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FINAL REPORT: OCEANIC SURVEILLANCE AND NAVIGATION ANALYSIS, FY 71

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16. Abstract

This report summarizes the oceanic surveillance and navigation analysis performed at Transportation Systems Center under PPA FA-04 for FY 71. Three major efforts are reviewed and discussed herein: (1) a tutorial summary of the NAT/SPG collision risk model; (2) a study of the impact of inertial navigation on air safety; and (3) an investigation of the modeling techniques required to assess the effect of ATC satellite surveillance on separation standards in the North Atlantic region.

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PREFACE

The author wishes to thank those individuals whose counsel, support and technical guidance proved invaluable in the success of the work effort detailed in this report. They included Messrs. Chester Dunmire and Sidney Hirshon of the Federal Aviation Administration, Dr. Kenneth R. Britting of the MIT Measurement Systems Laboratory, and Dr. Diarmuid O'Mathuna and Mr. Daniel L. Brandel of the Transportation Systems Center.

Special thanks is reserved for Mr. Leo M. Keane for his contributions.

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1. INTRODUCTION AND SUMMARY

This report reviews and summarizes the analytical work completed for the Federal Aviation Administration (FAA) by TSC under Project Plan Agreement (PPA) FA-04 (Ref 1), entitled, "Oceanic Surveillance and Navigation Analysis". The scope of fiscal year 71's (FY 71) effort is discussed in terms of specific task requirements and accomplishments.

Section 2 outlines the intended work effort for FY 71, presented in terms of PPA FA-04 (Ref 1). With only minor exceptions, all tasks were completed successfully and within the alloted time constraints. This outline, then, is essentially not only one of initial plans but of completed projects as well.

The results of Task 1 effort, as specified in the PPA are reviewed in Section 3. Specifically, a nonsurveillance, time-invariant, parallel-track collision risk model, originated by Mr. P. G. Reich of the United Kingdom, (Refs 2, 3 and 4), is reviewed and discussed. The original analysis has been adopted, refined and extended by the North Atlantic Systems Planning Group (NAT/SPG) (Refs 5, 6, 7, 8 and 9). Their comments and additions are also noted. This section is essentially a summary of a technical report prepared for, and submitted to, the FAA in May, 1971 (Ref 10). The report originally appeared as an unofficial working paper in November, 1970.

The probable impact of inertial navigation systems (INS) on the aircraft collision risk in the North Atlantic (NAT) region is analyzed in Section 4. Terminal error data, obtained from Air France, are combined with an INS error model to yield simulated, en route error distributions. These error distributions are then used in conjunction with a time-dependent collision risk model - an adapted version of the NAT/SPG model discussed in Section 3 - to derive first-cut collision risk estimates for both easterly and westerly transoceanic INS carriers. The results of this relatively conservative analysis show that there is strong evidence that the widespread use of inertial navigators will lead to reduced separation standards in the NAT region, while maintaining present safety standards. A more complete description of the analysis and results of this study (specified in Task 2 of PPA FA-04) can be found in Ref 11.

A study of the effects of an Air Traffic Control Surveillance System on the safety and routing structure in the NAT region has been proposed. This analysis will be performed through both in-house efforts and outside contracts. Task 3 calls for the awarding of an initial study contract in the area of satellite surveillance systems. Section 5 discusses this proposed study effort in detail. The requirements, as expressed in the Request for Proposal (RFP) (Ref 12), are noted. The winning proposal, submitted by Systems Control, Incorporated, (Refs 22 and 23), is reviewed, with the emphasis placed on the proposed modeling procedure. Alternate approaches are briefly discussed and critiqued in Ref 13.

Section 6 contains a brief review of the FY 71 effort in terms of results and conclusions. Emphasis is placed on specifying those areas of investigation to be pursued during the next fiscal year (Ref 24).

2. PROJECT PLAN AGREEMENT

The scope of the FAA-sponsored project reviewed in this report was agreed upon, and formalized, under PPA FA-04 (Ref 1), entitled "Oceanic Surveillance and Navigation Analysis".

The effects on air safety of on-board INS and independent satellite surveillance ATC systems were considered to be the key elements of the initial investigations. This was noted in the work description itself. In regard to the investigation of INS, it was stated that: The influence of INS performance on the possible reduction of air route separation requirements will be studied in terms of safety criteria. The effort undertaken will include:

1. An evaluation of the data base of INS accuracy as established by the FAA and others.

2. A statistical analysis of the INS data.

3. An assessment of the effects of inertial systems on overall air traffic performance.

With respect to the second element, an ATC surveillance system, it was proposed that analysis will be conducted showing the impact of an ATC surveillance system on the flight patterns in the region. The performance of this system will be evaluated, stressing safety as affected by such factors as separation standards, surveillance systems, on-board navigation systems, fix rates, control procedures and all applicable error probabilities. Recommendations will then be made for meaningful future work, leading to specific system requirements.

In order to assess the effectiveness of either the navigation or surveillance system, it was first deemed necessary to establish a methodology for deriving air safety as a function of the routing system, aircraft dynamics and other pertinent system parameters. This required a careful study of the existing models used to assess safety. Particular emphasis was placed on those procedures which have gained widespread acceptance and have demonstrated extendibility to future systems.

Specifically, the following tasks were scheduled:

Task 1 - Study nonsurveillance systems, particularly those accepted by the NAT Special Planning Group; select a particular system to be carefully documented. This would ensure a solid in-house familiarity with current procedures and their possible extensions.

Task 2 - With the aid of an outside contract, analyze INS performance data and assess the probable impact of INS on the collision risk in the NAT region. The data base used for this study would consist of the terminal data on the Litton LTN 51 System collected by both American Airlines and Air France. En route statistics would be inferred from these terminal data by the use of an error simulation model to be devised. These en route statistics would then be used as inputs to the NAT/SPG model. In conjunction with other parameter estimates, the expected safety level would be calculated.

Task 3 - Study the effect of a satellite surveillance system on the routing structure in the NAT region. This study would be performed with the aid of an outside contractor. It will involve, in part, (1) the selection of an appropriate control strategy, (2) the analysis of modeling procedures, and (3) an investigation of the relationships among such parameters as separation standards, fix rates, threshold widths, navigation and surveillance accuracy, and safety levels. In addition, recommendations would be made for further investigation of the various ATC surveillance systems.

Task 4 - Coordinate efforts with other government agencies, industry and international committees, when appropriate, with a view towards providing feedback for future work.

Task 5 - Write a final report of all FY 71 work (as detailed above).

A review of the results of the first three tasks are discussed in the remainder of this report, in the order indicated in the introduction.

3. COLLISION RISK MODEL FOR THE NAT REGION

3.1 INTRODUCTION

This section reviews and summarizes the essential features of a collision risk model used to analyze the effects of separation standards on safety for the parallel tracking system currently employed in the NAT region. This model, derived by P. G. Reich (Refs 2, 3 and 4), has been accepted, refined and extended by the North Atlantic Systems Planning Group (Refs 5, 6, 7, 8 and 9), a study group set up under the auspices of ICAO. (A more complete summary of the model can be found in Ref 10.)

3.2 ASSUMPTIONS

3.2.1 Air Space. The model, as adapted by NAT/SPG, assumes a parallel-track system in which each aircraft is cleared to fly down "tubes", normally centered at specific vertical and lateral coordinates.

The separation distance between the center lines of these usable tracks are chosen in order to maintain safety standards in the vertical and lateral directions. Similarly, entry times on each track are set in accordance with along-track safety requirements.

The procedures which follow can be applied to any parallel-track system. Of particular importance, however, is the track system used in a principal region of the North Atlantic defined to extend from 18^{o}W to 50^{o}W longitude and from 45^{o}N to 61^{o}N latitude. The results presented will specifically apply to this region.

There are two parallel systems which are discussed: (1) the conventional or rectangular system, and (2) the composite system (which has seen modified use since April 1, 1971). A cross-sectional view of these parallel systems is represented in Figure 3.1.

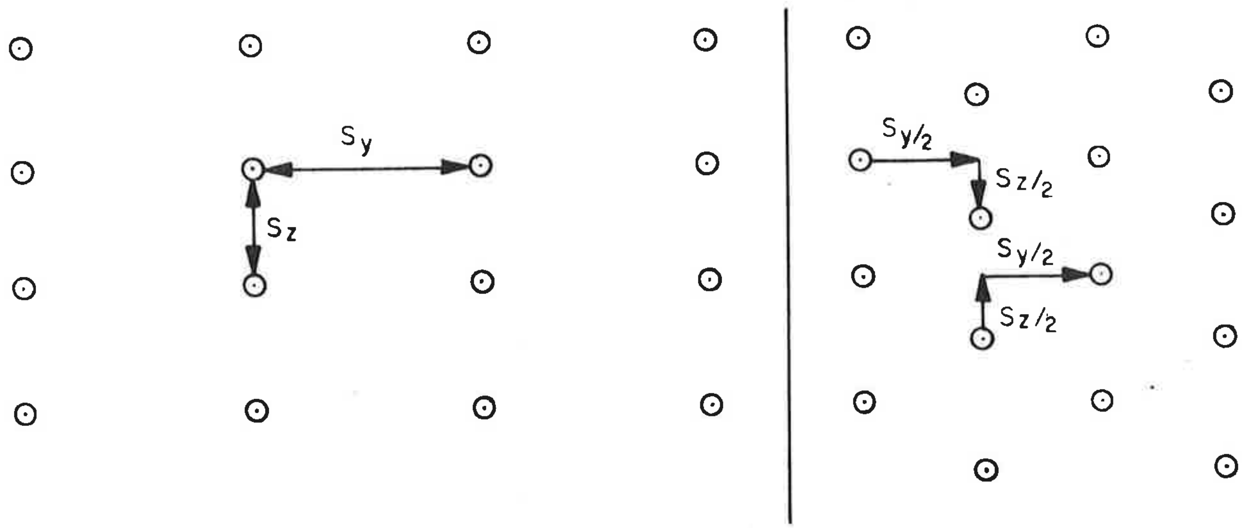

Figure 3.1. Tracking systems: Conventional (left) and Composite (right).

3.2.2 Collision Avoidance. No provision is made in the model for collision avoidance following visual or instrumental contact with another aircraft. Since pilot initiated evasive action is more likely to prevent an accident than cause one, this is considered a conservative assumption.

3.2.3 Dimensional Independence. The flying errors in each dimension are assumed to be independent of one another. A careful study of the data will be required in order to analyze the effect of this assumption on the final results.

3.2.4 Aircraft Independence. The flying errors between neighboring aircraft are assumed to be independent. There are a variety of ambient conditions (such as weather and atmospheric disturbances) which tend to cause correlated bias errors between neighboring aircraft. Hence, this assumption, too, is considered to be conservative in nature.

3.2.5 Position Control. The aircraft are uncontrolled and their position is unmonitored by any independent source. Section 5 considers an extended model which includes surveillance and positive control.

3.2.6 Time-Invariant Navigation. The on-board navigation system is considered time-invariant. This requirement is relaxed in Sections 4 and 5.

3.3 MODEL DERIVATION

Since it is assumed that the intended flight paths are designed to be nonintersecting, collisions result solely from flying errors. A collision will occur when one aircraft enters within a specific distance of another. To be mathematically precise, this distance is defined in terms of the collision slab (Figure 3.2).

The dimensions λ_x, λ_y and λ_z are nominally taken to be the metallic dimensions of the aircraft, although the slab can be extended to include other effects such as wake vortices.

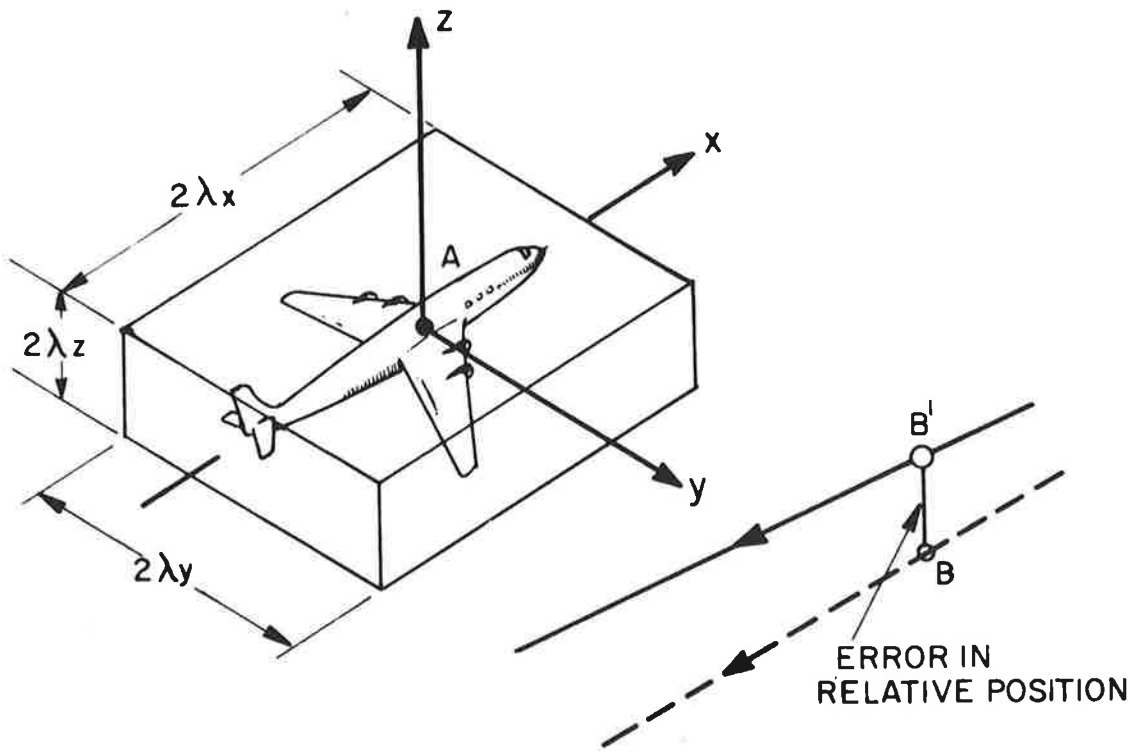

Figure 3.2. Collision Slab.

NAT/SPG found the collision risk function to be an inconvenient vehicle with which to establish target levels of safety. The expected number of accidents in 10 million hours of flying time, N_a, was considered a more appropriate figure.

Three modifications are necessary to transform the risk function into the accident function: (1) multiply CR by 2 to account for the two accidents per collision; (2) divide the result by the average number of flying hours over which the proximity time was calculated, producing the average accident level per hour of flight; and, finally (3) multiply this result by 10 million in order to scale the result to the stated number of flight hours. Therefore,

$$N_a = \frac{2 \times 10^7}{H} (CR). \tag{3-3}$$

The total number of accidents in 10 million flying hours due to the loss in lateral separation, N_{ay}, therefore, can be written as:

$$N_{ay} = \frac{2x10^7}{H} P_y(S_y) \left[\frac{T_y(\text{same})}{S_x} \left(\frac{\overline{\Delta V}}{2} P_z(0) + \lambda_x N_z(0) + \frac{\lambda_x \overline{|\dot{y}(S_y)|}}{2\lambda_y} P_z(0) \right) \right.$$

$$\left. + \frac{T_y(\text{opp})}{S_x} \left(\overline{V} P_z(0) + \lambda_x N_z(0) + \frac{\lambda_x \overline{|\dot{y}(S_y)|}}{2\lambda_y} P_z(0) \right) \right] . \tag{3-4}$$

where equation (A-2) was used to relate $N_y(S_y)$ to $P_y(S_y)$.

Similarly, the accident functions for the vertical and composite cases can be shown (Ref 10) to equal, respectively,

$$N_{az} = \frac{2x10^7 P_z(S_z)}{H} \left[\frac{T_z(\text{same})}{S_x} \left(\frac{\overline{\Delta V}}{2} P_y(0) + \lambda_x N_y(0) + \frac{\lambda x \overline{|\dot{z}(S_z)|} P_y(0)}{2\lambda_z} \right) \right.$$

$$\left. + \frac{T_z(\text{opp})}{S_x} \left(\overline{V} P_y(0) + \lambda_x N_y(0) + \frac{\lambda_x \overline{|\dot{z}(S_z)|} P_y(0)}{2\lambda_z} \right) \right] , \tag{3-5}$$

$$N_{ayz} = \frac{2x10^7}{H} P_z\left(\frac{S_z}{2}\right)P_y\left(\frac{S_y}{2}\right)\frac{\lambda_x}{S_x}\left[T_{yz}(\text{same})\left(\frac{\overline{\Delta V}}{2\lambda_x} + \frac{\overline{|\dot{y}(S_y/2)|}}{2\lambda_y} + \frac{\overline{|\dot{z}(S_z/2)|}}{2\lambda_z}\right)\right.$$

$$\left. + T_{yz}(\text{opp})\left(\frac{\overline{V}}{\lambda_x} + \frac{\overline{|\dot{y}(S_y/2)|}}{2\lambda_y} + \frac{\overline{|\dot{z}(S_z/2)|}}{2\lambda_z}\right)\right] . \qquad (3\text{-}6)$$

The calculation of the longitudinal or along-track accident function requires a number of new considerations, including: (1) a re-evaluation of longitudinal overlap probability, and (2) an assumption of one direction - one track. These factors are discussed in detail in Ref 10. The result is shown to be

$$N_{ax} = 2x10^7\left[\frac{\overline{|\dot{x}|}}{2\lambda_x} + \frac{\overline{|\dot{y}(0)|}}{2\lambda_y} + \frac{\overline{|\dot{z}(0)|}}{2\lambda_z}\right]P_y(0)P_z(0)\sum_t \tilde{P}_x(t)\tilde{E}_x(t) , \qquad (3\text{-}7)$$

where $\tilde{E}_x(t)$ is the average number of aircraft pairs flying the same path with an initial time separation of t minutes (plus an allowance for Mach number difference), and $\tilde{P}_x(t)$ is the probability that the along-track separation of a pair of aircraft, initially separated by time t minutes (plus an allowance for Mach number difference), will be less than λ_x.

The total number of accidents in 10 million flying hours is found by summing the results of equations 3-4 through 3-7:

$$N_a = N_{ax} + N_{ay} + N_{az} + N_{azy}. \qquad (3\text{-}8)$$

There are a number of additional modifications which are often found in the literature. Primary among them is the specification of the occupancy, E. The occupancy represents the average number of aircraft that are proximate to one another per hour of flight and is given by,

$$E_r(\text{same/opp}) = \frac{2\ T_r(\text{same/opp})}{H} , \qquad (3\text{-}9)$$

where r can represent y, z or yz. The occupancy function, E, was used in the NAT/SPG data analysis and represents a convenient function for obtaining experimental data.

