes place between the two roads.

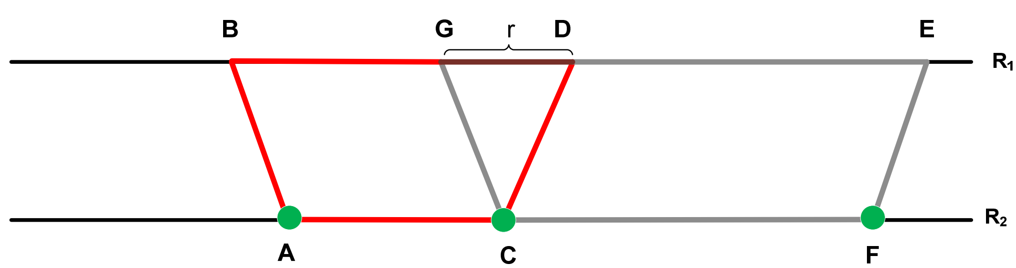

Figure 6: Overlap between Two Consecutive Bernoulli Regions.

According to the Bernoulli process, the random number of gaps on R_2 (or, Bernoulli regions) until the first gap out follows a Geometric distribution, whose mean is $1/p_r$. In other words, if the number of gaps before a gap out is denoted by p_r, then we have $E[N]=1/p_r-1$. In order to get the expected propagation distance on road R_2, we need to calculate the expected length of the gap before a gap-out. By the language of random geometric graphs, we treat H_i as i.i.d. random variables (in Equation 1) for the associated nodes in one connected component of the graph.

As vehicles' presence on both roads are independent of each other, the probability of gap out, p_r, is calculated as the product of two probabilities: one for a gap g larger than L on road R_2 and the other for a gap larger than L on road R_1 within the range $g+r$, where g is the vehicle gap on road R_2. Since the distance headway (or gap) of vehicles is assumed homogeneous along the roads, the probability of a gap larger than L on road R_1 within a range y, denoted by $p(y)$, can be calculated recursively by using conditional probability. If we denote by g_{next} the distance headway between the starting point on R_1 and next vehicle along the direction, by noting that $g(y\,|\,\{g_{next}>L\})=1$, we have

$$
\begin{aligned}
p(y) &= E(p(y\,|\,g_{next})),\\
&= \int_0^L f_1(t)p(y-t)dt + \int_L^\infty f_1(t)\cdot 1 dt.
\end{aligned}
\tag{2}
$$

Obvious when $y>L, p(y)=0$. Hence, we have

$$
p(y)=\begin{cases} \int_0^L f_1(t)p(y-t)dt + 1 - F_1(L), when\ y \ge L,\\ 0, when\ y<L. \end{cases}
\tag{3}
$$

Where, $f_i\ (\square)$ is the probability density function of the distance headway on road i, whose cumulative function is $F_i\ (\square)$. For the gap-out probability of a distance y on road R_1, Equation 3 means that if no vehicle is present in [0, L], whose probability is $1-F_1\ (L)$, there is a gapout; if a vehicle is present at distance t in [0, L], the failure of detour on R_1 then depends on gapout over the remaining length $y-t$, the probability for which is therefore $f_1(t)p(y-t)dt$. Note here that

t is the location of the first vehicle from the beginning of the Bernoulli region on road R_1. Figure 7 illustrates this recursive relation.

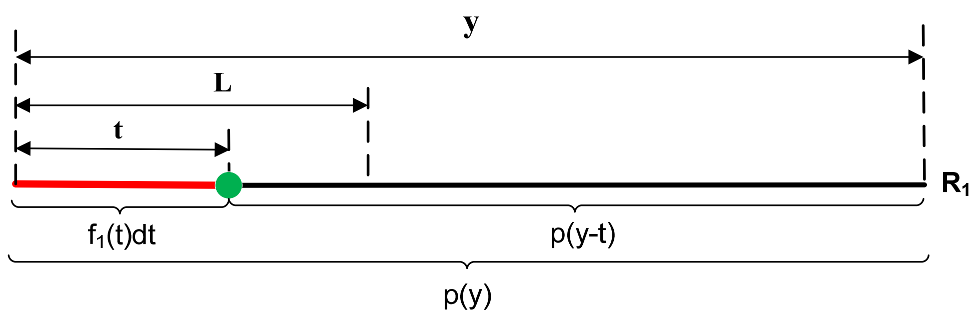

Figure 7: Illustration of Equation 1 – Conditional on Next Vehicle.

The failure (gap out) probability of a Bernoulli region is $p_r = \int_L^\infty f_2(t)p(t+r)dt$, meaning that there is vehicle distance headway larger than L on both roads. If we denote the event of success Bernoulli region by S , the expected length of each *distance headway* or *gap* associated with one Bernoulli region, denoted by $E[g \mid S]$, can be calculated as follows.

$$E[g \mid S] = \frac{\int_0^L tf_2(t)dt + \int_L^\infty tf_2(t)(1-p(t+r))dt}{1-\int_L^\infty f_2(t)p(t+r)dt}. \quad [4]$$

The denominator is the success probability of a Bernoulli region. The probability density function $f_2(\cdot)$ in the numerator divided by the denominator leads to the conditional probability density function on no gap-out. The first term in the numerator corresponds to the case of a gap smaller than L on R_2; and the second term refers to the case of a larger than L gap on road R_2 but of a success of the according Bernoulli region. If we denote $E[g]$ as the expected distance headway on road R_2, Equation 4 can be simplified as follows:

$$E[g \mid S] = \frac{E[g] - \int_L^\infty t f_2(t) p(t+r))dt}{1 - \int_L^\infty f_2(t) p(t+r) dt}. \quad [5]$$

Therefore, the expected propagation distance can be approximated by the following:

$$\begin{aligned} E[D_2] &= E[g \mid S](1/p_r - 1) \\ &= \frac{E[g \mid S](1-p_r)}{p_r} \\ &= \frac{E[g] - \int_L^\infty t f_2(t) p(t+r) dt}{p_r}. \end{aligned} \quad [6]$$

In addition, by using the Bernoulli process, we are able to find the variance of propagation distance D_2, $V[D_2]$. Recall the Equation 1, $D_2 = \sum_{i=1}^{N} H_i$, where H_i are distance headways indexed from the initial transmitting vehicle. According to previous analysis, H_i is i.i.d. with H, whose distribution is the same with g under the event S. By the law of total variance, the variance of propagation distance can be obtained. Then the next proposition becomes obvious.

Proposition 2. The mean and variance of information propagation distance on road R_2 may be expressed as follows.

$$E[D_2] = \frac{E[g] - \int_L^\infty t f_2(t) p(t+r) dt}{1 - \int_L^\infty f_2(t) p(t+r) dt},$$

and

$$\begin{aligned} V[D_2] &= V[E[\sum_{i=1}^{N} H_i \mid N]] + E[V(\sum_{i=1}^{N} H_i \mid N)] \\ &= E^2[H]V(N) + E[N]V(H) \end{aligned} \quad [7]$$

Where, H has a probability density function as follows:

$$f(y) = \begin{cases} f_1(t)/(1-p_r), \mathit{for}\ t \le L, \\ f_2(t)(1-p(t+r))/(1-p_r), \mathit{for}\ t > L. \end{cases} \qquad [8]$$

With Equation 8, we are able to numerically get $E[H]$ and $V(H)$. Therefore, Equation 7 can be evaluated with numerical method easily. The proposed model can also be applied to the special case when there is only one road. Setting $\lambda_1 = 0.0$, one can show that Equations 6 and 7 lead to the following results.

