Statistical Policy Working Paper 1

Report on Statistics for Allocation of Funds

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Office of
Federal Statistical
Policy and Standards

Joseph W. Duncan
Director

Statistical Policy Working Paper 1

Report on Statistics for Allocation of Funds

Prepared by
Subcommittee on Statistics for Allocation of Funds
Federal Committee on Statistical Methodology

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Preface

This working paper was prepared by the members of the Subcommittee on Statistics for Allocation of Funds, Federal Committee on Statistical Methodology. The Subcommittee was chaired by Wray Smith, Office of the Assistant Secretary for Planning and Evaluation, Department of Health, Education, and Welfare. The members of the Subcommittee are the authors of this report and their names are listed below. It is hoped that this report will aid administrators and drafters of future legislation in recognizing some characteristics of data and formulas used in distributing Federal funds to State and local governments. The Subcommittee plans to discuss these results with many interested parties to further disseminate the findings of this report.

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Chapter I, II, III, and IV were written by Eli Marks, Charles Troob, and Wray Smith on the basis of Subcommittee outlines and discussions, as well as on the five case studies of formula programs. Chapter V (Subcommittee recommendations) is a joint product of the Subcommittee. The appendixes were prepared by individual members of the Subcommittee and their names are given both on their papers and in the table of contents. Research assistance was provided to the Subcommittee in 1977 by Henrietta Hyatt.

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Executive Summary

The Subcommittee on Statistics for Allocation of Funds prepared five case studies selected from the ten largest grant-in-aid programs that use data on population and per capita income. These five programs were then analyzed in terms of the variables "Need", "Capability", and "Effort". These factors were selected by the Subcommittee as the key elements to be considered in analyzing both the formulas and data employed by grant-in-aid programs for allocation of funds. The report discusses the types of formulas used for allocation purposes, the required statistical data, and the impact of errors in the data on the actual allocation of funds. Based on the review of the case studies, the recommendations are as follows:

(1) That program goals be specified as clearly and completely as possible in the statement of purpose of each grant-in-aid act and that program drafters guard against over-specification of the statistical data and procedures to be used.

(2) That provisions be made for an active, continuous interface between legislative program drafters and the statistical community.

(3) That statistical and program agencies provide to program drafters an analysis of the sensitivity over time of formulas and of the statistics they incorporate so that possible effects on allocations can be anticipated. Also, that provisions be made for testing, monitoring, and assessing by program agencies of the performance of each specific formula or allocation rule prior to enactment.

(4) That legislative drafters and program designers be advised of data problems and the existence of statistical practices, as exemplified in the five case studies, which may lead to formulas with consequences that are generally recognized as undesirable.

(5) That a limited program of applied research and development be initiated to attack some critical problems and fill certain identifiable gaps in the present state-of-the-art of formula design.

(6) That the Office of Federal Statistical Policy and Standards, with the assistance of the statistical agencies, designate a limited number of additional official statistical series for use in fund allocation. These would be kept as current and as accurate as possible for States and for local areas.

(7) That in tiered allocation programs comparable data be used for allocation to States, but policy flexibility be allowed for sub-State allocations. When the Federal Government allows this flexibility it should be subject to the formulation of specific Federal statistical and administrative guidelines, concerning the designation of the responsible governmental unit for choosing among statistical series, for declaring the specific types of statistical series from which such a choice is permitted to be made, and for establishing administrative mechanisms for consideration of appeals from area governments.

(8) That since data errors are inevitable and since statistical resources are necessarily limited, priority be given to minimizing the very large errors which may occur in data used for the allocation of funds.

(9) That, to minimize the effects of data errors, eligibility cutoffs be such that there is a gradual transition from receiving no allocation to receiving the full formula amount.

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APPENDIXES

Overview and Description of Allocation Techniques

Introduction

This report examines the formulas used in allocating Federal funds to States and local areas. To understand the behavior of these formulas, one must understand the various aspects of the data, such as definitions, methods of collection, and methods of analysis. The objective of the Subcommittee was to study from the statistical standpoint, possible principles or guidelines which could be used to insure that the intent of Congress is fulfilled in the allocation of Federal funds. For the purposes of this study it is assumed that whatever Congress specifies in the authorizing legislation for a grant-in-aid program on the manner of allocation of Federal funds is in principle an equitable distribution, although anomalous and unanticipated results may emerge in some instances. In connection with the guidelines, the Subcommittee was also to identify possible improvements in statistical data and allocation processes that might be made either by better selection of the data, changes in data collection or tabulation methods, or statistical adjustments to compensate for known errors. The report is organized as follows: Chapter I gives an overview and description of allocation techniques; Chapter II examines the consequences of using existing data in allocation formula techniques; Chapter III presents the findings; Chapter IV discusses ways to reduce allocation errors; and Chapter V presents the recommendations of the Subcommittee based on its study of the deficiencies of existing data and allocation formulas and of possible alternatives.

We will now elaborate on some specific topics in these chapters. Chapter I and Chapter II are based on the five case studies presented as Appendixes A–1 to A–5 of this report. These five cases were selected from the ten largest grant-in-aid programs that use data on population and per capita income. In FY 1975, total formula grants for all programs amounted to nearly 36 billion dollars. Fiscal year 1975 grants for the five case study programs range from 1.6 to 6.2 billion dollars and account for 47 percent of the total of formula grants.[1]

[1] The amount of grant-in-aid funds has increased in the past two years, and a number of new programs have also been added.

Some of the findings of Chapter III are tentative and many of the recommendations of Chapter V are long-term goals which may never be achieved in the exact form presented. However, as an interim measure, some standard practices and guidelines are needed to aid policymakers and statisticians involved in constructing or revising allocation schemes for grant-in-aid programs. At the very least, such guidelines should warn practitioners away from some of the more dangerous practices with disagreeable consequences that may be found in some existing formula programs. For example, under some circumstances such guidelines might advocate the use of a particular population or economic statistic that was neither the most recent nor the most adequate from the standpoint of geographic detail but which had other statistical properties, such as stability from one time period to the next or uniform quality across

TABLE 1. TEN LARGEST FORMULA GRANTS USING POPULATION OR PER CAPITA INCOME DATA
(Fiscal Year 1975 Obligation)

Program Title and Agency		Amount Obligated in FY 75 (dollars in millions)	Percent of Grants
Medical Assistance (Medicaid)	HEW	6,944	19.5
General Revenue Sharing (GRS)	TREAS	6,205	17.5
Aid to Families with Dependent Children (AFDC)	HEW	4,549	12.8
Comprehensive Employment and Training Programs (CETA)	DOL	2,457	6.9
Public Assistance	HEW	1,952	5.5
Elementary and Secondary Education Act (ESEA)	HEW	1,874	5.3
Highway Construction Funds	DOT	1,573	4.4
Community Development Block Grants (CDBG)	HUD	1,563	4.4
National School Lunch Program	USDA	1,461	4.1
Lower Income Housing Assistance Program	HUD	792	2.2
Total Ten Largest Programs		29,370	82.6
Total Formula Grants		35,568	100.0

Source: "Use of Data on Population in Federal Grants-in-Aid to State and Local Government in Fiscal 1975" prepared by Charles A. Ellett of the Statistical Policy Division of OMB.

geographic areas. The Subcommittee believes that the development of some state-of-the-art guidelines will lead to a general simplification and increase in the transparency of allocation schemes to be adopted in the future.

The case studies show that many of the allocation formulas also contain constraints and special rules. For example, for administrative reasons it is necessary to impose some type of limitation on how often the allocations can be recomputed. Also, since the States and local areas must be able to prepare their own budgets and decide upon tax levies, capital investments, hiring, etc., some constraints may be imposed to prevent extreme year-to-year fluctuations in the allocation to individual jurisdictions. Sometimes, the restraints may prevent even moderate fluctuations in individual allocations.

Many of the formulas contain implicitly or explicitly a restriction designed to insure that every State or local area gets some amount. Sometimes this is coupled with a restriction on the maximum amount to be allocated to any area. The limitation is usually not distinguishable from the limitations designed to damp or prevent fluctuations in individual allocations over time. To some extent, the restrictions may represent a well-justified distrust of the behavior of the allocation formula and of the appropriateness of the statistics used in it.

The Nature of Federal Grant-in-Aid Formulas [2]

All of the allocation formulas studied deal with activities which are recognized as functions of State or local government but over time a feeling has developed that Federal assistance is appropriate to insure more equitable handling of the problem among local jurisdictions. That is, while there is recognition that the given function must be carried out locally and adjusted to the realities of local conditions, it is also recognized that financial resources available for handling the problem vary considerably among State and local governments so that it is appropriate for Federal funds to be used to supplement local funds.

Informally it is possible to adopt a helpful statistical paradigm for allocation formula research, in which the allocation is taken to be a function of "Need, Capability, and Effort", each of which is assumed to be at least approximately observable at the State or local level. There are, however, serious definitional and interaction problems imbedded in this model—the fact that a Need may appropriately have different components in two geographic areas, that taxable real estate and personal income may not give an adequate basis for Capability, that local tax revenue Effort may need to be analyzed in terms of the purposes to which the revenues are applied, and so forth. Frequently, one or even all of the factors in the model are defined neither in the statute nor in the legislative history or, if all the factors are defined, the measures of Need, Capability, and Effort are inconsistent with the definitions or with each other. Thus, the terms are used to refer to statistical abstractions which apply only approximately (if at all) to the actual elements that make up a given allocation formula.

There are other elements of allocation formula problems, for example, the sensitivity of a particular formula to small, perhaps irrelevant, changes in the specified data over time. Some programs may require almost immediate reaction to the changes while, for other programs, insensitivity to short-term changes may be imperative. One wants the formula to respond fast enough to changing conditions but not too fast. Local government must be given some reasonable assurance of the general level of Federal funding they are to receive in future fiscal periods in order to keep local planning from becoming chaotic.

Another important question is the transparency of an allocation formula—can it be understood? Can citizens understand it? Politicians? Statisticians? Some formulas we have examined in existing Federal programs deserve to be called opaque—their behavior over time cannot be simply explained and may even exhibit some surprising and unanticipated results.

The general statistical approach used in this report conceives Federal grant-in-aid formulas as starting with some activity which the Congress perceives as properly a function of State or local government. In our statistical model we use the term Need to designate the activity required. For the purpose of the present report, Need is always to be understood in terms of the services (or goods) to be supplied—e.g., for food, shelter, etc., for AFDC (Aid to Families with Dependent Children); or police and fire protection, street and highway maintenance, etc., for General Revenue Sharing (GRS). While Need can be defined in money terms, this

[2] Portions of this section were adapted from an invited paper presented by W. Smith at the August 1976 meeting of the American Statistical Association held in Boston.

definition involves the total amount required, whether or not that amount is available at the State or local level (or even whether it is available at any level). Thus, the Need in Title I, ESEA (Elementary and Secondary Education Act) might be defined as the total amount required to attain a given educational level in a local area, regardless of whether the funds are available at the local, State or national level, or perhaps, not at all.

Capability is used for an area's prospective ability to meet a stated Need—i.e., the possibility of meeting the Need from local or State (or private) funds. For example, Capability might involve the amount that could be raised by some (standard) taxing program whether or not actual tax revenues reach this level. Finally, Effort is used for the actual amounts available for the Need from local revenues. Frequently, Effort is measured relative to Capability.

Measures of State and local (relative) Need, and Capability of meeting the Need, are components of almost all allocation formulas. The measure of Need is often stated (at least approximately) in terms of the population to be served. Many allocation formulas also recognize that there may be considerable variation in the proportion of the available local resources actually devoted to meeting the Need and include some measure of Effort.

An important (but usually implicit) aspect of all allocation formulas is the time reference. Some programs are dealing with *immediate* objectives—to provide adequate food and shelter *here and now*. Others are dealing with a more distant time reference—to equip all of the Nation's children with the education and skills necessary to their functioning effectively in the Nation's economy as it is in 1977 (let alone as it will be in 1990). There is also a time reference or ability to meet a given Need. The United States can, fortunately, meet our requirements for food and some sort of shelter immediately; but building sanitary, safe, and comfortable housing on the massive scale required in many communities takes at least 3 years and building even a partial rapid transit system for a major metropolitan center takes at least 6 years (from the time the system is designed and approved in principle).

Structurally, the formulas vary considerably. General Revenue Sharing (see Appendix A–1 of this report, "The General Revenue Sharing Program") uses the ratio of a measure of Effort (taxes as a proportion of aggregate personal income) to a measure of Capability (per capita money income) and multiplies this by (total) population. Essentially, this says that the share of a State or local area increases proportionally with the increasing population, increasing Effort, and decreasing Capability. In the General Revenue Sharing formula, per capita income serves as an indicator of Capability and total population as a measure of Need. This is equivalent to assuming that all jurisdictions have an equal Need per capita for the services covered by General Revenue Sharing. The General Revenue Sharing formula is complicated for sub-State distributions by lower and upper limitations on the per capita share of any locality (not less than 20 percent and not more than 145 percent of the average per capita share for the State). GRS allocations are also complicated at all levels by options relating to the specific measures to be used, but these do not affect the basic formula structure.

Like the General Revenue Sharing formula, the formulas of the other programs also involve measures of the basic factors (Need, Effort, Capability—with Capability entering inversely). However, the other formulas usually show some measure of Need explicitly. Often total Need in (dollars required) is used, so that population does not appear explicitly in the formula. Also, the formula may ignore Capability or use a single measure which reflects both Effort and Capability (with results which the Congress has found at times quite frustrating).

Thus, in the ESEA formula (see Appendix A–3 of this report, "The Authorization and Allocation of Funds Under Title I, ESEA") there is a measure of Need (the number of economically disadvantaged children [3]) multiplied by (a percentage of) the State average expenditure per pupil. Unlike General Revenue Sharing, this measure of Effort (per capita expenditure for the specified Need) does not relate it to Capability. However, an adjustment for low Capability is provided by substituting 80 percent of the national expenditure per pupil for the *State* expenditure per pupil, whenever the State expenditure per pupil is less than 80 percent of the national expenditure per pupil (presumably on the basis that low State expenditure per pupil characterizes the poorer areas, and was, therefore, a reflection of low Capability rather than of lower Effort or of lower unit costs for education of a given quality level). Corresponding to this floor on the allowance for per

[3] The method of estimating this number has varied, originally taking into account children in poverty families plus children in families receiving AFDC payments greater than the poverty cutoff, and subsequently altering the mix of these two factors and adding children in institutions for neglected and delinquent children and in publicly supported foster homes.

pupil expenditure, the (present version of) ESEA also provides for a ceiling of 120 percent of the national expenditure per pupil. There is, however, no allowance below the State level for variation among school districts in either Effort or Capability.

The AFDC formulas (regular and alternate) (see Appendix A–4 of this report, "Aid to Families with Dependent Children (AFDC) as a Formula Grant-In-Aid Program") resemble the ESEA formulas in starting with a measure of total Need times Effort—i.e., the total of money payments to families with dependent children plus payment for foster care. These payments are multiplied by the complement of a measure of Capability.[4] However, in the regular AFDC formula there are provisions for using a fixed multiplier for part of the Federal payment (5/6 of the first $18 per recipient) and a maximum ($32 per recipient) above which no Federal reimbursement is made. As with the floor on per pupil expenditures in the ESEA formula, the use of a fixed multiplier for the first $18 has the effect of increasing the payments to States with very low Capabilities (measured by the State per capita income). Payments to States with very high Capabilities tend also to be decreased by the maximum of $32 per recipient in the regular formula. Since States have a choice between the two formulas, all but a few States with very low payments per eligible child elect to use the alternate formula based on actual payments and the computed percentage (Federal Medical Assistance Percentage, FMAP).

The formulas for CETA (see Appendix A–2 of this report "The Comprehensive Employment and Training Act") and CDBG (see Appendix A–5, "The Community Development Block Grant Program") are complicated by (a) a provision for a substantial proportion of the funds to be allocated on a discretionary basis and (b) so called hold harmless provisions for preventing sudden and drastic changes in an area's allocation. For CDBG, the hold harmless clauses provide for a gradual changeover from the previous (average annual) allocation level to the Basic Grant amount determined by the new CDBG formula. Communities whose new allocations would exceed their prior level would receive the full new allocation in the third year and the higher of the previous allocation and one-third or two-thirds of the new grant level during the first two years. Communities whose new allocation is less than their previous allocation would receive the previous allocation for the first three years of the program and would be cut back to the higher of the new level and two-thirds or one-third of the prior level during the fourth and fifth years, getting only the new allocation for the sixth year.

For CETA, the hold harmless provision involves use of a moving average of the current formula results and the previous period's allocation. This is similar to the exponential smoothing techniques used in economic predictions (for market planning, production inventory control, etc.) to obtain results which will reflect the real changes in basic economic conditions but will be insensitive to temporary fluctuations and disturbances. These averages are of the form:

$$y_t = by_{t-1} + (1-b)\,z_t$$

where
- y_t = current (time t) estimate
- y_{t-1} = last period (time $t-1$) estimate
- z_t = estimate for time t based on current data only
- b = weight of y_{t-1} (vs. z_t) in the formula for y_t $[0 \leq b \leq 1]$

The value of b is frequently determined (as is apparently the case for CETA) on the basis of expert opinion. There are methods for using past experience to determine an improved value of b (where a criterion for improved performance can be established).

CETA is further complicated by the existence of three titles with three different allocation formulas. All three formulas use unemployment level but one also considers adults in low-income families and has the hold harmless provision for smoothing short-term fluctuations; one has a lower limit for eligibility (6.5 percent unemployment rate for three consecutive months); and the third formula is a mixture of the other two.

While the primary effect of the smoothing (hold harmless) provisions of CETA is to reduce the effects of short-term fluctuations in unemployment, the smoothing provisions of the CDBG program are really phase-out/phase-in provisions designed to make a gradual transition to CDBG from the various housing and community development programs it replaces. After the fifth year of the CDBG program,

[4] In the AFDC formulas the measure of Capability is the State Percentage or the State Medical Assistance Percentage both based on the State's relative per capita income (State per capita income/national per capita income). This measure is subtracted from 100 percent to get the complements (Federal Percentage or FMAP) used in the formulas. The Federal Percentage has a floor of 50 percent and a ceiling of 65 percent. The FMAP has a floor of 50 percent and a ceiling of 83 percent.

the previous allocations are no longer considered in the formula.

The CDBG formula is different from the others considered in that it is additive. The basic allocation involves the weighted sum of three measures of Need, one of which is the total population of the area. The population measure receives a weight of ¼ in the formula and the two other measures of Need (poverty count and number of overcrowded dwelling units) receive weights of ½ and ¼.

The Advisory Commission on Intergovernmental Relations [5] reviewed the allocation provisions of all federal formula-based categorical grants to State and local governments existing in 1975. Formula-based programs then numbered 146 out of 442 categorical programs. A review of the 146 programs shows that about 130 include some measure of Need; 41 programs include a measure of Effort; and 24 programs include a measure of Capability. These data show that there are few formula-based allocation programs that include all three measures: Need, Effort, and Capability. More than half of the programs include only a measure of Need. However, there are many programs which combine two kinds of measures.

Definition and Measurement of Need in Grant-in-Aid Programs

As mentioned above, the term Need is used here to refer to the services which a given program is designed to provide. The measure of Need would usually be something proportional to the total cost of providing the services in a given jurisdiction.

The specificity of the Need to be met varies considerably between Federal grant-in-aid programs. In the examples of the Appendixes, AFDC probably has the most specific Need, that of providing adequate food, shelter, medical care, etc. for (non-institutionalized) children whose families are financially unable to provide for these needs adequately. At the other end of the spectrum is General Revenue Sharing where the Federal funds are to provide fiscal assistance for the general functions of local govenment.

The other programs fall in between AFDC and General Revenue Sharing with respect to the specificity of the Need to be met, but are, in general, nearer to AFDC than to General Revenue Sharing. For example, the ESEA is directed at establishing special education programs to help educationally deprived children. Most of the assistance is concentrated on improving basic skills such as reading, writing, and arithmetic but ESEA also includes funding for a wide variety of programs designed to meet other educational needs of educationally deprived children. There was also in ESEA, as originally conceived by Congress, the idea of a general antipoverty program to help poor people and poor school districts—e.g., the stated purpose of providing funds to school districts "whose ability to operate adequate educational programs is impaired by concentrations of low-income families."

The specificity of the aims of AFDC make it fairly easy to develop a measure of Need—i.e., the amount required to provide food, shelter, medical care, etc., to a child multiplied by the number of children in families who are financially unable to provide this care. The actual AFDC program accepts as the measure of Need, the individual State's definition of how much is needed per child and which families are too poor to provide this amount for their children.

General Revenue Sharing assumes a general Need based on the level of per capita income and the level of taxes collected. That is, it is implicitly assumed that the amount a State or local government requires for general governmental functions is reflected in how heavily it is taxing its residents. The amount received under General Revenue Sharing is a direct function of the level of adjusted taxes and inversely related to per capita income squared. Population is only brought in at the upper and lower constraint levels.

ESEA uses as its primary measure of Need (1) the number of children in poverty families, (2) two-thirds of the children in non-poverty families receiving AFDC payments and (3) the number of children in institutions for neglected and delinquent children and in foster homes supported by public funds. This is directly in line with the purposes stated above. The measure of Need originally excluded the children described in (3) above but included 100 percent of the children in nonpoverty families receiving AFDC payments. AFDC uses, to measure Need, the total payments made for children in poverty families or foster homes (also see Appendix B—1, "AFDC Counts and ESEA Title I").