3.5 PARAMETER EVALUATIONS

A great deal of effort has been expended in obtaining quantitative results based upon the NAT/SPG collision risk model. Most of this effort has been concerned with the techniques required to adequately estimate the various parameters of importance. On one hand, we can select a single "optimum" estimate for each parameter in question, and proceed to solve a deterministic equation. Alternately, each parameter can be modeled as a random variable by assigning to it a probability density function obtained from either empirical data or theoretical considerations. (See Agenda Item 4, Appendix C of Ref 5.)

In general, the solution method used will combine both techniques. These are certain parameters we will be able to model by assigning a single estimate. Other parameters, such as the flying densities used in calculating overlap probabilities, will require a distributed variable approach.

The reader is referred to Ref 10 for a more complete discussion of the problem of assigning parametric values. Among other items, the aforementioned reference contains are: (1) a general discussion of parameter assignments and its inherent problems; (2) a complete set of parameter definitions; and, (3) a detailed discussion of the procedure and issues involved in calculating the probability of overlap, $P_y(S_y)$. Refs 3 and 5 discuss the overlap probability in even greater detail. Refs 5 through 9 and 14 are excellent sources of information on the assignment of parametric.

Table 3.1 lists those parametric values used by NAT/SPG-4 (1968) in analyzing the results of the collision risk model. A source list is also included for those who wish to further investigate the problem.

These values are under constant scrutiny and are sensitive functions of current state-of-the-art techniques.

3.6 RESULTS

The principal results derived to date were the direct results of the data collection and reduction effort undertaken by NAT/SPG in conjunction with their fourth annual meeting. Deviations from track, occupancy rates, longitudinal separations and relative velocities were obtained for a variety of carriers. Appendix A of Ref 7 presents the data obtained. Item 1 of the same report details the methods used to process this data.

TABLE 3.1.-NAT/SPG PARAMETER VALUES

Parameter	Units	Values	Comments	References
λ_x	nmi	0.025	Measured metallic distance of Average A/C (Vortex ignored)	Ref 7 (p.1-23)
λ_y	nmi	0.025	Measured metallic distance of Average A/C (Vortex ignored	Ref 8 (p.2-A-3)
λ_z	nmi	0.0066	Measured metallic distance of Average A/C (Vortex ignored)	Ref 8 (p.2-A-4)
S_x	nmi	120; 240	Mathematical tool only	Ref 7 (p.1-A-19) Ref 7 (p.1-A-20)
S_y	nmi	75-135	Variable for Problem	-
S_z	nmi	0.33 (2000 ft)	Variable for Problem	-
$\overline{\lvert \dot{z}(0) \rvert}$	nmi/hr	1	Derived from FAA data (Report No. RD-64-4; 1/64)	Ref 7 (p.4-7)
$\overline{\Delta V}$	nmi/hr	13	Average of observed data	Ref 7 (p.1-22, p. 1-A-21)
$\bar{V}$	nmi/hr	480	Speed of average A/C considered	Ref 5 (p.4-8)
$P_z(0)$	—	0.25	Based on RAE study of IATA data (1951); cautious	Ref 5 (p.4-10)
$N_z(0)$	cycle/hr	20 or 40	20 used for N_{ay}, 40 used in N_{ax} case	Ref 7 (p.4-7)
H	hr	1.5×10^6	Taken from 1966-1971. Used in Ref 6, data collection scheme	Ref 6 (p.2-A-1)
$\overline{\lvert \dot{y}(0) \rvert}$	nmi/hr	20	Correlation study of lateral speed vs. deviation from track	Ref 7 (p.1-A-19)
$\overline{\lvert \dot{y}(60) \rvert}$	nmi/hr	47	Correlation study of lateral speed vs. deviation from track	Ref 7 (p.1-A-19)
$\overline{\lvert \dot{y}(90) \rvert}$	nmi/hr	60	Correlation study of lateral speed vs. deviation from track	Ref 7 (p.1-A-19)
$P_y(0)$	—	0.0012	-	Ref 8 (p.2-A-1)
$P_y(60)$	—	11×10^{-6}	-	Ref 7 (p.1-20)
$P_y(90)$	—	1×10^{-6}	-	Ref 7 (p.1-20)
E_y(same)	—	0.61	Based on data and assumption of 280 daily flights	Ref 5 (Fig. 3)
E_y(opp)	—	0.01	Based on data and assumption of 280 daily flights	Ref 5 (Fig. 4)
E_z(same)	—	0.73	Based on data and assumption of 280 daily flights	Ref 5 (Fig. 5)
E_z(opp)	—	0.02	Based on data and assumption of 280 daily flights	Ref 5 (Fig. 6)
E_{yz}(same)	—	1.46	Assumed to be twice E_z(same); cautious	Ref 8 (p.3-A-4)
E_{yz}(opp)	—	0.04	Assumed to be twice E_z(same); cautious	—
$E_x(t)$	—	0.014 for all $t > t_{min}$	Very rough estimate	Ref 8 (p.4-A-6)

In general, the central estimates were used to estimate the parameters. The body of the position error density was modeled by a first Laplacian.* The tail, shown to be overestimated by the "level tail" assumption and underestimated by the "exponential decay" assumption, was assigned a compromise distribution shape between these extremes.

The overlap probability was obtained by calculating individual overlap probabilities for various regions of the ocean, and taking a weighted sum of the results in accordance with the relative flying times in each region.

The results of interest are shown in Figures 3.4 through 3.6. Figures 3.4 and 3.5 were derived for the two groups of carriers indicated by directly convolving the position-error histograms to obtain the individual $P_y(S_y)$ values. This was referred to as the central (or "actual") estimate. In Figure 3.6 the individual $P_y(S_y)$'s were obtained by averaging the results obtained for the "level tail" and "exponential decay" cases.

Additional results, based on the NAT/SPG-4 data collection effort, were tabulated in NAT/SPG-6, Agenda Item 3, for a composite system. Here, values were presented for N_{ay}, N_{az}, N_{ayz} and N_{ax}, using both optimistic and pessimistic assumptions. In addition, the effect of flight level changes on the number of expected accidents was analyzed. These results are presented in Table 3.2.

TABLE 3.2. - NAT/SPG RESULTS

Accident Functions (10^7 - Hour Flying)	Best Estimate	Cautious Estimate
N_{ay} (120 nm)	0.11	0.18
N_{az} (2000 ft)	0.02	0.05
N_{yz} (60 nm + 1000 ft)	0.03	0.04
N_{ax} (t_{min} = 15 min)	0.05	0.15
Flight Level Changes	0.01	0.02
TOTAL, N_a	0.22	0.44

* The reader, unfamiliar with the issues regarding the specification of position error densities, is referred to Refs 3,5 and 10.

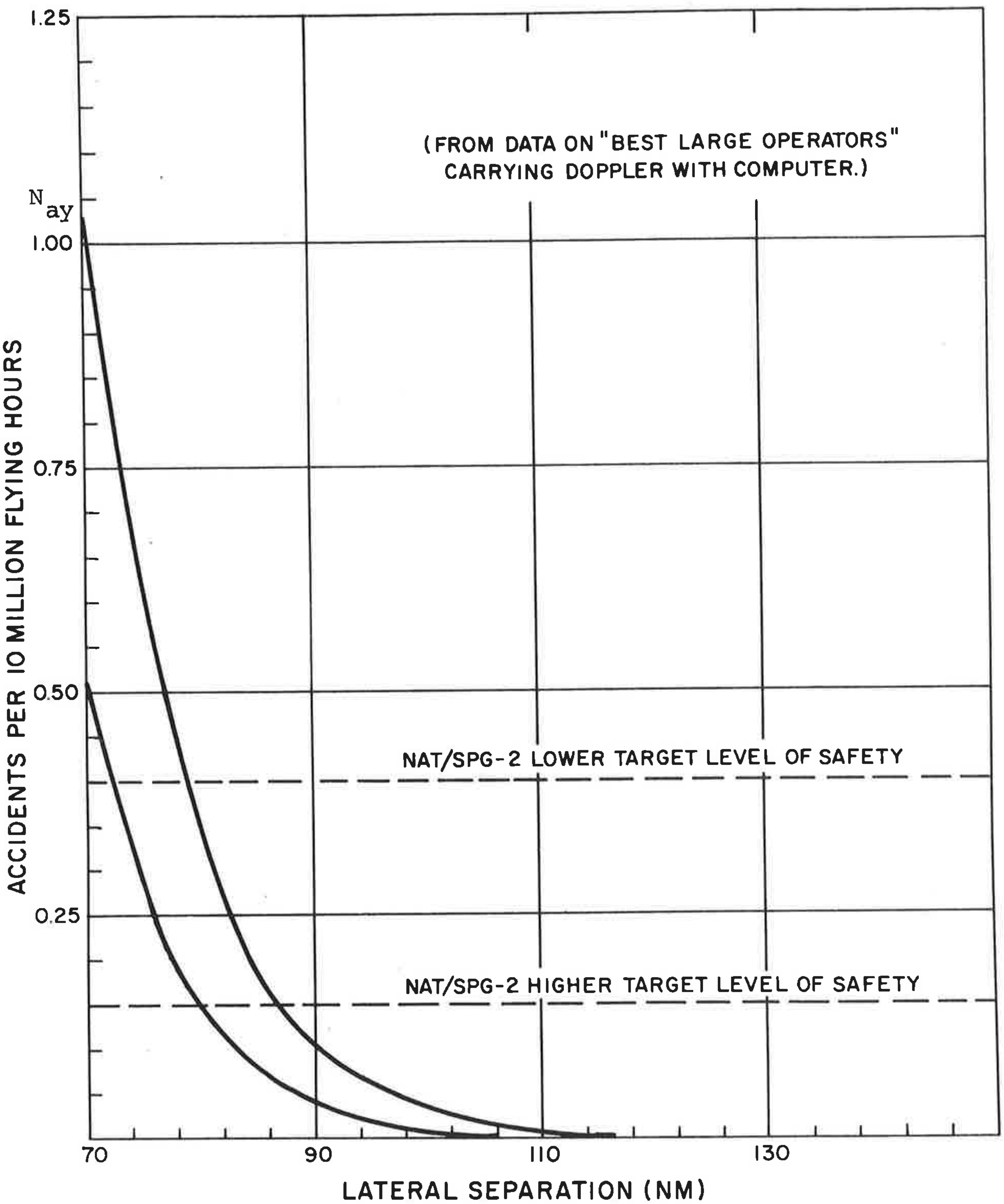

Figure 3.4. Accident Level vs. Lanewidth using "central" estimates (for the best large operators).

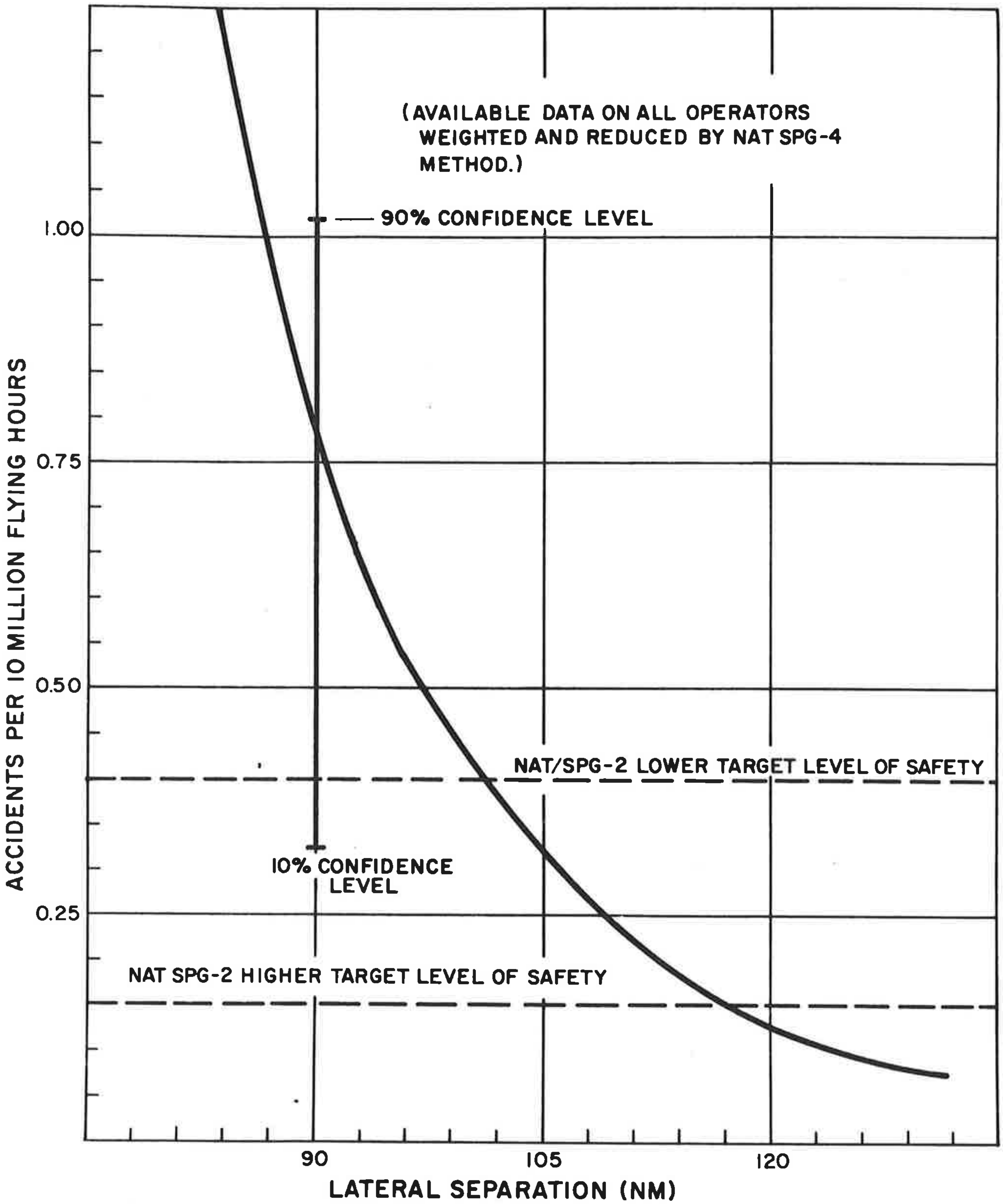

Figure 3.5. Accident Level vs. Lanewidth using "central" estimates (for all doppler operators).

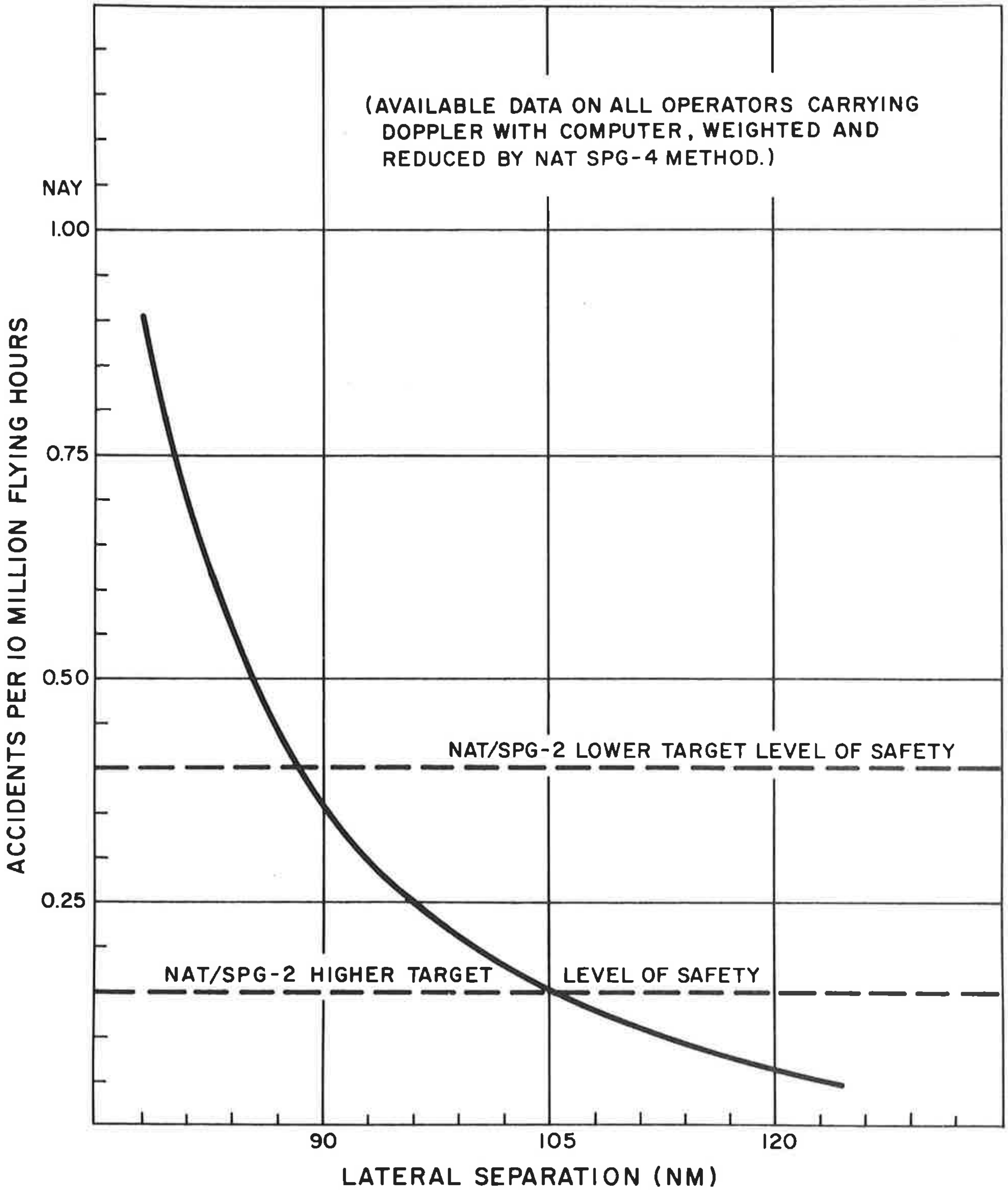

Figure 3.6. Accident Level vs. Lanewidth using NAT/SPG-4 Method (for all operators).