Proposition 3. Information propagation along a single road has an expected propagation distance and variance as follows.

$$E[D_2] = \frac{\int_0^L t f_2(t)dt}{1-F(L)},$$

and

$$V[D_2] = \frac{\int_0^L t^2 f_2(t)dt}{1-F(L)} + (E[D])^2, \qquad [9]$$

Where, $f(\cdot)$ and $F(\cdot)$ are the respective density and cumulative functions of distance headway with two roads combined, and D is the information propagation distance.

We provide a short explanation. Note that $p(t+r) = 1.0$ always holds when $\lambda_1 = 0$. Substitution of $D = D_2$ in Equation 6 gives $E[D_2] = \frac{\int_0^L t f_2(t)dt}{1-F(L)}$, and since N has a Geometric distribution, the following equations are easy to obtained:

$$E[N] = \frac{F(L)}{1-F(L)}, \mathit{and} \quad V(N) = \frac{F(L)}{(1-F(L))^2}.$$

Hence, by Equation 7, we have the expression of $V(D)$.

Proposition 3 was developed in Wang *et al.* (2010). Here by a Bernoulli process, we have provided an alternative proof.

CASE II

As stated earlier, Case II satisfies such a condition: $d > \frac{\sqrt{3}}{2}L$.

In Figure 8, the gap between two consecutive vehicles A and C is larger than L on road R_2 as we are interested in the gap out probability. B is the leftmost point on road R_1 that the vehicle at point A on road R_2 can reach within a communication range L; i.e., $AB = L$. E is the furthest point on road R_1 horizontally left of point C that is able to directly reach point C; i.e., $EC = L$. D is the furthest reach to the right directly from point E on road R_1; i.e., $ED = L$. In addition, G is the rightmost point on road R_1 directly reachable by vehicle A, and clearly $BG = 2\sqrt{L^2 - d^2} \neq r$. F is the leftmost point reachable on road R_2 by a vehicle at point E with $CF = BG$. E_1 is the leftmost point to the right of vehicle C horizontally that can reach vehicle C. C_1 is the rightmost point reachable on R_2 by point E_1. C_2 on road R_2 corresponds to point D on road R_1. We have $CC_2 = L$. Obviously, $BE = AC$ and $BD = AC + L$. We still refer to $ABDC$ as a Bernoulli region, although with slight notational abuse about points B and D defined earlier. Note that the Bernoulli region in Case I has $BD = AC + r$, which is different from Case II due to the different situations. In order to take a detour on road R_1 to C on R_2, there are two necessary conditions. First, there must be vehicle presence within the range $[B, G]$ on road R_1; otherwise, no vehicle within the section of a length $L - BG$ left of B is capable of propagating further till beyond G. This first condition implies ignorance of information coming from left of point B on road R_1. Second, vehicles cannot gap out within $[G, D]$ in order to assist the propagation. Note that vehicles have probabilistic presence on road R_1, given the vehicle gap AC on road R_2.

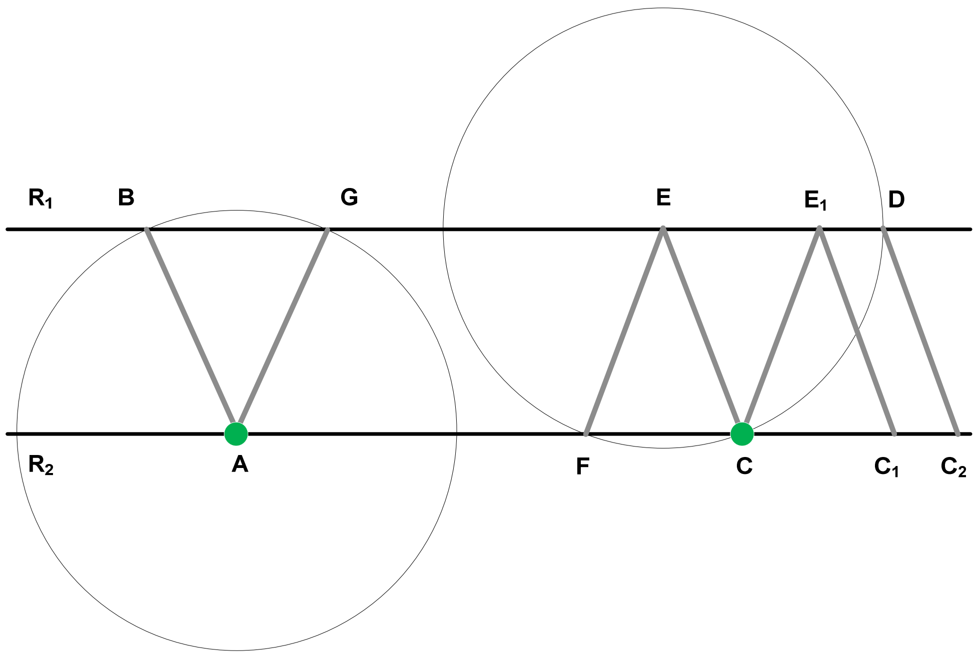

Figure 8: An *Approximation* Bernoulli Region in Case II.

There are two cases for one vehicle within the segment $[B, G]$ to propagate information to beyond point E, as follows:

- Information is propagated to a vehicle within the range $[E, E_1]$. In this case, information is able to reach vehicle C on road R_1.

- There is no vehicle within the range $[E, E_1]$, but information is propagated to a vehicle within the range $[E_1, D]$. We consider this case as not gapping out on road R_2 (approximately). Note that in this case, the vehicle at location C may actually be skipped.

The above discussions conclude that *there should be no gap of L or larger over the distance BD in order for information to detour to the vehicle at point C*. As a matter of fact, information getting through BD could have come from the vehicle at point A or from vehicles prior to location B on road R_1. For maneuverability, we simplify the process by ignoring the information that could have come from vehicles on road R_1 left of point B. The effect of this ignorance is minimized by choosing the larger vehicle density for road R_2.

The above discussions simplify the process by assuming that information getting through to a vehicle at point within E_1D successfully propagates to the vehicle at point C at a probability 1. The assumption could partially compensate for the underestimate of information getting through BG by ignoring that coming from left of point B , as in Figure 8.