For CETA, the main measure of Need is the number unemployed. For States, the (expanded) CPS (Current Population Survey) estimate of unemployment can be used. Below the State level, unemploy-

[5] *Categorical Grants: Their Role and Design.* Washington, D.C.: U.S. Government Printing Office, forthcoming, 1978, Chapter VII.

ment must be estimated mostly from unemployment insurance data. A supplementary measure of Need for CETA is the number of adults in low-income families. The estimate of such adults currently used is derived from the 1970 Census of Population with no updating to reflect change since that time.

For CDBG the measures of Need are the poverty count and the number of overcrowded dwelling units. Both measures are derived from the 1970 Census of Population and Housing. The poverty count is the number of persons in poverty families as shown in the 1970 census. An overcrowded dwelling unit is defined as one with 1.01 persons or more per room.

Measurement of Population, Capability and Effort

In the General Revenue Sharing and other formulas, total population as a measure of size enters implicitly in the use of a measure of total Need rather than per capita Need multiplied by population. Population is also used (explicitly) in the computation of per capita income which is the measure of Capability in the General Revenue Sharing and AFDC formulas.

Actually, in the CDBG formula, population is used as part of a measure of relative total Need rather than as a simple measure of the size of the area. That is, at each step, the allocation to an area is the average of its relative standing (ratio of the measure to the total for all areas in the class being allocated) with respect to population and number of overcrowded units (given weights of 1) and persons in poverty families (given a weight of 2). Since these three statistics are averaged in the formula, they must all be taken to represent measures of total area need for housing (relative, of course, to the total Need for all areas in the class). The AFDC and ESEA formulas use total population implicitly in the form of a measure of total Need (total amount of AFDC payments or number of 'educationally underprivileged' children for the area).

Per capita income as a measure of Capability is used by General Revenue Sharing and AFDC. The General Revenue Sharing formula uses the reciprocal of per capita money income so that an area's allocation is inversely proportional to this measure of its Capability of raising the needed funds locally. The AFDC formulas use per capita income (squared) to determine the percentage of AFDC payments to be met by State (or local) funds. This is called the State percentage and is subtracted from 100 percent to give the percentage to be reimbursed to States by Federal funds (subject to an upper and lower limit on the Federal Government's share of the AFDC costs).

In the Title I ESEA formula, per pupil expenditure is used as a measure of both Capability and Effort. Using per pupil expenditure as a measure of Effort, the formula provides for an area's share to go up proportionally to this Effort measure. However, using per pupil expenditure as a measure of Capability, there is a provision for increasing the allocation in States with low Capability—i.e., where the State expenditure per pupil is lower than 80 percent of the national average, 80 percent of the national figure is used in place of the State figure. At the other end, for States with high Capability the per pupil expenditure is reduced to 120 percent of the national figure.

Capability and Effort do not appear in the CDBG formula. As already noted, per pupil expenditure is used as a measure of Effort and of Capability in the ESEA formula. In the AFDC formula, payments made to poor families with dependent children and to foster homes are, in effect, taken as a measure of both Need and Effort. The General Revenue Sharing formula uses, as a measure of Effort, State and local tax revenues divided by aggregate personal income. This attempts to relate taxing effort to taxing capability.

Constraints and Time References

Formula constraints tend to be aimed either at obtaining a more equitable distribution of Federal funds (either between States or between localities within States) or at preventing large sudden changes in the amount a State or local area receives. Both types of constraint represent an attempt to balance an allowance for real differences (in Need or Effort or Capability) represented by the main formula, against a concern that extreme values may represent peculiarities due to random occurrences (or temporary conditions) and defects in the formulas or the statistical data used in them.

General Revenue Sharing does not apply restrictions to the formula or data for allocations among the States but does provide for upper and lower limits on the allocation below the States level. The logic of this distinction is that (a) figures for States are probably subject to distortions for all States whereas there may be considerable variation in the

quality of the available data below the State level and (b) in the State allocation one is dealing with the entire range of non-Federal (general) government functions while local general government units may have a restricted range of functions.

AFDC places restriction on the Federal Percentage and the Federal Medical Assistance Percentage which apply to all jurisdictions. These operate to curb extremely high Federal payments to States with low per capita income as well as extremely low Federal payments to the richer States. In the regular formula, the restrictions are further buttressed by providing that the State will be reimbursed 5/6 of the first $18 per recipient paid out, regardless of the Federal Percentage (limited to a maximum of 65 percent), and will get zero reimbursement for amounts paid out in excess of $32 per recipient. For AFDC also the use of constraints can be justified on the basis of the failure of the statistics and the formula to properly reflect a balance of Need, Effort, and Capability that is equitable for all States.

The constraints imposed to prevent large sudden changes in the allocation to an area frequently take the form of exponential smoothing; i.e., using an allocation which is a weighted average of the current computation and the allocation for the previous period. A constraint with a similar purpose (distinguishing between permanent changes and temporary aberrations) is the provision that, to be eligible for an allocation under Title II of CETA, an area must have an unemployment rate of 6.5 percent or more for three *consecutive* months.

Not previously mentioned are constraints on eligibility for a given program designed mainly to prevent the administrative nuisance and waste of handling a large number of extremely small grants, thus dissipating the available funds in areas where the amount allocated is too small to get an effective program going. This appears to be a relatively rare constraint but the provision of CETA Title II just cited appears to be motivated as much by this consideration as by the time series smoothing objective.

The question of short-term fluctuations in Need, and the techniques adopted to reduce their effects upon Federal allocations is closely related to the question of updating (keeping the statistics used in a formula current) and to the question of what is the appropriate time reference for a formula.

Time reference refers to the amount of updating which is appropriate to the particular program. Only one of the five programs examined in the Appendixes requires an immediate (i.e., month-to-month) time reference. This is the AFDC and, even here, since this is primarily a question of the Federal Government providing partial reimbursement to the States for money already spent, the only question is the Federal vs. State Percentage. In determining the Federal Percentage, the formulas and the data used in them are such that a redetermination once a year using per capita income figures for the preceding year should be quite adequate.

At the opposite extreme from AFDC with respect to time reference is the CDBG program. Here, the problem to be met is primarily an *accumulated* shortage of adequate housing and community facilities. For example, the rate at which such housing can be planned, gotten into construction and completed, is such that there is probably a *minimum* of three years from initiation of a housing project to occupancy of the completed project. Only one of the components of the CDBG formula, the number of persons in poverty families, is likely to show very substantial changes over a three-year period and, even if one could obtain figures on this factor for the current year in order to recompute the CDBG entitlement of each area, changes in work already underway would not be possible; by the time housing based on the new formula is underway more current data would again be available to require a change in plans. Overcrowding has also diminished but the measure is not available for small areas on a current basis. Actually, the five-year period for transition from the old to the new housing formula is probably not excessive (it is, in fact, desirable to permit completion of work contracted on the basis of the old formula grants). At present, 1970 figures are being used for housing overcrowding and poverty in the CDBG formula along with 1973 population estimates. Some updating for future computations may be desirable but may not be as urgent for CDBG as for some other programs.

The appropriate time reference for General Revenue Sharing, CETA and ESEA is somewhat greater than for AFDC and considerably less than for CDBG. For General Revenue Sharing, figures for the preceding year probably provide a satisfactory base (from the standpoint of time reference) for the current year's allocations. These can be provided for the GRS formula at the State level (probably with an accuracy almost as good as the 1970 figures). Below the State level, problems of providing current figures for all the GRS jurisdictions eligible becomes somewhat more questionable. Actually, it has been suggested that fluctations in GRS allocations from

one entitlement period to the next may influence unfavorably the fiscal policies of some local governments. Last year's figures are probably also satisfactory for the ESEA formula and would also be satisfactory for CETA, except for the hold harmless provisions of the program. These provisions, it is claimed, are so severe that allocations for a large part of the CETA money are based primarily on 1970 data, even where satisfactory current figures are available.

Allocation to Small Areas

All the programs mentioned here address a Need. In each program, there is a different governmental or quasi-governmental agency which is responsible for administering the funds and meeting the Need: State, county, and local governments in GRS, local education agencies in ESEA, prime sponsors in CETA, county welfare agencies in AFDC, and cities and counties in CDBG. Each program must devise a way of determining the fund level for these agencies and each program has a different method. GRS allocates to all eligible governments by formula. For sub-State areas, ESEA allocates by formula to counties. States then divide county allocations among the school districts within each county. The State procedure must follow Federal guidelines. CDBG allocates to SMSA, cities, and counties by formula. Other areas compete for funding, with total State and SMSA allocations determined by formula.

CETA has different procedures under each Title. In general, CETA is distinctive in that recipients of funding need not be preexisting governmental units: consortia of governments and agencies representing areas of substantial unemployment may apply for funding. Once applications are accepted, the money is divided up by formula.

In AFDC, unlike the other programs discussed, there is no ceiling on the Federal contribution. County agencies expend whatever is appropriate under State law; the reimbursement rate varies by State. Caseload data primarily determines the level of Federal contribution to each area.

Why Existing Allocation Formula Techniques Do Not Fully Achieve the Stated Objectives of Federal Programs

Problems of Choice of Formula Structure and Constraints

In view of the examples of formula creation and use found in the five case studies, it is clear that the typical allocation formula has a complex structure entailing the identification and selection of various options. For this reason, a decision to adopt a specific formula involves—at least implicitly—a series of distinct prior choices. An inappropriate decision at any of these choice points may lead to a formula which results in allocations that do not reflect congressional priorities. We realize that such choices are, as a result of the interaction of individuals and committees, often judgmental and sometimes not made in a fully logical order. Nonetheless, there are some necessary elements in any such specification process which we feel need to be made explicit as a basis for understanding problems and limitations of formula selection.

The first choice involves the definition and measurement of Need. As discussed in Chapter I, any proposal for a Federal grant-in-aid program that is to involve a formula mechanism is motivated in some fashion by a perception of a Need. A working definition and some measure of that Need must be adopted, whether or not there is full understanding or agreement on all of the dimensions of Need. For example, in the first enactment of Title I of the Elementary and Secondary Education Act it was recognized that school districts serving large numbers of low-income children were in some need of special assistance. While there was general agreement that such school districts needed more money, there existed by no means any fully consistent statement of the nature of special burden which low-income children represented. In fact the statute related the level of funding to the number of low-income children but left it up to individual school districts to assess the requirements of their children and to plan programs accordingly.

A measure of Need that is perfectly congruent with the definition of that Need is almost never available. As a consequence the program designer must resort to some proxy indicator, and the choice of a suitable proxy is by no means trivial. Surrounding Title I, for example, there was considerable debate over the proper measure of low-income status, and the measure was in fact improved in 1974. Yet, the 1974 debate did not resolve all questions concerning the appropriate measurement of the target population or even settle its definition. Dissatisfaction with the criteria of disadvantage embodied in the present formula led the Congress to commission a study at HEW on the measure of poverty (which was completed in 1976) and a related set of studies to be carried out by the National Institute of Education of the feasibility and probable impact of using measures of educational rather than economic disadvantage for Title I ESEA fund allocations.

As noted in Chapter I, in adopting allocation formulas Congress frequently takes into account some measures of what we have termed in this report Capability and Effort. These are, if anything, more difficult to define than Need, and may involve problems of measurement as well. After the program designer has set forth a working definition and measure of each of these elements the actual process of formula construction properly begins. At that point there is a wide range of possible allocation formulas which might be constructed as well as a variety of possible constraints and special rules.

A central question that must be answered by the program designer is in what way the resulting allocations should vary over the range of possible values of the measures of Need, Capability, and Effort, and also reflect considerations not accounted for by these concepts. In some cases the difficulty of ameliorating a social problem may be proportional to the measure of Need, so that a linear allocation formula would be appropriate. In other cases a nonlinear relation between the allocation and the Need measure may be called for. If the designer wishes to take into account Capability or Effort, then the maximum and minimum allocations for a given Need must be decided in relation to the expected range of measured Capability and Effort. There may also be other desired patterns of allocation to meet policy purposes other than those reflected in the measures of Need, Capability, and Effort.

Once these issues are settled, the formula can be

constructed. This process necessarily includes both policy and technical considerations. The central technical problem is the choice of a mathematical structure which in some sense best utilizes available data to produce the desired allocation pattern. As discussed further below, there are additional issues of data limitations, of interactions between the formula and data, of the dynamic properties of the formula, of its understandability to the public, and of its computational efficiencies.

The essential elements in the choice of a mathematical structure are as follows: (1) the class of the formula (e.g., additive as in the CDBG program or multiplicative as in General Revenue Sharing); (2) the weights or scale factors to be applied to each of the terms in the formula (e.g., giving unit weight to relative population and to overcrowding, double weight to poverty in CDBG); and (3) the specifications of constraints, if any, on either particular variables or on the resulting allocation (e.g., floors and ceilings on the cost factor and hold-harmless levels on the allocation in Title I ESEA). The statistical consequences of these choices are often not fully understood by either statisticians or program designers. Although the design sequence can be described as a set of logical choices, the sequence and timing of such choices will vary from program to program. In addition, both the valid demands of the political process and the primitive state-of-the-art of formula practice lead to choices at every stage of the program design whose full statistical and distributional implications cannot be foreseen at the time they are made.

For example, floors and ceilings or other types of constraints involve in some sense a distortion of the ideal allocation. As noted in Chapter I, the setting of such constraints is sometimes an attempt to limit annual variations in allocation levels and sometimes an attempt to modify a less-than-ideal formula by making sure that no one gets too much or too little. In either case, constraints may influence allocations more strongly than they were intended to. A striking example of this effect is seen in General Revenue Sharing under which townships with minor governmental functions are guaranteed a sizable minimum payment—a consequence that was not generally anticipated at the time the law was passed.

The complexity of the task of selecting a formula structure leads in practice to other problems. Every allocation formula represents a simplification of the real world. We have just pointed out that constraints distort an ideal allocation, but the very notion that an ideal allocation could be described for reference purposes implicitly assumes that we are willing to determine just how much of the fine-grain complexity of the real world should be captured in such an ideal formula. While technicians might reach some consensus on the attributes of an optimal degree of simplification, no statement of principles based on such a technical consensus would be immune from criticisms that some important aspect of reality was omitted from a formula designed according to such principles. This point serves to reinforce our recognition that formula building—if it is to be successful in implementing legislative goals—should not be the exclusive province of either the technician or the politician.

An important implication of the need to accommodate both political and technical considerations is that an allocation formula should be comprehensible to all parties involved. The policymaker needs to understand more about an allocation formula than just how much money it allocates to various jurisdictions this year. The formula should be transparent enough to support direct analysis of its distributional effects—across States and within States—at a point in time and as well as over time. The recipient—whether local official or ultimate beneficiary—should at the very least be able to verify the correctness of his allocation. For example, the General Revenue Sharing formula is extremely complicated, both in the determination of State allocations and in the division of funds within States. The procedure for allocation to States, which resulted from a compromise between House and Senate, combines two formulas to give each State the higher of two computed allocations. Because there is a fixed total appropriation for the program, the actual computation must be carried out iteratively, and only expert analysts can estimate the impact of even very simple changes in the existing formulas. Thus we see that lack of transparency in the formula for an ongoing program can be an important deterrent to meaningful attempts at reform of existing programs.

Problems Arising From the Nature of the Data Used and From Interaction of Data and Formulas Over Time

However difficult it may be to understand and evaluate the performance of a formula at one point in time, the task of foreseeing and assuring good performance through time is even more difficult. There seem to be three issues: (1) The formula may

require data which cannot be updated frequently, and the degree of distortion caused by the use of obsolescent data can neither be bounded closely in advance nor estimated precisely at the time current allocations are made; (2) Statistics which can be updated for formula use may slowly or suddenly depart from their historical behavior and from their assumed stable relationships with other variables; and (3) The social or economic problem to which the program is directed may evolve in such a way that the measures chosen to represent Need, Capability, and Effort may cease to be the most relevant measure available.

All of these issues are illustrated by the history of the measure of economic disadvantage used in Title I, ESEA. This measure has been and continues to be the sum of counts obtained from various sources. Census counts for 1960 were a major component in the Title I measure from 1965 until 1973, by which time they were hopelessly out of date. Annual counts of children in families receiving a high level of AFDC payments departed from their historical behavior shortly after Title I was enacted, as a result of an unprecedented increase in the AFDC caseload and of the onset of an unforeseen price inflation. While in 1965 the AFDC counts represented about ten percent of the total Title I measure, by 1974 they were sixty percent of the total measure. While some growth in the importance of the AFDC counts might have been expected in 1965, it was not anticipated that they would become the dominant component. While it could have been predicted that the fixed dollar family low-income threshold specified in Title I ($2,000) would become quite inappropriate upon the introduction of 1970 income data, Congress took no action to revise this specification until the effects of the use of the old cutoff with 1970 data were evident in the 1974 Title I allocations (see also Appendix B–1, "AFDC Counts and ESEA Title I").

Our third issue is illustrated by the rapid expansion of in-kind transfer programs, such as Food Stamps and Medicaid, whose income equivalent is not currently counted in family income statistics from the decennial census and the Current Population Survey (CPS). Depending upon the distribution of in-kind benefits, they might bias relative measures of low-income status across geographical areas. The degree to which they depart from such a uniform relationship with money income is not fully known, but the magnitude of these in-kind programs raises the possibility of serious bias.

Both here and at earlier points in this chapter we have reviewed issues which demonstrate that data and measurement limitations may dominate all other considerations in formula design and assessment. As we have stressed before, no measure can be perfect in all respects. One of the most difficult tasks in program design is to determine in advance whether a measure will prove to be at least minimally acceptable. A recapitulation follows of the different ways in which an operational measure may fail to fulfill the objectives of the program drafter.

(1) *Lack of fit between a measure and the real world phenomenon it is intended to portray.*

An inappropriate measure may be chosen because of its familiarity or its intuitive appeal. Within CETA, for example, the local unemployment rate is used both to measure the need for public employment, of which it is probably a satisfactory indicator, and to measure the need for job training, for which there may well be more appropriate though less familiar measures. The overcrowding index used in the CDBG program is a good example of a measure, the intuitive plausibility of which may exceed its suitability to the program in question. What makes the index attractive, however, is that it conveys some information about whether the inadequacy of housing leads to hardship. This possible relationship is certainly something one would want to measure in a Federal housing program. The overcrowding index, though, may be inferior as an indicator of the quality of the kind of housing generally available to the poor when compared to some possible physical measure of housing stock quality which contains no overt reference to occupancy. However, no simple measures of the physical quality of housing is available at this time. Perhaps CDBG should consider developing a more comprehensive measure of housing needs in which the overcrowding index is only one of the factors.

As the case study on General Revenue Sharing indicates, the use of per capita income as a measure of Need has been criticized despite its obvious virtues of familiarity and general plausibility.

(2) *Accuracy of a measure for the geographical area it applies to.*

This presents a problem for all programs which require formula allocation to small areas. The unemployment data for CETA and the poverty data for Title I, ESEA are pertinent examples. In the case of CETA, the flexible definition of labor market areas, although perhaps desirable for policy reasons, is

made less desirable because of the inadequacy of the statistics from which area Need must be calculated.

In the case of Title I ESEA the congressional intention to allocate directly to school districts was thwarted by the inadequacy of school district poverty data, and instead allocations were made to counties, with the States being responsible for subcounty allocation to school districts.

(3) *Stability of a measure in relation to the frequency of updates.*

Data which are expensive to gather as well as subject to considerable variability through time may not be cost-effective for allocation purposes. This is the chief obstacle to the generation of small area price deflators which could be used to adjust grant levels to local price differences.

CHAPTER III

Subcommittee Findings

In this chapter, we will present four major findings together with some illustrations.

Finding No. 1

There are very real difficulties in translating congressional intent into statistical terms.

We will illustrate this finding by reference to the Community Development Block Grant program authorized by the Housing and Community Development Act of 1974.

a. Section 101(c) of the Act states that "The primary objective of this title is the development of viable urban communities, by providing decent housing and a suitable living environment and expanding economic opportunities, principally for persons of low and moderate income."

b. The section goes on to say that the CDBG Federal assistance is for the support of community development activities directed toward certain specific objectives, including "the elimination of slums and blight and the prevention of blighting influences and the deterioration of property and . . . facilities . . .; the elimination of conditions which are detrimental to health, safety, and public welfare, through code enforcement, demolition, . . .; the conservation and expansion of the Nation's housing stock . . .; the expansion and improvement of the quantity and quality of community services . . .; a more rational utilization of land and other natural resources . . .; the reduction of the isolation of income groups within communities and geographical areas . . .; the restoration and preservation of properties of special value for historic, architectural, or esthetic reasons."

c. As described in the CDBG case study, the allocation and distribution of funds is specified in the Act on the basis of a three-term additive formula counting population, poverty (weighted twice), and housing overcrowding—where the count for, say, a metropolitan city is entered as the numerator in each of three ratios with the denominators being the counts for all metropolitan areas. In the framework of our report this is a Needs formula with no explicit components for Capability or Effort.

d. Congress apparently felt that the extent of poverty and housing overcrowding were reasonable surrogates for its target population (persons of low and moderate income) and for the conditions it hoped to alleviate (slums, blight, inadequate services, etc.). They did not try to legislate the use of some direct measure of housing quality or service adequacy. But a paradox remains: Two communities of the same size, poverty count, and overcrowding index might have, to an impartial observer, two quite different levels of adequacy of housing stock and services.

e. As can be seen from the above discussion, it would be very hard to construct a formula that would adequately operationalize the goals of the Act. It should be noted that Congress is expected to reconsider the CDBG formula during the 1977 session, partly in recognition of some of the problems outlined above (1.d).

f. The CDBG program illustrates the potential conflict between policy objectives and the rationalization of formula and data requirements. In this case, the broad objectives make it difficult to define and measure Need in the program formula. Congress set up CDBG to consolidate a number of categorical programs. One objective of CDBG was to allow for considerable local discretion in the specific purposes to which the allocated funds would be applied. Accordingly, a large number of program goals were recognized, and, purposely, there was no ranking of the various possible objectives.