4. THE IMPACT OF INERTIAL NAVIGATION ON AIR SAFETY

4.1 INTRODUCTION

The expected increase in traffic density in the NAT region (Ref 15) will create pressure for reduction in the current separation standards. One way to effect the reduction, without causing an undue increase in the risk of collision and, hence, of operating cost to the airlines, is through the use of inertial systems.

In order to investigate this potential reduction more thoroughly, Task 2 called for an analysis of INS performance data to assess the probable impact of INS on collision risk in the NAT region. The results of that analysis were presented to the Institute of Navigation Air Safety meeting on April 14, 1971, and represent the combined efforts of both TSC personnel and Dr. K. R. Britting of the MIT Measurement Systems Laboratory. Section 4.2 summarizes the essential elements of that study, as reflected in Ref 11.

4.2 OVERVIEW

An analysis of INS performance data was carried out to assess the probable impact of inertial navigation on the aircraft collision risk in the NAT region. These data were used to calculate the collision risk between two aircraft flying at the same nominal flight level on adjacent tracks. The inertial system's error sources were treated in a statistical sense to infer the en route error behavior from the terminal error data. Collision risk estimates were derived for easterly and westerly transatlantic flights. The results of this relatively conservative analysis indicate that the widespread use of inertial navigators will lead to reduced separation standards in the NAT region while maintaining present safety standards. Only lateral separation was considered in this initial study.

In Section 4.3, the INS input error model is presented along with a description of a terminal data obtained by Air France. This model and the terminal data are then combined in order to infer the en route inertial error characteristics. The collision risk model, adopted by the NAT/SPG (Refs 2 through 5 and 10), was discussed in Section 3. An adjusted formula for the expected number of accidents, which considers time and direction dependent navigation, is combined with the en route errors to yield an estimate on the risk associated with specific separation standards. Section 4.4 includes a summary and discussion of the principal results.

4.3 ENROUTE INS ANALYSIS

4.3.1 Data Base. The data were collected by Air France, over its NAT routes, between July, 1968 and April, 1970, with 29 INSs (Ref 25). A total of about 24,000 hours of navigation time was logged during 1528 flights. The INS was the Litton LTN-51, a free wander-azimuth, two-dimensional navigator, two of which were installed in each aircraft. Since no en route navigational fixes were available, the navigational accuracy was determined at the terminal point only. (The American Airlines study, alluded to on page 3 of this report, was used in an earlier working paper (Ref 16) to help develop some of the techniques detailed in this section.)

Figures 4.1 and 4.2 show the distribution of radial errors for the easterly-westerly flights, respectively. Two distributions for the operational navigation errors are shown for each flight path: an average distribution, R_A, and a maximum distribution, R_M.

In cases where an inflight failure or a large deviation (defined as a terminal radial error greater than 50 n.m.) of only one of the two inertial systems occurred, the operational radial error was taken to be the radial error associated with the in-spec. system, i.e., $R_A = R_M$. Neglecting the out-of-spec. errors in the statistical evaluation was justified on the basis that the flight crews were, in all cases, able to detect that the system had failed or was exhibiting large errors.

In situations where both of the systems had radial errors at arrival greater than 50 n.m., R_A and R_M are calculated from the formula,

$$R_M = R_A = \tfrac{1}{2}\left[\left(X_1+X_2\right)^2+\left(Y_1+Y_2\right)^2\right]^{\frac{1}{2}} , \qquad (4\text{-}1)$$

where

X_k = lateral error associated with system k (k = 1, 2)

Y_k = longitudinal error associated with system k (k = 1, 2)

Finally, for the case of nominal operation, the radial errors were calculated using

$$R_M = \text{maximum } (R_1, R_2) \qquad (4\text{-}2)$$

$$R_A = \tfrac{1}{2}\left[\left(X_1+X_2\right)^2 + \left(Y_1+Y_2\right)^2\right]^{\frac{1}{2}} . \qquad (4\text{-}3)$$

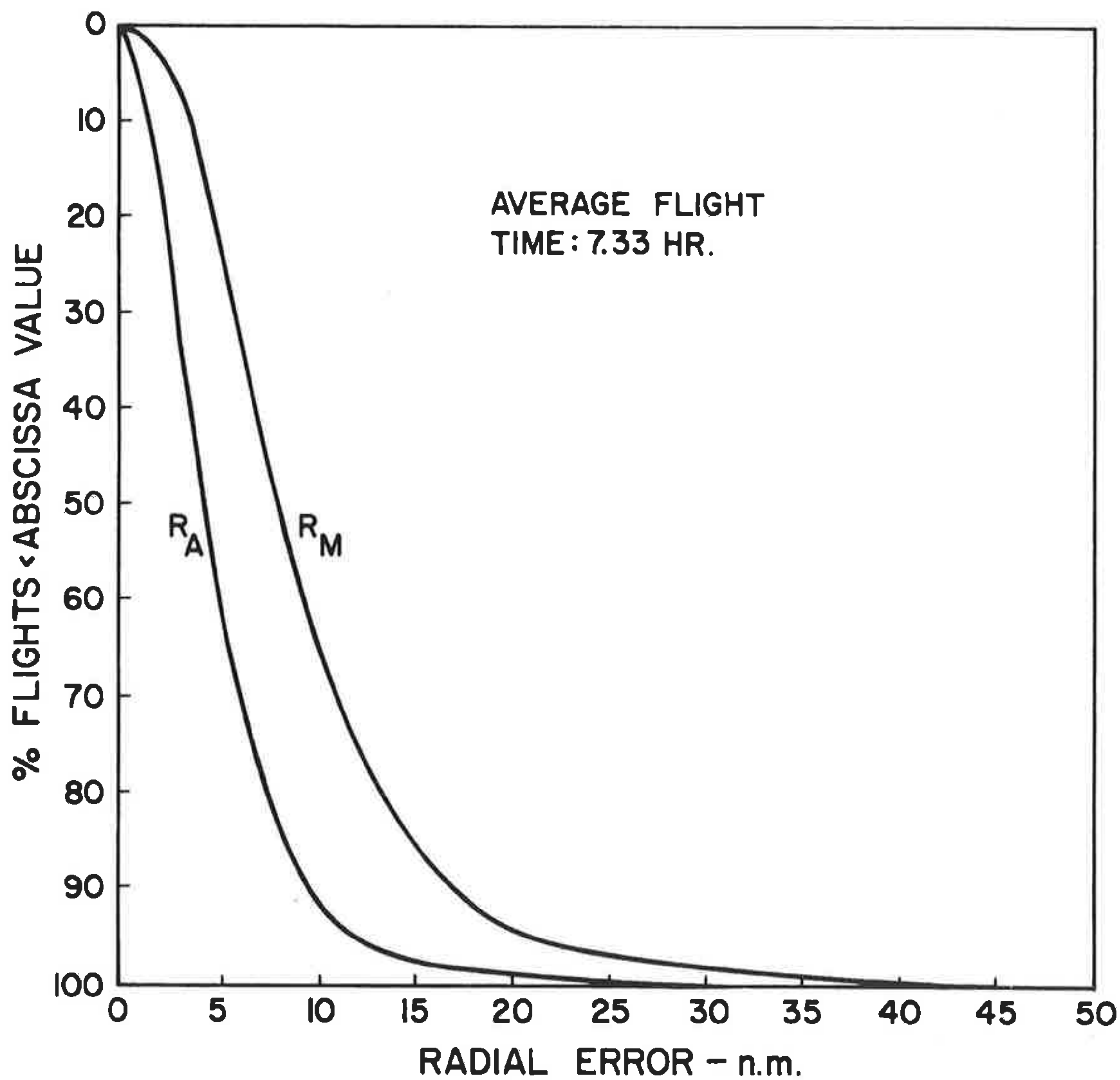

Figure 4.1. INS radial terminal error distributions: easterly flights.

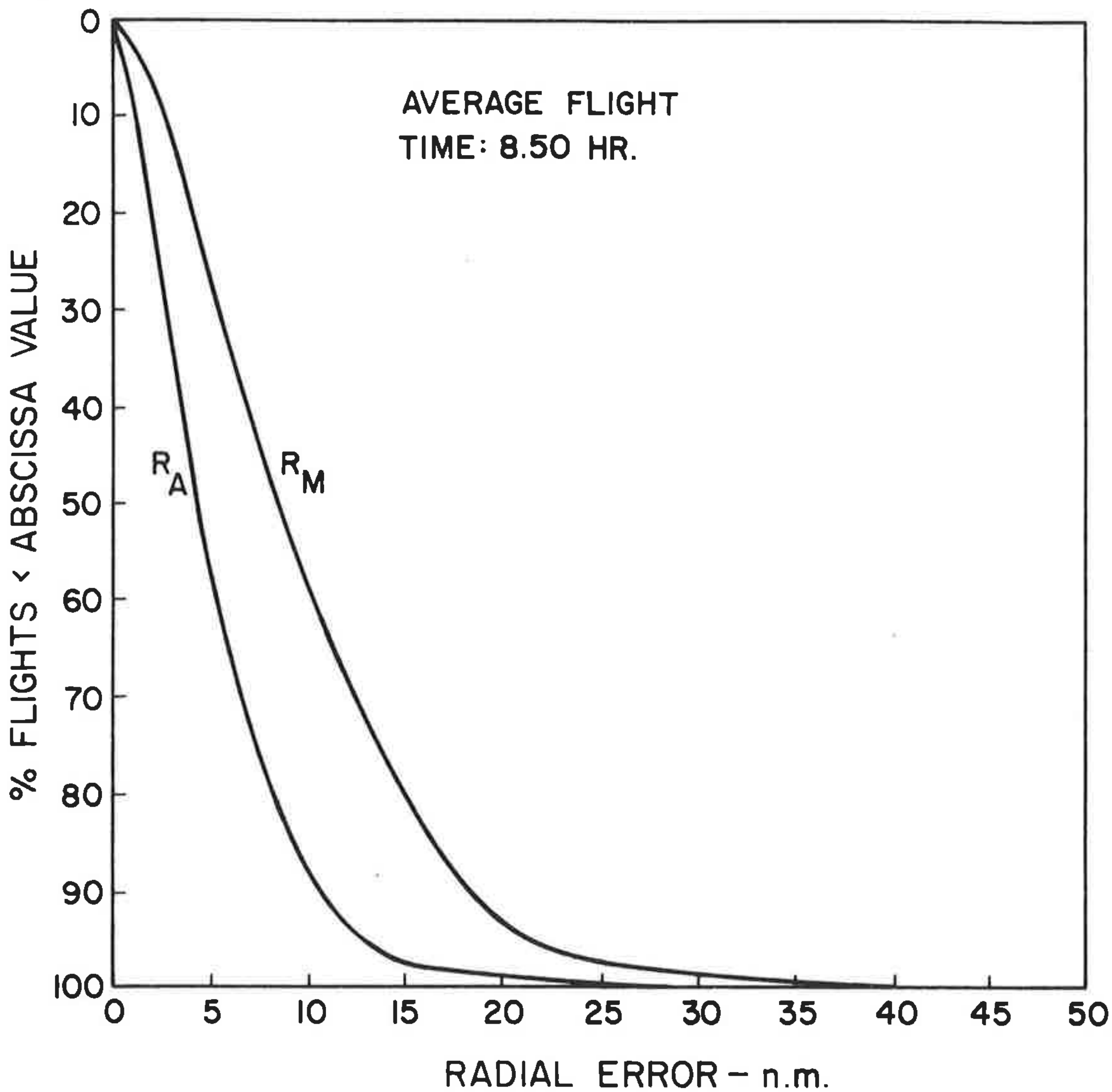

Figure 4.2. INS radial terminal error distributions: westerly flights.

For each point on the distribution curves, the ordinate, when divided by 100, can be interpreted as the probability that the radial error for a given flight will not exceed the abscissa value. Table 4.1 presents values for the radial error in nautical miles for several important probabilities. The decay of the terminal distribution curves can be shown to lie between exponential and Gaussian.

TABLE 4.1.- EASTERLY-WESTERLY INS RADIAL ERROR COMPARISON

Probability radial error is less than significance	Easterly		Westerly	
	R_A (n.m.)	R_M (n.m.)	R_A (n.m.)	R_M (n.m.)
0.683 (1σ-Gaussian)	5.8	10.6	6.2	12.0
0.757 (1σ-First Laplacian)	6.6	12.0	7.7	14.8
0.941 (2σ-First Laplacian)	11.7	20.0	12.4	20.4
0.950 (2σ-Gaussian)	11.9	20.8	12.7	21.0
0.990	23.0	36.0	20.0	30.0

Comparison of Figures 4.1 and 4.2 reveals that, although the average time-of-flight for the westerly route is 1.17 hours longer than the time of flight for the easterly route, the westerly radial errors are only slightly larger than the easterly radial errors.

The better performance for westerly flights is probably achieved because the systems' inertially referenced angular velocity is smaller than for easterly flights, resulting in a lower system sensitivity to gyro torquer uncertainty.

4.3.2 Inference of En Route Errors from the Terminal Errors. As previously discussed, the data base consisted exclusively of terminal error statistics. In order to calculate the collision risk between two aircraft occupying the same flight level and flying on adjacent tracks, it was necessary to know the error distributions for the entire time of flight. It is, therefore, necessary to infer the en route errors from the terminal errors.

For the purpose of this study, a simplified, albeit conservative approach was taken in order to expeditiously obtain the en route error statistics. The collision risk formulae were then utilized to yield a tentative conclusion as to the effect of inertial system technology on air safety. A more complete discussion of optimum procedures can be obtained from other sources, including Refs 11 and 17.

Specifically, the INS is simulated for the situation where the error uncertainties are modeled as being members of an ensemble-of-constant functions (Ref 18). Furthermore, the error equations are solved for the case of constant east-west velocity at constant latitude, a reasonable assumption given the NAT traffic structure. As discussed in Appendix C, the constant velocity assumption results in the inertial system's error differential equation having constant coefficients.

4.3.3 Inertial System Simulation. The Litton LTN-51 system was simulated using the error model shown in Appendix C for the case of constant east-west velocity of 637 knots at a constant latitude of 45°. As indicated, the major error sources consist of the gyro drive uncertainties, $(u)\underline{\omega}$, the accelerometer uncertainties, $(u)\underline{f}$, the gyro torquer scale factor uncertainties, τ_x and τ_y, and the initial platform misalignments, $\varepsilon_N(0)$, $\varepsilon_E(0)$ and $\varepsilon_D(0)$. The system response to each of these error uncertainties was separately determined.

The following error-source magnitudes were considered:

Gyro drive: $(u)\omega_k = 1$ meru (0.015 deg./hr.); $k = x,y,z$

Accelerometer: $(u)f_k = 10^{-4}g$; $k = x,y$

Gyro torquing: $\tau_k = 10^{-3}$; $k = x,y$

Platform misalignment: $\varepsilon_k(0) = 1$ arc-min; $k = N,E,D$,

where the x,y and z subscripts refer to uncertainty components occurring along the platform's x,y and z axes. (Note that since two-degrees-of-freedom gyroscopes are used, the three-dimensional gyro drift vector is associated with only two instruments.) Figure 4.3 shows the resulting across-track position error statistics for the above error source magnitudes.

The theoretically derived en route error curves were then scaled to match both the empirically determined terminal data and the assumption as to its distribution shape. This was a two-step process. First, the radial errors were scaled in accordance with the ratio of the terminal errors to the theoretical errors at the terminal point. Then, since the data consisted exclusively of radial-error statistics, the radial error had to be apportioned into equivalent latitude and longitude errors. This apportionment was performed on the basis of the error simulation, which showed that the latitude and longitude errors were approximately equal at the terminal points.

Figure 4.4 represents the across-track velocity error statistics obtained from the R_A data, using the above method and

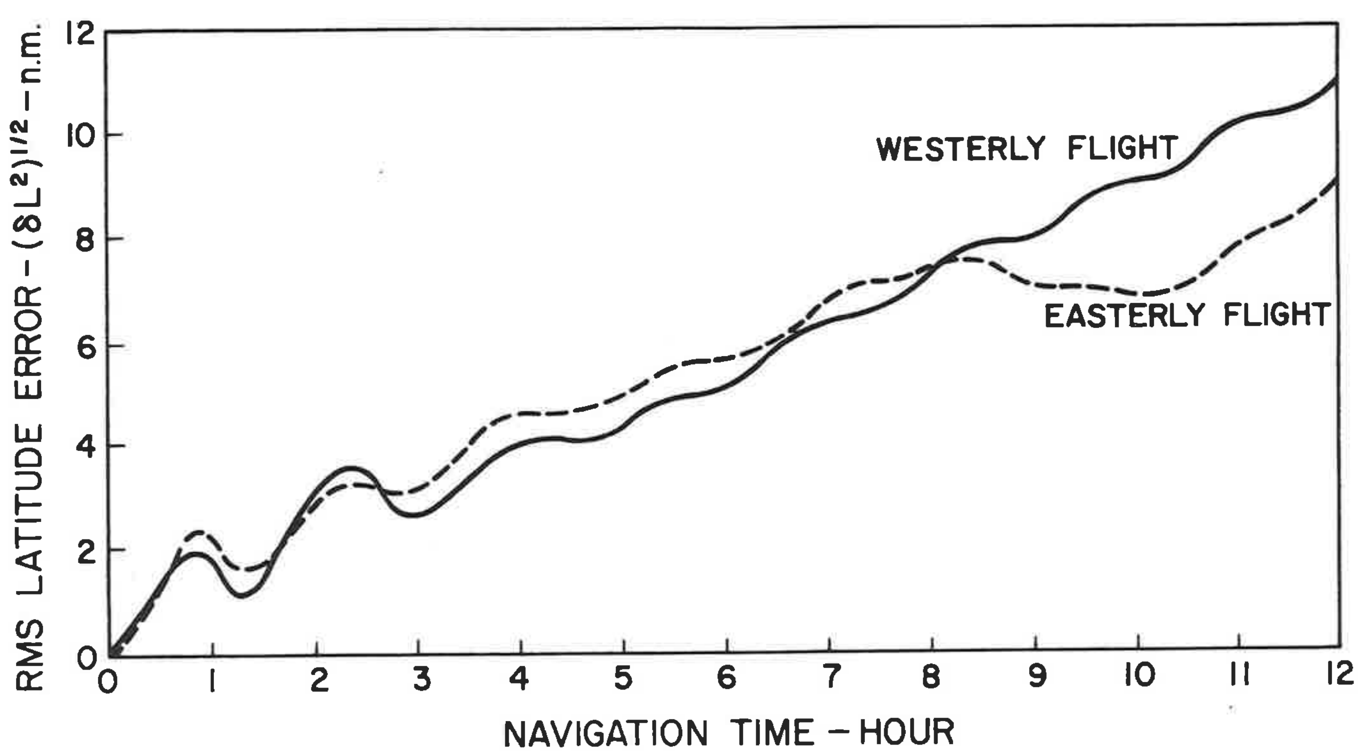

Figure 4.3. RMS position latitude error for free azimuth INS.