Therefore, the probability of getting through R_1 from vehicle A to C is approximated as follows by slightly modifying Equation 3, using the fact that $BG = 2\sqrt{L^2 - d^2}$ and $BD = AC + L$.

$$p(y) = \begin{cases} \int_0^{2\sqrt{L^2-d^2}} f_1(t)p(y-t)dt + 1 - F_1(2\sqrt{L^2-d^2}), \mathit{for}\ y \geq L, \\ 0, \mathit{for}\ y < L. \end{cases} \qquad [10]$$

In calculation for Case II, the according new formulas in Proposition 2 are as follows:

$$E[D_2] = \frac{\int_0^L tf_2(t)dt + \int_L^\infty tf_2(t)(1-p(t+L))dt}{\int_L^\infty f_2(t)p(t+L)dt},$$

and H has a probability density function as follows:

$$f(y) = \begin{cases} f_1(t)/(1-p_r), \mathit{for}\ t \leq L, \\ f_2(t)(1-p(t+r))/(1-p_r), \mathit{for}\ t > L. \end{cases}$$

CHAPTER 4:
NUMERICAL TESTS

The numerical tests are designed to show the propagation distance in relation to the system parameters such as communication range, vehicle density, and road separation distance. A discrete numerical method is applied to solve the equations for the expectation and variance of successful propagation distance. A small step-length h is used for discretization to estimate the integrals of function $f(x)$; i.e., the integral $\int_0^a f(x)dx$ is approximated by $\sum_{i=0}^{n} f(ih)h$. When a is sufficiently large; for example, when a includes a vehicle distance headway at a probability of *almost* 1.0, one gets the integration $\int_0^a f(x)dx$. In our case, we take $a = 30L$. The $p(t)$, seen in Equations 2 through 8, can be estimated recursively by discretizing the integral and using the boundary value $p(0) = 0$.

The simulation only takes into account the statistical property of distance headway without the consideration of a real mobility. And the communication connectivity is also simulated to be instantaneous. The more sophisticated simulation will be studied in the future.

It is worthwhile to note that when a test instance is constructed, it only reflects the relativity of distance and vehicle density. The test instances constructed should cover a range of the relative magnitude of parameters. We set the communication range to be a standard unit 1.0. All the other lengths are measured against this unit, including the vehicle density.

This study has tested both the case of Poisson vehicle distribution and other independent vehicle distance headway distributions using the formulas developed. 4000 runs of simulation are conducted for each instance to benchmark results from the analytical formulas.

POISSON DISTRIBUTION OF VEHICLES ON THE ROADS

In testing the formulas for Case I, the density for road R_2 was set to 0.2, 0.6, 1.0, 1.5, and 2.0, respectively, each corresponding to a set of lower densities on road R_1. Each pair of road densities for both roads had a set of varying road separation distances of 0.1, 0.5, and 0.8, respectively. Except for a particular case with a road R_1 density of 0.2, the expectation and variance of propagation distance were highly accurate in all instances of Case I.

The test results for Case II show high accuracy, but slightly poorer than in Case I. For each test in Case I, by increasing the road separation distance to 0.9, 0.94, and 0.98, we got corresponding instances in Case II. Table 3 through Table 7 in Appendix A provides these results.

The analytical approximation in case II is generally very close to the simulation results if the vehicle density on R_1 is smaller than that on R_2 However, we have a few cases in which both roads have high densities, which gives rise to large errors in the calculated numbers compared with simulation results, a distinct example of which is seen at $\lambda_1 = \lambda_2 = 2.0$ and $d = 0.98$. See Table 3 for details. This is most likely attributed to the violation of our assumptions in the development of the formulas for Case II. In Case II, the derivation implies that road 2 has a longer propagation distance. Based on this assumption, no vehicle left of point B on road 1 in Figure 8 is able to transmit the information to vehicles beyond point B . The magnitude of errors in Case II may be interpreted as a result of the extent to which this assumption is violated. For example, when road separation distance gets closer to the critical value $\sqrt{3}/2$, the approximation errors in Case II become smaller because in this instance; the formulas for Case II depend less on the assumption. The reason for less dependence is due to $ABDC$ being closer to the Bernoulli region defined in Figure 5.

GAMMA DISTANCE HEADWAY DISTRIBUTION

This study also tested a select number of cases in which the vehicle distance headway follows Gamma distributions on both roads. We set μ_2 to 2.0, and μ_1 to 1.2, 1.0, and 0.8, respectively, where μ_i is the mean distance headway on road R_i , $i \in \{1,2\}$. The road separation is 0.01, 0.1, 0.3, 0.5, 0.7, and 0.8, respectively. In these cases, we set the variance of vehicle headway on R_2 to 0.5 times its mean, and R_1 had a variance of vehicle headway equal to its mean. Note that Gamma distribution became exponential when the mean and variance were equal. Table 8 and

Table 9 in Appendix B show these test results. It appears that formulas in Case I provide good estimates for the mean. Analytical variance tends to be larger than from simulation, though the difference between them increases with road separation.

TRUNCATED GAUSSIAN DISTANCE HEADWAY DISTRIBUTION

This study tested on traffic with distance headway following the truncated Gaussian distribution, whose density function was as follows:

$$f(x;a) = \frac{\phi((x-\mu)/\sigma)}{1-\Phi((x-\mu)/\sigma)}, x \geq a,$$

Where, $\phi(\cdot)$ is Gaussian density function, $\Phi(\cdot)$ Gaussian cumulative function, a the truncation point, μ the mean, and σ the standard deviation of Gaussian distribution. Here a can be set to a reasonable value to control the minimum headway allowed. Generally, the smaller the μ reaches, the higher the density becomes. As stated previously, L was set to 1.0. The coefficient of variation, ratio between standard deviation and mean of the distribution was set to 0.8. Table 10 in Appendix C provides the results of each instance with the analytical values followed by simulation values.

INFORMATION PROPAGATION ALONG TWO ZIGZAG ROADS

In real application, two roads might not be suitable to for consideration as parallel lines. There is a need to test the proposed model in a different geometric structure of the roads. In this section, we apply our developed formulas to two parallel zigzag roads, as illustrated in Figure 9.

The center lines of the two zigzag roads are apart from each other by a distance, d. The actual zigzag roads have an angle α with the center line. We tested a series of instances with α varying from 10 to 20 degrees. Each zigzag section was set to $0.8L$, where L was the length of communication range and was standardized at 1.0. The headway on both roads follows Poisson distributions and the successful information propagation distance was measured by two different kinds of distance: 1) horizontal distance, measured horizontally from the sender and the last receiver on the same road; and 2) actual curve distance between the sender and the furthest receiver. Table 11 through Table 14 in Appendix D, λ_1 and λ_2 denote the density on road R_1 and R_2, respectively; for the simulation results, 'Horiz' and 'Curve' denote the horizontal and curve distance, respectively. The theoretical results based on the proposed model, denoting by 'Theoretical (Condensed),' was calculated based on the curve density divided by $\cos(\alpha)$. The motivation to do so was that we wanted to check in what manner the approximation worked well.

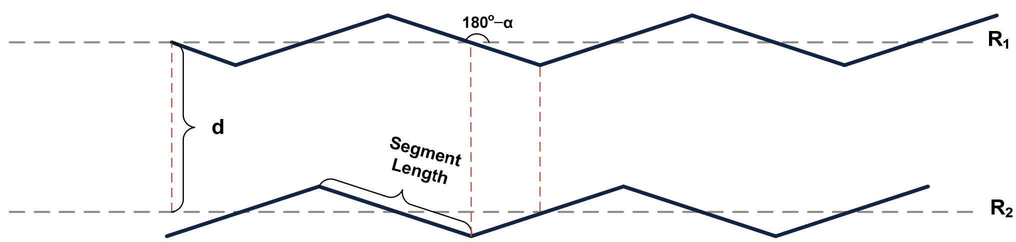

Figure 9: Propagation along Zigzag Roads.