Finding No. 2

Current administrative and statistical practices do not always deal adequately with the problems that have been identified in Chapter II.

a. A good example of "why . . . existing allocation formula techniques do not fully achieve

the stated objectives of Federal programs" may be found in the methods for counting Title I ESEA eligibles.

With regard to the problems arising from the nature of the data used, the law specifies a determination of the "number of children aged five to seventeen, inclusive, from families below the poverty level on the basis of the most recent satisfactory data available from the Department of Commerce for educational agencies (or . . . counties) . . . utilizing the criteria for poverty . . . in the 1970 Decennial Census." There is a parallel provision for counting some disadvantaged children (AFDC recipients, etc.) above the poverty level.

(1) The "most recent satisfactory data" may not in fact be recent enough to be satisfactory. Furthermore, in spite of the age of the data, no provision has been made for a reinterpretation of the counts in a way that might constitute a partial adjustment for time effects. For example, instead of the cohort aged 5 to 17 in 1970, the cohort aged 0 to 12 in 1970 (which was aged 5 to 17 in 1975) might be considered as a relevant reference group for current allocations.

(2) The argument is sometimes made that the Title I formula is partly protected from obsolescence by the inclusion of the AFDC factor which in some sense can update the eligibility counts, even if the poverty counts cannot be updated. As pointed out in Appendix B-1, the AFDC component is only about 7 percent of the total and is distributed among States and counties very differently from the poverty count—in either 1970 or, say, 1975.

b. Another example is provided by the General Revenue Sharing program, which has been operational for more than six years. Much of the criticism of the program has been focused on how well the formula structure reflects the needs of the recipient localities. The GRS program distributes funds to approximately 39,000 jurisdictions, the great majority of which are areas of population less than 2,500 in the 1970 Census of Population. Because of the complexity of dealing with different kinds of local governments, and the severely limited data available for this purpose, GRS has used a uniform procedure that treats similarly governmental units with very different sets of responsibilities.

In addition, the use of GRS as a countercyclical device is hampered by considerable data lags. Despite the procedures involved for updating census money income (one of the elements of the formula mandated by the Act), based on the more current IRS wage data (used in conjunction with BLS county and State wage data) and the BEA county personal income data, there is still a lag of several years between the reference year of the data used in the formula and the year in which the allotment is made. Even if the currentness of the inputs could be improved enough to appreciably narrow the gap it could not be done without introducing other difficulties. Although improvements in the formula have been proposed, introducing other elements purported to be better indicators of Need, these other elements also can be measured only with several years' lag, and may not even be available for smaller areas or only with some sacrifice of precision.

A further criticism has been that occasional sharp fluctuations in the size of the allotment for a given area from one period to the next, caused by unusual variations in the data inputs, tend to hamper long-range planning by the recipient governments for efficient use of the revenue sharing funds. However, changing a formula structure which has been in operation and has come to be generally accepted by all levels of government could be more disruptive than the occasional random fluctuations in allotments encountered with the present formula.

Finding No. 3

The nature of the statistical problems arising in formula programs is such that present knowledge does permit the identification of at least some interim principles. There are some existing programs for which the existing formulas or allocation rules appear to be satisfactory from a statistical standpoint.

a. One example may be found in the AFDC case study. Whether or not the resulting reimbursement levels to the individual States are completely appropriate is a matter for Congress to consider from time to time. But there are

no apparent statistical bases for concluding that the resultant reimbursements are inappropriate. There is an inverse relationship between per capita income (PCI) and reimbursement rates. If this were adopted simply as a fair relationship it would be hard to argue that it is not. By that standard there would appear to be no serious problems with the current practice. If the inverse relationship were interpreted as an incentive device to get the poorer States to set up programs comparable to those of the richer (higher PCI) States with higher benefit levels, then that Federal purpose would have to be seen as not fulfilled by the matching rate rules, since the poorer States have not so responded.

b. Another example concerns the Comprehensive Employment and Training Act of 1973. The major portions of the funds allocated by formula under Title I of that Act are distributed in a manner that incorporates several elements that are sound from a statistical standpoint:
   (1) The units to which funds are allocated, the prime sponsors, are large (100,000 or more population) and thus avoid the problems associated with the preparation of estimates for very small units.
   (2) The prime sponsors are defined in terms of units of general local government. While these may be combined into various configurations, this eliminates the difficulties associated with the development of estimates for neighborhoods or other parts of cities or counties that do not have an established geographic definition.
   (3) The unemployment data used in the allocation is based on annual averages. It is, therefore, not subject to seasonal influences and the distortions that they can inflict on the allocations. The use of annual averages is, in a sense, an example of the use of the best available data from a single standard source—the Current Population Survey (CPS). However, the CPS is used only for the States and 30 SMSA's and 10 central cities; a problem remains for large counties and large cities. Moreover, the formula incorporates legislative determination that while all areas need manpower services, the need is greater where the number of unemployed is higher. The distribution is therefore based on the number of unemployed.
   (4) The problems of administering a continuing program of manpower services with a shifting financial base are recognized by providing a floor based on the preceding year's allocation below which the funding of the current year cannot fall, and a ceiling above which the allocation cannot go. Title I, CETA avoids wide year-to-year swings in the allocations received by prime sponsors. It does this both by distributing funds largely on the basis of the previous year's allocation, and also by providing floors and ceilings, based on the previous allocation, beyond which the current allocation cannot go. This facilitates the chief objective of Title I—to provide a continuing program of manpower services—by keeping funding levels relatively constant and predictable.

c. The third example concerns the sub-county allocation system in Title I ESEA. This is a creative approach to the problem of allocation to small areas, in this case to school districts. The data used in the formula to allocate to counties—1970 poverty counts, special AFDC tabulations, and counts of neglected, delinquent, and foster children—are not currently available at the district level to the Federal Government. States therefore have been given the right to allocate county funds to the school districts in each county, using the most recent appropriate data. The Federal guidelines recommend census and AFDC data, but States may choose among a number of data series. While not without problems, the system appears to work relatively smoothly. One benefit of this system is that questions about the correctness of the data for very small districts can be raised as well as resolved locally, by people familiar with the actual conditions.

Finding No. 4

The present state-of-the-art will not permit formalization of a fully definitive or wholly acceptable set of statistical rules for formula programs. In view of the present gaps in our knowledge there is a need

for some short-range applied research on problems of allocation statistics. For example, while the use of quadratic loss functions (minimizing the mean squared error) is well established, there appears to be a need in formula research for the use of asymmetric loss functions. At present there is little readily applicable theory and some research is needed soon on this topic as well as on related problems in approximation theory (also see Appendix B-5, "An Agenda for Basic and Applied Research Problems").

Ways to Reduce Allocation Errors and Inequities

Introduction

It is usually easy to arrive at a consensus that the allocation of funds under any given program is inequitable. However, it is often difficult to get any agreement on the nature and location of the inequities and even more difficult to get agreement on how to correct the inequities. There are, though, some aspects of allocation formulas and the data used in them which lead to substantial discrepancies from the intent of the original legislation. This chapter addresses this type of problem.

1. *Problems arising from the data used.* In connection with data used for allocation, there are rather complicated trade-offs among five factors, three of which are relatively well understood (at least we think we know what they mean), namely bias, variance, and cost. The other two are the timeliness of the data (the time-frame of the data) and the appropriateness. The appropriateness can be defined as the extent to which the concept one is using (no matter how well or poorly it is measured), approximates the thing that one really wants to measure.
   a. *Updating.* Before discussing the interaction of the five factors, a few observations are in order on timeliness and the question of updating statistics for use in allocation formulas. As noted in Chapter I (p. 6) the appropriate time reference (timeliness) varies from program to program. In the field of government action, one can distinguish between programs to meet immediate (and very time-dependent) requirements and those designed to deal with situations which change relatively slowly over time. In the first category are those welfare and unemployment insurance programs designed to deal on an emergency basis with immediate problems. The impact of this type of problem on any given area at any specified time is largely unpredictable. Here one is dealing primarily with questions of accounting for funds after they have been spent, rather than of allocating funds to specific areas. This type of problem is best handled by providing for a central pool of Federal or State funds which is drawn upon as required locally. To the extent allocation of Federal funds is involved, the statistical problem becomes one of determining the amount of allocations appropriate to maintaining the State or local pools of funds at (legislatively) specified levels over a time period of a year or more.

      Thus, even when there is the requirement for immediate action that varies locally from month-to-month (and even week-to-week), updating of data used for allocations is not necessary more frequently than once a year. Where the basic economic and social conditions at which a program is aimed change slowly, updating statistics every 2 or 5 or even 10 years may be adequate. In the case of programs involving massive training or building programs (highway and mass transportation programs, slum clearance, teacher training or retraining of individuals in declining industries) frequently updated figures, even for rapidly changing situations, may be of little appropriateness for fund allocation, since a large portion of the work in progress must be completed even though plans for future work may need drastic revision.

   b. *Trade-offs.* The total population of an area is a factor in many allocation formulas and the problem of making estimates of population illustrates the trade-off among bias, variance, cost, timeliness, and appropriateness. The cheapest estimate that might be in any sense acceptable is, of course, the population of the area based on the most recent decennial census. However, for some allocations, the decennial census figures are out-of-date by the time they are published. Even if one uses the hand counts (announced locally immediately after completing the census field work) and takes the risk of major differences from the final revised figures, decennial census figures are at least 10 years old by the time the figures for the next decennial census are available.

      The recent authorization of quinquennial

censuses will somewhat reduce the problem of updating population census figures; but the cost of taking a 100 percent census will almost certainly mean that the 1985 enumeration will be on a sample basis. While the sampling biases and variances of a sample census will be small for most states and for major metropolitan areas, the sampling errors for small areas will be, at a minimum, a source of considerable controversy (e.g., claims that "my city or county was 'robbed' in GRS allocations"). Even for the largest areas there can be considerable dispute since, while the relative sampling errors will be small, the *absolute* errors and the *absolute* sums of money involved may be substantial.

For some uses, updating population figures every five years will be considered unsatisfactory; there is pressure for annual and biennial sample surveys and for the use of more current statistics derived from administrative records (birth and death registrations, income tax returns, school enrollments, etc.). The unit costs of a sample survey are high and, for a number of quite valid reasons (difficulties with privacy, confidentiality, public resistance, availability of satisfactory personnel), are increasing, in spite of improved survey techniques and generally improved overall efficiency in the conduct of sample surveys. Even well funded and well conducted sample surveys (e.g., the 1976 Survey of Income and Education) are restricted to small samples and also require the use of clustering in order to minimize travel time and other nonproductive expenditures. Small, highly clustered samples mean large sampling variances even for some relatively big areas, and also mean that many small areas will have no sample households at all.

Using administrative records to update the population involves major problems and can involve serious biases. Applying statistics from birth and death registration records to the previous census should produce reasonably good figures for areas which have had very little in-or-out-migration since the census. For the areas with relatively heavy (net) migration in the 1950's and 1960's (e.g., California, Florida, Arizona, Nevada, Alaska, most metropolitan areas west of the Mississippi, rural areas of the South Central and West North Central States), estimates based on births and deaths tend to be improved by making an adjustment based on past migration trends—e.g., using the average population change in any area due to migration (total population change less births plus deaths) from 1960 to 1970 as an estimate of the annual change due to migration since 1970. Adjustment for past migration trends usually gives improved estimates for the areas with substantial past in- or out-migration but it does not allow for the second (and higher) order derivatives of the population change curve for an area. Such an allowance can be made by using a curvilinear regression on past migration trends but this involves either using still earlier censuses (e.g., the 1950 and the 1960 censuses) and intercensal births and deaths to estimate net migration since 1970 or obtaining estimates of intercensal populations. While the use of past migration trends (linear or curvilinear) will improve most estimates of current population based on births and deaths, it results in poorer estimates for some areas because of the biases and variances of the estimates of past migration trends as well as changes in the shape of the population growth curve since the last census. While sudden and drastic changes in the shape of the population growth curve of an area are rare, they occur (e.g., the decrease in California population growth rates between 1960 and 1970) and in these cases there may be serious biases in the population estimates in spite of the adjustment.

Similar difficulties of bias and variance occur in the use of estimates based on other administrative records. For example, population estimates derived from income tax returns do not provide for persons who did not file a tax return for the given year. Partial adjustments for these omissions could be made by using supplementary sources (e.g., W-2 files, files of welfare families) but adjustments (e.g., determining how many persons are represented by W-2 forms to adjust for the cases where the income recipient did not file a 1040 return) are difficult and the estimates will still be deficient for other reasons (e.g., individuals may not be shown as dependents or income recipients in any source).

The estimates can be improved by using the administrative records to estimate *change* in an area since the last census (rather than the current population level) and by applying this estimated change to the census figure for that area. Similarly, the percent change since the census in school enrollments can be applied to the census population to produce a current estimate. One can also use a combination of change in income tax and enrollment statistics to estimate current population by applying the regression of the census population on census-year tax returns and school enrollments to current-year tax returns and school enrollments.

For some allocations updating the census population counts may be unnecessary. However, even for these cases, there is a question of biases in the counts. The Census Bureau estimates that, even after very vigorous efforts (and very large expenditures) to obtain 100 percent coverage in the 1960 and 1970 Censuses, there were undercounts of 2.7 and 2.5 percent. It is likely that census techniques in 1980 will have to be improved, and efforts and expenditures per capita will have to be increased even to attain the 97.5 percent coverage level of 1970.

The trade-offs of cost, bias, variance, and appropriateness are particularly evident in the area of control and estimation of census coverage error. There is, for example, the question of trying to reduce differentials in coverage among areas and subgroups. For several reasons Black, other minority, and low income groups are more difficult to enumerate completely than the rest of the population. The coverage problem is particularly acute for certain types of areas, e.g., sparsely settled rural areas and ghetto areas in large cities. Frequently improving coverage of the poorly enumerated groups and areas requires very much higher census expenditures per household, and this, in turn, raises the question of reducing expenditures elsewhere or increasing total census costs. Reducing expenditures elsewhere may mean slightly higher overall bias in order to decrease the *differentials* in coverage bias.

The handling of imputations in a census also provides an example of the problem of balancing variance, bias and cost. Because of imperfections in the most well-designed census, problems of imputing for known errors always arise. Thus, discrepancies between the area hand counts and the initial machine counts have existed for every census where tabulation machines or computers have been used. These may be due to errors in addition, to failure to count some census sheets or lines, to errors in the hand count, to questionnaires lost in transit to the processing center, to questionnaires misfiled and lost in the sheer mass of paper, to failure to punch or film questionnaires or groups of questionnaires, to errors in punching or optical sensing of the questionnaires, etc.

In the 1970 Census possible errors in the census counts were also signaled by the Vacancy Recheck and PEPOC (the Post-Enumeration Post Office Check). These involve checking units reported as vacant to determine whether they were, in fact, vacant, and having the local post office check the census listing for possibly missed households for those areas where a post office check was not done before the enumeration.

Possible census errors detected by discrepancies between hand and machine counts or by a vacancy recheck or by PEPOC can be met by:

(1) ignoring the possible hand count or vacancy recheck or PEPOC results,

(2) tracing the errors and making corrections,

(3) reenumerating areas or units where errors are detected, and

(4) imputing more correct values.

All of these methods were used in the 1970 Census. Small discrepancies between hand and (initial) machine counts were ignored; some misfiled questionnaires were detected and the appropriate counts corrected; a sample of vacant units and a sample of the enumeration districts where PEPOC showed possibly missed households was reenumerated; the results from the Vacancy Recheck and PEPOC samples were used to impute corrections for the nonsample vacant units and the nonsample enumeration districts of PEPOC; and imputations were also made where the initial machine count was well below the hand count and investigation

confirmed that the hand count was more accurate.

With respect to updating and coverage error and imputations, possible solutions involve some compromise among the five factors. Thus, in a quinquennial *sample* census there may be a satisfactory compromise between the low cost, low variance, and poor timeliness of using decennial population figures, and the increased cost, high variance and bias, and good timeliness of using annual sample survey estimates. Essentially making imputations based on a sample check (as was done for the Vacancy Recheck and PEPOC) is a compromise between the bias and low cost of not correcting for the known census error, and the lower bias and higher cost of trying to follow-up and (re)enumerate all of the questionable cases.

A form of compromise which seems particularly desirable for the problems of updating and adjusting for undercoverage is the use of low bias and high variance data from a small sample study to adjust higher bias but low variance estimates from a larger scale study. Thus, for updating population data we could use the high variance and low bias of changes measured from a small annual or biennial sample survey to correct the bias of (zero variance) statistics derived from administrative records. By substituting regression of the changes on other characteristics, we decrease the variance of the resultant estimates with some (hopefully small) increase in bias. In estimating undercoverage, we can correct biased estimates from a large sample survey by the low bias results of small samples of administrative records (from IRS, Medicare, driver's license files, etc.) matched to the census. By using regression techniques, we can obtain considerable reduction in the biases of the estimates from the large sample source and avoid the high variances of the estimates for individual areas in the small sample study. The impact of errors on allocation is discussed in Appendix B–2, "Technical Notes on Sensitivity Analysis". Raking as a statistical adjustment procedure may be used to reduce error in data (see Appendix B–4).

c. *Data Comparability.* Where different areas (States, counties within a State, school districts within a county) are in competition for a share from the same pot, equity dictates that the allocation data for the competing jurisdiction be as nearly comparable as possible. Comparability is usually attained by taking the estimates for all competing jurisdictions from the same source. Thus, the population estimates for all States might come from the census, and adjustments for updating would all be computed in the same way—e.g., from the regression of data from a national sample survey on the numbers of taxpayers and dependents (determined from Federal income tax records) and current school enrollments.

The fact that comparability between competitive jurisdictions is frequently best served by taking the data for these governmental units from the same source, has been extended into a rule that data for all jurisdictions, competitive or noncompetitive, must come from the same source. Such a rule can acutally lead to less rather than more comparability. It may, in fact, force the use of grossly inadequate data because the only source available for *all* jurisdictions is a very inferior source. In tiered allocation systems [6], it may be better to use a common data source at any one level but not to insist on using it at all levels. Thus, sample survey estimates of current State populations might be the best estimates for the allocation of funds to States, but, for allocating the total for a State among cities and counties, we might use estimates based on adjusting 1970 Census populations for changes in school enrollments and in the number of income tax payers and dependents.

It may even be desirable to use different data series for allocations within different States. Thus, one State may be able to get a quite good estimate of the population of each city and county (and also of each township and city ward) in the State from the regression of census population on the number of registered voters plus school enrollments, while the voter registration and school enrollment statistics would be much inferior

[6] These are systems where an allocation is made to each State and each State total is, in turn, allocated among the jurisdictions within the State.

for another State in projecting past intercensal population increases.

The use of different data series for different levels and for different allocations within a level is a case where the use of nonidentical data actually helps to maintain comparability. A much more difficult problem is the availability of better data for a few jurisdictions in a set of competing jurisdictions. For example, one city in a State has a special census taken which shows a population increase for the area of 30 percent since the previous census, as against the estimates of population growth of five to nine percent obtained for this city and other cities and counties of the State by projecting population trends shown by the last three decennial censuses. Is it proper to use the population figure from the special census for this particular city when no comparable figures are available for the other cities and counties of the State? One could argue that using the special census estimate gives an unfair advantage to this city since other cities or counties may have had similar or greater population growths. On the other hand, it could be argued that there are, at most, two other areas in the State that had more than nine percent population growth and that it is unfair to penalize this city because the other areas of the State had no reason to take a special census. Solutions to the problem might be:

(1) to try to find some method using already existing data which would properly reflect post-censal population growth for all areas of the State, or

(2) to execute a small sample survey to determine whether any other city or county has had unusual population growth and follow up by larger sample surveys of those jurisdictions which do show large population changes.

Problems arising from the formula. There are many alternatives in the construction of allocation formulas. For some of these alternatives (e.g., the use of an additive versus a multiplicative formula) the pros and cons are pretty evenly balanced and the choice becomes a matter of purposes to be served, the data available, and individual tastes. There are a few alternatives which are clearly inferior from both a statistical and policy standpoint. The handling of cutoffs is one of these.

a. *Additive versus multiplicative formulas.* In a multiplicative formula the allocation is automatically, equally sensitive to variation in any of the factors. That is, a 10 percent change (or a 10 percent difference between two areas) in any factor will mean a 10 percent difference in the allocation (unless the formula includes a cutoff provision). In an additive formula weighting is needed to determine the relative sensitivities of the allocation to the different factors in the formula.[7] Weights are frequently arbitrary and poor choice of weights can lead to serious dissatisfaction with the operation of an additive formula. On the other hand, if a multiplicative formula is used, a small error in one factor can throw the whole allocation seriously off. Thus, one is damned if one does, and damned if one doesn't. The choice of the formula type must, then, depend upon judgments of the accuracy of the various data to be used versus the availability of suitable weights for an additive formula. It is important to provide for constant monitoring of the allocation system so that major errors in the data can be promptly detected and corrected for multiplicative formulas, or so that a poor choice of weighting factors (or a major shift in the underlying causal system) can be promptly detected and corrected for additive formulas.

b. *Cutoffs.* Undesirable discontinuities may be introduced into an allocation system by cutoffs, especially by eligibility cutoffs. For example, if an area must have an unemployment rate of five percent before it can receive *any* funds, a very trivial error in the estimation of the unemployment rate can easily throw an area from under five percent or from over five percent into the other group. Here a very small error can make a tremendous difference and lead to continual complaints about the accuracy of the data on the part of governmental units which feel the cutoff operates to their disadvantage.