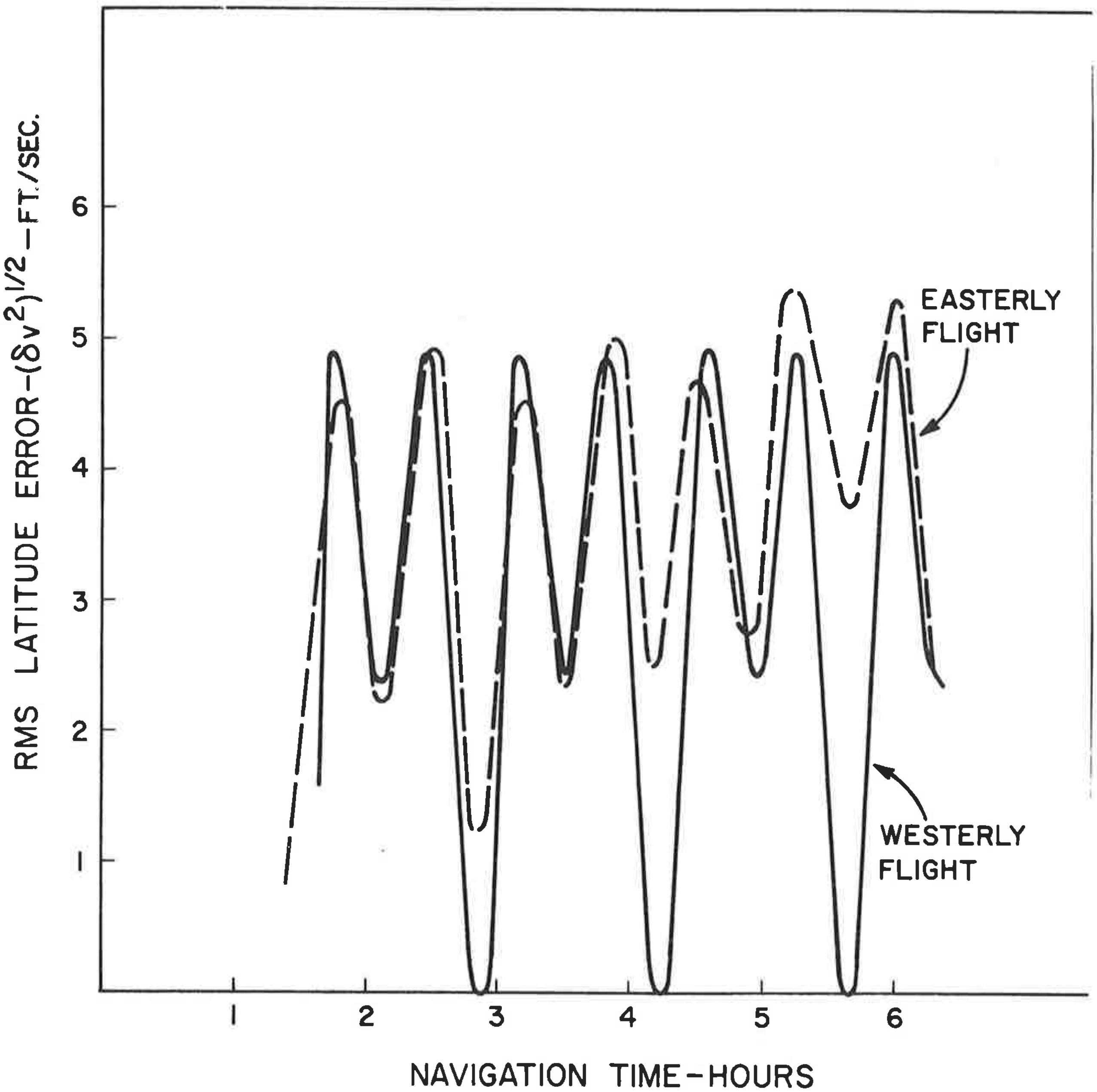

Figure 4.4. RMS velocity latitude error for free azimuth INS (Gaussian: Average case).

assuming a Gaussian distribution. Similar plots can easily be derived for the three other cases (i.e., R_A - First Laplacian, R_M - Gaussian and R_M - First Laplacian) by simply rescaling the theoretically derived en route error curves.

For the westerly flight, the dominant error for the first several hours is caused by the initial platform misalignments, while the long term error is dominated by effects due to gyro drift. For the easterly flight, on the other hand, the long term error is dominated by a combination of gyro drift and torquing uncertainty. Obviously, the shape of the latitude error curves depends on the assumptions made as to the relative weighting of the error sources.

It is to be emphasized that the en route determination used herein tends to be conservative since modeling the gyro drift as a member of the ensemble of constant functions results in an approximately linear error growth. Other, more accurate, gyro drift models involving random walk processes, result in navigation errors which grow proportional to the square root of time.

4.4 COLLISION RISK CALCULATIONS

4.4.1 Collision Risk Equation. The effects of the en route inertial navigation statistics are assessed by considering the collision risk associated with specific lateral separation standards. An adaption of the NAT/SPG collision risk model, discussed in Section 3, has been used to analyze the number of accidents expected to occur in an airspace containing only inertially-equipped air carriers. This adaptation consists of the admission of time and direction dependences to the original formulation. For the number of accidents due to loss of lateral separation, the adjusted formula, discussed in more detail in Appendix D, is given by,

$$N^*_{ay} = 10^7 \Bigg\{ E_y(\text{opp}) P_y(\text{opp}) \left[\frac{1}{S_x} \left(\overline{V} P_z(0) + \lambda_x N_z(0) + \frac{\lambda_x \overline{|\dot{y}(\text{opp})|}}{2\lambda_y} P_z(0) \right) \right]$$

$$+ E^e_y(\text{same}) P^e_y(\text{same}) \left[\frac{1}{S_x} \left(\frac{1}{2} \overline{\Delta V} P_z(0) + \lambda_x N_z(0) + \frac{\lambda_x \overline{|\dot{y}^e(\text{same})|}}{2\lambda_y} P_z(0) \right) \right]$$

$$+ E^w_y(\text{same}) P^w_y(\text{same}) \left[\frac{1}{S_x} \left(\frac{1}{2} \overline{\Delta V} P_z(0) + \lambda_x N_z(0) + \frac{\lambda_x \overline{|\dot{y}^w(\text{same})|}}{2\lambda_y} P_z(0) \right) \right] \Bigg\}^* \qquad (4-4)$$

where * denotes the average over the time-of-flight for the quantities concerned. In the above, the superscripts e and w denote east and west directed flights, respectively. The other parameters are defined in Section 3 and Appendices A and B.

4.4.2 Remarks on the Treatment of Large Flying Errors. "It is the large, rare errors (rather than those of moderate size which form the bulk of observations) which determine the risk of collision." (Ref 2) The fact that the treatment of these errors are critical to the analysis of expected accident levels explains the emphasis placed in the literature (Refs 3 and 5) upon the careful modeling of the tails of the error distribution. Of particular importance, in this regard, is the inclusion of all significant sources of such error.

There are two general sources of large error to be considered in investigating the flying density of inertially equipped carriers. The first type, which we will refer to as "blunders," might arise either from a system breakdown, such as a specific mechanical or electrical failure, or from an incorrect set of input instructions, such as faulty way-point or initial position information. The second type, occurring in the absence of two first-type errors, is assumed to be a statistical characteristic of the system itself; namely, there is a finite, albeit small, probability that an operating system will, upon occasion, exhibit large errors.

The Air France data excludes certain "blunders" and inherently includes others. This fact, coupled with the lack of sufficient data, precludes a thorough analysis of the effect of large errors at this time. Therefore, for this investigation we shall assume that the navigation errors arise solely from the characteristics of the navigation system itself. From the viewpoint of assessing nominal INS accuracy this is a conservative assumption. However, for the broader question of estimating risk, this assumption is likely to lead to optimistic conclusions. This matter is discussed further in the Summary and Discussion of Results, Section 4.5

4.5 SUMMARY AND DISCUSSION OF RESULTS

4.5.1 Presentation of Results. N_{ay}^{*} is calculated for each of the four cases in Section 4.3. The details are presented in Appendix D. The relationship between the risk, as reflected in the value for N_{ay}^{*}, and the lateral separation standard, S_y, are graphically presented in Figures 4.5 and 4.6. These values are compared with the target levels of safety specified by NAT/SPG for the assessment of future separation standards over North Atlantic (Ref 3).

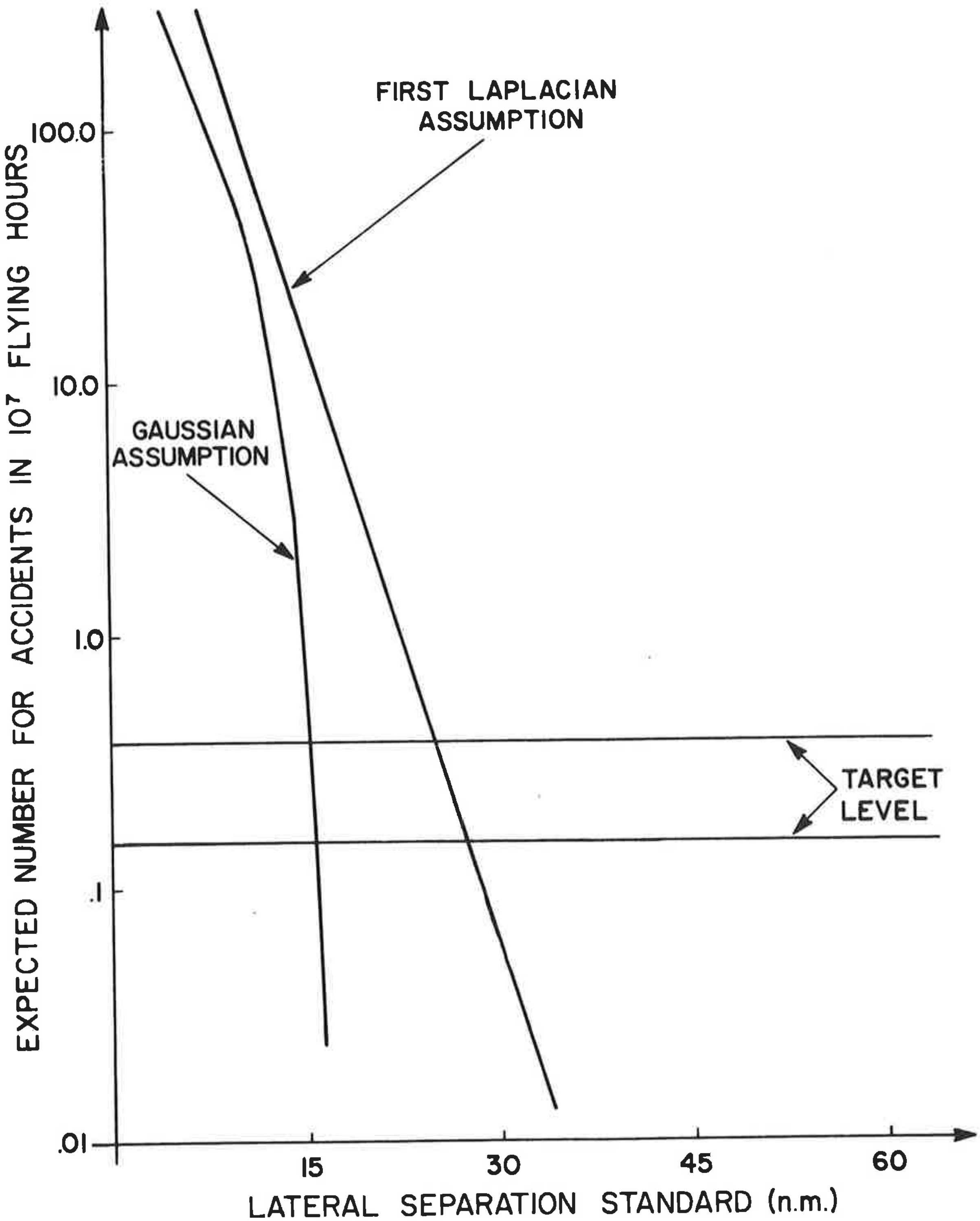

Figure 4.5. Accident level vs. lanewidth using average distributions.

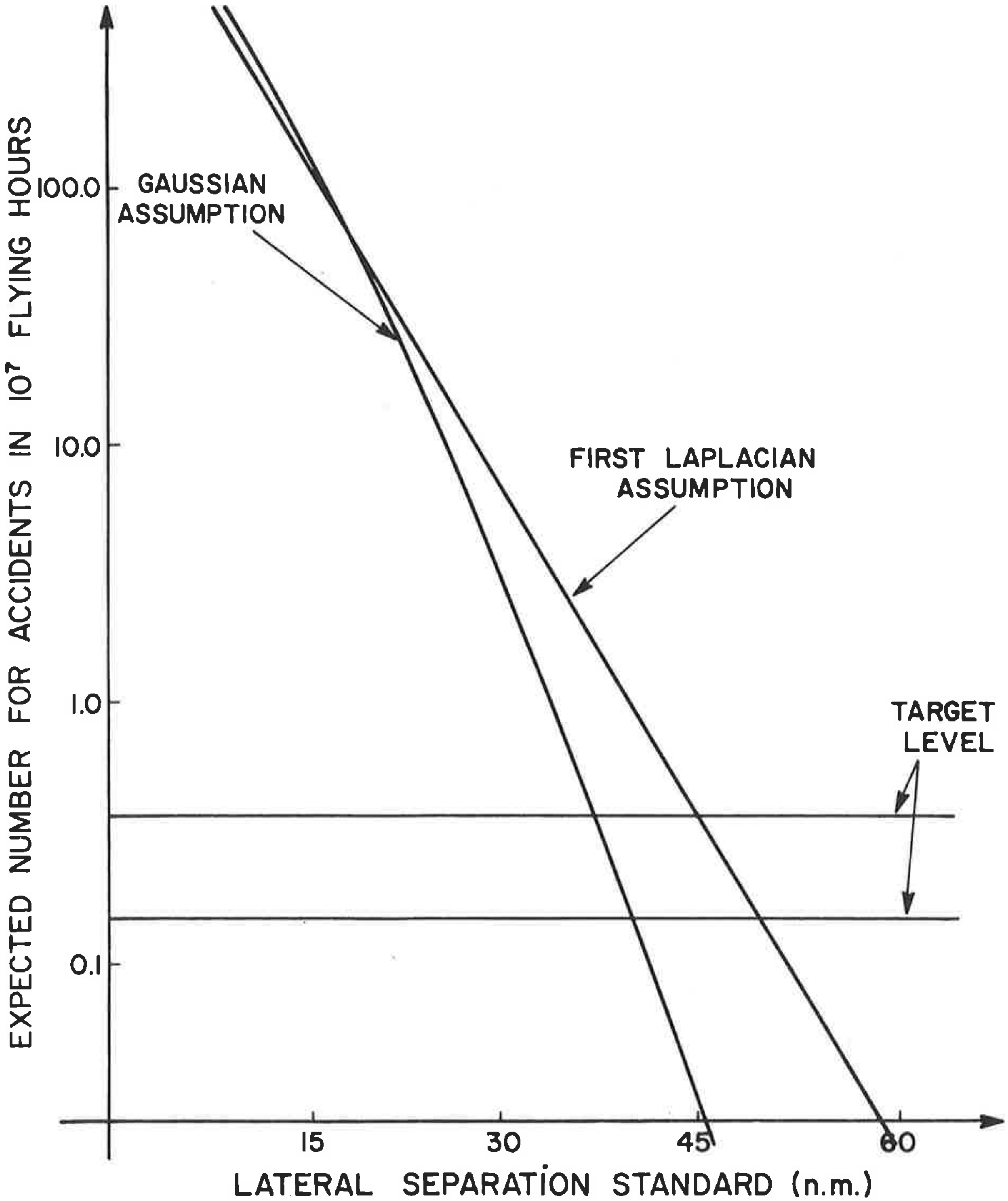

Figure 4.6. Accident level vs. lanewidth, using maximum distributions.

In the most optimistic case, namely where the average error is associated with a Gaussian distribution, the target level of safety can be achieved with a separation of approximately 15 n.m. On the other hand, for the most pessimistic case, where the maximum error is associated with an exponential distribution, the safety target requires approximately a 45 n.m. separation. The intermediate assumptions of maximum with Gaussian and average with exponential yield required separations of 37 n.m. and 25 n.m., respectively.

In view of the conservative nature of the navigation model and the presence of some blunder statistics in the data, it appears plausible that inertial navigation systems, in the absence of blunders, are accurate enough to meet safety requirements with a separation standard of 30 n.m. or less. It also appears reasonable that, even with the inclusion of blunders in the analysis, INS technology will allow for a substantial reduction in the present 90 n.m. set for safety.

This possible reduction in separation standards, afforded by the introduction of inertial systems, has been anticipated. The present analysis provides some quantitative corroboration. A more precise and reliable estimate of this reduction will require further studies.

Further statements concerning the results of this analysis and the need for future work are presented under Conclusions, Section 6.

5. SATELLITE SURVEILLANCE SYSTEMS

5.1 INTRODUCTION

Section 4 dealt with the introduction of INS as the primary means of navigation for oceanic carriers. Quantitive evidence was presented, indicating that inertial navigation can lead to a substantial reduction in lane separation without causing a simultaneous increase in the hazard of collision.

This section considers an additional approach for optimizing the safety-cost criteria: that of an independent satellite surveillance system used in conjunction with an inertial navigator. Such a system would have several obvious advantages: it would provide a source-of-position information independent of the carrier navigation system; the surveillance errors would not grow with time-of-flight, as do INS and Doppler errors; the satellite position information could be used to update the on-board navigation system; a ground station, able to monitor the position information of all aircraft in the region of interest, would, with proper communication links, be able to effect positive Air Traffic Control (ATC).