From the results, one can figure out that if the horizontal successful propagation distance was measured, by using the condensed density, the model worked very well in estimation of both expectation and variance for all instances in Case I. However, if the curve distance was measured, as α became larger, say, up to 20 degrees, the model underestimated the successful propagation distance. If the curve density was used, the theoretical results tended to be smaller than those when α got larger. For Case II, it was also obvious that when the density on R_1 became relatively smaller, the theoretical results became more accurate.

APPROXIMATION TO PROBABILITY DISTRIBUTION OF SUCCESSFUL PROPAGATION

In application, researchers are concerned with the probability distribution of the propagation distance. In light of the result in Wang (2007), one may suspect whether the probability distribution of propagation distance follows a Gamma type. Therefore, we constructed for each case a Gamma distribution by setting the Gamma parameters in such a way that the resulting Gamma distribution had the same mean and variance as calculated with our formulas. We then graphically compared this approximate Gamma distribution with the simulated results. Although such approach was too heuristic, it helped understand the 'approximate property' of the information propagation.

The simulated frequency distribution and the constructed Gamma distribution showed a good fit in general. Two cases are presented as examples. In the first case, the distance headway follows an exponential distribution; in the second, the distance headway follows a Gamma distribution.

The Case of Exponential Distance Headway

In this case, the headway on each road was assumed to follow an exponential distribution. All the parameters are shown in Table 1. The results are shown in Figure 10, where the approximate Gamma function is denoted by the blue line and the simulation result frequency distribution is denoted by the red line.

Table 1: Parameters for Gamma Approximation to Exponential Headway.

Parameters for Poisson Headway Distribution	
$\lambda_1 = 1.2$	$d = 0.5$
$\lambda_2 = 1.5$	$L = 1.0$
Moments of Successful Propagation Distance	
Theoretical Mean = 3.1859	Simulation Mean = 3.2927
Theoretical Variance = 12.9437	Simulation Variance = 13.8804
Parameters for Gamma Approximation	
Mean of Gamma Distribution = Theoretical Mean of Propagation Distance	
Variance of Gamma Distribution = Theoretical Variance of Propagation Distance	

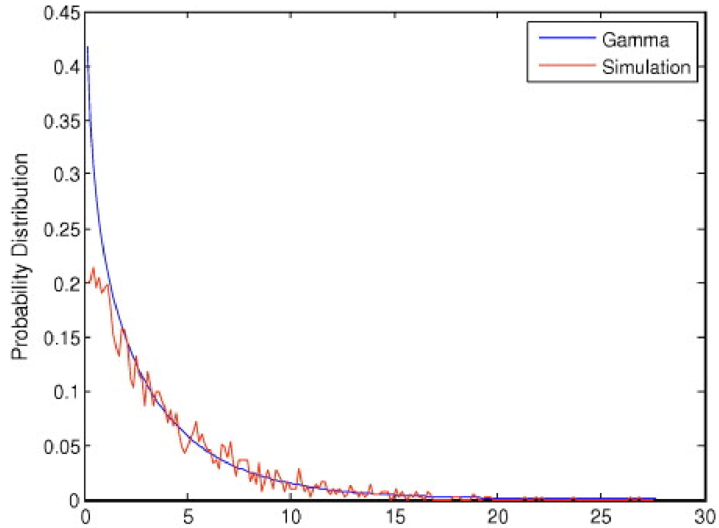

Figure 10: Gamma Approximation with Exponential Distance Headway.

Case of Distance Headway with Gamma Distribution

The headway on each road is assumed to follow a Gamma distribution to make a more general case, and the variance of headway on road R_2 equals 0.5 times its mean, and the variance of headway on road R_1 equals its mean. The values of parameters can be found in Table 2. The results are shown in Figure 11 where the Gamma distribution is denoted by the blue line and the simulation is denoted by the red line.

Table 2: Parameters for Gamma Approximation.

Parameters for Gamma Headway Distribution	
Mean Headway on R_1 = 1.0	$d = 0.5$
Mean Headway on R_2 = 1.0	$L = 1.0$
Moments of Successful Propagation Distance	
Theoretical Mean = 1.9108	Simulation Mean = 1.8816
Theoretical Variance = 5.5412	Simulation Variance = 5.3965
Parameters for Gamma Approximation	
Mean of Gamma Distribution = Theoretical Mean of Propagation Distance	
Variance of Gamma Distribution = Theoretical Variance of Propagation Distance	

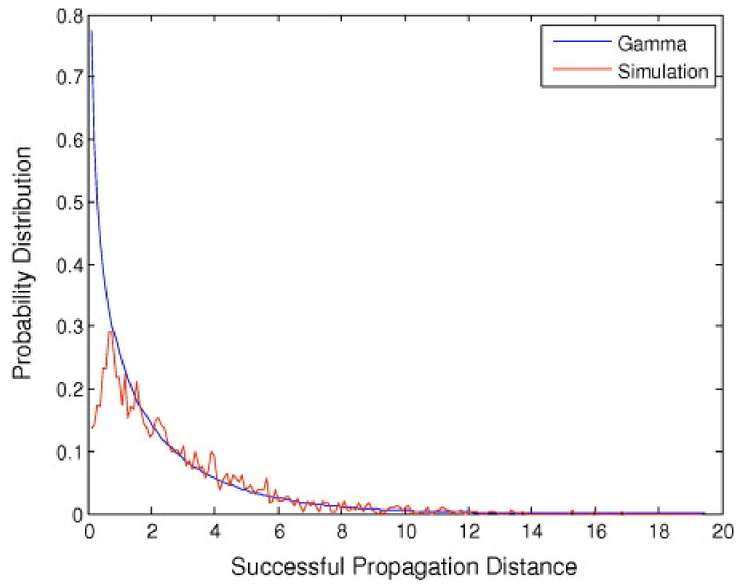

Figure 11: Gamma Approximation when Headway Follows Gamma Distribution.

CHAPTER 5:
CONCLUSIONS

Information propagation along traffic streams through inter-vehicle communication is an important process in the VANETs. This study examined a special case in which a network of two parallel highways was present as a step toward addressing information propagation on a discrete network.

This study developed an approximate method based on the Bernoulli process to characterize the process of information propagation in terms of their expected value and variance. The simulation is used to evaluate the quality of this approximation. According to simulation, our developed formulas were highly accurate in almost all cases. The developed formulas were also robust, especially in Case I with a road separation distance below $\sqrt{3}/2$ times the transmission range, e.g., $d \le \sqrt{3}L/2$. The derivation in Case I was accurate for Poisson vehicle distribution, except for dependence on a weak assumption of independent Bernoulli regions. This weak assumption was *roughly* satisfied in almost all cases if the higher density of the two roads was on road R_2. The numerical tests indicated that the correlation between Bernoulli trials was normally negligible. Furthermore, when other vehicle headway distributions were applied, the formulas showed great robustness and still yielded results of high accuracy. An interesting result was that the formulas proposed in Case I took one road as a special case. The proposed formulas for Case II in which $d > \sqrt{3}L/2$ also performed well, except for a small number of instances that had two comparable high densities for both roads and the road separation distance was large. Case II will be studied further in future work.

The numerical test also showed an encouraging fit of Gamma distribution to the propagation distance distribution. The Gamma curve was defined by the calculated mean and variance through our proposed formulas. Another interesting observation was that the ratio between the mean and standard deviation of propagation distance was close to 1.0 in cases of large vehicle densities, an evidence of the two road case to support a conjecture in Wang (2007) that an exponential distribution is the limiting distribution for propagation distance. Note that the presence probability of vehicles may be taken as the final probability after considering hopping failure, market penetration, signal conflict, and other factors. Users need to calibrate the probability according to practice. In addition, when significant transmission delay takes place, explicit consideration of multiple lanes on each road would be of interest.