A common solution to controversies over cutoffs is to provide alternative formulas and to permit each jurisdiction to select the formula which is most advantageous. While this

[7] Of course, one can provide for differential sensitivities in a multiplicative formula by attaching exponents to the different factors.

works moderately well, it has the disadvantage of making it difficult to predict in advance (and budget for) the amount required for the program if no fixed overall sum to be allocated is specified. If an overall sum is specified but each jurisdiction may choose which formula it will use in determining its share (with the computed amounts totaled over all competing jurisdictions, so that the percent of the total allocated to each jurisdiction can be determined), one gets a floating cutoff, where the amount one jurisdiction gets depends upon the decisions made by other jurisdictions.

For eligibility cutoffs it is almost always possible to devise a formula such that there is a gradual approach to zero (or to some cutoff point lower than the existing absolute cutoff). Here small errors in the data lead only to small changes in the allocation and the tendency to prolonged (and insoluble) arguments over minor errors is removed. Of course, major errors will and should continue to be the subject of controversy but one will be spared the waste of time and effort involved in the use of a formula which requires data of unattainable accuracy.

c. *Sensitivity to change.* In most cases it is desirable for allocation rules to be relatively insensitive to short-term fluctuations in the data but responsive to long-term changes. However, short-term and long-term are in the eye of the beholder. How short is short-term and how long is long-term? The answer varies from one program to the other. The CDBG obviously needs at least a four or five year period even to permit contemplation of a building project or the planning of any substantial building program. What one needs is something that will not be thrown totally off the target by short-term fluctuations. On the other hand, gradual change in response to changing needs is desirable and some type of damped dynamic system (for example, an exponential smoothing type of function) is required.

CDBG appears to be the only one of the case studies which tried to use such a damped dynamic system (for bridging the transition to a drastically changed allocation system). The CDBG formula used for this purpose involves a so-called hold harmless provision. However, it should be noted that the hold harmless provisions of most allocation formulas are the reverse of damped dynamic systems. At one end, hold harmless clauses create a totally static situation, permitting an area to claim its allocation of last year (and possibly of several years previously) although conditions may have changed permanently so that a considerably reduced allocation would be quite adequate. At the other end, an area can claim a sharply increased allocation due to a temporary change in the local situation.

When responsiveness to short-term problems is desired, fixed annual allocations for every funding level usually are not appropriate. Switching of funds as needed, both from one time period to another and from one jurisdiction to another, may be required. In AFDC, funds are allocated for a year so that jurisdictions can determine approximately what to expect. The specific allocations are determined as the money is spent and can vary from month to month.

3. *Setting feasible accuracy goals.* A major question is to what extent should one adjust the data to fit the accuracy requirements, and to what extent should accuracy requirements be adjusted to fit the data. Some people tend to think in terms of statistics that are literally correct and in terms of an absolute truth which must be met in fund allocations. Many law suits deal with errors in the data and with other errors which cannot possibly be avoided at a reasonable cost. We need to learn to accept the fact that the function of the statistician is not to provide error free data but to pick out those errors which are largest, and try to control them. As for the smaller errors, we must learn to live with them.

Recognizing that errors in the data and resultant inequities in the allocations are inevitable, major attention must be given to deciding which errors need to be reduced. As mentioned above, a subject of considerable controversy is whether one should try to minimize the sum of the absolute errors or of the relative errors (or of something in between) in the data for individual areas.[8] When sample data are used, minimizing the sum of the relative sampling errors of the individual area figures leads to allocating the

[8] Since more effort devoted to reducing error in one area means less effort and bigger error in another area, we have to talk in terms of reducing the sum of the errors rather than of each individual error.

same number of sample cases to each area (e.g. to each State); minimizing the sum of the absolute errors leads to allocating a number of sample cases proportional to the total population of the area.

A commonly-used compromise between the two allocation rules mentioned above (minimizing the sum of the absolute errors vs. minimizing the sum of the relative errors) is

a. to minimize the sum of the absolute errors by assigning cases proportional to the area population;
b. if this would give any area a relative error larger than the predetermined error limit, increase the sample for the area(s) to the level necessary to give the desired relative error; and
c. reallocate the residual sample for the areas not changed by (b) above, proportionally to area populations.

For fund allocation, this sampling design fits the logic that a big relative error for a small area leads to a serious error in the amount allocated to that area, but cannot have an appreciable effect on fund allocation to the other areas (since the amount of funds going to the area is small in any event), while, for the larger areas, even a small relative error can involve a substantial sum of money and thus lead to inequities in the allocations to all areas when the total amount to be allocated to all subdivisions is a fixed sum.

The technique of proportional allocation with the overall sample set to give a predetermined maximum relative error for an individual area has some limitations. For example, the amount budgeted for the survey may not permit a sample large enough to achieve the predetermined maximum relative error. An alternative is to use proportional sampling for larger areas but to take a sample sufficient to achieve the maximum relative error limit for the smaller areas. Further discussion of these issues may be found in Appendix B-3, "Some Considerations in Designing Samples to Obtain Data for Use in Allocation Formulas."

Finally, there is no such thing as an ideal formula or ideal data. Therefore, one may have to sacrifice something in the formula and something in the data in order to reach a reasonable compromise between an ideal formula with poor data or a poor formula with ideal data.

CHAPTER V

Subcommittee Recommendations

In Chapter II of this report, a number of causes were identified that contributed to a phenomenon encountered in the five case studies—that "existing allocation formula techniques do not fully achieve the stated objectives of Federal programs." Our review identified problems of formula structure and constraints, problems of implicit and explicit assumptions, problems arising from the data used, and some effects of the interaction of formulas and data. In Chapter III the Subcommittee has presented some general findings on the basis of the five case studies and in Chapter IV has identified some specific ways to reduce allocation errors and inequities. On the basis of these general and specific findings, the Subcommittee has formulated the following set of recommendations to improve the Federal process for specifying and administering the formula aspects of grant-in-aid programs, for dealing with statistical considerations in formula construction, and for relating programmatic measures to ongoing statistical series.

The Subcommittee recognized in its review of the five case studies that there were pervasive problems in the obsolescence of key data, particularly where decennial census data were required to be used, and in the choice of statistics to represent small geographic areas. The Subcommittee feels that it is quite important to recognize these elements as important problems early in the program design process so that sufficient attention can be devoted to the generation of at least partially satisfactory solutions. The specific recommendations on these points are as follows:

RECOMMENDATION 1. *Program Goals and Statistics:*

That program goals be specified as clearly and completely as possible in the statement of purpose of each grant-in-aid act and that program drafters guard against over-specification of the statistical data and procedures to be used.

Comment:

Vague specification of program goals and over specification of statistical procedures are common problems. Providing flexibility to program administrators in the choice of statistics for allocation is sometimes desirable for a variety of reasons, but in the absence of reasonably clear and complete goal statements, administrative decisions which involve use of that flexibility will necessarily be arbitrary to some degree, and may run counter to the intent of Congress. The AFDC counts in Title I, ESEA are an example of highly specified statistical procedures written into authorizing legislation. The Education Amendments of 1974 describe with some precision how to determine the number of AFDC children counted for Title I, ESEA funding, which year's poverty standard to use, which of the many poverty cutoffs (nonfarm family of four), and which month's caseload data. What is lacking is a clear statement of what the resulting total is supposed to represent.

The Subcommittee has recognized in its review of the five case studies that some Federal programs have an extensive list of specific purposes and amount to a form of special revenue sharing, or are directed toward some broad categorical objective in, say, education or community development. The Subcommittee does not expect legislative drafters to alter markedly the kind of purposes set forth in future allocation legislation, but rather to recognize the problem of translating such statements of purpose into programmatic measures. If goal statements can be made clear then there will be less necessity to build into legislation in rigid form the specification of the statistics and techniques to be used. For example, Congress might decide to specify a certain mechanism for allocation to, say, the State level, but might leave to Federal-State negotiations and administrative determination the mechanisms for making allocations to lower levels. It should be recognized that sound, flexible administration depends on clear and distinct statutory goals. When goal statements are not clear, then an administering agency which exercises discretion may be subject both to political pressure and to litigation.

RECOMMENDATION 2. *Legislative-Statistical Interface:*

That provision be made for an active, continuous interface between legislative program drafters and the statistical community.

Comment:

This recommendation by the Subcommittee is motivated in part by a recognition that Recommendation 1 will be most difficult to achieve without

sustained professional interchange between program and statistical staff, both executive and legislative.

RECOMMENDATION 3. *Formula Performance Testing and Monitoring:*

That statistical and program agencies provide to program drafters an analysis of the sensitivity over time of proposed formulas and of the statistics they incorporate so that possible effects on allocations can be anticipated. Also, that provisions be made for testing, monitoring, and assessing by program agencies of the performance of each specific formula or allocation rule prior to enactment.

Comment:

An example of the type of analysis that might be provided, is that given in the Bureau of the Census report "Coverage of Population in the 1970 Census and Some Implications for Public Programs," [9] which describes some possible effects on the distribution of General Revenue Sharing funds of adjusting the 1970 Census of Population for the estimated undercount and for error in income reporting.

Before an allocation procedure is adopted, it should if possible be subjected to a test. In some cases this could be done by using data from prior years to determine whether or not the proposed procedure would have allocated funds for each prior year in accordance with Congressional intent. In cases where data from prior years are not available the testing would have to rely on simulation techniques. It is important that allocations be neither unduly sensitive to short-term fluctuations nor lacking in sensitivity to long-term changes in programmatic measures. Once a program is in place, a built-in monitoring mechanism is needed to provide early warning to the executive branch and the Congress that a particular formula or allocation rule may not be behaving as expected.

RECOMMENDATION 4. *Undesirable Formula Practices:*

That legislative drafters and program designers be advised of data problems and the existence of statistical practices, as exemplified in the five case studies, which may lead to formulas with consequences that are generally recognized as undesirable.

Comment:

CETA allows ASU's (Areas of Substantial Unemployment) considerable freedom in drawing their own boundaries. They need not follow jurisdictional lines. While it may (or may not) be going too far to say political jurisdiction boundaries should be followed, the current procedure may be too free, offering substantial opportunity for drawing boundaries in an artificial way. In addition, ASU's in order to qualify for CETA Title II funds must experience an unemployment rate of 6.5 percent or more for three consecutive months. This specific eligibility cutoff introduces the problem that small errors close to the cutoff of 6.5 percent may have serious effects on the distribution of funds. These two factors substantially complicate the data collection and may lead to possible inequities as well. An alternative might be to base the amount allocated on the difference between the unemployment rate and some lower cutoff, for example 5 percent, arranging the formula so areas above some upper limit point (say eight percent) get the allocation provided by the present formula.

The GRS program distributes funds to approximately 39,000 jurisdictions, the great majority of which are areas of population less than 2,500 in the 1970 Census of Population. For these areas the problems of obtaining intercensal estimates of population and per capita income are very serious.

RECOMMENDATION 5. *Needed Formula Research:*

That a limited program of applied research and development be initiated to attack some critical problems and fill certain identifiable gaps in the present state-of-the-art of formula design.

Comment:

As discussed further in Appendix B-5, "An Agenda for basic and Applied Research on Allocation Formula Problems," the identification and characterization of key technical problem areas involves the following elements: equity considerations, structural aspects and the nature of the data required for the computational formula, performance criteria, presence or absence of constraints and other specification or modeling problems. Furthermore, relevant methodological tools and relevant areas of substantive theory need to be brought together if we are to achieve a coherent approach to allocation problems. Some of the statistical research issues of allocation procedures can be illuminated by theoretical principles from other fields. We need to bring together into a concerted research effort knowledge and tools, not only from theoretical statistics, but also from applied areas such as decision theory, welfare economics, data adjustment techniques, and income measurement procedures.

[9] *Current Population Reports*, P-23, No. 56, August 1975, Washington, D.C.

RECOMMENDATION 6. *Designation of Official Statistics:*

That the Office of Federal Statistical Policy and Standards, with the assistance of the statistical agencies, designate a limited number of additional official statistical series for use in funds allocation. These would be kept as current and as accurate as possible for States and for local areas.

Comment:

Official statistics presently designated in the Directives for the conduct of Federal statistics are:

(1) Directive No. 13, Standard Data Source of Total Population Used in Distributing Federal Benefits, designates as current data on total population those published by the Bureau of the Census in *Current Population Reports,* P-25, P-26, except where data from a decennial census are more current.

(2) Directive No. 11, Standard Data Source for Statistical Estimates of Labor Force and Unemployment, specifies that the Federal executive branch departments, agencies, and establishments shall use the most current national, State, or local area labor force or unemployment data published by the Bureau of Labor Statistics. These data shall be used for all program purposes including the determination of eligibility for and/or the allocation of Federal resources.

(3) Directive No. 14, Definition of Poverty for Statistical Purposes, designates the poverty statistics published in Census Series P-60 as official. This series is frequently designated as the series to be used in allocation formulas as a proxy for economic deprivation.

(4) Other general-purpose statistics are now only available in fine geographic detail at each decennial census (and prospectively at the planned mid-decade census).

RECOMMENDATION 7. *Data Comparability:*

That in tiered allocation programs comparable data should be used for allocation to States, but policy flexibility may be allowed for sub-State allocations. When the Federal Government allows this flexibility it should be subject to the formulation of specific Federal statistical and administrative guidelines, concerning the designation of the responsible governmental unit for choosing among statistical series, for declaring the specific types of statistical series from which such a choice is permitted to be made, and for establishing administrative mechanisms for consideration of appeals from area governments.

Comment:

Unique statistical series may be available in some States that would objectively improve the precision and equity of sub-State allocations and those States should not be penalized simply because other States might not have access to such unique series for their own sub-State areas. At any one level of distribution of funds (for example, to counties within a State) one and only one formula should be used. However, two different States might properly distribute funds to counties using different formulas, and similarly two different counties within the same State might use different formulas for subcounty allocations if the Federal legislation authorizes this flexibility. For example, in Title I ESEA, individual States select the data sources to be used for subcounty allocation to school districts under guidelines established by and with the oversight of the Office of Education.

RECOMMENDATION 8. *Data Accuracy Goal:*

That since data errors are inevitable and since statistical resources are necessarily limited, priority be given to minimizing the very large errors which may occur in data used for the allocation of funds.

Comment:

Data used to distribute funds to competing areas need to be evaluated differently for large areas versus small ones. To the extent that error measurements are available for small geographic areas one should check that relative errors are no greater than a prespecified maximum, but one should not be overconcerned with small errors since their effect on the total distribution is relatively minor. For a large area a relatively small error may represent a substantial absolute error and have a large impact on the total amount of funds distributed, and therefore it is necessary to keep the absolute errors to a minimum. In the case of administrative record data, edits and cross checks should be applied to surface major changes in the relative distributions, but efforts to develop better methods of measuring the errors not detected by these means should continue (e.g., matching studies such as the completeness of birth registration studies).

RECOMMENDATION 9. *Eligibility Cutoffs:*

That, to minimize the effects of data errors, eligibility cutoffs be such that there is a gradual transition from receiving no allocation to receiving the full formula amount.

Comment:

CETA Title II provides that an area must experience an unemployment rate of 6.5 percent or more for three consecutive months to qualify for benefits under this title. When an area has an unemployment rate near 6.5 percent, data errors will frequently lead to its being wrongly classified as eligible or ineligible (with substantial sums turning on the classification). Recognizing this problem, when CETA Title VI was added, it provided that part of the amounts allocated under the title be based on the difference between the unemployment rate and a lower cutoff of 4.5 percent. Areas with an unemployment rate above 4.5 percent would receive some funds, the amount received being proportional to the amount of unemployment in excess of 4.5 percent. Note that the policy underlying both titles is the same, namely to make minimal allocations to areas with relatively low unemployment rates and use the money saved on these areas for the areas with high unemployment rates. However, Title II allocations magnify the effects of small data errors in the neighborhood of the desired cutoff, while the errors in Title VI allocations are proportional to the errors in the data.

* * * * * *

Much has been learned about generating and implementing formula-based grant-in-aid programs at the Federal level. No allocation procedure, we now know, can come very close to an ideal. The diversity of local conditions, and the limited amount of data reflecting these conditions, prevent a tight match between a theoretical model of what a program is intended to accomplish, and a practical rule for giving out the funds. But though the programs we studied are far from ideal, they are also far from unacceptable. Much imaginative work has already gone into program design. If the unresolved problems are addressed seriously, we can expect considerable improvement in the future.

The General Revenue Sharing (GRS) Program

prepared by

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Introduction

The General Revenue Sharing (GRS) Program allocates funds in sequence to States, county areas, Indian Tribal Councils and Alaska Native Villages, county governments, townships and other local governments. Basically, however, the revenue sharing allotments are derived at two levels, the State level and the sub-State level. Each level has a separate formula.

At the State level there are five possible factors which are used in the allocation of funds:

1. Population
2. Per capita total money income
3. Personal income
4. Urbanized population
5. State income tax collections.

These factors interact in what is referred to as 3-factor and 5-factor formula to determine the State allocation. The effective formula varies from State to State determined by which of the two yields the higher allotment.

In the sub-State allocation, only a 3-factor formula is used. The data elements are:

1. Total population
2. Per capita total money income
3. Adjusted taxes (tax revenues excluding those earmarked for education).

The sub-State allocation procedure also includes three constraints:

1. No unit below the county level may receive less than 20 percent of the State average per capita payment.
2. No sub-county unit may receive more than 145 percent of the State average per capita payment.
3. No unit may receive more money in revenue sharing funds than 50 percent of its adjusted taxes plus intergovernmental transfers.

Moreover, the Secretary of the Treasury has, at his discretion, the ability to use other source data provided he determines that such data will provide for equitable allocation.

The interaction of the two sets of factors at the State level and the constraints at the sub-State level make the formula specified for revenue sharing indeterminant in the sense that no exact equation may be written for its operation. Rather, its operation must be simulated and its answer derived through computation until a desired element is reached and the allocation is said to be final.

The objectives of the revenue sharing program are (1) that the size of the allotments should be responsive to relative need in terms of the degree of dependency of the area's constituency on governmental services (determined by how income is distributed among the residents of the jurisdiction); the local fiscal capacity to service the needs of the constituency; and the division of responsibility for provision of services among the various levels of government within each State and (2) that the allocation procedure be equitable.

Per capita (money) income, the variable in the allocation formula which serves as indicator of constituency need or capacity to pay, is subject to error because of underreporting of income and misreporting of income. Since it is a *per capita* measure, it is also affected by the nationwide undercount of Blacks in the census population estimates. There has been a growing concern that the impact is disproportional, resulting in a Net loss of revenue sharing funds to central cities. It is not possible, however, to assess the validity of such a concern since the Bureau of the Census has not yet developed a procedure for allocating the undercount below the national level.

The underreporting of income is well documented. However, there is little known about the geographic distribution of the underreporting and, therefore, its impact cannot be assessed. If the underreporting is proportional throughout the country, then obviously the impact of the allocation of funds would be small. On the other hand, if it is not proportional, then biases in the allocation of funds are likely to result.

The misreporting of income is the reporting of income of one type incorrectly as that of another. Although distorting the type of income, it has no effect on the revenue sharing formula.

Per capita income has also been subjected to criticism related to concept and currency of data. Conceptually, questions have been raised as to the adequacy of per capita income as a measure of need. Areas having the same per capita income, but whose cost of living differ, may vary as to degree of need. Moreover, per capita income does not indicate the type of services needed nor does it indicate the income mix. Finally, income as a measure of well-offness has its limitations.

The problem of currency relates to the fact that data on money income are collected by the Bureau of the Census only in the decennial census of population. In order to update the benchmark, a variety of sources and statistical procedures are used. Wages are updated on the basis of data from the Federal individual income tax returns. Other types of income are updated, at the State and county levels, on the basis of special money income estimates made by the Bureau of Economic Analysis (by adjusting its estimates of personal income, by type of payment, to conform to the money income concept). Below the county level, the updating procedures are more tenuous since there are practically no relevant income data for small areas.

Fiscal capacity in the allocation formula is measured in part by aggregate personal income. This measure also has its limitations since it does not reflect the additional tax base of those jurisdictions that levy taxes on nonresidents (such as tourists, corporations, and commuters).

The fiscal capacity of local governments is measured, in the 3-factor allocation formula, by adjusted taxes. By ignoring assessments and user charges, local variations in sources of revenue are not reflected. The smaller and newer municipalities tend to rely upon current charges as a source of revenue, relative to taxes, to a greater extent than do the larger and older cities.

The formula structure itself fails to reflect variations among States in relation to taxing authority and to responsibility for services by level of government and discourages States from taking over some local government responsibilities even though a more centralized provision of some services would be more economical.

The tiered structure of the allocation procedures, so designed that there are different allocation formulas for the several sub-county area levels of government (county governments, townships, and all other units of local government), tends to lessen the likelihood of equitable allotment of funds among subcounty area governments because of the lack of reliable data for small areas.

The constraints built into the allocation procedure also affect the equitableness of the allocation. The 20 percent rule tends to keep alive some essentially dormant governments. Moreover, it tends to increase the allotments of many of the relatively high income suburbs. The 145 percent rule, on the other hand, puts a limit on money that areas receive even though they are in need of the money. The 50 percent budget limit tends to reduce the size of the allotment to rural areas.

Finally, fluctuations in the amount of the allotment from one entitlement period to the next, engendered by the sometimes large, unusual variations in the data inputs into the allocation formulas, tend to influence the fiscal policies of local governments. The uncertainty caused by these fluctuations undermines the recipient government's ability to plan for the efficient use of the Federal funds and to accomplish the implied goals of the revenue sharing program. They tend to deter the inauguration of new programs and, instead, encourage the use of the allotments for capital expenditures.