The philosophy of control, the system requirements, the parametric trade-offs involved and the degree to which the separation standards could be reduced while still maintaining safety, are all questions open to analysis. (A more complete discussion of the general nature of satellite ATC surveillance systems can be found in Refs 19, 20 and 21).

Task 3 specifically addressed the satellite surveillance problem: (Ref 1).

> The effect of a satellite surveillance system on the routing structure in the NAT region will be undertaken. This study will be performed by an outside contractor. It will involve (1) the selection of an appropriate control strategy, (2) the analysis of modeling procedures, and (3) an investigation of the relationships among such parameters as separation standards, fix rates, threshold widths, navigation, and surveillance accuracy and safety levels. In addition, recommendations will be made for the further efforts required in investigating various ATC surveillance systems.

5.2 CONTRACT STATEMENT

A RFP was issued and appeared in the Commerce Business Daily on March 4, 1971, with a contract code designated of TSC/PS-0029. The important technical items to be considered are given below as they appeared in the RFP (Ref 12):

Item 1 Conduct a study and investigation of both the nature of present day operations in the North Atlantic (NAT) and the future developments forecast for the region and present a brief review to the TSC Technical Monitor. The review will indicate those areas of investigation considered pertinent to this current study with particular emphasis on how the analysis will relate to the future requirements and design of a satellite ATC surveillance system. The contractor shall select the particular control philosophy technique to be considered in the analysis, and briefly discuss and justify this choice in terms of: (i) its impact on the system, (ii) it operational requirements (i.e., required data and control procedures), and (iii) its ease of implementation. Indicate those parameters required to fully characterize the system; and discuss proposed procedures for modeling the relevant input parameters.

Item 2 Choose the model and solution technique to be initially considered, and justify the decision on the basis of: (i) the expected input-output format, (ii) the assumptions and approximations to be made, (iii) the analytical and/or computational requirements, and (iv) all other information deemed appropriate. Indicate, where possible, the advantages of the procedure chosen over other possible approaches.

Item 3 Fully discuss the total input-output format to result from the analysis, and justify this format in terms of: (i) its information content, (ii) its applicability in answering meaningful questions concerning the future procedures to be employed in the NAT region, and (iii) its ability to interface with future studies designed to define the final system requirements. The contractor shall conduct a trade-off, sensitivity analysis among the parameters of interest, which may include, among others: (i) surveillance accuracy, (ii) on-board navigation accuracy, (iii) fix rates, (iv) alarm rates, and (vii) error probabilities.

Item 4 As a result of the work performed under Items 1 through 3, the contractor shall provide discussions in the Final Technical Report which will include conclusions on: (i) the reducibility of separation standards, and (ii) recommended sets of parameter values. Recommendations shall be made for possible future areas of research related to the design and specification of a satellite Air Traffic Control Surveillance System.

The original schedule was given by:

A. Commencement Date: June 1, 1971

B. Completion Dates:

Item 1	June 30, 1971
Item 2	July 31, 1971
Item 3	September 15, 1971
Item 4	October 15, 1971

with a contract completion date of October 31, 1971. The issuance of this contract was delayed by a number of factors, thereby moving the actual commencement date back to July 1, 1971. Therefore, each of the dates given above are all delayed by approximately one month.

The evaluation criteria to be used in evaluating the proposals were carefully stated in the RFP and are repeated below as they appeared:

1. <u>Understanding of System Requirements</u>

   a. Show an understanding of the procurement objectives.

   b. Indicate an understanding of the present mode of operation in the North Atlantic.

   c. Define those parameters you deem important in analyzing the performance of an Air Traffic Control surveillance system in the NAT region.

   d. Discuss the scope of this current effort. What do you hope to accomplish in this initial study? How would you interface the results of this contract with previous and future studies?

2. Technical Approach

a. Model selection - Indicate the mathematical model you intend to employ in analyzing the ATC surveillance problem. Specify the advantages of your technique over other possible approaches. (For example: (1) Do you favor a probabilistic or deterministic approach? What is your opinion of extending the NAT/SPG collision risk model?, etc.)

b. Solution method - Indicate the general techniques envisioned as necessary to solve the problem at hand. As examples of possible questions to be answered: Will you be using a computer? If so, will you use a Monte Carlo technique? Can the solution be done in a closed form analytical manner? Will a hybrid approach be necessary?,etc.

c. Indicate the parameter trade-offs to be performed and the general input-output format to be used.

d. In short, present a detailed summary of how you would approach the problem on a first cut basis. Make sure this summary is not at variance with the opinions asked for in paragraph 1 above.

3. Technical Team Qualifications

a. Team background must correlate with the technical approach chosen. For example, a highly theoretical mathematical approach may call for an expert in Kalman filtering or random process, whereas a "common-sense" deterministic approach may call for a systems and operationally-oriented individual, perhaps with flying experience. This matching of approach to individual should be indicated.

b. The contract manager must have the time to become actively involved in the project.

c. Show previous work in the Air Traffic Control surveillance field. Indicate the team's general appreciation of the problems involved.

4. Company Qualifications

a. Indicate the support and guidance which can be provided to the technical team when required.

Two additional remarks on evaluating policy were considered appropriate:

1. The proposal will be expected to contain specific details in regard to the points raised above. Unduly long proposals that do not primarily address themselves to the specific problems discussed in this RFP are discouraged.

2. The purpose of this contract is to initiate an organized systems approach towards the analysis of a future ATC surveillance system in the North Atlantic. These tasks, as outlined above, represent the present attitude of TSC as to how this requirement may be best fulfilled. In this proposal, the prospective contractor is at liberty to comment on the general scope of the contract as well as on the specific items indicated. In particular, the prospective contractor should not feel constrained from expressing disagreement with the judgement expressed by TSC and is encouraged to recommend and justify an alternate approach. Carefully considered deviations from the detailed order here specified will be given full consideration and, in the evaluation of proposals, the degree of in-depth understanding of the problems involved will be significantly weighted.

5.3 CONTRACT AWARD

Twenty-six companies and institutions were sent copies of the RFP and 11 responded. They were, in alphabetical order: (1) ARCON Corporation, (2) Autonetics, (3) Bell Aerospace Company, (4) Boeing Company, (5) Computer Sciences Corporation, (6) Dynamics Research Corporation, (7) IBM Corporation, (8) General Electric Company, (9) Software Sciences Limited, (10) Systems Control, Incorporated (SCI), and (11) TRW Systems Group. All companies responding were considered to have submitted acceptable proposals.

Three companies were thought to have exhibited superior qualifications - as adjudged by the evaluation criteria detailed in Section 5.2. A careful comparison of their proposed technical approaches to the problem led to the following ratings in descending order: (1) SCI, (2) ARCON Corporation, and (3) TRW Systems Group.

A complete review of the evaluation process is contained in a memo, written by technical evaluators, Ronald Hershkowitz and Daniel Brandel, to the TSC Procurement Division (Ref 13).

5.4 MODELING APPROACH

The surveillance collision risk model, proposed by Systems Control, Incorporated, is discussed in this section. A more detailed mathematical description appears in Appendix E.

Refs 22 and 23 contain additional information on SCI's concept of (1) the background required, (2) the philosophical and technical issues involved, (3) the initial approach to modeling the navigation surveillance and ATC systems, (4) the modeling extensions to be investigated, and (5) the input-output format to be presented.

The overall problem considered in the proposal is how to relate surveillance system parameters to route capacity and safety and, thereby, to determine the extent to which the separation standards can be reduced for a given type of surveillance system. To summarize the details of this complex problem and to isolate its significant parts, SCI's discussion is organized as follows:

1. The need for surveillance is first developed by summarizing the present NAT route structure, separation standards, summary of past attempts at lane reduction and projected traffic densities.

2. The complex relationship between route capacity (separation standards) and safety (collision risk) for a given type of surveillance system is considered, with specific reference to the NAT/SPG approach of separate collision risk and surveillance system models. The advisability of retaining these separate models is indicated, and the collision risk model is discussed in terms of assumptions, computational problems and alternatives.

3. Each of the major elements of the ATC surveillance system, including the navigation system, the satellite surveillance system and the ATC procedures, is discussed in detail to summarize the modeling problems.

In the development of the necessary methodology, SCI mentions the essential requirement, that of considering both the long-range objectives and the immediate scope of the proposed effort.

SCI states that the long-term objectives should be to develop a generalized, but accurate, methodology for determining the route structure and ATC system parameters that will be consistent with established safety standards. This methodology could then be used to evaluate the quantitative impact on the oceanic ATC system of the following: (1) different stages in the evolution of an oceanic surveillance system, (2) present and future nativation systems, (3) varying ATC procedures, levels of automation and aircraft mix, (4) more accurate and detailed descriptions of the physical systems and procedures, and (5) complete sensitivity analysis and trade-off studies of the system elements, parameters, procedures and assumptions.

Clearly, there are many steps in the development of a broad, generalized modeling methodology. As a first step in this development, SCI proposes to define the basic model structure so that succeeding steps will (1) extend the accuracy and level of detail of the model elements (navigation systems, surveillance systems and ATC procedures), (2) develop more powerful computational techniques, and (3) extend the scope of the sensitivity studies that can be performed.

The scope of work for the proposed effort should clearly keep in mind the long-term objectives. However, SCI realizes that it must consider the most essential problems and develop the method of solution that will produce meaningful near-term results, and yet be able to achieve the above-stated long-term objectives. For these reasons, SCI proposed the following short term objectives:

1. To thoroughly understand the ATC surveillance system requirements for the NAT region. This understanding should provide the background for development of a systems analysis approach. It will include the physical elements of the navigation and surveillance system, the ATC procedures, the operating environment, and the results of previous studies, models and reports on the subject.

2. To develop a basic model structure for assessing the quanititative impact of the ATC surveillance system on the separation standards of the NAT routes. This effort should concentrate on the development of a mathematical model of the ATC surveillance system that would be compatible with the Reich collision risk model. The model should include the essential elements of the ATC system, such as navigation systems, satellite surveillance systems, ATC control procedures and environmental effects. This model should be (1) sufficiently flexible to describe the many different system elements; (2) sufficiently accurate to provide an adequate description of the essential characteristics of any one system element; and (3) adaptable enough to accept

statistical data from actual flights (e.g., INS data), as well as simulated input data (e.g., satellite ranging errors).

3. To perform a sensitivity analysis of the ATC surveillance system parameters, such as surveillance accuracy, on-board navigation accuracy, fixed rates; lane spacing, safety levels and alarm rates; to define future ATC surveillance system requirements; and recommend safe separation standards and compatible surveillance systems parameters.

SCI has developed a new modeling technique for describing navigation, surveillance systems and ATC procedures, incorporating these elements to obtain a time-dependent description of aircraft position errors in each of the three dimensions.

The SCI method includes:

1. Describing the new ATC surveillance model which contains:

    a. Navigation, surveillance and ATC system models that are flexible enough to be used for any of the proposed physical systems and yet retain the important characteristics of each system.

    b. An efficient modeling technique for obtaining a time-varying description of aircraft position errors in the ATC surveillance environment.

2. Describing how the SCI surveillance model and the Reich collision risk model would be used to perform trade-off studies for such parameters as surveillance accuracy, on-board navigation accuracy, fix rates, lane spacings, safety levels and alarm rates.

The major elements of system control's ATC surveillance model are described in Section 4 of Ref 22. A qualitative discussion follows.

5.4.1 <u>SCI/ATC Surveillance Modeling Technique</u>. The SCI/ATC surveillance modeling technique incorporates the navigation and surveillance system models and the ATC procedures in a computationally efficient manner in deriving a closed-form, probabilistic, time-dependent description of aircraft position errors. This expression for the position errors can be used in the Reich collision risk model to obtain the safety level of the NAT routes as a function of time.

The principal features of the SCI modeling technique include:

1. The inputs are time-dependent descriptions of the navigation position errors, the surveillance position errors, and either tactical (time-dependent, traffic-dependent) or strategic ATC procedures.

2. The modeling technique includes a detailed procedure for the longitudinal dimension as well as for the lateral and vertical dimensions.

3. The inputs are used to derive closed-form expressions for the time-dependent behavior of an aircraft, under the influence of a surveillance system, in each of the three dimensions.

4. The closed-form expressions for aircraft position errors can be numerically evaluated very efficiently.

5. The closed-form expressions for aircraft position errors are also used to derive time-dependent probabilities of lateral, vertical and longitudinal overlap for aircraft traveling in the same or opposite directions (except in the longitudinal case).

6. The time-dependent probabilities are combined in the Reich collision risk model to obtain a time-dependent description of the safety level of the NAT routes.

The SCI approach to ATC surveillance system modeling included time-dependence. This is because navigation and surveillance system error sources, environmental effects and aircraft behavior all vary with time. Any approach that attempted to eliminate this time-dependence would have to use worst-case errors and, therefore, result in needlessly conservative estimates of collision risks. The SCI approach also results in closed-form expressions for positive errors as opposed to Monte Carlo methods or even a computer simulation of aircraft behavior.

The initial modeling technique includes only two assumptions on aircraft behavior:

1. The new heading command, issued to the aircraft after it appears to have exceeded the surveillance threshold, will return the aircraft to the desired track before the next surveillance fix.*

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* This assumption has since been modified. Relationships have been derived for the case where the aircraft do not necessarily return within one surveillance interval. These will replace the expressions presented in Appendix E.

2. The navigation system errors (e.g., gyro drift rate) are assumed to change in value only at surveillance fix time.

The succeeding paragraphs summarize the SCI surveillance system modeling technique, including its advantages over previous approaches and its computational requirements. A detailed mathematical development of the closed-form expression for the lateral aircraft position error distributions is included in Appendix E. Ref 22 includes detailed discussions of (1) the longitudinal and vertical aircraft position error distributions, (2) the overall computational requirements, and (3) the advantages of the System Control approach.

5.4.2 Computation of Lateral Position Errors. The derivation of the time-varying position error distribution proceeds in distinct time steps: (1) from time zero to the first surveillance fix time, (2) from the first to the second fix time, (3) from the second fix time to the third, and so forth. Note that this structure will easily accommodate varying fix rates, i.e., the time between fixes need not be constant.

Between time zero and the first surveillance fix, the position errors result only from the initial position error distribution and the navigation errors. The surveillance system errors do not enter until the first surveillance fix. At that time, those aircraft appearing to be beyond the surveillance threshold are issued a position update and a new heading. The position errors between the first and second fix times are, therefore, a function not only of the navigation errors but also of the surveillance positioning errors.

The position errors at any time between the first and second surveillance fixes can be written as a function only of the position errors at the first surveillance fix time. This is due to the two modeling assumptions outlined previously. No prior information is necessary. By repeating this procedure, it is possible to show that the position error distribution between the second and third surveillance fix times requires only a determination of the position errors at the second surveillance fix. This procedure can be carried on repeatedly for the duration of the flight. In this manner, the time-dependent position error density function will evolve in time. This property arises because, under the imposed behavior, the ensemble of aircraft trajectories between any two surveillance fixes is a function only of the distribution and drift rates at the previous fix time. This "Markov type" behavior has been known for some time to simplify trajectory calculations.

It is evident that this technique can be readily adapted to different aircraft types, surveillance and navigation systems, and ATC procedures. All that is required is the distributions of the navigation position errors and the surveillance position errors. These distributions are obtained as outputs from the navigation and surveillance system models.

The probability of lateral overlap can easily be calculated from the convolution of the position error density function for two adjacent aircraft. Since these density functions are functions of time, this calculation will involve a time-delay factor to indicate differences in departure times for the two aircraft, and whether they are going in the same or opposite directions.

The combination of the SCI surveillance system model and the NAT/SPG collision risk model provides the required methodology for initiating an organized systems analysis of a future ATC surveillance system in the NAT region. (Ref 22 contains an excellent critique of the NAT/SPG model and a procedure for its use.)

A more detailed mathematical development of the procedure for obtaining the lateral overlap probability can be found in Appendix E.

5.4.3 Input-Output Format. A great deal of emphasis has been given to the form and content of the study's output. SCI presented a full discussion of their proposed input-output format. The inputs to their combined surveillance-collision risk model include:

1. The ATC surveillance system parameters: fix rate and alarm threshold. These parameters are of primary interest since they will have a major effect on the capacity of the NAT routes and the workload for the OCC controllers.

2. The navigation system elements: type of navigation system, physical errors sources and time-dependent error source distributions (e.g., INS navigation system with gyro drift and accelerometer bias error sources; exponential distribution for the gyro drift). These parameters will specify the accuracy with which the aircraft can maintain its desired track without the help of the surveillance system.

3. The surveillance system elements: type of surveillance system, physical error sources, and time-dependent error source distributions (e.g., satellite surveillance

system with ranging bias error source; uniform distribution). These parameters will specify the accuracy with which the surveillance system can monitor the position of the aircraft.

4. The ATC procedure: type of control philosophy, NAT route structure (e.g., tactical position threshold for a composite track structure) and separation standards. These parameters will specify when and how often a surveillance alarm is to be sent and the physical alignment of the aircraft when they are adjacent to each other. These parameters will be obtained through discussion with FAA personnel and from fourth-generation ATC studies.

5. The Reich collision risk model parameters: percentage of time in which two aircraft are proximate, average relative velocity of two aircraft during lateral overlap and similar factors.

The output of this combined surveillance system/collision risk model will be the safety level (in terms of fatalities per 10 million flying hours) associated with a set of parameters from the above categories.

The format itself depends on the parametric trade-offs thought to yield maximum information in assessing the quantitative impact of various ATC surveillance systems on the capacity and safety of the NAT routes.

SCI notes that in performing these trade-off studies, it is important to keep in mind the categories of parameters which describe the ATC surveillance system. Some parameters are known quantities, such as aircraft dimensions and nominal speeds. These will take on specific values (e.g., aircraft nominal speed is 500 knots). Other parameters, although known quantities, will take a specified range of values, such as separation standards of 30, 45, 60, or 90 n.m. The third category includes those parameters that may take on any value, such as the navigation and surveillance accuracies.

Examples given by SCI of some specific parameter trade-offs that should be performed are:

1. For specified safety levels, navigation accuracy and surveillance accuracy, vary lateral separation against surveillance fix interval for several different values of alarm threshold. This will determine, for a given navigation and surveillance accuracy, the range of ATC surveillance parameters values that result in a "safe" collision risk level.

2. For specified safety levels, navigation accuracy and alarm threshold, vary surveillance alarm rate against surveillance accuracy for several values of fix rate. This will determine, for a given navigation accuracy, how many surveillance alarms the controllers must respond to per hour as a function of surveillance position accuracy.