REFERENCES

Balcioglu,Y., F. Ozguner, U. Ozguner, and K. Redmill, A multiple intersection integrated wireless and vehicular network simulator, in Proc. of IEEE Vehicular Networking Conference, pp. 1-5, Tokyo, Japan, October 2009.

Bettstetter, C., and J. Eberspacher, Hop distances in homogeneous ad hoc networks, in Proc. of 57th IEEE Semiannual Vehicular Technology Conference, pp. 2286-2290, Jeju, Korea, April 2003.

Briesemeister, L., L. Schafers, and G. Hommel, Disseminating messages among highly mobile hosts based on inter-vehicle communication, in Proc. of IEEE Intelligent Vehicles Symposium, pp. 522-527, Dearborn, MI, USA, 2000.

Chen, R.,W.-L. Jin, and A. Regan, Broadcasting safety information in vehicular networks: issues and approaches, IEEE Network, vol. 24, pp. 20-25, January-February 2010.

Diggavi, S., M. Grossglauser, and D. Tse, Even one-dimensional mobility increases the capacity of wireless networks, IEEE Transactions on Information Theory, vol. 51, pp. 3947-3954, November 2005.

Dion, F., J.-S. Oh, and R. Robinson, Virtual testbed for assessing probe vehicle data in Intellidrive systems, IEEE Transactions on Intelligent Transportation Systems, Accepted for publication 2010.

Du, L., S. Ukkusuri, W. F. Y. D. Valle, and S. Kalyanaraman, Optimization models to characterize the broadcast capacity of vehicular ad hoc networks, Transportation Research Part C: Emerging Technologies, vol. 17, pp. 571-585, December 2009.

Franceschetti, M., O. Dousse, D. N. C. Tse, and P. Thiran, Closing the gap in the capacity of wireless networks via percolation theory, IEEE Transactions on Information Theory, vol. 53, pp. 1009-1018, March 2007.

Grossglauser, M., and D. Tse, Mobility increases the capacity of ad hoc wireless networks, IEEE/ACM Transactions on Networking, vol. 10, pp. 477-486, August 2002.

Gupta, P., and P. Kumar, The capacity of wireless networks, IEEE Transactions on Information Theory, vol. 46, pp. 388-404, March 2000.

Haenggi, M., Outage, local throughput, and capacity of random wireless networks, IEEE Transactions on Wireless Communications, vol. 8, pp. 4350-4359, August 2009.

Harri, J., F. Filali, and C. Bonnet, Mobility models for vehicular ad hoc networks: A survey and taxonomy, research report RR-06-168, Institure Eurecom, France, March 2006.

Hoogendoorm, S.P., and P. L. Bovy, State-of-the-art of vehicular traffic flow modelling, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 215, no. 4, pp. 283-303, 2001.

Jarupan, B.,Y. Balcioglu, E. Ekici, F. Ozguner, and U. Ozguner, An integrated wireless intersection simulator for collision warning systems in vehicular networks, in Proc. of IEEE International Conference on Vehicular Electronics and Safety, pp. 340-345, Columbus, OH, USA, September 2008.

Jin, W.-L., and W. Recker, An analytical model of multihop connectivity of inter-vehicle communication systems, IEEE Transactions on Wireless Communications, vol. 9, pp. 106-112, January 2010.

Kawashima, H., Japanese perspective of driver information system, Transportation, vol. 17, pp. 263-284, September 1990.

Kendall, W. S., and I. E. Molchanov, New Perspectives in Stochastic Geometry, ch. 16, pp. 520-554, New York: Oxford University Press, 2010.

Kesting, A., M. Treiber, and D. Helbing, Connectivity statistics of store-and-forward intervehicle communication, IEEE Transactions on Intelligent Transportation Systems, vol. 11, pp. 172-181, March 2010.

Killat, M., The Impact of Inter-Vehicle Communication on Vehicular Traffic. PhD dissertation, University of Karlsruhe (TH), Germany, July 2009.

Mukherjee, S., and D. Avidor, Connectivity and transmit-energy considerations between any pair of nodes in a wireless ad hoc network subject to fading, IEEE Transactions on Vehicular Technology, vol. 57, pp. 1226 -1242, March 2008.

Mullen, J., Robust approximations to the distribution of link distances in a wireless network occupying a rectangular region, Mobile Computing and Communications Review, vol. 7, pp. 80-91, April 2003.

Nebat, Y., R. Cruz, and S. Bhardwaj, The capacity of wireless networks in nonergodic random fading, IEEE Transactions on Information Theory, vol. 55, pp. 2478-2493, June 2009.

Ohyama, T., S. Nakabayashi, Y. Shiraki, and K. Tokuda, A study of real-time and autonomous decentralized DSRC system for inter-vehicle communications, in Proc. of International IEEE Conference: Intelligent Transportation Systems, pp. 190-195, Dearborn, MI, USA, October 2000.

Orriss, J., and S. Barton, Probability distributions for the number of radio transceivers which can communicate with one another, IEEE Transactions on Communications, vol. 51, pp. 676-681, April 2003.

Park, B. , and J. Lee, Assessing sustainability impacts of route guidance system under cooperative vehicle infrastructure environment, in Proc. of IEEE International Symposium on Sustainable Systems and Technology, pp. 1-6, Tempe, AZ, USA, May 2009.

Papadimitratos, P., A. La Fortelle, K. Evenssen, R. Brignolo, and S. Cosenza, Vehicular communication systems: Enabling technologies, applications, and future outlook on intelligent transportation, IEEE Communications Magazine, vol. 47, pp. 84-95, November 2009.

Penrose, M., Random Geometric Graphs, p. 13. New York: Oxford University Press, 2003.

Piorkowski, M., M. Raya, A. L. Lugo, P. Papadimitratos, M. Grossglauser, and J.-P. Hubaux, TraNS: Realistic Joint Traffic and Network Simulator for VANETs, ACM SIGMOBILE Mobile Computing and Communications Review, vol. 12, January 2008.

Shladover, S. E., X.-Y. Lu, and D. Cody, Development and evaluation of selected mobility applications for VII: concept of operations, working paper, California PATH, UCB-ITS-PWP-2009-3, March 2009.

Shladover, S. E., Development Path Analysis for Cooperative ITS Systems, working paper, UCB-ITS-PWP-2009-4, California PATH Program, University of California, Berkeley, 2009.

The website of IntelliDrive(SM). "http://www.intellidriveusa.org", accessed on August 1st, 2010.

Vehicle-Infrastructure Integration (VII) Initiative benefit-cost analysis. "http://www.intellidriveusa.org", accessed on May 10th, 2008.

Wang, X., Modeling the process of information relay through intervehicle communication, Transportation Research Part B: Methodological, vol. 41, pp. 684-700, July 2007.

Wang, W., F. Xie, and M. Chatterjee, Small-scale and large-scale routing in vehicular ad hoc networks, IEEE Transactions on Vehicular Technology, vol. 58, pp. 5200-5213, November 2009.