Modifications to the General Revenue Sharing formulas that have been suggested include:

1. Eliminating or modifying the 20 percent and 145 percent per capita limits and the budget constraint.
2. The inclusion of current charges as eligible revenue in measuring fiscal capacity.
3. Modifying the tier structure so that allotments for small areas are not based on inadequate or unavailable data.
4. Addition of percent of families below poverty level to the 3-factor formula.
5. Setting limits on the amount of fluctuations in the size of allotment from one entitlement period to the next to encourage fiscal planning in keeping with the goals of the program.

Other suggested alternatives such as including a cost-of-living index in the allocation formula, substituting taxable property values for aggregate income as a measure of fiscal capacity, or including a measure of wealth as an indicator of well-offness are hampered by the problems of availability and validity of data.

Since the revenue sharing program has been in

operation for a number of years, changes in formula or allocation procedures that produce moderate improvements in equity and responsiveness to need would be more acceptable than those that would radically alter the distribution. The latter could be more disruptive than the current problems caused by fluctations of allotments between entitlement periods.

The Office of Revenue Sharing in the U.S. Department of the Treasury, through its experiences in implementing the General Revenue Sharing Program, has offered several suggestions that should be considered in the development of future programs that involve the distribution of Federal funds with regard to data that are specified in the program for use as the basis for allocation:

1. Data upon which funds are to be allocated should be comparable and readily available for all participants (eligible recipients) in the program prior to its inauguration. The use of data from different statistical sources that purport to measure the same factor often results in irreconcilable biases for or against some of the participants.
2. When drafting a program, comments on the availability and quality of data required should be solicited from the statistical agency that will have the responsibility of providing the data, prior to the enactment of the legislation. Data specified in a statute may not be the most comprehensive available to accomplish the goals of the program, and statistical methodogy cannot overcome such deficiences.
3. The data and statistical methodologies used in a program should be generally understood and accepted by both administrators and participants. This would engender wide acceptance of the program itself as well as confidence in the individual allocation of funds.

The Comprehensive Employment and Training Act (CETA)

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Introduction

The Comprehensive Employment and Training Act of 1973 was enacted "to provide job training and employment opportunities for economically disadvantaged, unemployed, and underemployed persons, and to assure that training and other services lead to maximum employment opportunities and enhance self-sufficiency by establishing a flexible and decentralized system of Federal, State and local programs."

The approach embodied in CETA represents a vast shift in administrative procedures for Federal attempts to solve and anticipate manpower problems. In the preceding 12 years the Federal Government gained considerable experience in dealing with the specific problems of employment and underemployment, with the economically disadvantaged persons and with different target groups such as youth and minorities. Each of these was treated under different legislative authorities involving several Federal agencies. These programs were funded, administered, and planned directly by the Federal Government. State and local governments had little or no decisionmaking power. As a result, this patchwork of programs began to overlap, were seldom coordinated, and began to be viewed as an inefficient use of public funds.

Several years of debate in Congress resulted in a significant change in the orientation of Federal manpower programs. The primary goal—Federal committment to improving the earnings and employability via manpower services—did not change with CETA. The major change was to provide, for the first time, for the meaningful involvement of State and local elected officials in the analysis and design of programs to meet the needs of their populations and job markets. The law is predicated on the belief that State and local governments can be more responsive to the particular problems facing individual areas of the country. It was also felt that local governments can be held more accountable to the people of the area than can the Federal Government. While striving to meet national goals, Federal attempts often failed to deal effectively with the unique local manifestations of manpower problems. Under CETA, therefore, the basic responsibility for planning, administering and evaluating programs is placed on the State and local governments, under broad Federal direction.

This is accomplished through decategorized funding, that is, Federal funds are transferred to prime sponsors with only general guidelines as to how to spend them. Prime sponsors are generally defined as State and local governments, or combination of governments called consortia, having 100,000 population or more.

The second major thrust of CETA was to consolidate most of the growing number of Labor Department programs under one roof. Thus, rather than a large number of sometime overlapping contracts from separate Federal agencies confusing control and administration, a small number of block grants to State and local governments leave officials relatively free to institute a coordinated set of programs for greater effectiveness in solving local problems.

Provisions

CETA is made up of seven titles, of which only I, II, and VI are relevant for discussion here.

Title I is entitled "Comprehensive Manpower Services." The primary focus is to institute and administer employment and training programs. Prime sponsors must submit plans to the Secretary of Labor for review and acceptance. These plans must explain in detail the programs and services to be provided and indicate how these will work to solve the problems of the area.

Upon acceptance of the plans, funds are allocated by a formula whose effect is to avoid great fluctuations in the level of funding each prime sponsor receives from year to year. Eighty percent of Title I

funds are allocated according to the following formula: 50 percent of the funds are allocated according to the relative share of the prime sponsor's previous year's funds; 37.5 percent are allocated according to the relative share of U.S. unemployment; and 12.5 percent according to the relative number of adults in low income families.

Thus, half of the formula provides a measure of stability. This can be important if sponsors use part of their funds to institute long-term programs. Additional provisions state that no prime sponsor receive less than 90 percent nor more than 150 percent of the previous year's level of funds.

Of the remaining 20 percent, 6 percent is for the Secretary of Labor to use at his discretion, but first for ensuring that no sponsor has received less than 90 percent of his prior year grant; 5 percent is for consortium incentives (the bulk of this is not needed for consortia; the excess reverts to the Secretary as discretionary funds); 5 percent is for supplemental vocational education grants to governors; and 4 percent is for governors to use at their discretion.

Title II, entitled "Public Employment Programs," is "to provide unemployed and underemployed persons with transitional employment in jobs providing needed public services in areas of substantial unemployment and, where feasible, related training and manpower services to enable such persons to move into employment or training not supported under this title."

All funds under Title II are allocated to areas of substantial unemployment (ASU), which are defined by law as areas experiencing an unemployment rate of 6.5 percent or more for three consecutive months. The Employment and Training Administration (ETA) has interpreted the definition of an ASU to include any area (including Census tracts) which has a population of 10,000 or more. Eighty percent of the funds are allocated according to the number of unemployed in each area compared to the number of unemployed in all such areas, and 20 percent are allocated at the discretion of the Secretary of Labor taking into account the severity of unemployment.

Title II is aimed at aiding areas with high unemployment by providing funds to hire unemployed and underemployed persons in temporary public service jobs.

The formula has raised some controversy. First of all, there is some question as to whether there is sufficient weight given to degrees of unemployment greater than 6.5 percent. Thus, depending on population size, an area of 8 or 10 percent unemployment may not receive substantially greater funding than an area of 6.5 percent unemployment.

The three-month period for defining ASU's is another problem in that it allocates funds by relying on a factor subject to seasonal fluctuations.

Title VI, added to CETA by the Emergency Jobs and Unemployment Assistance Act, is designed to react quickly to cyclical unemployment through creating temporary public service jobs. For this purpose, the allocation formula, covering 90 percent of Title VI funds, is based entirely on unemployment: 50 percent are alloted based on total unemployment in the prime sponsor area; 25 percent based on unemployment in ASU's; and 25 percent generally on the basis of unemployment in the sponsor area above 4.5 percent. The remaining 10 percent of funds under Title VI are reserved for discretionary use of the Secretary of Labor.

Statistics Used in the Allocation Formula

Unemployment (Titles I, II and VI)

The monthly State and local area unemployment estimates are the product of a Federal-State cooperative program in which State employment security agencies prepare labor force and unemployment estimates using concepts, definitions and technical procedures established by the Bureau of Labor Statistics. The estimates are developed from a complex formula which makes use of unemployment insurance (UI) data and certain assumed statistical relationships between the unemployed covered by UI and those not eligible to receive benefits. In the 10 largest States, New York City and the Los Angeles–Long Beach, California SMSA unemployment estimates are obtained from the monthly Current Population Survey. In the remaining 40 States and the District of Columbia the preliminary estimates derived from UI data are benchmarked to annual data derived from the Current Population Survey. The benchmark procedure also provides bias adjustment factors for use in extrapolating the estimates to the current month.

The estimates which are derived from this program are of varying quality. The variation reflects the fact that the underlying UI data which are assembled as a byproduct of the operations are not yet standardized for statistical use.

In addition, the unemployment estimates for portions of labor market areas are often based on fixed decennial census relationships which may change over

time. The Bureau of Labor Statistics has made an assessment of the quality of the estimates and has grouped the areas into three broad categories. The best data are for States and SMSA's for which there are independent estimates for both employment and unemployment components, and for which on the employment side there are BLS approved estimates of nonagricultural employment. Within this group, the greatest confidence can be placed in those States which are independently benchmarked to the Current Population Survey or for which monthly CPS data are used directly.

The second group, in which less confidence can be placed, consists of individual cities and counties that are frequently parts of labor market areas for which estimates are derived synthetically by using UI claims and current population estimates or fixed census ratios. CETA prime sponsors (Title I) and program agents (Title II) are included in this group. Also included in this group are cities and counties that are not SMSA's for which independent estimates may be prepared, but for which there is no BLS approved nonagricultural wage and salary employment estimate.

The third group, in which the least confidence can be placed, consists of smaller cities and counties and the estimate for areas that are parts of other geopolitical units. Areas of substantial unemployment (Title II of CETA) are often part of this category.

The Bureau of Labor Statistics has taken steps to improve the methods to be used in the preparation of the unemployment estimates and to assist the States to upgrade the quality of the UI data available for use in developing the estimates. In addition, as of February 1977 the BLS (under contract with the Bureau of the Census) has expanded the CPS sample to cover all States.

Low Income Data

The method for determining the relative number of adults in low income families for Title I allocations was changed by the Employment and Training Administration in 1975 and used in FY 1976 allocations. The low income definition was raised from the $7000 used in 1970 to $8000 in 1973 to reflect the change in the Consumer Price Index over the same period, as mandated in the CETA legislation. The use of 1973 data in FY 1976 allocations was due to the delay in obtaining the data.

CPS data for March 1970 and March 1974 (reporting 1973 income data) provided an estimate of the change in the number of low income adults in each State for the 19 States where the CPS sample was considered adequate by the Bureau of Labor Statistics for 1973. For other States, the overall regional change, for census regions, was used. Prime sponsor and consortium component estimates were developed from State totals using a census-share method.

The actual calculation process is as follows for each State: The CPS derived proportion of all adults (18 and over) with low family income for 1973 is divided by the 1970 proportion. The resultant factor is multiplied by the 1970 census proportion of low income adults to obtain an updated proportion. Then, this figure is multiplied by the 1974 census population estimate of all adults in each State resulting in a numerical estimate of low income adults incorporating the most current estimate of population.

Within-State relationships are maintained for prime sponsor estimates by the census-share method using 1970 and 1973 CPS data. The resulting State estimates are modified by controlling regional estimates to the national total and controlling State estimates to the regional totals.

Since these data are based on the most recent March CPS, they provide adequate reliability for national and regional estimates. The CPS is also reasonably reliable for the 19 larger States separately, but even there the data are subject to considerable sampling variability. The overall regional change is assumed to be applicable for each of the smaller States within a region and is used as a basis for deriving their current estimates. The procedure for the smaller States involves an assumption of homogeneity and may be less reliable than the estimates for the larger States. It should also be noted that by comparison with estimates developed from the National Income Accounts, the CPS survey based estimate of aggregate income (as do all survey based estimates of aggregate income) tend to be underreported overall; nonwage and nonsalary income tends to be an income more underreported than wages and salaries.

The Authorization and Allocation of Funds Under Title I, Elementary and Secondary Education Act (ESEA)

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Objectives of Title I, ESEA

Title I of the Elementary and Secondary Education Act of 1965 established the major program of Federal aid for elementary and secondary schools. It provides funds to local school authorities for the establishment of special programs to help educationally deprived children. The law requires that local school authorities assess the special needs of their educationally deprived children and that they design programs to meet those needs with Title I funds. The local authorities submit applications for funds which are reviewed by State educational agencies; programs which are approved are then monitored and evaluated by the State agencies. Because of the emphasis on local response to individual needs, a great variety of programs are funded with Title I aid. Most of the assistance is concentrated on improving basic skills such as reading, writing and arithmetic. School districts however, also fund science and social science programs, cultural activities and other programs designed to meet the health, psychological and nutritional needs of educationally deprived children.

Annual appropriations under Title I increased from about $1.0 billion in 1966 to about $2.3 billion in 1978. About 6 million children were served by Title I programs in 1974 and annual appropriations in that year were about $1.8 billion. Therefore, in 1974, Federal assistance under Title I amounted to about $300 per child. This amount is small relative to the average expenditures per pupil; but, it is significant in the poor school districts where expenditures per pupil tend to be quite low.

The Elementary and Secondary Education Act was originally conceived by Congress as an antipoverty program designed to help educationally disadvantaged children in low income areas. The major instrument for achieving this objective was Title I of this Act.[1]

Development of Title I Formula to Meet Objectives

The clear intent of Title I was to distribute substantial Federal aid to school districts which were too poor to provide adequate educational programs on their own.[2] To achieve this objective, Congress developed a formula for authorizing funds to counties. Authorizations were made to counties because adequate data were not available at the school district level. This formula defined the eligible population and the payment rate. In 1965, two groups were included in the eligible population: (1) all children in families with incomes under $2,000 in the 1960 census; and (2) all children in families with AFDC payments of $2,000 or more. The payment rate was set at 50 percent of the State expenditure per pupil or 50 percent of the national average expenditure per pupil, whichever was higher. The following formula describes these Title I authorizations:

$$G_{ij} = .5D_j(P_{ij} + A_{ij})$$
$$D_i = \text{Max}\,[N, Q_i]$$

[1] "Title I can be considered as another very potent instrument to be used in the eradication of poverty and its effects. Under Title I of this legislation the schools will become a vital factor in breaking the poverty cycle by providing full educational opportunity to every child regardless of economic background. The major thrust of this legislation is contained in Title I where it is proposed that approximately $1.06 billion be provided to local school districts for the purpose of broadening and strengthening public school programs in the schools where there are concentrations of educationally disadvantaged children." Quoted from U.S. Congress, House of Representatives, *Elementary and Secondary Education Act of 1965*, Report Number 146, 89th Congress, 1st Session, April 6, 1965, p. 5.

[2] U.S. Congress, House of Representatives, *Elementary and Secondary Amendments of 1974*, Report Number 93-805, 93rd Congress, 2nd Session, p. 5.

Where:

- i = Suffix denoting State within United States
- j = Suffix denoting county within State
- A_{ij} = Children in families with AFDC payments >$2,000
- D_i = Per pupil expenditure for State i used in authorization
- G_{ij} = Authorization to local education agency
- N = National per pupil expenditure
- P_{ij} = Children in families with income <$2,000 (1960 census)
- Q_i = State per pupil expenditure

When the Title I formula was prepared in 1965 (and even at present) the decennial census was regarded as the best sources for estimating the count of poor children in each county. This is the only source which provides income distributions for the entire population for small geographic areas throughout the country. The major shortcoming of these data is that they are available only at the beginning of each decade. If they are to be used for this purpose, therefore, a procedure must be developed for updating them periodically. "That update," according to a congressional report prepared in 1974, "was written into the original law as the portion of the formula which counts AFDC children."[3]

If the cost of providing educational services were the same throughout the country, the count of poor children alone would have provided an adequate basis for allowing Title I funds to counties. These costs, however, do vary considerably. In the interest of equity, Congress decided to adjust the payments to reflect differences in the cost of providing educational services. The current expenditures per pupil (CEPP) in each State were used for this purpose. There is no explicit statement that Congress had this in mind when it adopted the use of CEPP in the allocation formula. This conclusion however, can be inferred from the congressional report for 1974. In discussing a change in the payment rate, this report states that payments based on CEPP "reflect much more accurately the differences in providing compensatory education throughout the country."[4] Presumably, therefore, both the original and the revised payment rates were intended to adjust Title I payments for differences in the cost of providing educational services.

After several years, Congress found that both key elements on the allocation formula—the count of poor children and the payment rate—were not working according to expectations. The count of children in AFDC families with grants above $2,000 proved to be a very poor substitute for the actual count of all poor children in each school district. As a result of sharp increases in AFDC payments and mushrooming caseloads the number of AFDC children counted under the formula increased very rapidly, whereas the count of poor children based on the 1960 census remained fixed. As a result, the AFDC component of the eligible population increased from 10 percent of the total in 1966 to over 60 percent in 1974.[5] These changes did not occur uniformly throughout the country. AFDC payments tend to be highest in the large, high-income, urban States and these States also contain most of the children in families with AFDC payments above $2,000. As a result, these States made the greatest gains in the number of eligible children to be counted under the formula. In 1965, for example New York had 5.4 percent of all the children in the Nation eligible to be counted under the Title I formula. By 1972, this proportion more than doubled to 13.4 percent. Similar changes took place in California and New Jersey. The greatest relative losses in eligible population were in States with low AFDC payments. Most of these States are in the South where reductions of 50 percent in the eligible population were typical. After reviewing similar data, Congress concluded "clearly, the present Title I formula, because of its great reliance on AFDC statistics, has become skewed heavily in favor of the wealthier States in the Country. That result is completely contrary to one of the principal purposes of Title I: To provide assistance to school districts and States whose ability to operate adequate educational programs is impaired by concentrations of low-income families."[6]

As noted above, in the interest of equity, Congress decided to adjust the payments to each State to reflect differences in the cost of providing education. Under the formula adopted in 1965, counties were eligible to receive either one-half of the State or national average expenditure per pupil, whichever was higher for each State. Although the national average was used as the minimum payment rate, no upper limit was set on the amount each county could receive. After several years of operation, Congress decided that this aspect of the formula "also contributed to a distortion in the distribution of Title I funds among States."[7] Particularly onerous was

[3] *Ibid.*, p. 9.
[4] *Ibid.*, p. 13.
[5] *Ibid.*, p. 9.
[6] *Ibid.*, p. 11.
[7] *Ibid.*, p. 13.

the large amount received by New York under this formula. The congressional report for 1974 points out that New York was eligible to receive $772 per child as compared to $465 per child for California and it concludes that "there are few who would contend that it costs that much less to live in California than it does in a similar area in New York."[8] As a result, Congress decided to change the payment rate in such a way as to bring the payment rate among States closer to the national average.

In 1974, the present authorization formula was adopted after considerable debate. An attempt was made to correct some of the more important defects in the earlier formula; but, the basic allocation procedure remained much the same. The eligible population was redefined to include the following three groups: (1) children 5-17 in poor families as defined in the 1970 census,[9] (2) two-thirds of the children in families receiving AFDC payments which exceeded the poverty line; and (3) children residing in institutions for neglected and delinquent children and children in foster homes supported with public funds.

The payment rate was also revised. The new payment rate is based on 40 percent of the State current expenditure per pupil. The minimum was set at 40 percent of 80 percent (i.e., about one-third) of the national average expenditure per pupil and the maximum rate was set at 40 percent of 120 percent (i.e., about one-half) of the national average. In addition, each county was guaranteed an allotment of at least 85 percent of the allotment received the preceding year, a provision referred to as the "floor" and as the "hold harmless provision."

The Rule for Authorizing and Allocating Funds to States and Counties Under Title I

The following formulas describe the current authorization and allocation procedures.

Define as follows:

- i = Subscript denoting State within United States
- j = Subscript denoting county within State
- k = Subscript denoting number of iterations in the ratable reduction
- A_{ij} = Eligible AFDC population
- B = Minimum administration allowance for a State
- C_i = Authorization for children in State operated institutions, such as mentally retarded, delinquent, etc.
- D_i = Per pupil expenditure for State i used in authorization formula
- E_{ij} = Total number of eligible children
- F_c = County "floor" percent
- G_{ij} = Authorization for grant to local education agencies
- H_i = Previous year's authorization for children in State operated institutions
- L_{ij} = Previous year's allotment for grants to local education agencies
- N = Per pupil expenditure for Nation
- O_{ij} = Other eligible population, such as children residing in institutions for neglected and delinquent children and children in foster homes
- P_{ij} = Poverty population
- Q_i = Per pupil expenditure for State
- R_A = Ratio for eligible AFDC population, currently set at 0.67
- R_B = Ratio for administrative costs, currently set at 0.01
- R_O = Ratio for eligible other population, currently set at 1.0
- R_p = Ratio for poverty population, currently set at 1.0
- S_i = Eligible population in State operated institutions
- T = Total funding available.
- U_{ik} = Allocation for administration
- V_i = Authorization for administration
- W_k = Reduction ratio for the Nation
- X_{ijk} = Intermediate allocation amount used to compute Y_{ijk}
- Y_{ijk} = Allocation for grants to local education agency

$$(1)\quad E_{ij} = R_p P_{ij} + R_A A_{ij} + R_o O_{ij}$$

$$(2)\quad D_i = \text{Med}\,[.4(.8)N,\ .4Q_i,\ .4(1.2)N]$$

$$(3)\quad G_{ij} = \text{Max}\,[F_c L_{ij},\ E_{ij} D_{ij}]$$

$$(4)\quad C_i = \text{Max}\,[S_i D_i,\ H_i]$$

$$(5)\quad V_i = \text{Max}\,[B,\ R_B(C_i + \sum_j G_{ij})]$$

However, Title I has never been fully funded and therefore:

$$\sum_i \sum_j G_{ij} + \sum_i V_i > T$$

[8] *Ibid.*, p. 13.

[9] The measure of poverty used in the authorization formula was originally developed by Mollie Orshansky of the Social Security Administration in 1964. The measure is built around the Department of Agriculture's economy food plan of 1961 and the national average ratio of family food expenditures to total family after-tax income as measured in the 1955 Household Food Consumption Survey. The measure consists of 124 separate poverty cutoffs differentiating families by size, number of children, age and sex of head, and farm or nonfarm residence.