3. For specified safety levels, navigation and surveillance accuracies and separation standards, vary alarm rate against alarm threshold for different values of surveillance fix rate. This will specify, for a given load, the range of ATC surveillance parameters that is safe.

5.5 CONCLUSIONS

SCI has demonstrated an excellent understanding of the nature of the problems raised in the RFP. They have presented a detailed, complete and justified approach to investigating the qualitative and quantitative questions surrounding a satellite ATC surveillance system. A more complete discussion, including a detailed work statement, can be found in their proposal (Ref 22).

6. CONCLUSIONS

6.1 GENERAL COMMENTS ON FY 71

The essential elements of the proposed FY 71 effort, as outlined in Section 2, were completed as required: (1) the NAT/SPG collision risk model was thoroughly researched and discussed (Section 3 and Ref 10); (2) work was begun on introducing time-dependent inertial navigation into the collision risk model (Section 4 and Ref 11); and, finally, (3) a contract was awarded, calling for a system's study of a satellite ATC surveillance system (Section 5 and Refs 12 and 22).

The principal results derived by NAT/SPG for non-INS carriers are reviewed in Section 3.6, as well as Ref 7. The INS results are presented and discussed in Section 4.5 and Ref 11. Specific results are not yet available from the surveillance study. They will be generated in the first half of FY 72.

The effort reviewed in this report is the initial phase of TSC's involvement in oceanic navigation and surveillance analysis.

6.2 CURRENT OBSERVATIONS AND FUTURE WORK

Future studies, as detailed below, are required before making definite statements on future routing structures and system parameters. Certain observations and recommendations are obvious, however:

1. There are three aspects of the inertial, non-surveillance problem (Section 4) requiring futher investigation:

   a. A study to determine the source, magnitude and likelihood of blunder errors; estimate their significance, particularly in relation to system equipment reliability, checkout procedures and operational procedures; investigate error detection procedures using multiple INS.

   b. A study of externally aided inertial systems is called for; such systems appear to promise greater reliability, in terms of independent position checks, and greater accuracy, in terms of the en route updating of inertially derived positions.

c. A more detailed analysis of the en route navigation statistics will eventually be required.

2. Notwithstanding the need for the above modeling extentions, it is nonetheless reasonable to state that INS technology will allow for a substantial reduction of the present standards. Based on present studies which look at crosstrack initial errors, a separation standard of about 30 n.m. would appear reasonable, provided proper safety and/or redundancy procedures are followed, thus limiting the occurrence of blunders.

3. An in-house effort on satellite ATC surveillance modeling should be performed in parallel with the contract effort. This would serve as an independent check on System Control's modeling technique.

4. The original NAT/SPG model, as derived by Mr. P.G. Reich (Refs 2,3 and 4), and its extensions (Refs 5,6 and 8), require additional refinements which include:

   a. Updated parameter estimates in terms of (i) state-of-the-art improvements in existing systems; (ii) new data sources, such as the IATA study of inertial systems to be conducted; and (iii) revised estimation procedures, including confidence level and sensitivity analyses.

   b. Solutions which pertain to mixed airspaces (i.e., takes into account varying types of aircraft and navigation systems).

6.3 PROJECTED PLAN AGREEMENT FOR FY 72

A continued study of the effect of navigation and ATC performance on collision risk for the routing structure in oceanic regions will be undertaken in FY 72. Various levels of ATC will be investigated, and the effects of each carefully analyzed. Navigation work will be extended to include the investigation of (1) hybrid inertial systems; (2) more sophisticated methods of INS terminal and en route data analysis; and (3) the effects of blunders.

Specifically, the following tasks are contemplated (Ref 24):

1. Extend the FY 71 study of INS en route statistics to include an analysis of the time-varying nature of the input errors.

2. Study the magnitude and likelihood of blunder errors, their effect on lane spacing, and their implications on the need for improved system equipment reliability, checkout and operational procedures.

3. Analyze the future impact of aided inertial systems on minimum allowable separation standards in the oceanic regions. These aided inertial systems would include both INS systems combined with external radio aids (such as satellites, Loran or Omega) and INS systems combined with a non-external aid (such as Doppler).

4. Continue and extend the FY 71 effort in modeling and analyzing ATC surveillance systems; investigate ATC strategies and techniques; relate their impact on separation standards and safety levels in the Pacific and the NAT regions, in terms of parametric studies of future system requirements; study the relationships between safety, lane widths, threshold distances, fix times, alarm rates, on-board navigation accuracies and surveillance accuracies; perform a detailed tradeoff analysis between the various system parameters of interest; make recommendations for future ATC surveillance studies.

5. Prepare a final report detailing the FY 72 effort discussed above; prepare interim reports when deemed appropriate (such as following the completion of each task).

APPENDIX A

LATERAL COLLISION RATE

The collision rate is defined as the rate at which one aircraft will enter the collision slab (Figure 3.2) of another aircraft, provided it has entered, or is close to, its proximity shell (Figure 3.3). The rate, then, is a function of the intended paths of the respective aircraft.

By considering relative motion, the collision process may be looked upon as a particle bombarding a slab. The collision rate is given by the expected number of times the relative position shrinks to within collision slab dimensions. There are three ways in which one aircraft can enter the protective volume of another: (1) through the sides, (2) through the ends, and (3) through the top or bottom. The total rate can be computed by summing the effects of these three contributions.

The frequency with which each of these events occur is equal to the probability that the aircraft overlap in two dimensions simultaneously, multiplied by the frequency with which they overlap in the third dimension. These considerations lead up to a collision rate, CR, that can be shown to equal (Refs 2 and 10),

$$CR_{[AB]} = (N_x P_y P_z)_{[AB]} + (N_z P_x P_y)_{[AB]} + (N_y P_z P_x)_{[AB]}, \tag{A-1}$$

where

N_x is the expected frequency with which the along-track separation shrinks to less than λ_x, the collision slab dimension along-track;

N_y and N_z are similarly defined for, respectively, the across-track and vertical directions;

P_x is the probability that the along-track separation is less than λ_x (i.e., the proportion of time the aircraft spends in this condition);

P_y and P_z are similarly defined for, respectively, the across-track and vertical direction;

[AB] denotes that the above quantities are evaluated during periods when the planned separation may be assumed to be a constant vector [AB]. In general, this planned separation is time-varying and each [AB] vector will result in a different collision rate.

The relative frequency of overlap in the r^{th} direction, N_r, can be expressed as a function of the probability of overlap in the r^{th} direction, P_r. This relationship is derived in Refs 3 and 10 and is repeated below:

$$N_{r_{[AB]}}(S_r) = \overline{|\dot{r}(S_r)|} P_{r_{[AB]}}(S_r)/2\lambda_r \qquad \text{(A-2)}$$

where

$\dot{r}(S_r)$ is the relative velocity in the r^{th} dimension's upon overlap between two aircraft which are nominally separated by S_r in the r^{th} dimension; and r represents one of the dimension under consideration (x,y or z).

Thus far, the derivation has been quite general. It provides the framework for calculating the collision rate between two aircraft, regardless of the orientation of their intended tracks. Specializing the problem, consider the parallel tracking system presently in operation over NAT. Furthermore, concentrate on the collision rate between two laterally separated aircraft flying at the same nominal altitude (i.e., lateral neighbors in Figure 3.1 (left)).

The parallel-track system has a special feature, namely that the intended separation between proximate aircraft in the lateral (y) and the vertical (z) direction remains constant throughout their respective flights. Therefore, the P_y, P_z, N_y, and N_z dependence on the vector [AB]can be suppressed.

In the case under consideration, a collision will occur as a result of a loss in the vertical separation, between two aircraft occupying the same flight level. Combining equations A-1 and A-2, and explicitly specifying the intended separations, the lateral collision rate, $(CR_y)_{[AB]}$, can be written as,

$$\begin{aligned}(CR_y)_{[AB]} &= (N_x)_{[AB]} \left[P_y(S_y)P_z(0)\right] \\ &+ (P_x)_{[AB]} \left[P_y(S_y)N_z(0) + N_y(S_y)P_z(0)\right]. \end{aligned} \qquad \text{(A-3)}$$

In order to obtain the average CR, it would appear necessary to evaluate equation A-3 for all possible separation vectors, [AB], and form a weighted sum average. This approach is not practically feasible. Instead, we analyze the aggregate behavior of the traffic in a statistical manner.

The longitudinal separation between aircraft on adjacent tracks is assumed to be independent. Given proximity, the probability of overlap in the along-track (or x) direction is simply the ratio of the x dimension of the collision slab, $2\lambda_x$, to the x dimension of the proximity shell, $2S_x$.

Therefore,

$$(P_x)_{[AB]} = \frac{\lambda_x}{S_x} \quad . \tag{A-4}$$

The relative frequency, given in equation A-2, is therefore,

$$(N_x)_{[AB]} = \frac{|\overline{\dot{x}}|}{2S_x} \quad , \tag{A-5}$$

where $\dot{x}$ is the average relative along-track velocity between two aircraft on adjacent paths.

Substituting equations A-4 and A-5 into equation A-3, the CR due to loss in lateral separation can be expressed as,

$$CR_y = \frac{1}{S_x}\left\{\frac{|\overline{\dot{x}}|}{2} P_y(S_y)P_z(0) + \lambda_x P_y(S_y)N_z(0) + \lambda_x N_y(S_y)P_z(0)\right\}. \tag{A-6}$$

Further, by introducing equation A-2 into the above relationship, the rate function can be expressed directly in terms of the relative velocities and hazard distances in the lateral dimension.

One additional comment on the lateral collision rate is in order: it concerns directivity. The value of $\dot{x}$ (defined above) will depend on whether the two aircraft are travelling in the same or opposite directions. For same-direction traffic,

$$|\overline{\dot{x}}| = \overline{\Delta V}, \tag{A-7}$$

where $\overline{\Delta V}$ is the average difference in their velocities. For opposite-direction traffic,

$$|\overline{\dot{x}}| = 2\overline{V}, \tag{A-8}$$

where $\overline{V}$ represents the velocity of each aircraft which, for the moment, is assumed to be equal. Substitution of these values for $|\dot{x}|$ into equation A-6 results in different rate functions for same and opposite-direction flow.

APPENDIX B

PROXIMITY TIMES

Lateral proximity occurs when two aircraft which are assigned to adjacent lateral lanes at the same flight level fall within longitudinal proximity of each other. This condition is shown in Figures B.1(a) and B.1(b).

(The analysis in this appendix will be restricted to the lateral case. However, an example of vertically proximate pairs is included in Figure B.1(c) and (d) for purposes of comparison.)

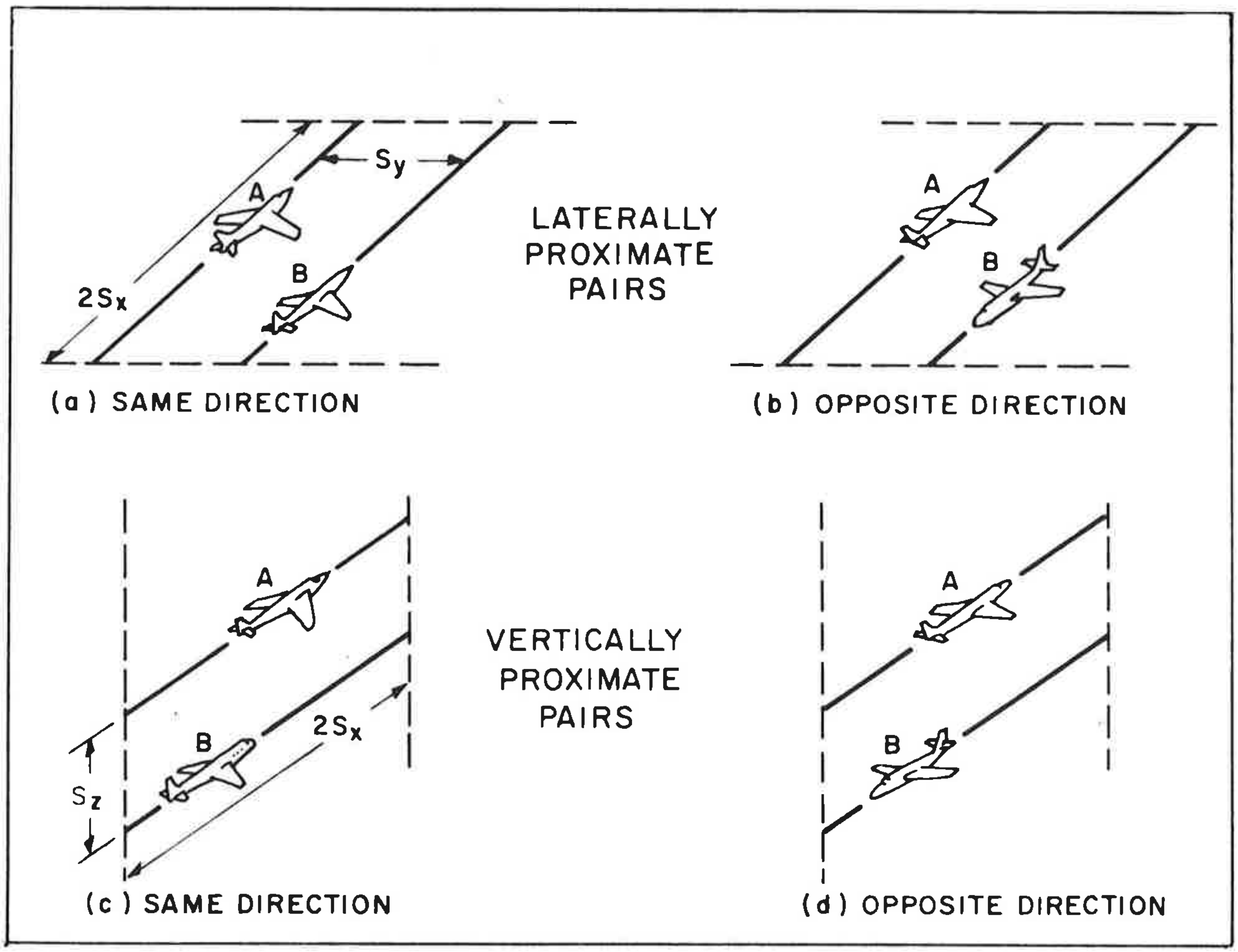

Figure B.1. Proximity Pairs.

A cross-section of a typical tracking section is presented in Figure B.2.

The $(i,j)^{th}$ track has associated with it a flow rate of m_{ij} aircraft per hour. Therefore, the average density of aircraft (aircraft per n.m.) on the $(i-1, j)^{th}$ track is

$$\frac{m_{i-1,\, j}}{\overline{V}} ,$$

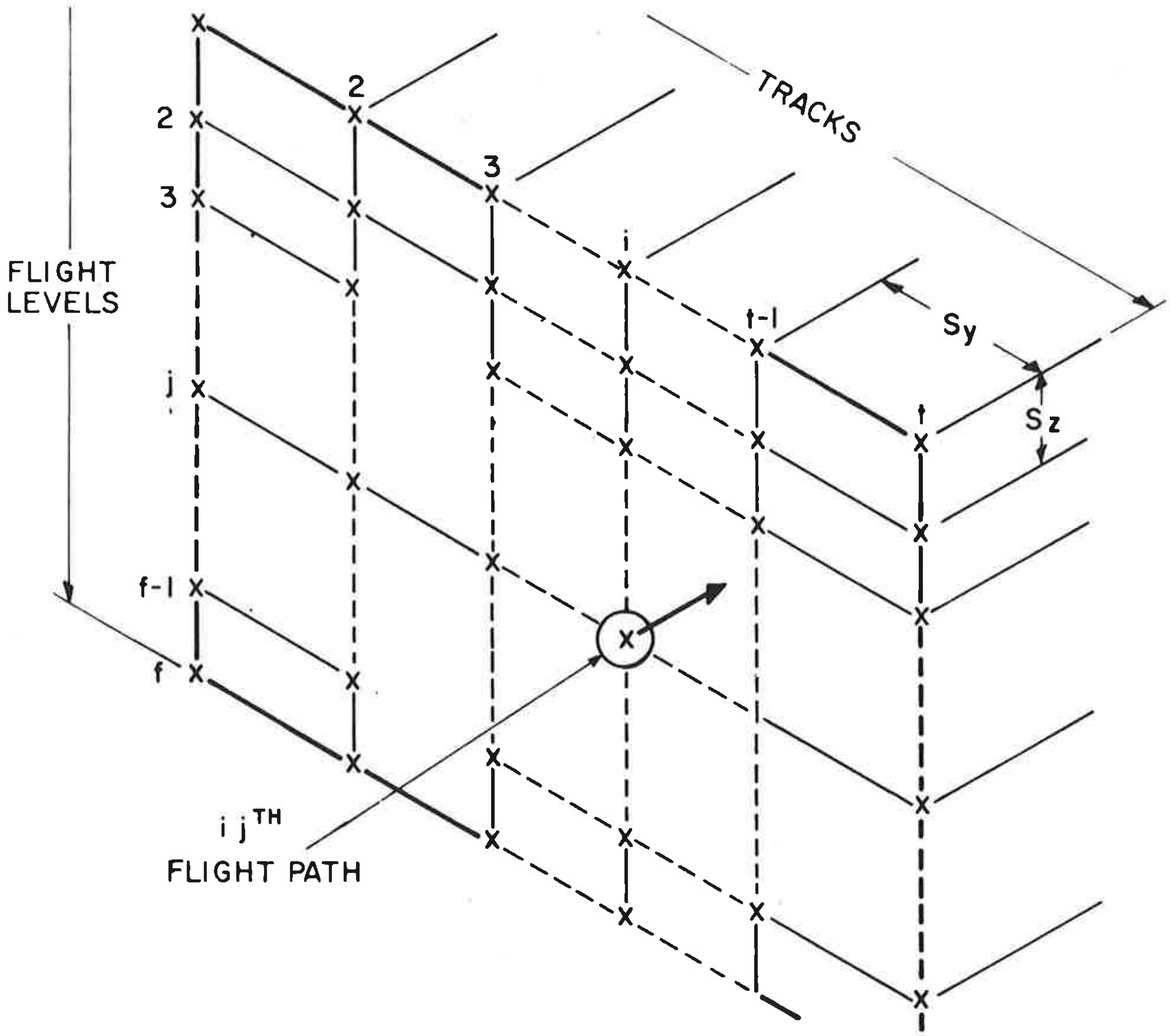

Figure B.2. Tracking System (3-dimensional view).

where $\overline{V}$ is the average velocity of the aircraft. At any instant of time, the expected number of aircraft on track (i-1, j) that are proximate to, or within S_x of, a given aircraft on track (i,j) is given by,

$$\frac{2S_x m_{i-1,\,j}}{\overline{V}} .$$

The time required to travel an L n.m. tracking system is $L/\overline{V}$. Therefore, the expected length of time during which an $(i,j)^{th}$ track aircraft is proximate to aircraft in the $(i-1,\,j)^{th}$ track per trip is simply,

$$\frac{2S_x m_{i-1,\,j}}{\overline{V}} \cdot \frac{L}{\overline{V}} .$$

In order to establish the total number of proximate pairs existing between lateral lanes i and i-1 on flight level j, multiply the above expression by the flow rate for track (i,j). Therefore,

$$\left\{\begin{array}{l}\text{Mean time of exposure of}\\ \text{proximate pairs on tracks}\\ \text{(i-1,j) and (i,j)/unit of}\\ \text{time of operation}\end{array}\right\} = \frac{2LS_x}{\overline{V}^2}\; m_{i-1,j}m_{i,j} \quad . \qquad \text{(B-1)}$$

Summing the result of equation B-1 over all pairs of adjacent lateral lanes yields,

$$\dot{T}_y = \frac{2L\;S_x}{\overline{V}^2}\sum_{i=2}^{t}\;\sum_{j=1}^{f} m_{i-1,j}m_{i,j} . \qquad \text{(B-2)}$$

The lateral proximity time, T_y, is obtained by multiplying $\dot{T}_y$ by the total number of flying hours, H, under consideration:

$$T_y = \frac{2HLS_x}{\overline{V}^2}\sum_{i=2}^{t}\;\sum_{j=1}^{f} m_{i-1,j}m_{i,j} . \qquad \text{(B-3)}$$

Similar expressions can be obtained for: (1) the vertical proximity time, T_z, by summing over vertically proximate flight levels; and (2) the composite proximity time, T_{yz}, by summing over all neighboring, diagonally separated, paths.