Wang, X., T. Adams, W. Jin, and Q. Meng, The process of information propagation in a traffic stream with a general node headway: a revisit. Transportation Research Part C, vol. 18, pp. 367-375, 2010.

Weber, S., X. Yang, J. Andrews, and G. de Veciana, Transmission capacity of wireless ad hoc networks with outage constraints, IEEE Transactions on Information Theory, vol. 51, pp. 4091-4102, December 2005.

Weber, S., J. Andrews, X. Yang, and G. de Veciana, Transmission capacity of wireless ad hoc networks with successive interference cancellation, IEEE Transactions on Information Theory, vol. 53, pp. 2799-2814, August 2007a.

Weber, S., J. Andrews, and N. Jindal, The effect of fading, channel inversion, and threshold scheduling on ad hoc networks, IEEE Transactions on Information Theory, vol. 53, pp. 4127 - 4149, November 2007b.

Willke, T., P. Tientrakool, and N. Maxemchuk, A survey of inter-vehicle communication protocols and their applications, IEEE Communications Surveys Tutorials, vol. 11, no. 2, pp. 3-20, 2009.

Wu, H., R. Fujimoto, G. Riley, and M. Hunter, Spatial propagation of information in vehicular networks, IEEE Transactions on Vehicular Technology, vol. 58, pp. 420-431, January 2009.

Xie, L.-L., and P. Kumar, On the path-loss attenuation regime for positive cost and linear scaling of transport capacity in wireless networks, IEEE Transactions on Information Theory, vol. 52, pp. 2313-2328, June 2006.

Zanella, A., M. Stramazzotti, F. Fabbri, E. Salbaroli, D. Dardari, and R. Verdone, Comments on "probability distributions for the number of radio transceivers which can communicate with one another," IEEE Transactions on Communications, vol. 57, pp. 1287-1289, May 2009.

APPENDIX A:
POISSON DISTRIBUTION OF VEHICLES

The distance headway distribution was assumed exponential with parameters λ_1 and λ_2 on road R_1 and road R_2, respectively. Clearly, the two parameters reflect the traffic densities on the two roads, respectively. In the following tables, both analytical results based on the equations and simulation results are provided. In the tables that follow, d denotes the distance between the two roads, E the expected propagation distance, and V variance of the propagation distance. Furthermore, the numbers for E and V in each instance below start with the analytical value followed by its according simulation counterpart.

Table 3: Instances of Exponential Headway at λ_2 =2.0.

<table>
<tr><td colspan="8" align="center">λ_2 =2.0</td></tr>
<tr><td colspan="2"></td><td colspan="3" align="center">Case I</td><td colspan="3" align="center">Case II</td></tr>
<tr><td colspan="2" align="center">d</td><td>0.1</td><td>0.5</td><td>0.8</td><td>0.9</td><td>0.94</td><td>0.98</td></tr>
<tr><td rowspan="2">λ_1 =0.2</td><td>E</td><td>3.1113
3.0062</td><td>2.8018
2.6694</td><td>2.3877
2.4599</td><td>2.2633
2.2434</td><td>2.2134
2.2808</td><td>2.2009
2.1916</td></tr>
<tr><td>V</td><td>11.6432
10.8335</td><td>9.5446
9.1231</td><td>7.0530
7.2428</td><td>6.3727
6.4460</td><td>6.1081
6.1823</td><td>6.0438
6.5348</td></tr>
<tr><td colspan="2"></td><td colspan="3" align="center">Case I</td><td colspan="3" align="center">Case II</td></tr>
<tr><td colspan="2" align="center">d</td><td>0.1</td><td>0.5</td><td>0.8</td><td>0.9</td><td>0.94</td><td>0.98</td></tr>
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6.1823</td><td>6.0438
6.5348</td></tr>
</table>

Table 3: Instances of Exponential Headway at λ_2 =2.0 (Continued).

λ_2 =2.0							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.6	E	3.6960	3.2129	2.5862	2.3517	2.2398	2.2029
		3.7140	3.1669	2.6131	2.4177	2.3431	2.2929
	V	16.1372	12.3752	8.2113	6.8543	6.2467	6.0544
		17.1476	11.8683	8.1146	7.8916	6.4712	6.2594
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.8	E	4.3863	3.7093	2.8551	2.4821	2.2834	2.2054
		4.3901	3.7136	2.8347	2.5968	2.4435	2.3348
	V	22.3380	16.2562	9.9120	7.5969	6.4797	6.0681
		22.4843	16.0576	9.7043	8.3828	7.4897	7.3756
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.0	E	5.2010	4.3036	3.1991	2.6579	2.3468	2.2090
		5.1607	4.2211	3.1708	2.7975	2.6367	2.5097
	V	30.8972	21.5632	12.3064	8.6551	6.8265	6.0867
		30.9821	19.6625	12.2637	9.6766	8.8439	7.9201
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.2	E	6.1623	5.0113	3.6258	2.8827	2.4323	2.2138
		6.1833	4.9362	3.6066	3.1974	3.0978	2.6989
	V	42.7192	28.8155	15.6130	10.1024	7.3078	6.1126
		39.9055	29.6488	15.7733	12.8741	11.9535	9.1578

Table 3: Instances of Exponential Headway at λ_2 =2.0 (Continued).

λ_2 =2.0							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.4	E	7.2966	5.8511	4.1455	3.1609	2.5418	2.2206
		7.0643	5.6454	4.1856	3.4504	3.2973	2.9280
	V	59.0621	38.7316	20.1396	12.0378	7.9472	6.1480
		58.6426	38.3815	20.6432	14.7272	14.1258	11.3427
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.6	E	8.6356	6.8452	4.7712	3.4979	2.6772	2.2296
		8.6578	6.8334	4.6095	4.1060	3.7288	3.3272
	V	81.6785	52.3054	26.3137	14.5944	8.7735	6.1959
		86.7528	52.5455	26.1259	23.6484	18.1198	15.8284
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.8	E	10.2167	8.0206	5.5190	3.9003	2.8407	2.2415
		10.0141	8.0166	5.5345	4.6526	4.3466	3.6814
	V	113.0139	70.9145	34.7279	17.9493	9.8216	6.2589
		105.0044	69.7134	34.2559	26.4321	22.4226	17.2950
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =2.0	E	12.0844	9.4093	6.4087	4.3760	3.0344	2.2567
		11.9310	9.4989	6.4212	5.2115	4.9744	4.1341
	V	156.4857	96.4719	46.2018	22.3378	11.1355	6.3401
		148.8270	94.0844	48.9465	33.7764	30.0973	22.5572

Table 4: Instances of Exponential Headway at λ_2 =1.5.