The following ratable reduction procedure is then followed:

First ratable reduction

$$W_1 = T/(\sum_i \sum_j G_{ij} + \sum_i V_i)$$

$$X_{ij1} = W_1 G_{ij}$$

$$Y_{ij1} = \text{Max}\,[F_e L_{ij}, X_{ij1}]$$

$$U_{i1} = \text{Max}\,[B, R_B(C_i + \sum_j Y_{ij1})]$$

If

$$\sum_i \sum_j Y_{ij1} + \sum_i U_{i1} - T > 0$$

The ratable reduction continues as follows:

Second and subsequent ratable reductions

For $k = 2, 3, \text{---------}, n$

$$W_k = T/(\sum_i \sum_j Y_{ijk-1} + U_{ik-1})$$

$$X_{ijk} = W_k Y_{ijk-1}$$

$$Y_{ijk} = \text{Max}\,[F_e L_{ij}, X_{ijk}]$$

$$U_{ik} = \text{Max}\,[B, R_B(C_i + \sum_j Y_{ijk})]$$

This is repeated until on the nth iteration:

$$\sum_i \sum_j Y_{ijn} + \sum_i U_{in} - T = 0$$

Examination of the Impact of Alternative Authorization Procedures and Alternative Poverty Populations

In 1974, when Congress became concerned that Title I funds were not being fairly distributed, they mandated under Public Law 93-380 that an examination be made of the impact on the allocation of Title I funds of (1) a change in the poverty definition; and (2) an updating of the 1970 census estimates of the number of children in poverty.

An analysis of the impact of changing the poverty definition [10] was carried out by calculating the allocation of $1.5 billion in Title I funds in 1975 under the 13 definitions of poverty defined in Chapter V of the report, *The Measure of Poverty*.[11] (Data were from the one percent sample of the 1970 census.) A concomitant change was made in the AFDC population above the poverty line to reflect the change in the level of the poverty definition. All 13 poverty concepts were tested, and 5 of them are discussed here: the current measure, 125 and 150 percent of the current measure, a single poverty threshold based on half of the national median family income, and a single poverty threshold based on the poverty threshold for a nonfarm family of four. The results for most of the other poverty definitions fall somewhere within the range of the 5 presented here.

There is good reason to be concerned about distributing Title I funds in 1975 on the basis of the 1970 census estimates of the number of poor children in each State. During the past few years, the Nation has suffered a recession which has undoubtedly affected some parts of the country more than others. The current allocation formula assumes that the distribution of poor children by State is the same today as it was in 1970, which is unlikely. To test this assumption, allocation based on the 1970 census estimates were compared with the allocations based on estimates of the number of poor children by State for 1973, the most recent year for which such estimates could be made. Two estimates for 1973 were used: one by the Bureau of the Census and the other by the Bureau of Economic Analysis, both in the Department of Commerce.

With the exception of 1973 BEA estimates, alternative poverty populations are not available at the county level. Therefore, authorization and allocation procedures were performed at the state level. Although the results obtained from State allocations differ from the results obtained from county allocations, the state analysis gives good insight into the effect of using alternative poverty populations.

In addition, the hold harmless provision was not taken into account in allocating funds because this provision tends to minimize differences in allocations based on alternative poverty population and alternative Title I authorization formulas.

In analyzing the impact of revised poverty definitions and of updating the count, the basic tabulations were performed assuming that the current allocation formula was unchanged. In order to identify separately the effects of various components of the formula, additional tabulations were made to explore the impact of: the hold harmless provision (the 85 percent floor); omitting the AFDC children; and omitting the AFDC children and the CEPP factor.

Chapter VIII of the report, *Measure of Poverty*,

[10] Preliminary analyses from U.S. Department of Health, Education, and Welfare, *The Measure of Poverty, Technical Paper XVI, Implications of Alternative Measures of Poverty on Title I of the Elementary and Secondary Education Act.*

[11] U.S. Department of Health, Education, and Welfare, *the Measure of Poverty, A Report to Congress as Mandated by The Education Amendments of 1974* (Washington, D.C.: U.S. Government Printing Office, 1976).

shows in detail the impact of alternative authorization procedures and alternative poverty populations. The discussion of the general conclusions of that study follows.

Impact of changing the poverty definition

For each of the 50 States and the District of Columbia, a comparison was made of the Title I funds received under the current poverty concept with the funds that would be received if the poverty line were increased by 25 percent or 50 percent, and commensurate changes were made in the number of AFDC children above this new poverty line. A 25 percent increase in the poverty line would produce a reduction in the funds going to several of the largest States and, with a few exceptions, would redistribute these funds to the rest of the country. With few exceptions, the pattern described above would prevail if the poverty line were raised by 50 percent. In most cases the changes resulting from a 50 percent increase are in the same direction, but larger than those resulting from a 25 percent increase.

The largest States would lose funds primarily because as the poverty line is increased, the influence of the AFDC add-on becomes negligible.

The two other alternative poverty populations studied are two different single thresholds: one-half the U.S. median family income in 1969 ($4,795) and the poverty threshold for a nonfarm family of four persons in 1969 ($3,748). If the higher threshold were used, the results would be very similar to those obtained using the current concept. The allocations to only 7 States would differ by more than 10 percent of the present allocation, and most of these differences would be in the smaller States, representing relatively small amount of money. If the lower threshold were used, most of the largest States would have gains in funds, largely at the expense of southern States. The reason for this change, as previously explained, is that the lower poverty line would include more of the AFDC children among the eligible population under the Title I allocation formula. Most of these children live in the larger northern States.

Impact of updating the poverty count

Comparisons were made of the amount of Title I funds each State would receive in 1975 with no change in the authorization formula, with a replacement of the 1970 census estimate of the number of school-age children in poverty with the census estimate for 1973, and with a replacement of the 1970 census estimate with the estimate for 1973 prepared by the Regional Economic Analyses Division (READ).

These comparisons show that the substitution of current estimates of children in poverty for the 1970 census estimates, with few exceptions, transfers funds from the smaller rural States to the larger industrial States. The impact of updating the poverty count is greater than the impact of changing the poverty definition.

Although there are some differences between the Census Bureau and the READ estimates, both sets of data support this conclusion. This change undoubtedly reflects the fact that the slow economic growth experienced in the United States between 1969 and 1973, had a much greater negative impact on the larger industrial States than it had on the smaller ones. As a result, relatively more of the Nation's poor children in 1973 were located in the large States than was the case in 1969.

Impact of changing the authorization formula

Comparisons were made of the amount of Title I funds each State would receive in 1975 under the current authorization formula with the funds that would be received if the authorization formula itself were changed.

If the current authorization formula were replaced with a formula that authorized Title I funds solely on the basis of the number of children in poverty as reported in the 1970 census, most large industrial States would receive a sharp reduction in Title I funds, and most smaller rural States would receive a sharp increase in such funds. This change is due largely to the elimination of current expenditures per pupil from the allocation formula.

Using CEPP to determine funding tends to transfer funds from those States with large proportions of poor children to those that make relatively large expenditures per capita on education.

Of all the factors considered, it appears that the allocation formula itself, and particularly current expenditures per pupil, exerts the greatest impact on the allocation of Title I funds. The greatest change in the allocation of funds among States would take place if the funds were allotted on the basis of the number of children in poverty rather than according to the present formula. If the present formula is retained, an increase in the poverty line would have a relatively minor impact on the allocation of Title I funds; however, an updating of the

number of children in poverty would appreciably increase the funds going to the larger States and would decrease those funds to the smaller States.

Joint impact of changing the poverty definition and updating the poverty count

In the preceding sections attention was focused on the impact of a change in the definition of poverty or an update in the count of poor children. We shall now examine the impact of a joint change in these variables. Title I allotments to each State in 1975, assuming a 25 percent increase in the poverty line and using the 1973 estimated number of poor children were compared with the amounts each State would receive if the current formula were used with the 1969 estimate of poor children.

The change in both variables would, with some important exceptions, have the same impact as that previously described for updating of the poverty count alone. That is, there would be a transfer of funds from the small States to the large ones. New York, an important exception, would have gained considerably from an update of the poverty count alone, but would lose slightly if both variables were changed at the same time, due to the AFDC factor. The gains for the other large States were largely offset by declines in most of the 12 moderately large States and in nearly all of the moderately small States. On the other hand, most of the 13 States with less than 1 million inhabitants would gain as a result of this change; however, these changes are subject to large errors of estimation.

Aid to Families with Dependent Children (AFDC) as a Formula Grant-In-Aid Program

Prepared by

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Introduction

The Aid to Families with Dependent Children program involves a Federal contribution to welfare programs legislated and run by States. It is a variable matching grant program: the Federal Government matches State expenditures at a rate which is higher for States with low per capita income. The Federal contribution to each State depends on the State's caseload, on the State's payment rates, and on the Federal matching rules appropriate to that State.

Because AFDC is aimed at a recipient population, it is only partially appropriate to think of it as an intergovernmental transfer. In other words, in judging the equity of the AFDC formula, it is appropriate to consider interpersonal equity as well as interstate equity. At present, the AFDC recipient in a wealthy State is likely to get far more from both the State and the Federal Government than the AFDC recipient in a poor State. (Since this outcome is largely a matter of State law, it is unclear whether *interstate* "equity" is violated.)[1]

As for our Need, Effort, Capability paradigm, the major issues are:

1. Is it appropriate for the Federal support of a State's payment to an individual to be related to the Effort of the State in which he resides?
2. Does the funding formula take proper account of interstate differences in fiscal capacity?

The Two Formulas

There are two reimbursement formulas in the AFDC program. In general, States may use the one which is more favorable to them in each quarter, although there are certain restrictions concerning how frequently they can shift from one to the other. The regular formula is used by a small number of States with low levels of AFDC payments, while the alternate formula is more generous to high-spending States. The alternate formula reimburses all payments at the Federal Medical Assistance Percentage (FMAP). The regular formula reimburses regular maintenance expenditures differently from foster care, and places a ceiling on reimbursement. For maintenance expenditures, the regular formula reimburses 5/6 of the first $18 (per recipient per month), and the Federal Percentage (FP) of the next $14, with no reimbursement on payments over $32. The regular formula was used, in 1976, in Alabama, Arizona, Georgia, Mississippi, South Carolina, Tennessee, and Texas.

More precisely, the monthly reimbursement under the regular formula is a function of five parameters:

N_M = number of recipients receiving money payments

N_{FC} = number of children receiving foster care

P_M allowable money payments (vendor payments in excess of 10% of total are disallowed)

P_{FC} = payments for foster care

FP = the Federal Percentage (see below)

Reimbursement = 5/6 A + FP (B–A), where A and B are defined as follows:

A is $18 times the money payment caseload up to a ceiling of total payments in both categories. That is, $A = \min (18\, N_M, P_M + P_{FC})$.

B is the total reimbursable. It is the sum of reimbursable money payments and reimbursable foster care payments. Reimbursable money payments are all money payments up to $32 times the caseload in this category. Reimbursable foster care payments are all foster care payments up to $100 times the caseload in this category. That is, B_1 =

[1] This nontechnical use of the term "equity" is meant to reflect notions of fairness which inform congressional debate. It does not imply that existing arrangements, once approved by Congress, can be judged "inequitable."

min (P_M, 32 N_M); B_2 = min (P_{FC}, 100 N_{FC}); $B = B_1 + B_2$.

The Federal Percentage is related inversely to State per capita income, with a floor of 50% and a ceiling of 65%. The Virgin Islands and Puerto Rico have FP's of 50%.

It is computed as follows: Department of Commerce estimates of personal income are divided by census estimates of population to get per capita income estimates. These are averaged over three years. The figures for 1971–73 were used in August 1974 to calculate the rates for the period July 1, 1975 to June 30, 1977 (extended to September 30, 1977). In August 1976, data for 1973–75 were used to calculate rates for October 1, 1977 to September 30, 1979. After the per capita income estimates are averaged, the ratio between the State average PCI and the Federal average PCI is squared. This is multiplied by 50 percent to get the State percentage. The result is subtracted from 100 percent to get the Federal percentage. FP's below 50% are raised to 50 percent. FP's above 65% are lowered to 65 percent.

Example:

Idaho—average PCI = $3864
50 States + DC—average PCI = $4595
Ratio squared = $(3864/4595)^2 = .7072$
State percentage = $.7072 \times 50\% = 35.36\%$
Federal percentage = 64.64%

A State with FP of 50 percent receives a maximum of $22.00 by the regular formula.

A State with FP of 65 percent receives a maximum of $24.10.

The alternate formula for AFDC reimbursement has no maximum. It is also decidedly simpler—reimbursement is total money and foster care payments times the Federal Medical Assistance Percentage.

The Federal Medical Assistance Percentage is similar to the Federal Percentage except:

1. (PCI_s/PCI_n) is multiplied by 45%.
   For Idaho, $.7072 \times 45\% = 31.82\%$.
   FMAP = 100% − 31.82% = 68.18%.
2. The ceiling is 83%. There are no States at the ceiling. Mississippi's FMAP is 78.28%.

Analysis of the Two Formulas

The amount of money received by a State (to give to its residents) depends on (a) reimburseable AFDC payments, and (b) the Federal matching rate.

The State, through law and administrative practice, can strongly influence the total level of AFDC payments (and therefore its own share) by (a) setting eligibility standards—including choosing whether or not to participate in the AFDC-unemployed parent program; (b) adjusting payment standards; and (c) making it easy or difficult for eligibles to get on the AFDC rolls.

The Federal contribution to a State's AFDC program therefore reflects: Need, in the incidence of poverty in the population of potential eligibles; Effort, in the willingness of the State to put people on welfare and to support them generously; population, in that larger States are likely to have more poor individuals; Capability, in that matching rates vary with PCI. The formula does not take into consideration cost-of-living differences or other factors affecting individual need, except insofar as the State considers these factors in its AFDC standards.

The alternate formula uses a wide range of matching rates. In this sense it is progressive. But the absence of a ceiling on reimbursement in this formula turns out to be very important. Despite the difference in matching rates, States with high per capita incomes tend to have higher payment levels than poorer States, and this means that Federal AFDC payments are given to wealthier States out of proportion to their caseload. Note that any formula which rewards State Effort adds a second disadvantage to potential recipients living in low Effort States. This is a problem in all programs that use Effort rewards as incentives. In principle, one might wish to reward Effort unrelated to State fiscal capacity, but not reward Effort which reflects a greater ability to pay. In practice, it is hard to sort these things out.

At any rate, there is not clear justification for setting the floors at 50% in the current formulas. The reimbursement rates under the regular formula are identical for all States at or above mean PCI. Under the alternate formula, the FMAP floor sets rates the same for all States for which

$$(.45)\,(PCI_s/PCI_n)^2 \geq .50$$

$$(PCI_s/PCI_n)^2 \geq 1.11$$

$$(PCI_s/PCI_n) \geq 1.054$$

i.e., all States with PCI greater than 5% above the national average.

Other Questions and Policy Issues

The role of PCI in the reimbursement formulas

1. *Use of lagged values of PCI.* The reason for the legally mandated lag seems to be the desire to set and announce reimbursement rates substantially in advance of the period to which they will apply. How necessary is this to State budget planning?
2. *Use of squared values of PCI.* This tends to exaggerate the impact of statistical errors. If the purpose of squaring is to create a strongly negative relation between PCI and FP, is it preferable to devise a linear formula which would achieve the same effect?
3. *Is PCI the best attainable measure of State Capability?* Should it be modified by the addition of a variable representing those revenue sources available to the State which are not represented in income statistics? Should it be modified by a price deflator? Should it be modified by the elimination of transfer income from the income definition?

Caseload error

1. *What is the appropriate way to measure caseload error?* The current procedure is to review a sample of cases at the shared expense of the State and Federal Governments. The size of the sample varies with the size of the caseload: from 150 to 1200. Approximately 90 percent of the funds go to 27 States with samples of 1200. The number 1200 seems to have been chosen to create a 95 percent confidence interval of .01 around an error rate of .03. True error rates are much higher, so that the actual confidence intervals are much wider and wider still for States with smaller samples. Error rates are computed for eligibility errors (ineligibles mistakenly on the rolls), overpayment errors, underpayment errors, and, effective July 1, 1977, the number wrongly rejected. There is no continuing attempt to estimate the number of eligibles who did not seek assistance.
2. *What is the appropriate penalty for caseload error?* Implicit in the quality control system put into effect by HEW has been that the Federal Government would refuse to reimburse the States for all errors in excess of tolerance limits, and for some percentage of erroneous payments. In effect, reimbursement would be based on a revised estimate of the true AFDC reimbursable level, after subtracting out estimates of erroneous payments. The current procedure intends to penalize States by refusing to reimburse them for the difference between the error (considered to be the lower bound of the 95 percent confidence interval around the estimated error rate) and the tolerance rate. This penalizes more heavily States with small confidence intervals (large sample). There has been some discussion about replacing the lower bound with the point estimate.
3. *What is the appropriate tolerance level for caseload error?* Assuming that the States will be denied reimbursement for errors in excess of tolerance, the Federal Government might set these levels by:
   1. examining analogous Federal programs
   2. examining model State AFDC programs
   3. examining analogous private programs.

   Whatever method we choose, different levels might be set for different demographic groups.

   The current system, which was recently rejected by the Courts, set limits of 3 percent on ineligibility errors and 5 percent on overpayment errors.

   The courts have enjoined HEW from assessing disallowances in 13 States on the grounds that the tolerance levels are capricious, arbitrary and unreasonable.
4. *How should errors other than caseload errors be dealt with?* HEW has an accounting and auditing procedure, which examines State accounts to determine whether inappropriate or excess administrative activities, support services, vendor payments, etc. are charged to AFDC. If so, payment is routinely refused. The States must bear the full cost of any error or malfeasance—at least in principle. After payments are refused, the State has the option of entering a reconciliation process.

The Community Development Block Grant (CDBG) Program

Prepared by

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Introduction

The CDBG program is basically a program that allocates funds to local areas on the basis of a formula, which is based on population, poverty (counted twice), and overcrowded housing.[1] It was designed to replace several categorical programs, such as model cities and urban renewal, and to allow for greater local control over community development funds.

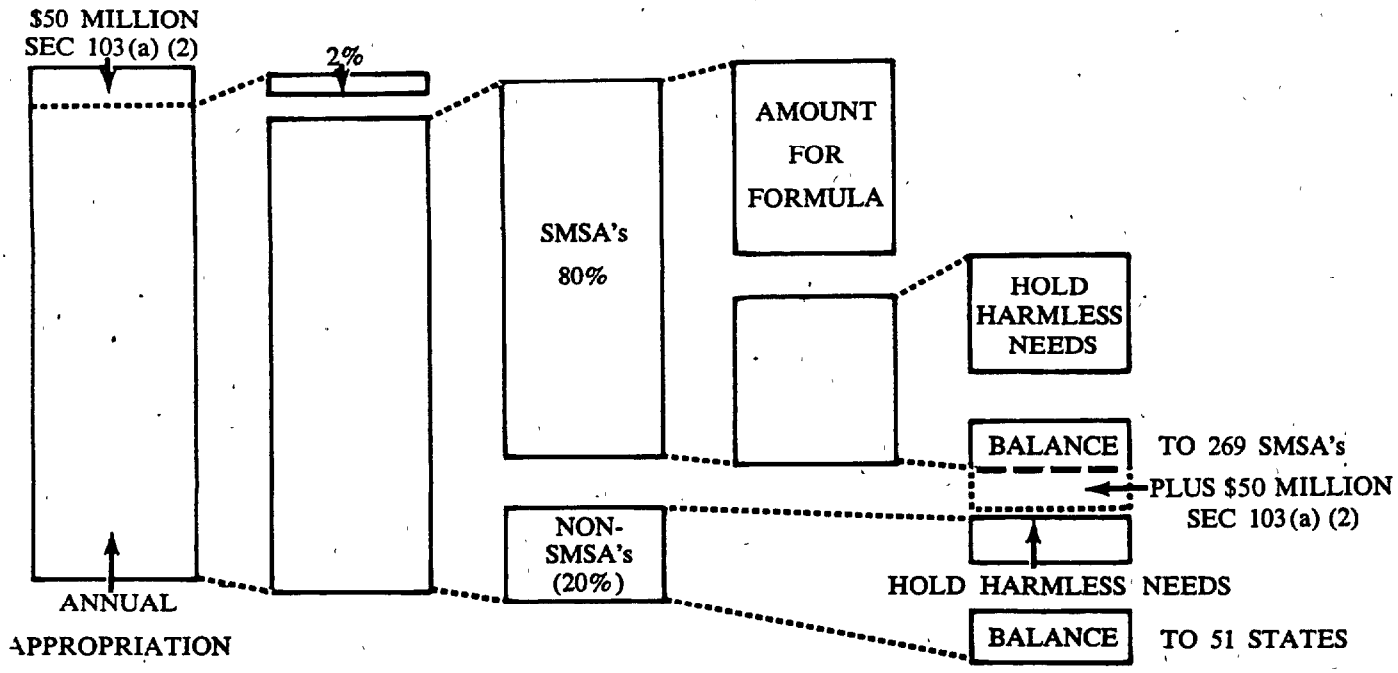
In the words of Section 101 of the Housing and Community Development Act of 1974, "The primary objective of this title is the development of viable urban communities, by providing decent housing and a suitable living environment and expanding economic opportunities, principally for persons of low and moderate income." This is to be achieved through "(1) the elimination of slums and blight and the prevention of blighting influences and the deterioration of property and neighborhood and community facilities of importance to the welfare of the community, principally persons of low and moderate income; (2) the elimination of conditions which are detrimental to health, safety, and public welfare, through code enforcement, demolition, interim rehabilitation assistance, and related activities; (3) the conservation and expansion of the Nation's housing stock in order to provide a decent home and a suitable living environment for all persons, but principally those of low and moderate income; (4) the expansion and improvement of the quantity and quality of community services, principally for persons of low and moderate income, which are essential for sound community development and for the development of viable urban communities; (5) a more rational utilization of land and other natural resources and the better arrangement of residential, commercial, industrial, recreational, and other needed activity centers; (6) the reduction of the isolation of income groups within communities and geographical areas and the promotion of an increase in the diversity and vitality of neighborhoods through the spatial deconcentration of housing opportunities for persons of lower income and the revitalization of deteriorating or deteriorated neighborhoods to attract persons of higher income; and (7) the restoration and preservation of properties of special value for historic, architectural, or esthetic reasons."