The example leading to equation B-3 assumed all flights were unidirectional, as indicated in Figure B.2. This assumption is now modified.

Just as directivity was an issue in calculating CR, it is, likewise, an issue with the calculation of the proximity time. Each proximity is classified as either a same-direction or an opposite-direction proximity, depending on the respective flows on the adjacent tracks being considered. (The flow in any given track is, of course, unidirectional.) The proximity calculations for each case are segregated and summed separately.

In general, then, equation B-3 can be specialized as indicated below:

$$T_y(\text{same}) = \frac{2HLS_x}{\bar{V}^2} \sum_{\substack{\text{Same level} \\ \text{Adjacent lanes} \\ \text{Same-direction traffic}}} \sum m_{i-1,j} m_{i,j} , \qquad \text{(B-4)}$$

$$T_y(\text{opp}) = \frac{2HLS_x}{\bar{V}^2} \sum_{\substack{\text{Same level} \\ \text{Adjacent lanes} \\ \text{Opposite-direction traffic}}} \sum m_{i-1,j} m_{i,j} . \qquad \text{(B-5)}$$

Similar statements can be made for the vertical and composite proximities.

There are two conclusions that can be drawn from equations B-4 and B-5: (1) the proximity time is approximately proportional to the square of the traffic intensity; and (2) the collision rate is inversely proportional to S_x. Since CR is directly proportional to S_x, the consequence of conclusion (2) is that the total lateral collision risk is independent of S_x.

APPENDIX C
INERTIAL SYSTEM MODEL

The Litton LTN-51 INS is a free-azimuth, two-dimensional navigator. This system utilizes a local level platform with a space-stabilized azimuth channel, i.e., the azimuth or vertical gyro is untorqued. The error equations for the free azimuth system are obtained by specializing the generalized theory in Ref 18. In particular, the error equation for this type of system is given by,

$$\underline{\Lambda}\ \underline{x} = \underline{Q}, \tag{C-1}$$

where $\underline{x}$ is the system's error state vector, composed of the system's attitude and position errors. The attitude error is defined to be the orthogonal transformation error between platform and geographical coordinates. The error state vector is written as,

$$\underline{x} = \left\{\varepsilon_N, \varepsilon_E, \varepsilon_D,\ \delta L,\ \delta \ell\right\} \tag{C-2}$$

where

ε_N = North component of attitude error;

ε_E = East component of attitude error;

ε_D = Vertical component of attitude error;

δL = Latitude error;

$\delta \ell$ = Longitude error.

The lefthand side of the above error differential equation is written as,

$$\underline{\Lambda} = \left[\begin{array}{ccc|cc} & & & \dot{\lambda}\sin L & -\cos L\ p \\ & p\,\underline{I} + \underline{\Omega}_{in}^{n} & & p & 0 \\ & & & \dot{\lambda}\cos L & \sin L\ p \\ \hline 0 & f_D & -f_E & \delta f_N/\partial L & \delta f_N/\partial \ell \\ -f_D & 0 & f_N & \delta f_E/\partial L & \delta f_E/\partial \ell \end{array}\right], \tag{C-3}$$

where

p = Differential operator, d/dt;

$\underline{I}$ = Identity matrix;

f_N, f_E, f_D = North, East and vertical components of the specific force vector, respectively;

$\underline{\Omega}_{in}^{n}$ = Skew-symmetric form of the angular velocity of the geographic framing relative to the inertial frame, resolved in geographic axes, and having the components

$$\left\{ \dot{\lambda}\cos L, \ -\dot{L}, \ -\dot{\lambda}\sin L \right\},$$

where

ℓ = Terrestrial longitude;

$\dot{\lambda}$ = Celestial longitude rate: $\left(\dot{\lambda} = \dot{\ell} + \omega_{ie}\right)$;

L = Geographic latitude.

The forcing function, $\underline{Q}$, which reflects the effects of the inertial system's errors, is given by:

$$\underline{Q} = \begin{bmatrix} \underline{C}_p^n (u)\underline{\omega}^p + \underline{C}_p^n \left\{ \tau_x \dot{\lambda}\cos L, \ -\tau_y \dot{L}, 0 \right\} \\ \\ \underline{C}_p^n (u)\underline{f}^a \end{bmatrix}, \qquad \text{(C-4)}$$

where

$\underline{C}_p^n$ = coordinate transformation between platform and geographic axes or

$$\underline{C}_p^n = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

with

$$\phi = \int_0^t \dot{\lambda}\sin L \, dt;$$

$(u)\underline{\omega}^p$ = gyro drift uncertainty vector;

$(u)\underline{f}^a$ = accelerometer uncertainty vector;

τ_x, τ_y = torquer scale factor uncertainty associated with the platform's x and y gyros, respectively.

As seen from equation C-3, the equations of motion are time varying, except for the case of constant east-west velocity at constant latitude where $\dot{\lambda}$ = constant and $\dot{L} = 0$. Also note that this system is insensitive to azimuth gyro torquing uncertainties, since, of course, the azimuth gyro is untorqued.

APPENDIX D

ADAPTATION OF THE NAT/SPG COLLISION RISK MODEL

The number of accidents in 10 million flying hours, between aircraft occupying identical flight levels and adjacent lateral tracks, was previously given by N_{ay} in equation 3-4. The introduction of inertial systems as the primary source of navigation necessitates two immediate modifications. The error statistics are functions of (1) flight direction, and (2) time into flight. Therefore, the overlap probability, P_y, and the relative velocity upon overlap, $\dot{y}$, are also time and direction-dependent.

Directional dependence is indicated by the superscripts e and w in equation 4-4. The opposite-directed case requires no such specification as it considers one aircraft in each direction. The rate function for each case is then weighed by the appropriate exposure function, E_y.

The function within the parenthesis is now a function of time. (For notational convenience, the time dependence of both P_y and $\dot{y}$ has been suppressed.) This function is evaluated at a number of time intervals and averaged (signified by the *).

Proximate aircraft flying in the same direction are assumed to have equal elapsed navigation times and, therefore, identical flying statistics; on the other hand, proximate aircraft flying in opposite directions have distinct navigation time associated with each aircraft and, hence, different flying statistics.

D—1 PROBABILITY OF OVERLAP

The probability of lateral overlap, $P_y(S_y)$, can be obtained by convolving the across-track flying error densities of laterally proximate aircraft (Refs 3,5 and 10). Symbolically, this can be written as,

$$P_y(S_y) = 2\lambda_y \int_{-\infty}^{\infty} f_1\left(y - \frac{S_y}{2}\right) f_2\left(y + \frac{S_y}{2}\right) dy , \qquad \text{(D-1)}$$

where $f_i(y)$ is the flying density of aircraft i about the center of its track, and λ_y is the lateral dimension of the aircraft.

The distributions of terminal errors, graphically represented in Figures 4.1 and 4.2, decay at a slower rate than a Gaussian and more rapidly than an exponential. The en route error distributions are assumed identical, in form, to the terminal distributions. Therefore, we have chosen a Gaussian random variable and a first-Laplacian (double-sided exponential) random variable as the optimistic and pessimistic models, respectively, of the en route errors. Both of these random variables are completely specified in terms of their second-order statistics. They are assumed to have zero mean. Their standard deviations, which are time dependent, were obtained by combining an error simulation model with the terminal data as explained in the text. (An example of a standard deviation, σ, in time, t, curve is shown in Figure 4.3.)

Applying the Gaussian assumption to equation D-1 leads to,

$$P_y(S_y) = 2\lambda_y \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_1(t_1)} e^{-\left(y - \frac{S_y}{2}\right)^2 / 2\sigma_1^2(t_1)} \cdot \frac{1}{\sqrt{2\pi}\sigma_2(t_2)} e^{-\left(y + \frac{S_y}{2}\right)^2 / 2\sigma_2^2(t_2)} \, dy \,, \tag{D-2}$$

where $\sigma_i(t_i)$ is the standard deviation of the en route Gaussian navigation errors in aircraft i at time t_i. The final result is very much a function of the times t_1 and t_2, although this dependence has been notationally suppressed. Equation D-2 reduces to,

$$P_y(S_y) = \frac{\lambda_y e^{-\frac{1}{2}\left\{\frac{S_y^2}{\left[\sigma_1^2(t_1) + \sigma_2^2(t_2)\right]}\right\}}}{\sqrt{\frac{\pi}{2}\left[\sigma_1^2(t_1) + \sigma_2^2(t_2)\right]}} \,. \tag{D-3}$$

In the case of opposite-direction proximity, the two flight times can be related to one another. Based upon the reported flight times and a number of simplifying assumptions, it can be shown that, (Ref 11),

$$T_w = 510 - \left(\frac{51}{44}\right) T_e \,, \tag{D-4}$$

where T_e represents the time-into-flight of an easterly directed aircraft and T_w represents the time-into-flight of a westerly directed aircraft at the time of along-track overlap.

In the case of same-direction proximity, make the simplifying, first-order, assumption that $t_1 = t_2$; thus, since their intended direction is assumed identical, $\sigma_1(t_2) = \sigma_2(t_2) = \sigma(t)$. The overlap probability is then reduced to,

$$P_y(S_y) = \frac{\lambda_y e^{-\frac{1}{4}\frac{S_y^2}{\sigma^2(t)}}}{\sqrt{\pi}\,\sigma(t)} \,. \tag{D-5}$$

The exponential case can be treated in a completely analogous fashion:

$$P_y = 2\lambda_y \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}\sigma_1(t_1)} e^{-\frac{|y - \frac{1}{2}S_y|}{\sigma_1(t_1)}} \cdot \frac{1}{\sqrt{2}\sigma_2(t_2)} e^{-\frac{|y + \frac{1}{2}S_y|}{\sigma_2(t_2)}} dy \,,$$

we now obtain,

$$P_y(S_y) = \frac{\lambda_y}{\sqrt{2}} \left\{ \frac{1}{\sigma_1(t_1) + \sigma_2(t_2)} \left[e^{-\sqrt{2}\frac{S_y}{\sigma_1(t_1)}} + e^{-\sqrt{2}\frac{S_y}{\sigma_2(t_2)}} \right] + \frac{1}{\sigma_1(t_1) - \sigma_2(t_2)} \left[e^{-\sqrt{2}\frac{S_y}{\sigma_2(t_2)}} - e^{-\sqrt{2}\frac{S_y}{\sigma_1(t_1)}} \right] \right\} \,. \tag{D-7}$$

For the same direction, this reduces to,

$$P_y(S_y) = \frac{\lambda_y}{\sigma(t)} e^{-\sqrt{2}\frac{S_y}{\sigma(t)}} \left[\frac{1}{\sqrt{2}} + \frac{S_y}{\sigma(t)} \right] \,, \tag{D-8}$$

a result derived by Reich in Ref.3.

D–2 RELATIVE OVERLAP

The relative velocity upon overlap was obtained by scaling the results of Figure 4.4 by appropriate ratios of the aircraft's actual position error upon overlap to its expected error.

D–3 PARAMETER VALUES

Table D.1 shows the parameters used in the analysis.

TABLE D.1 - PARAMETER VALUES FOR INS ANALYSIS

Parameter	Assigned Value	Explanation/Source
E_y^e (same)	0.417	Figures 3 and 4 in Ref 14, using: (1) an average daily traffic forecast (for 1975), obtained from Ref 15, and (2) a longitudinal proximity of 120 nm.
E_y^w (same)	0.417	
E_y (opp)	0.014	
S_x	120 nm	Assumed longitudinal proximity distance.
$N_z(0)$	20 cycle/ hour	Ref 7
$P_z(0)$	0.25	Ref 7
$\overline{V}$	560 knots	Approximate average of carriers; increase over value used in Ref 7; slightly inconsistent with the 637 knots used for mathematical convenience in the INS analysis (results are essentially unaffected).
$\overline{\Delta V}$	15 knots	Slightly more conservative than value given Ref 7.
λ_y	0.033 nm	Approximate average of carriers; larger than value of Ref 7.
λ_y	0.033 nm	Approximate average of carriers (ignoring vortex); larger than value of Ref 7.

D-4 GENERATION OF RESULTS

The en route INS errors were simulated on the IBM 7094 Computer. The program was set up and directed by Dr. K.R. Britting and was coded and run with the aid of the Service Technology Corporation (STC).

The solutions for N_{ay} (equation 4-4) were programmed and run on the time-sharing facilities of the Tymshare Corporation by R.M. Hershkowitz.

APPENDIX E
MATHEMATICAL DESCRIPTION OF SCI LATERAL OVERLAP PROBABILITY MODEL

As outlined in Section 5.4, the SCI surveillance system model relates the different surveillance parameters (fix rate, lane separations and alarm thresholds) to a time-varying probability density function of position errors in each of the three dimensions: $F_Y (y,t)$ for cross-track, $F_X (x,t)$ for in-track, and $F_Z (z,t)$ for vertical. This appendix presents a detailed preliminary development of the SCI surveillance system modeling technique for the lateral dimensions.

E.1 GENERAL ASSUMPTIONS

The general assumptions include:

1. Aircraft enter the surveillance region with a distribution of lateral, vertical and in-track position errors.

2. Aircraft are allowed to proceed unhindered as long as their position errors are within the thresholds (distance or time) at the surveillance fix times.

3. Aircraft that appear, to the surveillance system, to have exceeded the threshold have their position estimates updated by the surveillance system and are issued a return heading and velocity. This procedure is repeated at each surveillance fix time.

The alarm threshold is denoted by r_o, the fix interval by t_o, and the surveillance position error (a random variable) by ε_h.

The actual lateral position is assumed to be a sum of the intended position plus the navigation error. For the lateral case, then,

$$y_{actual} = y_{intended} + e_y (t), \qquad \text{(E-1)}$$

where e_y is the latitude error of the on-board navigation system. (The on-board system will be assumed to be an INS, though this is by no means a constraining factor in the analysis.)

Although the SCI surveillance system model will include delay terms and finite radius of turn, those are not included in this appendix. Since they only affect aircraft that receive surveillance alarms and since the SCI approach singles out the trajectories of those aircraft receiving surveillance alarms, these effects are easily included.

E.2 SURVEILLANCE SYSTEM MODEL FOR THE LATERAL DIMENSION

As described in Section 5, the navigation model will consist of the best possible one-dimensional relationship between the error sources and the position errors. For the purpose of this calculation, a single navigation error source will be assumed, namely random drift rate, $m_y(t)$. The inclusion of a second error source introduces a known amount of difficulty and is discussed in detail under Computational Requirements of Ref 22. The drift is described by a time-dependent density function, $f_{m_y(t)}(\cdot)$, not necessarily Gaussian, which includes the effects of the physical error source, such as gyro drifts and human blunders. (An alternative method would be to include the human blunder effect as a separate navigation error.) The gyro drift term, for example, might arise from a model in which the error grows proportional to the square root of time (Ref 11). Therefore, the accumulated latitude error at time t, (Figure E.1), is given by,

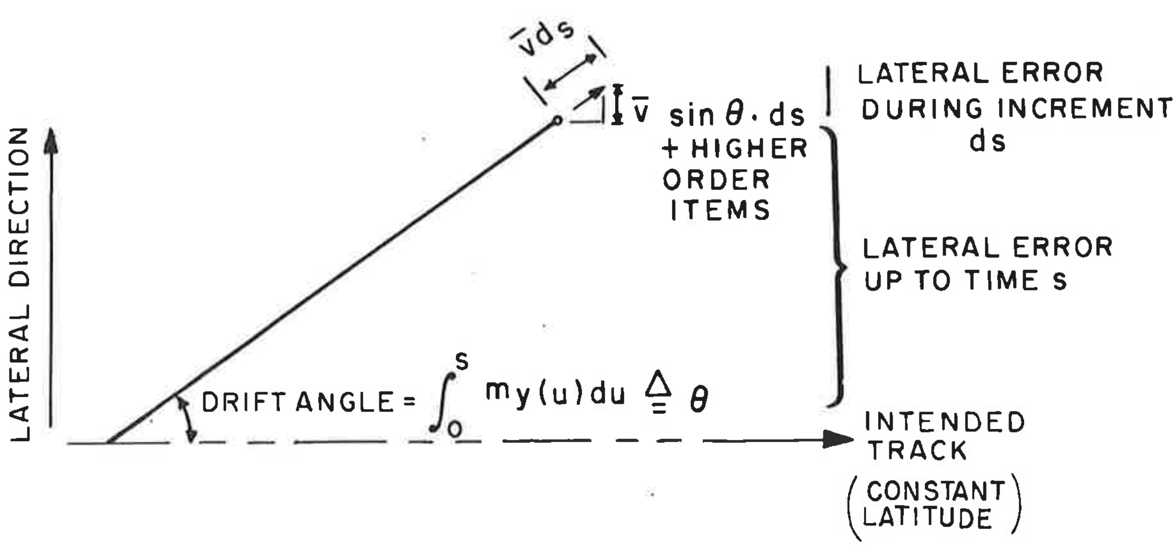

Figure E.1. Position error due to lateral drift rate.

$$e_y(t) = \int_o^t \bar{v} \sin\left(\int_o^s m_y (u)\, du\right) ds. \qquad \text{(E-2)}$$

If the drift rate is considered constant over the interval [o,t], this becomes,

$$e_y(t) = \int_o^t \bar{v} \sin (m_y(o)\ s) ds, \qquad \text{(E-3)}$$

and if $m_y(o)s$ is small (e.g., 1^o),

$$e_y(t) = \bar{v}\ m_y(o) \frac{t^2}{2} . \qquad \text{(E-4)}$$

More sophisticated models for m_y will also be investigated.