λ_2 =1.5							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.2	E	1.5909	1.4914	1.3601	1.3330	1.3248	1.3241
		1.5857	1.4734	1.3512	1.3444	1.2504	1.3641
	V	3.5498	3.1431	2.6463	2.5492	2.5197	2.5177
		3.5836	3.0704	2.6561	2.4956	2.3066	2.6697
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.4	E	1.9102	1.7079	1.4448	1.3650	1.3326	1.3254
		1.8504	1.7163	1.4866	1.3631	1.3638	1.3455
	V	4.9905	4.0507	2.9668	2.6661	2.5475	2.5229
		4.9656	4.0167	3.0003	2.4637	2.7275	2.5846
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.6	E	2.2898	1.9777	1.5776	1.4231	1.3493	1.3269
		2.2175	1.9910	1.6233	1.4202	1.4391	1.3648
	V	6.9873	5.3281	3.5051	2.8860	2.6080	2.5289
		6.7457	5.4012	3.7979	2.8366	2.9953	2.7851
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.8	E	2.7401	2.3078	1.7609	1.5108	1.3775	1.3288
		2.6796	2.2873	1.7607	1.5999	1.5299	1.4206
	V	9.7504	7.1047	4.3145	3.2334	2.7119	2.5362
		9.6620	6.8648	4.5104	3.5000	3.3558	2.8838

Table 4: Instances of Exponential Headway at λ_2 =1.5 (Continued).

λ_2 =1.5							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.0	E	3.2738	2.7070	1.9990	1.6308	1.4192	1.3313
		3.3438	2.6597	2.0260	1.7561	1.6595	1.4942
	V	13.5699	9.8603	5.4757	3.7386	2.8696	2.5457
		14.6653	9.3438	5.8894	4.7211	4.0426	3.1665
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.2	E	3.9053	3.1859	2.2977	1.7864	1.4767	1.3347
		3.7922	3.2927	2.3335	2.0241	1.8102	1.6269
	V	18.8475	12.9437	7.1038	4.4418	3.0924	2.5583
		17.5766	13.8804	7.7589	5.7811	5.0603	3.9668
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.4	E	4.6522	3.7593	2.6647	1.9812	1.5500	1.3392
		4.5355	3.6857	2.6114	2.3179	2.0486	1.8385
	V	26.1395	17.5985	9.3606	5.3964	3.3931	2.5752
		24.7830	16.5469	8.9771	7.9980	6.1810	5.4124

Table 5: Instances of Exponential Headway at λ_2 =1.0.

λ_2 =1.0							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.2	E	0.8789	0.8174	0.7407	0.7253	0.7203	0.7203
		0.8809	0.8219	0.7308	0.7316	0.6976	0.7084
	V	1.3878	1.2109	1.0085	0.9707	0.9575	0.9583
		1.4658	1.2166	0.9456	0.9976	0.9541	0.9878
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.4	E	1.0723	0.9488	0.7929	0.7448	0.7249	0.7214
		1.0714	0.9657	0.8019	0.7526	0.7396	0.7266
	V	2.0005	1.5995	1.1496	1.0219	0.9690	0.9623
		1.9719	1.7040	1.1348	1.0275	1.0360	1.0293
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.6	E	1.3057	1.1175	0.8781	0.7814	0.7348	0.7227
		1.2590	1.1373	0.8873	0.8314	0.8244	0.7585
	V	2.8618	2.1661	1.3994	1.1218	0.9947	0.9667
		2.6795	2.2642	1.4475	1.3145	1.2957	1.0434
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.8	E	1.5863	1.3288	0.9993	0.8380	0.7521	0.7243
		1.5416	1.3634	0.9877	0.9298	0.8625	0.7557
	V	4.0670	2.9756	1.7909	1.2856	1.0402	0.9718
		3.8586	3.1107	1.7473	1.6350	1.3956	1.1332

Table 5: Instances of Exponential Headway at λ_2 =1.0 (Continued).

λ_2 =1.0							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =1.0	E	1.9226	1.5890	1.1609	0.9177	0.7781	0.7262
		1.9858	1.5901	1.0878	0.9856	0.9231	0.8432
	V	5.7473	4.1179	2.3717	1.5320	1.1112	0.9778
		6.0198	4.1090	2.2898	1.8237	1.6483	1.3757

Table 6: Instances of Exponential Headway at λ_2 =0.6.

λ_2 =0.6							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.2	E	0.4616	0.4256	0.3826	0.3746	0.3714	0.3718
		0.4625	0.4349	0.3895	0.3808	0.3999	0.3742
	V	0.5626	0.4822	0.3852	0.3817	0.3736	0.3768
		0.5507	0.5260	0.4185	0.3912	0.4303	0.3642
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.4	E	0.5746	0.5027	0.4139	0.3864	0.3740	0.3732
		0.5752	0.4833	0.4115	0.3907	0.3795	0.3908
	V	0.8412	0.6615	0.4620	0.4084	0.3795	0.3821
		0.5266	0.6441	0.5065	0.4147	0.4103	0.3934
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.6	E	0.7142	0.6056	0.4673	0.4091	0.3799	0.3746
		0.7300	0.5955	0.4662	0.4205	0.4057	0.3754
	V	1.2436	0.9366	0.5912	0.4605	0.3921	0.3877
		1.2706	0.8762	0.6109	0.5146	0.4454	0.3619

Table 7: Instances of Exponential Headway at λ_2 =0.2.

λ_2 =0.2							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
λ_1 =0.2	E	0.2462	0.2380	0.2338	0.2370	0.2344	0.2362
		0.1366	0.1201	0.1125	0.1097	0.1113	0.1041
	V	0.1337	0.1118	0.0894	0.1053	0.0872	0.1058
		0.1369	0.1007	0.0874	0.0827	0.0829	0.0789

APPENDIX B:
GAMMA DISTRIBUTION OF THE VEHICLE HEADWAY

The headway on each road was assumed to follow a Gamma distribution with the parameters shown in Table 8. The theoretical and simulation results are shown in Table 9.

Table 8: Configuration of Gamma Distribution.

On Road 1	Mean of Gamma Distribution: μ_1
	Variance of Gamma Distribution: μ_1
On Road 2	Mean of Gamma Distribution: μ_2
	Variance of Gamma Distribution: $0.5\mu_2$

Table 9: Analytical and Simulation Results.

λ_2=1.0							
d		0.01	0.1	0.3	0.5	0.7	0.8
λ_1=1.2	E	2.0091 2.0073	1.9950 1.9935	1.8726 1.8924	1.6380 1.6563	1.3361 1.3195	1.1807 1.1819
	V	6.0299 6.3135	5.9556 5.7853	5.3284 5.6661	4.2145 4.1474	2.9533 3.0711	2.3791 2.3346
λ_1=1.5	E	1.6163 1.7259	1.6072 1.6868	1.5260 1.6146	1.3611 1.3781	1.1380 1.1591	1.0235 1.0294
	V	4.1071 4.8706	4.0663 4.6104	3.7099 4.4811	3.0324 3.2178	2.2139 2.3368	1.8365 1.8945
λ_1=1.8	E	1.3556 1.4862	1.3496 1.4275	1.2956 1.4163	1.1803 1.2126	1.0154 0.9954	0.9303 0.9105
	V	3.0120 3.8272	2.9883 3.4366	2.7773 3.3335	2.3519 2.5664	1.8012 1.8260	1.5426 1.6399

APPENDIX C:
TRUNCATED GAUSSIAN DISTRIBUTION OF THE VEHICLE HEADWAY

Table 10: Instances of Truncated Gaussian Headway.