"It is also the purpose of this title to further the development of a national urban growth policy by consolidating a number of complex and overlapping programs of financial assistance to communities of varying sizes and needs into a consistent system of Federal aid which (1) provides assistance on an annual basis, with maximum certainty and minimum delay, upon which communities can rely in their planning; (2) encourages community development activities which are consistent with comprehensive local and areawide development planning; (3) furthers achievement of the national housing goal of a decent home and a suitable living environment for every American family; and (4) fosters the undertaking of housing and community development activities in a coordinated and mutually supportive manner." According to the Conference Report accompanying the Act, the Senate version stressed the development of viable urban communities as being the prime objective, while the House version stressed national growth. Both provisions were contained in the final version.

Eighty percent of the overall appropriation for the program (after deductions for certain discretionary funds) is allocated to SMSA's and the remainder goes to nonmetro areas. The sequence of fund-allocation process of the overall appropriation is presented in Figure 1. Since the purpose of this report is to consider the effects of Federal statistics on the

[1] At the time of this writing, Congress was considering a revision in the allocation formula. While the form of the revision has not been determined, it could include other formula elements, such as loss of population or age of the housing stock. Also, the new formula may allow the local government to choose between two separate formulas. The description in this case study refers to the formula as it existed in March 1977, prior to any revisions.

FIGURE 1. SEQUENCE OF FUND ALLOCATION PROCESS



CREATE "SPECIAL" FUND FOR SECRETARY (SECTION 107)	ALLOCATE AMONG BASIC GEOGRAPHIC AREAS	DETERMINE PROPORTION TO BE ALLOCATED TO ENTITLEMENT COMMUNITIES	SATISFY HOLD HARMLESS NEEDS (BALANCES FORM DISCRETIONARY FUNDS)

allocation formulas, matters dealing with discretionary allocations will not be considered in any detail, except perhaps for the hold harmless provision of the Housing and Community Development Act of 1974.

A major adjustment to the formula amounts is the hold harmless (grandfather) allocations and phase-out and phase-in provisions of the program. After the fifth year of the program, however, these provisions will no longer apply, so the formula will bear more of the burden of fund allocation than at present. Generally, an area that made extensive use of the categorical programs replaced by CDBG will have a large hold harmless amount, which is determined by the average amount of grants or loans approved over the period FY 1968 to FY 1972. In brief, if the hold harmless amount is greater than the formula amount, then for three years the area receives the hold harmless amount, and then this is phased down to the formula entitlement over the next three years, in three equal steps. Thus, in the sixth year of the program, hold harmless amounts would have been phased out. If the formula amount is greater than the hold harmless amount, then the area receives the greater of 1/3 of the formula amount *or* the hold harmless amount in the first year, 2/3 of the formula amount (or the hold harmless amount) in the second year, and the formula amount in the third year, and in future years.

The Formula

The formula used at several steps of the allocation process is straightforward:

$$X = \frac{1}{4}\left(\frac{\text{Area population}}{\text{Larger area population}}\right) + \frac{1}{2}\left(\frac{\text{Area poverty count}}{\text{Larger area poverty count}}\right) + \frac{1}{4}\left(\frac{\text{Area overcrowded dwelling units}}{\text{Larger area overcrowded dwelling units}}\right)$$

Allocation of SMSA Funds

Ignoring hold harmless provisions, the metro-city share of the SMSA fund is determined by using the sum of metro-city statistics in the numerators, and the sum of all SMSA statistics in the denominators. Then, each metro city receives its share on the basis of the formula calculated using its statistics in the numerator and the sum of all metro-city statistics in the denominator.

Next, the urban county (plus central city) amount is determined by using the urban county plus metro-city statistics in the numerator, and the SMSA totals in the denominators. Then, the allocation to each urban county is determined by using its statistics in the numerators, and the metro city plus urban county statistics in the denominators. In effect, urban counties are treated as if they were metro cities, and the roundabout procedure above is followed because of statutory requirements.

After SMSA funds are allocated to metro cities and urban counties, the remainder, after allowance for hold harmless allocations, is available for distribution to other parts of the SMSA's on a competitive basis, not unlike the old categorical allocation process, except the proposals are not directed to the narrow categories as before. As it turned out, in Fiscal 1975, far more urban counties qualified for formula grants than had been anticipated, so discretionary funds were limited in the first year. Because of this, Congress appropriated an additional $54 million for discretionary grants to SMSA's.

Allocation of Non-SMSA Funds

Non-metro CDBG funds are allocated to States on the basis of the allocation formula, applied to non-SMSA population, poverty, and overcrowding. Localities within each State compete for these funds on the basis of their applications, so that after the State allocation is determined, the formula has no further bearing on allocation. Of course, as in the cases above, hold harmless provisions apply, but there are no grant entitlements at this level, so this precludes problems of phasing in or out with respect to a block grant entitlement. The phase-out provisions of hold harmless amounts would apply in this case, however.

Statistics Used in the Allocation Formula

The Housing and Community Development Act of 1974 directed HUD to use the most current data available in allocating funds under the CDBG program, and this translates to mean the poverty counts and overcrowding counts are available on an adequate geographic breakdown only in the decennial census of population and housing. On the other hand, population estimates are more current, and 1973 estimates are used in the current allocation. While this is determined by statute, there is still considerable controversy involved in whether statistics from different years ought to be used for the

allocation. On the one hand, central cities are losing population, so with the other statistics unchanged, they automatically lose entitlement funds. The problem is that such cities may be gaining in poverty population compared to the rest of the SMSA's, so a poverty count in a more recent year might leave their allocations unchanged. Existing data limitations would thus seem to suggest reverting to a single year, the census year until other adequate sources become available. This does not solve the problem, however. For one thing, we cannot be sure that the three variables in the formula will continue to move in opposite directions. If they move in the same direction, then an adjustment based on only one of the variables will at least be some improvement. Further, with population growth in certain areas, new metro cities and urban counties may qualify between census years. It would be quite difficult to ask a city to wait ten years for its entitlement, because the other variables in its allocation are not current. However, the following alternative may be suggested to improve the fund allocation.

The data on the three variables might be collected every two or three years and the estimates of the populations representing each variable should be obtained for intervening years. Regression techniques or other appropriate statistical techniques could be used to derive these estimates. Some smoothing of these estimates might be appropriate to avoid large annual changes in entitlements. The main drawback of this alternative is of course a fairly large additional cost of data collection, and a less ambitious approach might be to rely on mid-decade census estimates. On the other hand, local governments may desire to collect data in order to base their entitlements on more recent estimates. This might be allowed, and monitored by a Federal agency (such as HUD, Census, or BLS). Alternatively, local governments could request a special census, paid for by them. In this way, the objectivity of the data can be assured without excessive monitoring and local governments could receive an update, based on a new count.

Meaningfulness of the Formula

One of the goals of the CDBG program was to replace several categorical programs with a single program, simplifying problems of application and red tape on the part of the local jurisdictions, and transferring much of the control over the allocation to local governments. While these goals seem to have been adequately met, this is not germane to the particular formula selected, however, since other formulas might serve as well. When the formula was being debated in Congress, computer runs were available that indicated the probable allocations of funds under alternative weightings of the formula variables. In particular, a single weighting of poverty was considered. The legislative decision at that time was to go with the double weighting of poverty, but HUD was directed to study the formula in detail, and report to the Congress by March 31, 1977. HUD has completed an internal study of the formula, as well as contracting for an evaluation by the Brookings Institution on the topic.

Some issues that were raised are related to the items included in the formula. For example, it is well known that overcrowding does not correlate especially well with overall housing quality. For example, some families with very high income live in "crowded" dwellings according to the definition. Older dwellings may tend to be in need of repair to a greater extent than new, and a variable based on the age of the housing stock may be a better indicator of housing needs than relying totally on the overcrowding measure. It appears that an alteration of the formula might result in a better fit between allocations and goals of the program.

The stated goals of the CDBG program, other than to consolidate and replace existing programs, were to prevent slums and blight, and to conserve and expand the housing stock. Apart from these primary goals, there are many subgoals, including improvement of local services and encouragement of more rational land development patterns. The very generality of these goals leads one to consider whether an examination of the formula itself might provide a better indication of the legislative intent of the program than a study of the stated goals. To pursue this, we could conclude that in the overall context of community development, cities and urban counties with greater population, poverty, and overcrowding somehow need more funds. The population variable would seem to be a measure of need in that it is a close proxy for the housing stock, at least in terms of numbers. The poverty variable, which is of course weighted twice, is a proxy for inadequate housing, and need for community services. The overcrowding variable, already discussed, is another proxy for poor housing, but poor in the sense that there are not enough physical units, whereas the poverty proxy might measure another

aspect, inadequate housing due to the inability to afford well-maintained housing.

Conclusion

In one sense, the CDBG program is relatively simple in that it allocates funds on the basis of needs without consideration of local effort. The formula itself is very simple, with complexity arising only because of repeated applications of the formula, and the impact of hold harmless and phase-in and phase-out rules. Since the goals of the program are difficult to define in operational terms, it is difficult to evaluate the effectiveness of the formula in meeting the goals of the program. A development of a better proxy or set of proxies for housing condition than overcrowding (and perhaps poverty) might lead to an improved allocation of funds. Studies now under way should shed more light on this. We still have the question of whether to use the most current data, even though not all variables in the formula can be updated. Were Congress to fund adequate data collection to support the standards they place on programs, such problems would be resolved. In a context of continued inadequate data, the question of whether to update on the basis of only those variables where data is adequate remains an open issue.

AFDC Counts and ESEA Title I

Prepared by

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Introduction

The Title I formula includes three measures of eligible children:

1. children in families with incomes less than the poverty line (the poverty line varies with family size, other family characteristics, and farm-nonfarm residence);
2. children in families receiving high levels of assistance through Aid to Families with Dependent Children (AFDC); and
3. children in institutions for the neglected and delinquent, or maintained in foster homes at public expense.

In FY 77, these categories accounted for approximately 90 percent, 7 percent, and 3 percent of the total.

We are concerned with the second group of eligibles, the "high AFDC" children. They are measured as follows: the poverty line for a nonfarm family of four is divided by twelve, to convert it to a monthly figure. The States provide to HEW the number of children of ages five through seventeen whose families during a given month received AFDC payments greater in dollar amounts than this monthly standard. Note that this standard does not vary with family characteristics, but is uniform for all families. For this reason, AFDC children from large families (which receive the largest cash benefits) are most likely to be counted.

The number of high AFDC children is reduced by one-third when calculating a county's eligible total. The other two groups (the children in poor families and the children in institutions or foster homes) are fully weighted.

Four arguments are frequently heard as justifications for the counting of high AFDC children.

1. AFDC children should be eligible for Title I assistance whether or not they are poor, because the need for AFDC assistance reflects a social disadvantage likely to create special educational needs. The AFDC children who are in poor families are counted in the first group of Title I eligibles. The high AFDC measure includes the remaining AFDC children.
2. Families receiving high levels of assistance, which bring them over the poverty line, would have been poor had they not been aided by the State AFDC program. If children in these families are not counted for Title I purposes, the Federal Government is in effect reducing aid to the States which assist children generously through AFDC.
3. The poverty counts used to enumerate the first category of eligibles are available only at the decennial census. The 1960 census counts were used until 1973. AFDC counts are collected monthly, and their use adjusts the formula to reflect more recent population trends.
4. The AFDC counts direct additional funds to large cities. This is appropriate because:
   a. the poverty level is unreasonably low for central cities;
   b. educational problems in cities are particularly great, more than proportional to poverty counts; and
   c. urban budgets are particularly strained by the need to deal with a wide variety of social problems, and these problems are related to high rates of in–migration of low-income families from other areas.

These arguments are complex and controversial. It is for Congress to judge their validity and importance. In this paper I will simply describe the relationship between the actual AFDC measure, and the arguments advanced for its use. Whether or not these arguments are themselves valid, the AFDC measure currently in use bears only a loose relationship to them.

Categorical Eligibility

There are Federal programs for which eligibility is reserved to those who are either poor, or receiv-

ing AFDC or public assistance. Either condition is considered to be an adequate indicator of need. In view of commonly held attitudes about the impact of family structure on learning (attitudes that may have been confirmed by, for example, the Coleman Report), it might be appropriate to set a similar requirement for Title I eligibility.

Defining the Title I eligibles as those who are either poor or on welfare raises a double-counting problem: how can we determine the total number of those who are either poor or AFDC recipients, if the only data we have are poverty counts and AFDC counts?

One way of interpreting the high AFDC criterion—the enumeration of AFDC recipients only if they receive more than a certain amount—is a way out of this problem. Some people believe that the high AFDC recipients are unlikely to be poor, so that adding only them to poverty counts will minimize the risk of double counting.

Pre-Transfer or Post-Transfer Poverty

An eligibility criterion which excludes those who have been brought out of poverty by AFDC penalizes generous States and localities. It also introduces incentives to be less generous, though these are probably negligible. Of course, the generous States and localities are being generous with Federal money (at least 50 percent), which cuts somewhat the force of this argument. But only somewhat: had only the Federal share of AFDC been given to these families, nearly all of them would have remained poor.

The argument against reducing assistance to States with high assistance standards is entirely different from the argument for categorical eligibility for all AFDC children, but the two arguments lead to similar conclusions: to count the AFDC nonpoor. But the high AFDC children that are counted are not necessarily a good proxy for the AFDC nonpoor, for two reasons.

First, other sources of income are not counted in determining high AFDC status, though, of course, they are counted when totals of children in poverty are calculated. Thus, the high AFDC statistic omits the AFDC nonpoor who receive only a portion of income from AFDC. This problem is compounded by the fact that the AFDC rolls are examined only for one month. Certainly the largest group of AFDC nonpoor are those who are able to earn substantial income for part of the year, and must rely on AFDC assistance at other times. The size of the AFDC payment during the months they receive assistance may bear little relation to whether or not they are poor on an annual basis. In fact, large numbers of AFDC families have sufficient other income so that they are not poor even when their AFDC payments are disregarded.

Second, as mentioned in the introduction, the uniform cutoff produces a standard which can, in general, only be reached by large families, since AFDC payments are related to family size. Large families receiving high levels of assistance may still be poor, because the poverty line is higher for large families. Thus, large families may be counted twice, as poor families and as high AFDC families. Conversely, two- or three-person families who are nonpoor AFDC recipients may fail to be counted either as poor families or as high AFDC families.

Both these problems could be avoided if the AFDC nonpoor were counted directly. One way to do this is to use 1970 census data. Income data in the census is collected in disaggregated form. If transfer payments were excluded from the income definition when poverty status was determined, then many AFDC nonpoor would simply be counted as poor. Because it is known that transfer payments are more under-reported than wage and salary income, the pre-transfer poverty counts might well be more accurate than current poverty counts.

Of course, if 1970 census income data is used, the updating argument for AFDC counts is ignored. To this we now turn.

Updating

The need for an update arises because the poverty measure—the census low-income population—can be computed only once every ten years.[1] While AFDC data can perhaps be used to update poverty counts, it is not appropriate to use high AFDC data for this purpose. There is no reason to think that year-to-year changes in the high AFDC total help bring the poverty counts up to date.

In other words, the formula might use two measures derived from AFDC data: one estimating the AFDC nonpoor (which need not be up to date), the other estimating current shifts in poverty. The current AFDC measure does neither task very well.

Yet, although year-to-year changes in poverty are

[1] Intercensal surveys (such as the CPS) produce high-variance estimates, even at the State level, and have until now not been considered adequate for Title I purposes. The Survey of Income and Education (1976) was mandated to produce State estimates usable in Title I.

not well measured by year-to-year changes in the high AFDC measure, the high AFDC counts can be thought of as an updater in one important sense. Since 1970, a larger percentage of the poor live in the North and in metropolitan areas. As the AFDC eligibles are also concentrated in the North and in metropolitan areas, adding the AFDC counts to the poverty counts has to some extent offset the obsolescence of the 1970 counts. For this purpose, the currency of the AFDC data is irrelevant—1970 high AFDC counts would serve equally well as would a number of other similar adjustments to the poverty counts.

Urban Assistance

That poverty has become increasingly urban since 1970 is another argument for an urban adjuster, to be added to those given in the introduction: that the poverty levels are somehow inappropriate to the cities; that educational problems are more than proportional to poverty counts; that cities need particular assistance because of overburden and fiscal crises.

We cannot here evaluate these arguments. Let it be simply said that whatever the general merits of AFDC statistics as indicators of urban distress, high AFDC statistics are inappropriate because payment levels in AFDC vary extensively from State to State. There are currently no AFDC eligibles at all in Texas, Tennessee, Georgia, and nine other States, and there are small numbers in Ohio, Florida, North Carolina, Colorado, and Indiana. At present, if Congress wishes to aid large cities, it could be done effectively in a number of ways, most obviously by using a different formula for cities over a given size.

Technical Notes On Sensitivity Analysis

prepared by

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The Allocation Procedure

For each program separately, the allocation of funds to the individual States is computed by means of a formula which mirrors the Need, Capability, and Effort of each State with respect to the program. In a typical case, Need may be expressed in terms of the population of the State, Capability in terms of the State revenue, and Effort in terms of local tax revenue.

In what follows we will focus on one program to be denoted by P, and one State to be denoted by S. It is helpful to make a distinction between the *ideal* allocation and the *actual* allocation to that State under that program.

The ideal allocation A_{PS} is defined as some function F of three measures M(·), viz.:

1. M(N) = a measure of Need
2. M(C) = a mseaure of Capability
3. M(E) = a measure of Effort

and:

A_P = the total amount of dollars available for the program.

Symbollically, A_{PS} may be written as:

$$A_{PS} = F\,[M(N), M(C), M(E); A_P]$$

The actual amount allocated to State S for program P will be denoted by a_{PS}. Typically, it will be different from A_{PS}. The difference between the ideal and the actual allocation reflects the impact of one or both of two deviations between what may be termed the ideal and the actual allocation procedure. Thus, the measures M(·) may not be accurately known and thus have to be approximated by estimates m(·). Moreover, the ideal function F may not be available, in which case some other function, say G or H, has to be used. In summary, the actual allocation may be determined as:

1. $a_{PS} = F\,[m(N), m(C), m(E); A_P]$
2. $a_{PS} = G\,[m(N), m(C), m(E); A_P]$
3. $a_{PS} = H[m(N), m(C), m(E); A_P]$

or by some combination of these allocation formulas.

The Notion of Sensitivity Analysis

While it is typically not feasible to determine A_{PS} as defined above, the agency responsible for making the actual allocation may have a choice between alternative procedures to determine the actual allocation a_{PS}. It is clearly of interest to assess how sensitive the actual allocation is to this choice. We will use the term "sensitivity analysis" to denote this kind of assessment.

In the following section we will give a very simple example of sensitivity analysis.

A Simple Example

We will consider the program under Title I of the Elementary and Secondary Education Act (ESEA) of 1965 (see Appendix A-3 for further reference). The total amount of money available for this program is A_P = $1.5 billion.

In the interest of keeping the example simple, we will restrict the assessment to the impact of using alternative estimates m(N) of the ideal measure M(N) = the number of children in poverty. More specifically, we will consider three estimates, viz.:

1. Using 1970 Bureau of the Census estimates; this is in fact the approximation actually used.
2. Using 1973 READ estimates, i.e., estimates prepared by the Regional Economic Analysis Division, Department of Commerce.
3. And using 1973 Bureau of the Census estimates.

The allocations which result from using these three estimates are summarized in Table 1, col. 1-3; in col. 4 and 5, the allocations using 1973 READ estimates and 1973 Bureau of the Census estimates, respectively, are expressed in percent of the actual allocation.

As shown in Table 1, the actual allocation to California was (in millions) equal to $128.0. Had

Table 1. TITLE I FUNDS TO BE RECEIVED BY EACH STATE IN 1975 USING CURRENT ALLOCATION FORMULA AND ALTERNATIVE ESTIMATES OF CHILDREN IN POVERTY
(In Millions of Dollars; Assumes a Total Allocation of $1.5 Billion, Without Floor).