As stated in Section 5, two assumptions are made that simplify the description of the aircraft behavior in the lateral dimension. They are:

1. The drift rate will be allowed to change only at surveillance fix times. This change will account for all random perturbations throughout the previous interval.

2. The heading correction, issued by the surveillance system, is designed to return the aircraft to the desired track by the next surveillance fix.

Although this latter assumption has been explicitly assumed in several previous surveillance models, it can be removed from this model with a corresponding increase in complexity.* The former assumption inplies that the accumulated effect of the random perturbations, applied only at the fix times, will make the drift rates independent from fix interval to fix interval.

*See footnote on page 5-9.

The complete description of $F_Y(y,t)$ will proceed in separate time steps: (1) 0 to t, (2) t_o to $2t_o$, (3) $2t_o$ to $3t_o$, and so on.

Between time 0 and the first surveillance fix at time t_o, the lateral errors are due only to navigation and initial position errors. The surveillance system errors will not enter until t_o. The probability, therefore, that at any time between 0 and t_o an aircraft will have a lateral position error of y n.m. is,

$$F_Y(y,t)\Big|_{t\in[0,\,t_o)} = P_r\left\{e_y(t) = y\right\} = \int_{-\infty}^{\infty} \left\{ P_r \; \frac{\bar{v}m_Y\;(0)\;t^2}{2} = y - u \right\} F_Y(u,0)\;du \tag{E-5}$$

where $F_Y(u,0)$ is the density of position errors at time 0.

At time t_o, those aircraft that appear, to the surveillance system, to have violated the threshold will have their position estimates updated by the surveillance system's best position estimate. (The surveillance system's positioning error has already been defined as $\varepsilon_h(t)$.) In addition, a return heading and velocity, v_y, is prescribed. v_y is assumed to be positive and, in general, is a function of both t_o and ε_h. Immediately after the aircraft secures the position data, however, the navigation error appears again. There are then two factors influencing the position errors between time t_o and $2t_o$, the navigation error and the surveillance positioning error. The inclusion of time dependence for the surveillance positioning error is seen to be an easy task with this approach.

Five terms are required to completely specify the position error density function between times t_o and $2t_o$. They relate to the five possible ways an aircraft can pass through a lateral position, y, at some time $t\in\left[t_o,\,2t_o\right]$. The complete expression for $F_Y(y,t)$ is:

$$F_Y(y,t) = \int_{-\infty}^{\infty}(1 - P_A(r,t_o))\;F_Y(r,\,t_o)P_1\;dr + \int_{-\infty}^{\infty}F_Y(r,\,t_o)\;(P_2 + P_3)\;dr + \int_{-\infty}^{\infty}F_Y\;(r,t_o)\;(P_4 + P_5)dr;\; t\in\left[t_o,\,2t_o\right). \tag{E-6}$$

The first integral expression includes those aircraft which do not receive an alarm at t_o and drift to the lateral position y at time t. The second integral expression involves those aircraft which appear, to the surveillance system, to be beyond r_o at time t_o. This second integral is split into two parts, with probabilities P_2 and P_3 dependent upon whether, at time t, the aircraft's estimated return to the intended track is complete. The third integral expression involves those aircraft which appear, to the surveillance system, to be beyond $-r_o$ at time t_o. This expression is also divided into two parts, involving P_4 and P_5, with the same criterion as for P_2 and P_3.

A more complete description of the individual probabilities is presented below.

E.3 $P_A(r)$

$P_A(r)$ represents the probability that an aircraft, with a lateral position, r, receives a surveillance alarm (i.e., it appears to the surveillance system to be outside the thresholds, $\pm r_o$). Therefore,

$$P_A(r, t_o) = P_r\left[|r + \varepsilon_h(t_o)| > r_o\right] = \int_{-\infty}^{-(r+r_o)} f_{\varepsilon_h(t_o)}(s)\,ds + \int_{-(r-r_o)}^{\infty} f_{\varepsilon_h(t_o)}(s)\,ds \qquad \text{(E-7)}$$

E.4 P_1

P_1 is the probability that an aircraft, which is initially at position r at t_o and does not receive an alarm, passes through the lateral position, y, at time t. This occurs when the actual position at $t_o(r)$ plus the lateral drift, occurring between times t_o and t, is equal to y. Therefore,

$$P_1 = P_r \left\{ r + e_y\,(t-t_o) = y \right\}. \qquad \text{(E-8)}$$

or, combining equations E-4 and E-8,

$$P_1 = P_r \left\{ m_y(t_o) = \frac{2(y-r)}{\bar{v}\,(t-t_o)^2} \right\} \quad \text{(E-9)}$$

E.5 P_2

P_2 is the probability that an aircraft, appearing to be beyond r_o at time t_o, passes through y at time t while returning to its intended track. (Note that it is only the aircraft's "estimated" position which will actually return to the intended track (Figure E.2).

This case requires a surveillance error, ε_h, large enough so that the estimated position at time t, calculated by adding the original estimate at t_o, $r + \varepsilon_h$, to the expected returned distance, $-v_y\ (t-t_o)$*, has not yet returned to the intended track. Therefore,

$$P_2 = P_r \left\{ \text{aircraft at } y \mid \text{alarm, } \varepsilon_h > \varepsilon_h^+ \right\} Pr \left\{ \text{alarm, } \varepsilon_h > \varepsilon_h^+ \right\}, \quad \text{(E-10)}$$

where ε_h^+ is the critical surveillance error, such that the aircraft's intended position at time t is y = o. In general,

$$\varepsilon_h^+ = \min \left\{ \varepsilon_h \mid r + \varepsilon_h - v_y(t-t_o) = 0 \right\}$$
$$\triangleq \min \left\{ \varepsilon_h \mid \frac{r + \varepsilon_h}{v_y} = t - t_o \right\} \quad \text{(E-11)}$$

For the simple case in which v_y is a constant, independent of ε_h and r,

$$\varepsilon_h^+ = v_y\ (t - t_o) - r \quad \text{(E-12)}$$

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*v_y will, in general, be a function of $\varepsilon_h(t_o)$ and r, the notation will be suppressed in the relationships to follow.

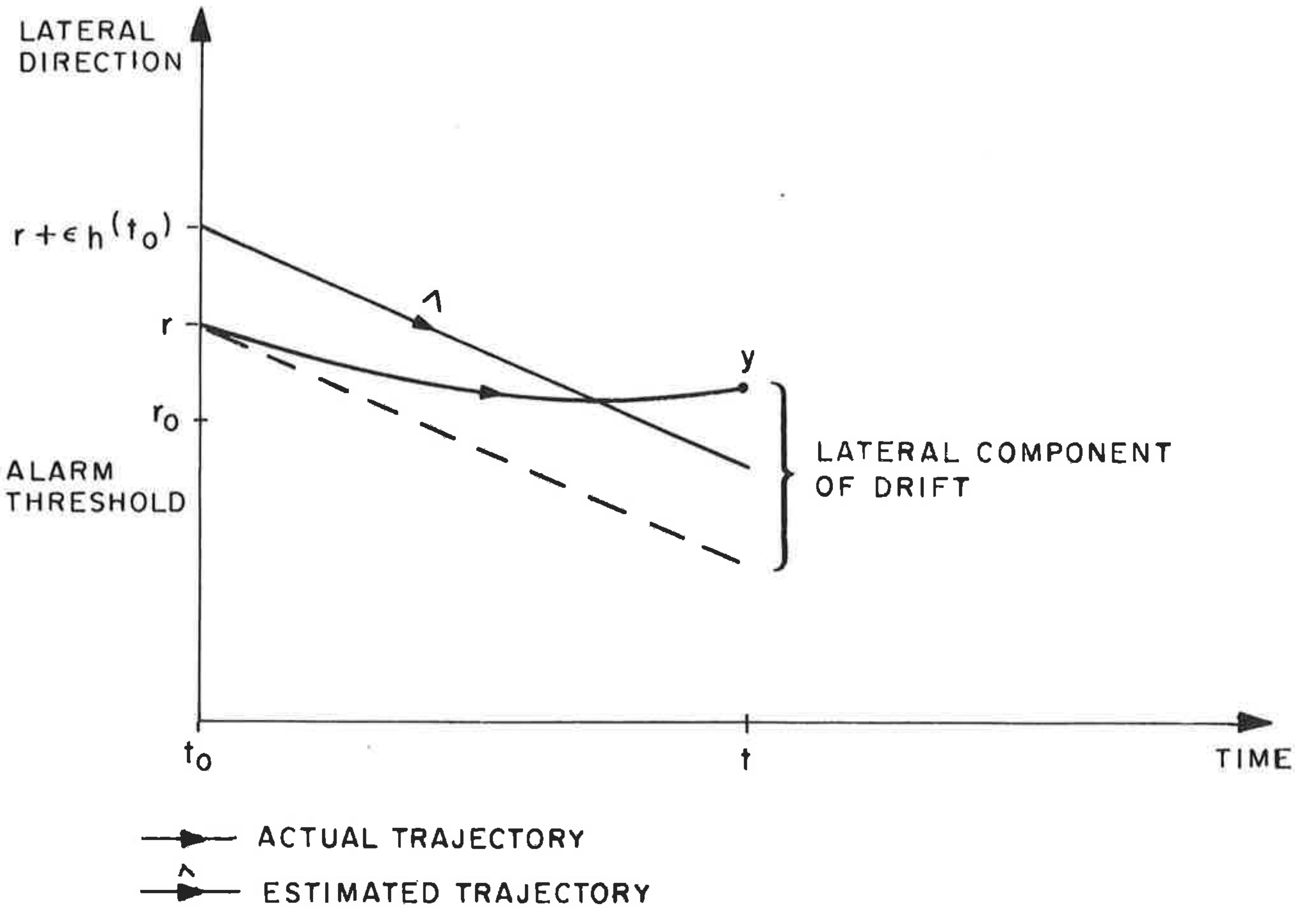

Figure E.2. Lateral drift vs. time (P_2).

Calculating the component expressions in equation E-10,

$$P_r \left\{ \text{alarm}, \epsilon_h > \epsilon_h^+ \right\} = P_r \left\{ r + \epsilon_h \geq r_o, \epsilon_h > \epsilon_h^+ \right\}$$

$$= \int_{\max(r_o - r, \epsilon_h^+)}^{\infty} f_{\varepsilon(t_o)} (u)du, \qquad \text{(E-13)}$$

where $f_{\varepsilon_{h(t_o)}}$ is the surveillance error density function. Since it is given that the aircraft has not returned to its intended position,

$$P_r \left\{ \text{aircraft at } y \mid \text{alarm}, \varepsilon_h > \varepsilon_h^+ \right\}$$

$$= P_r \left\{ r - v_y(t-t_o) + e_y(t-t_o) = y \right\}$$

$$= P_r \left\{ e_y(t-t_o) = y - (r - v_y(t-t_o)) \right\} \qquad \text{(E-14)}$$

Finally, considering equations E-4, E-13, and E-14 equation, E-10 for P_2 can be written as,

$$P_2 = \int_{\max\{\epsilon_h^+, r_o - r\}}^{\infty} P_r \left\{ \frac{\bar{v}}{2}(t-t_o)^2 m_y(t_o) = y - (r - v_y(t-t_o)) \right\} f_{\varepsilon_{h(t_o)}} (u)du$$

(E-15)

E.6 P_3

P_3 represents the probability that an aircraft, having appeared to be beyond r_o at time t_o and returned, according to its position estimate, to its intended track, passes through y at time t (Figure E.3). This case is similar to that of P_2, except that, now, $\varepsilon_h < \varepsilon_h^+$. Since the time at which the estimated position reaches the center line and then proceeds to change direction as indicated in Figure E.3, is a function of ε_h, the solution cannot be split up as conveniently as was the case with P_2 (equation E-10).

At some time, t_1, prior to the estimated return, the lateral position of the aircraft at time t_1 is given by,

$$r - v_y (t_1-t_o) + e_y (t_1-t_o). \qquad \text{(E-16)}$$

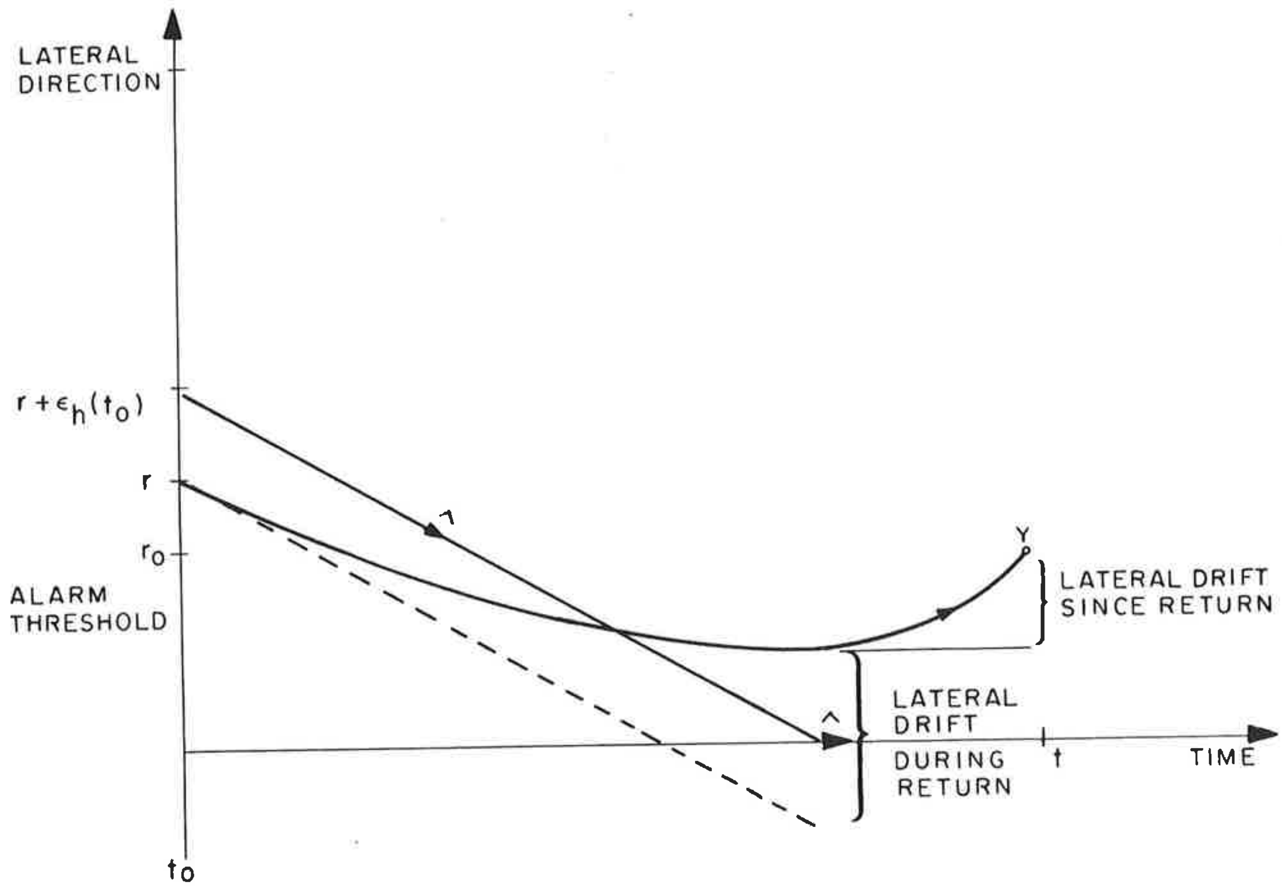

Figure E.3. Lateral drift vs. time (P_3).

To find the lateral position error density function at any time between $2t_o$ and $3t_o$, the same expression can be used but with $F_Y(y,t_o)$, $f_{m_{y}(t_o)}(\cdot)$ and $f_{\varepsilon_{h}(t_o)}(\cdot)$ replaced by $F_Y(y,2t_o)$, $f_{m_{y}(2t_o)}(\cdot)$ and $f_{\varepsilon_{h}(2t_o)}(\cdot)$. This identical procedure can be carried out indefinitely.

E.9 CALCULATION OF PROBABILITY OF OVERLAP

Since the lateral errors of aircraft on adjacent tracks are assumed to be independent, the probability of lateral overlap between two such aircraft with intended separation, S_y, is simply the probability that their relative lateral separation is less than λ_y, i.e.,

$$P_y(t) = 2\lambda_y \int_{-\infty}^{\infty} F(y + \frac{S_y}{2}, t)\, F(y - \frac{S_y}{2}, t-d)\, dy, \qquad \text{(E-28)}$$

where $\lambda_y << S_y$ and d is a time-delay factor indicating a difference in time origins. (See Ref 3,5, or 10 for a derivation of equation E-28 in the stationary case.)

The term d in equation D-28 is introduced to account for two facts: (1) that overlapping aircraft traveling in the same direction need not have the same velocities and, therefore, the same starting time, and (2) that overlapping aircraft traveling in opposite directions definitely do not have the same initial entrance times into the surveillance region. In each case, d is meant to be a random delay factor which may take on any value, with certain probability, which allows two aircraft to be adjacent at time t. In all practical situations, the range of d will be limited by the possible aircraft speeds. In addition, a complete formulation would require two P_y's, one to account for "same direction" aircraft and the other for "opposite direction" aircraft.

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