μ_2=0.5, α = 0.01							
		Case I			Case II		
d		0.2	0.5	0.8	0.9	0.94	0.98
μ_1=0.2	E	4.0664 4.3995	3.9808 4.2121	3.7870 3.8474	3.5697 3.7832	3.6107 3.7621	3.6720 3.6763
	V	19.1593 23.8971	18.4042 23.0550	16.7506 17.1167	14.9849 15.4219	15.3117 16.3400	15.8053 16.0258
μ_1=3.0	E	4.2193 4.5921	4.0975 4.4444	3.8167 3.7313	3.4963 3.6375	3.5556 3.6791	3.6446 3.7320
	V	20.5435 23.9911	19.4358 23.0739	16.9992 16.0244	14.4097 15.9636	14.8738 16.6468	15.5836 16.4426

μ_2=1.0, α = 0.01							
		Case I			Case II		
d		0.2	0.5	0.8	0.9	0.94	0.98
μ_1=3.0	E	0.5395 0.5732	0.5046 0.5185	0.4535 0.4518	0.3362 0.4178	0.3638 0.4334	0.4020 0.4471
	V	0.7169 0.8505	0.6353 0.6933	0.5268 0.5448	0.2559 0.4534	0.3186 0.4773	0.4072 0.5318
μ_1=2.0	E	0.6247 0.6858	0.5634 0.5716	0.4707 0.4660	0.2595 0.4461	0.3088 0.4360	0.3763 0.4557
	V	0.9238 1.2108	0.7701 0.8508	0.5633 0.5561	0.0896 0.5358	0.1961 0.5116	0.3473 0.5521

APPENDIX D:
INFORMATION PROPAGATION FOR TWO ZIGZAG ROADS

From Table 11 to Table 14, the simulation results of information propagation were measured in two ways: 1) horizontal distance, measured horizontally from the sender and the last receiver on the same road; and 2) curve distance, measured along the curve. They are denoted by 'Horiz' and 'Curve,' respectively. Theoretical results are also provided in terms of projected density; i.e., the curve density divided by $\cos(\alpha)$.

Table 11: Instances of Exponential Headway on ZigZag Roads for λ_1=0.5.

λ_1=0.5, λ_2=2.0						
			Case I			Case II
d			0.3	0.5	0.7	0.9
α =10	Horiz	E	3.3664	3.0594	2.7132	2.3175
		V	13.7026	11.6453	9.1897	6.5161
	Curve	E	3.4183	3.1066	2.7551	2.3532
		V	14.1286	12.0073	9.4754	6.7087
Theoretical (Condensed)		E	3.3689	3.1007	2.7431	2.3743
		V	13.5001	11.5346	9.1449	6.9555
α =15	Horiz	E	3.3509	3.1932	2.8206	2.5264
		V	13.4040	12.1114	9.3242	7.8947
	Curve	E	3.4691	3.3058	2.9201	2.6155
		V	14.3663	12.9809	9.9936	8.4616
Theoretical (Condensed)		E	3.5213	3.2382	2.8608	2.4696
		V	14.6347	12.4809	9.8666	7.4638
α =20	Horiz	E	3.6461	3.3592	2.9912	2.6634
		V	16.3080	13.5451	10.5167	8.3137
	Curve	E	3.8801	3.5747	3.1831	2.8344
		V	18.4684	15.3395	11.9098	9.4150
Theoretical (Condensed)		E	3.7539	3.4479	3.0401	2.6141
		V	16.4545	13.9953	11.0177	8.2688

Table 12: Instances of Exponential Headway on ZigZag Roads for λ_1=0.7.

λ_1=0.7, λ_2=2.0						
			Case I			Case II
d			0.3	0.5	0.7	0.9
α =10	Horiz	E	3.9288	3.5479	3.0228	2.5199
		V	18.0247	15.2847	11.3196	8.5763
	Curve	E	3.9894	3.6026	3.0694	2.5588
		V	18.5879	15.7599	11.6716	8.8429
Theoretical (Condensed)		E	3.9664	3.5758	3.0673	2.4898
		V	18.4081	15.1225	11.3109	7.6142
α =15	Horiz	E	4.0655	3.5859	3.1493	2.5967
		V	19.5763	15.0090	12.5803	8.3987
	Curve	E	4.2089	3.7124	3.2604	2.6883
		V	20.9818	16.0866	13.4514	9.0017
Theoretical (Condensed)		E	4.1572	3.7440	3.2068	2.5938
		V	20.0693	16.4498	12.2629	8.1962
α =20	Horiz	E	4.3199	3.8464	3.2776	2.9147
		V	21.7084	17.3916	13.4926	9.7362
	Curve	E	4.5971	4.1257	3.4879	3.1017
		V	24.5842	19.6955	15.2800	11.0259
Theoretical (Condensed)		E	4.4496	4.0015	3.4199	2.7521
		V	22.7546	18.5891	13.7913	9.1223

Table 13: Instances of Exponential Headway on ZigZag Roads for λ_1=1.0.

λ_1=1.0, λ_2=2.0						
			Case I			Case II
d			0.3	0.5	0.7	0.9
α=10	Horiz	E	4.9275	4.4419	3.7132	2.9614
		V	26.7850	22.0993	16.4029	11.0786
	Curve	E	5.0035	4.5105	3.7705	3.0071
		V	27.6178	22.7864	16.9129	11.4231
Theoretical (Condensed)		E	5.0783	4.4749	3.7125	2.7508
		V	29.4708	23.1713	16.2638	9.2081
α=15	Horiz	E	5.2089	4.6258	3.7804	3.1169
		V	30.5780	24.7879	16.7966	11.8781
	Curve	E	5.3927	4.7890	3.9138	3.2268
		V	32.7734	26.5676	18.0025	12.7309
Theoretical (Condensed)		E	5.3451	4.7042	3.8966	2.8745
		V	32.4157	25.4158	17.7728	9.9713
α=20	Horiz	E	5.6613	4.9242	3.9747	3.1638
		V	35.5937	27.7942	17.8093	11.2393
	Curve	E	6.0246	5.2402	4.2297	3.3669
		V	40.3090	31.4762	20.1686	12.7282
Theoretical (Condensed)		E	5.7563	5.0573	4.1793	3.0636
		V	37.2323	27.0738	20.2211	11.1960

Table 14: Instances of Exponential Headway on ZigZag Roads for λ_1=1.5.

λ_1=1.5, λ_2=2.0						
			Case I			Case II
d			0.3	0.5	0.7	0.9
α=10	Horiz	E	7.6225	6.3632	5.3606	3.8998
		V	63.4701	46.7651	33.3066	18.7436
	Curve	E	7.7401	6.4614	5.4433	3.9600
		V	65.4435	48.2191	34.3421	19.3263
Theoretical (Condensed)		E	7.6912	6.6145	5.3065	3.4539
		V	65.2747	48.8942	32.1199	14.2038
α=15	Horiz	E	7.9230	6.9048	5.5446	4.1859
		V	66.3784	52.8162	33.9887	20.7906
	Curve	E	8.2024	7.1484	5.7402	4.3336
		V	71.1441	56.6083	36.4290	22.2833
Theoretical (Condensed)		E	8.1539	7.0022	5.6079	3.6311
		V	72.9145	54.4267	35.6033	15.5652
α=20	Horiz	E	8.8628	7.3733	5.9583	4.4887
		V	88.2312	60.8332	42.0374	22.6167
	Curve	E	9.4316	7.8464	6.3407	4.7767
		V	99.9195	68.8921	47.6062	25.6128
Theoretical (Condensed)		E	8.8751	7.6049	6.0754	3.9042
		V	85.6677	63.6214	41.3629	17.7853