STATE	Using 1970 Bureau of the Census estimates	Using 1973 estimates		Column 2	Column 3
		READ	Bureau of the Census	(in % of Column 1)	
Col #	1	2	3	4	5
12 Largest States	$	$	$	%	%
California	128.0	139.3	146.2	109	114
New York	173.3	194.4	181.5	112	105
Pennsylvania	78.7	76.3	78.4	97	100
Texas	93.3	101.1	88.9	108	95
Illinois	82.4	77.0	90.0	93	109
Ohio	48.5	50.2	49.5	104	102
Michigan	61.8	62.6	68.3	101	111
New Jersey	44.2	54.1	54.0	122	122
Florida	48.7	47.8	51.0	98	105
Massachusetts	30.3	34.3	34.6	113	114
Indiana	19.5	22.3	20.9	114	107
North Carolina	44.3	39.7	40.2	90	91
12 Moderately Large States					
Missouri	28.2	25.8	27.5	91	98
Virginia	34.4	29.3	33.2	85	97
Georgia	44.4	37.3	40.2	89	90
Wisconsin	24.6	24.4	25.2	99	103
Tennessee	36.1	31.3	31.0	87	86
Maryland	25.2	29.4	28.8	117	114
Minnesota	24.0	20.9	24.1	87	100
Louisiana	48.5	48.1	41.1	99	85
Alabama	40.8	41.6	32.7	102	80
Washington	17.7	17.5	20.5	99	116
Kentucky	30.0	29.4	26.2	97	87
Connecticut	12.9	17.2	17.2	133	134
14 Moderately Small States					
Iowa	13.8	9.7	11.9	71	87
South Carolina	31.0	22.8	25.7	78	83
Oklahoma	17.7	16.6	16.7	94	94
Kansas	12.6	10.1	10.2	80	81
Oregon	13.6	13.4	13.1	98	96
Mississippi	37.3	31.0	30.7	83	82
Colorado	14.3	13.2	13.8	92	97
Arkansas	23.1	16.3	19.1	71	83
Arizona	13.1	14.9	14.1	114	107
West Virginia	16.4	14.4	12.7	88	78
Nebraska	8.9	7.5	7.2	84	80
Utah	5.0	4.0	4.0	79	80
New Mexico	12.7	11.4	10.4	90	82
Maine	5.5	8.3	5.1	152	92
13 Smallest States					
Rhode Island	6.4	4.7	6.3	74	99
Hawaii	5.0	4.2	5.1	84	102
New Hampshire	2.8	4.5	3.0	158	107
Idaho	3.5	5.2	3.2	146	90
Montana	4.5	4.1	3.9	93	86
South Dakota	5.0	4.8	4.1	95	81
North Dakota	4.5	2.4	2.9	52	64
Delaware	4.2	4.7	4.4	114	105
Nevada	1.9	1.0	2.6	54	137
Vermont	2.9	3.0	2.8	104	95
Wyoming	1.8	3.9	2.0	218	109
Alaska	2.3	3.6	3.7	152	158
Washington, D.C.	9.8	8.6	9.6	87	97

Source: Adapted from special tabulations prepared by the National Center for Education Statistics.

the 1973 READ estimates been used, California would have received $139.3, that is an additional 9 percent. Had the 1973 Bureau of the Census estimates been used, this State would have received $146.2, that is an additional 14 percent. For other States, for example Texas, the outcome would make the allocation less than the actual one. The table shows that the allocation is sensitive to the choice of an approximate measure, which is exactly the purpose of sensitivity analysis. It does not address, however, the question whether the procedure is too sensitive to that choice.

In summary, Table 1 shows that using 1973 READ estimates or 1973 Bureau of the Census estimates tends to transfer funds from the smaller, rural States to the larger, industrial States.

Some Additional Considerations

The discussion in the above section has been limited to the impact of using alternative measure $m(\cdot)$. As indicated in the section on The Notion of Sensitivity Analysis, the scope of sensitivity analysis may be broadened by encompassing, in addition, the impact of using alternative allocation formulas.

We will not enter upon this aspect here. We will, however, state that this specific aspect is a virgin area for applied formula research (for further reference see Chapter IV, Recommendation 5).

Selected References

The need for sensitivity analysis has generated a sizeable literature. With respect to analysis of the impact of errors in population statistics on allocation of public funds, reference may be given to Siegel (1975) and references given in that document. For discussion of additional techniques, reference is given to Cruz (1973), Garvin (1960), Gass (1975), and Taha (1971).

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Some Considerations In Designing Samples To Obtain Data for Use in Allocation Formulas

prepared by

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In this appendix we consider the problems of designing sample surveys when the data will be used in allocation formulas. Special attention will be paid to sampling errors; the discussion applies in principle to other types of error, such as errors due to nonresponse and incomplete coverage. The allocation of funds may be unsatisfactory either because the formula for distribution of funds is not well designed or because the data used contain errors large enough to affect the intended distribution of funds. The discussion in this appendix relates only to errors in the data.

Information is usually not free, so there will be some cost incurred to obtain the data elements required by the allocation formula. If we are willing to tolerate some error in the allocations resulting from sampling error, the cost will be less than if we require values based on complete counts.

The first question, then, is whether or not to sample. Second, if we do use sample estimates, how large should the samples be and how should they be distributed over States, SMSA's, etc.? Or more generally, how should the effort to reduce errors be distributed over various political subdivisions in order to approximate the allocation which would be obtained with error-free data?

For the potential beneficiary of the funds allocation, this is essentially a question of insurance. If we assume that the cost of estimating the formula elements must be deducted from the total amount available for allocation, then the question of sample size can be restated as follows:

> How much reduction in expected benefits is desirable in order to control the risk of an unsatisfactory result in the allocation actually made from the particular sample data used?

It will probably seem desirable to spend some amount to insure a reasonably accurate allocation. If the allocation is poor, then its objectives will not be met everywhere, and certain social costs will be incurred. On the other hand, we might find that the cost of a complete count would be equal to or more than the funds to be allocated, in which case we would almost certainly reject this alternative.

Best solutions probably lie somewhere between these extremes. We have not yet developed any neat way of finding a best solution in particular cases, and we suggest this as an area for research. Certainly as a first step we should compare the cost of complete count information with the size of the funds to be allocated.

Once we have decided how much insurance to buy, i.e., how much to spend to obtain sample estimates of the formula elements, there is a further question, namely, how should the data collection and other statistical resources be allocated to the political subdivisions participating in the funds allocation. Obviously, the allocation of statistical resources to political subdivisions should be such as to minimize the data errors and the resultant departures of fund allocations from those which would result if error-free data could be obtained. However, with fixed overall statistical resources, increased efforts to reduce the expected error in one political subdivision must inevitably mean decreased efforts and increased expected errors in other political subdivisions.

The answer to this allocation question (allocation of the sample, not of the funds) is by no means obvious. It will depend both on the formula being used for the allocation of funds and on the desired relation between expected errors in amounts allocated to different political subdivisions. We use the expression "error criteria" to describe such relationships. Thus, one error criterion might be to have the same expected squared error in relative terms

for the amount allocated to each subdivision. Another might be to have the same expected squared error in absolute terms for the amount for each subdivision. There are many other possibilities.

The choice of the error criterion to be used in any particular allocation is essentially a political one. However, it seems desirable that such decisions be made with knowledge of the consequences of alternative choices. In this appendix we describe the consequences and the sample allocations for selected error criteria.

Table 1 presents the sample allocations implied for two different fund allocation formulas and for several different error criteria.

Formula A is one which allocates funds to subdivisions in proportion to estimated numbers of eligibles. Algebraically:

$$\hat{D}_i = D \frac{N_i P_i}{\sum_i N_i P_i}$$

where D = the fixed total amount to be allocated

N_i = the population of the i^{th} subdivision (assumed known)

P_i = the sample estimate of the proportion of eligibles in the i^{th} subdivision.

and $\hat{D}_i$ = the amount allocated to the i^{th} subdivision

Formula B is one which gives each subdivision a fixed amount per eligible person, using the sample estimate of eligibles to determine the total funds allocated to the subdivision. Algebraically:

$$\hat{D}_i = K N_i P_i$$

where K = the fixed amount per eligible and the other symbols have the same meaning as in Formula A

For each of these formulas, the sample allocation formulas were derived for several different error criteria, some based on relative errors and some on absolute errors.[1] For those error criteria applicable to both funds allocation formulas, the indicated sample allocations are the same for both formulas. The result for equal absolute errors, i.e., to allocate the sample to areas in proportion to the square of their populations, is clearly not one which would be seriously proposed; it has been included merely for the purpose of showing sample allocation results for a wide array of possible error criteria.

The criterion "minimize error of sum" was used only for Formula B, since in Formula A, the total funds to be allocated are fixed. For both relative and absolute errors, this criterion leads to proportional allocation of the sample. Minimizing the error of the sum, i.e., in the total funds paid out, might be the criterion of choice for the Federal Government. However, it would not necessarily be equally attractive to the areas receiving the funds.

Several assumptions were used in deriving the formulas for allocation of the sample. These were:

1. Estimation of the P_i's using a simple random sample of individuals in each subdivision.
2. For each subdivision the sample would be small relative to the total population, so that finite correction factors could be ignored.
3. No advance knowledge concerning the relative values of the proportion of eligible individuals in each subdivision.

TABLE 1. ALLOCATION OF A FIXED TOTAL SAMPLE FOR SELECTED ERROR* CRITERIA
(See Text for Assumptions Used)

Funds allocation formula and error* criterion	Allocation of sample to subdivisions
A. Fixed total amount, allocated in proportion to estimated number of eligibles	
1. Criteria based on relative errors	
a. Same for each area	Equal
b. Minimize sum of errors	Equal
2. Criteria based on absolute errors	
a. Same for each area	Proportional to square of population
b. Minimize sum of errors	Proportional to population
B. Fixed amount per eligible person based on estimated number of eligibles in each area	
1. Criteria based on relative errors	
a. Same for each area	Equal
b. Minimize sum of errors	Equal
c. Minimize error of sum	Proportional to population
2. Criteria based on absolute errors	
a. Same for each area	Proportional to square of population
b. Minimize sum of errors	Proportional to population
c. Minimize error of sum	Proportional to population

* "Error" means expected squared error.

[1] Here, and in what follows, the term "errors" refers to expected squared errors.

4. The cost per individual of collecting the information is the same for all subdivisions.

Assumption 3 deserves particular attention. If historical values of the P_i's are available, it may be desirable to use these in determining the sample allocation needed to achieve the error criterion selected.

Even for the two funds allocation formulas treated in Table 1 there are many other error criteria that could be used. For example, under Formula A, one could establish a maximum acceptable relative error per subdivision [2] and then minimize the sum of the absolute errors over all subdivisons subject to this constraint.

Exhibit I presents part of a recent paper [3] that examined the implications of a different set of error criteria on the allocation of a fixed sample to obtain data for allocating funds under Formula A. In that paper, it was assumed that within each subdivision, each eligible individual would benefit (directly or indirectly) equally from the use of the funds allocated to that subdivision under the formula.

Under this assumption, the effects of sampling errors on the benefits to individuals were examined. Two error criteria for errors in individual benefits were used:

1. Equal errors for individuals in all subdivisions.
2. Minimize the sum of individual errors over all subdivisions.

Using assumptions similar to those described above, criterion 1 led to equal sample sizes for all subdivisions and criterion 2 led to allocation of the sample in proportion to the square root of total population in each subdivision.

[2] It would, of course, be necessary to set a maximum that could be met in all subdivisions, given the resources available for sampling.

[3] Jabine, Thomas B., "Equity in the Allocation of Funds Based on Sample Data," paper presented at the 136th Annual Meeting of the American Statistical Association, Boston, Massachusetts, August 25, 1976.

EXHIBIT 1

Allocations of Funds to Political Subdivisions Based on Sample Data [1]

Introduction

Federal funds are often allocated to political subdivisions on the basis of formulas using population counts and other statistics, such as numbers of school age children, per capita incomes, or unemployment rates. Complete count data are not available for some of these items and for others are available only once every 10 years from the decennial census. Hence, for allocations to reflect the current situation, estimates based on sample data must often be used in the allocation formulas.

Because the allocations are affected by sampling error, some individuals will receive less than they would have in the absence of sampling error and others will receive more. This consideration leads to a question of equity or fairness to individuals, namely, how should resources for the collection of sample data be allocated among the political subdivisions to which the funds are to be distributed?

In this section, a simple funds allocation model is used to examine the effects of alternative criteria for sample allocation. The alternatives considered are:

1. A sample allocation which insures equal treatment for eligible individuals in all political subdivisions.
2. A sample allocation which minimizes for all eligible individuals, the sum of squared differences between actual and "correct" per capita allocations, but does not necessarily insure equal treatment for individuals in different political subdivisions.

These alternative criteria lead to different allocations of the resources available for collecting sample data. The first alternative leads to approximately equal sample sizes for all States. The second alternative leads to allocation in proportion to the square root of total population.

Model and Assumptions

It is postulated that the Federal Government is to allocate a fixed amount of money among the States, in proportion to the number of residents of each State who have some specified characteristic, such as being of school age. Since current counts of persons with the specified characteristic are not available, a sample survey will be taken to obtain estimates. A fixed amount of money is available for the survey.

The problem is how to allocate the resources available for the sample among the States. The following assumptions are made:[2]

1. Sampling error is the only source of error in the allocations.
2. Counts, or good estimates of total population, are available for all States.
3. A simple random sample of persons will be used in each State.
4. The only cost attached to the gathering of the sample information in each State is a fixed cost per individual in the sample.
5. Within each State, the amount allocated will be shared equally by each person with the specified characteristic.

Notation—

N_i = population of the i^{th} State

$N = \sum_i N_i$

n_i = sample size for the i^{th} State

$n = \sum_i n_i$ (fixed)

Π_i = proportion of persons with the specified characteristic in the i^{th} State

p_i = proportion of persons with the specified characteristic in the sample for the i^{th} State

D = total amount of money to be allocated

The allocation formula is—

$$\hat{D}_i = \frac{N_i p_i}{\sum_i N_i p_i} D \qquad (1)$$

where $\hat{D}_i$ is the amount to be allocated to the i^{th} State

and $$\sum_i \hat{D}_i = D$$

[1] Part I of Jabine, T. B. "Equity in the Allocation of Funds Based on Sample Data", presented at the 136th Annual Meeting of the American Statistical Association, Boston, Massachusetts, August 1976.

[2] Most of the assumptions are not met in real allocation problems, but can probably be relaxed or removed without appreciably changing the main result.

Within a State, the amount to go to each individual, either directly or indirectly, through provision of benefits or services is—

$$\hat{D}_{ij} = \frac{\hat{D}_i}{N_i \Pi_i} = \frac{p_i}{\Pi_i} \cdot \frac{D}{\sum_i N_i p_i} \quad \text{(see assumption 5)} \quad (2)$$

The difference between the actual allocation to the j^{th} person in the i^{th} State and the "correct" allocation, which would have been made if complete counts were available, is given by—

$$\Delta_{ij} = \hat{D}_{ij} - D_{ij}$$

$$= D\left[\frac{p_i}{\Pi_i \sum_i N_i p_i} - \frac{1}{\sum N_i \Pi_i}\right] \quad (3)$$

Since $\Delta_{ij} = \Delta_i$ for all j, we will drop the j subscript.

Alternatives will be evaluated by examining their effects on Δ_i^2. This gives equal importance to errors in either direction, and gives greater weight to extreme departures from the "correct" value.

First Criterion—Equal Treatment to Residents of All States

To achieve this objective, we require that the *expected value* of Δ_i^2 be the same for every State.

We have

$$E\,\Delta_i^2 = D^2\,E\left[\frac{p_i}{\Pi_i \sum_i N_i p_i} - \frac{1}{\sum_i N_i \Pi_i}\right]^2$$

$$= \frac{A}{\Pi_i^2} E\left[p_i \frac{\sum_i N_i \Pi_i}{\sum_i N_i p_i} - \Pi_i\right]^2 \quad (4)$$

where $$A = \frac{D^2}{(\sum_i N_i \Pi_i)^2}$$

Since the factor $\frac{\sum_i N_i \Pi_i}{\sum_i N_i p_i}$ is the same for all States, and close to 1, we may expect that—

1. It will have only a minor effect on $E\Delta_i^2$.

2. It will have, at most, a second-order effect on the *allocation* of the sample between States.

Thus, we can say—

$$E\,\Delta_i^2 \doteq \frac{A}{\Pi_i^2}\sigma_{p_i}^2$$

$$\doteq A \frac{N_i - n_i}{N_i} \frac{(1 - \Pi_i)}{n_i \Pi_i} \quad (5)$$

if we assume sampling without replacement.

To fulfill our criterion, we must have: $E\,\Delta_i^2 = C$

which requires $$n_i = \frac{A(1 - \Pi_i)}{C\,\Pi_i + \frac{A(1 - \Pi_i)}{N_i}} \quad (6)$$

and $$\lim_{N_i \to \infty} n_i = \frac{A(1 - \Pi_i)}{C\,\Pi_i} \quad (7)$$

The interpretation of these results is that, to treat residents of all States equally, we must estimate the number, or proportion, of eligible persons in each State with the same *relative* reliability.

This in turn implies that we should select approximately the same size sample from each State. Departures from equality in sample size will depend on—

1. The finite population correction.
2. Advance information about approximate values of Π_i which are not the same for all States.

Departures from equality in sample size can be substantial if the range of assumed initial values of the Π_i's is large, especially if the smallest value is close to zero or the largest close to one. Using the subscripts "S" and "L" to denote the States with the smallest and largest assumed Π_i's, respectively, the ratio of the indicated sample sizes for the two States is (assuming $N_i \gg n_i$)

$$R_2 = \frac{n_S}{n_L} = \frac{1 - \Pi_S}{1 - \Pi_L} \cdot \frac{\Pi_L}{\Pi_S} \quad (8)$$

For example, with $\Pi_L = 0.100$ and $\Pi_S = 0.005$, we have $R_2 = 22.1$. The total populations of the two States do not enter into this formulation. Thus we could have the rather surprising result that a small State with a small proportion of persons in the target population could require a sample over 20 times as large as that needed for a large State with a considerably larger proportion of its residents in the target population.

Second Criterion—Minimizing the Expected Value of the Total Squared Difference Over All States

Another possible objective would be to minimize the expected value of

$$\theta = \sum_i \sum_j \Delta_{ij}^2$$

What does this imply in terms of sample allocation?

$$\theta = \sum_i \sum_j D^2 \left[\frac{p_i}{\Pi_i \sum_i N_i p_i} - \frac{1}{\sum_i N_i \Pi_i}\right]^2$$

$$= \frac{D^2}{(\sum_i N_i \Pi_i)^2} \sum_i \frac{N_i}{\Pi_i} \left[p_i \frac{\sum_i N_i \Pi_i}{\sum_i N_i p_i} - \Pi_i\right]^2 \quad (9)$$

and $$E(\theta) = B \sum_i \frac{N_i}{\Pi_i} E \left[p_i \frac{\sum_i N_i \Pi_i}{\sum_i N_i p_i} - \Pi_i\right]^2 \quad (10)$$

where $$B = \frac{D^2}{(\sum_i N_i \Pi_i)^2}$$

Making the same assumption as before:

$$E(\theta) \doteq B \sum_i \frac{N_i}{\Pi_i} \sigma_{p_i}^2$$

$$\doteq B \sum_i \frac{N_i}{\Pi_i} \frac{N_i - n_i}{N_i} \frac{\Pi_i(1 - \Pi_i)}{n_i} \quad (11)$$

$$= B \sum_i \frac{N_i - n_i}{n_i} (1 - \Pi_i)$$

To minimize $E(\theta)$, subject to the constraint that $\sum_i n_i = n$, we have:

$$n_i = n \frac{[N_i(1 - \Pi_i)]^{1/2}}{\sum_i [N_i(1 - \Pi_i)]^{1/2}} \quad (12)$$

As opposed to the first criterion, which called for approximately the same sample size in every State, this criterion calls for allocating the sample in proportion to the square root of the resident population, with departures from this allocation if there is advance information on approximate values of the Π_i's. The effect on the allocation of differing Π_i's is much less than when the first criterion is used. Using the same notation as before, we have:

$$R_2 = \frac{n_S}{n_L} = \frac{[N_S (1 - \Pi_S)]^{1/2}}{[N_L (1 - \Pi_L)]^{1/2}} \quad (13)$$

Using the same example as before, if we assume $N_S = N_L$, $\Pi_L = 0.100$ and $\Pi_S = 0.005$, we have $R_2 = 1.05$.

Comparison of the Two Criteria

For a fixed total sample of size n, the second criterion minimizes the expected value of θ, the total squared difference for all eligible persons in all States. How much larger will the expected value of θ be if we allocate the sample of size "n" according to the first criterion, which assures equal treatment for eligible persons in every State?

To simplify the comparison, first, let us consider the case where $\Pi_1 = \Pi_2 = \cdots = \Pi_s$ where s is the number of States and $N_i \gg n_i$ for all i.

Under these conditions we have:

$$E(\theta) \doteq B \sum_i \frac{N_i}{\Pi} \cdot \frac{\Pi(1 - \Pi)}{n_i}$$

$$= B (1 - \Pi) \sum_i \frac{N_i}{n_i} \quad (14)$$

For the first criterion, we have:

$$E \Delta_i^2 \doteq \frac{A(1 - \Pi)}{n_i \Pi} = C$$

so $$n_i = \frac{A(1 - \Pi)}{C \Pi}$$

and $$\sum_i^s n_i = n = \frac{s A(1 - \Pi)}{C \Pi}$$

so that $$C = \frac{s A(1 - \Pi)}{n \Pi}$$

and $$n_i = \frac{n}{s}$$

i.e., we should have the same size sample in each State.

Substituting in $E(\theta)$, we have for criterion one—

$$E(\theta)_{min} = B(1 - \Pi) \sum_i \frac{N_i}{n/s}$$

$$= \frac{B(1 - \Pi) N s}{n} \quad (15)$$

For the second criterion, we find that the expected value of θ is minimized when

$$n_i = n \frac{N_i^{1/2}}{\sum_i N_i^{1/2}} \text{ for all i.}$$

and the minimum value of $E(\theta)$ is

$$E(\theta)_{min} = B(1 - \Pi) \sum_i \frac{N_i}{n} \frac{\sum N_i^{1/2}}{N_i^{1/2}}$$

$$= \frac{B(1 - \Pi) [\sum_i N_i^{1/2}]^2}{n} \quad (16)$$

We may now examine the ratio, R_1 of $E(\theta)$ for criteria one and two

$$R_1 = \frac{E_1(\theta)}{E_2(\theta)} = \frac{Ns}{[\sum_i N_i^{1/2}]^2} \tag{17}$$

and if we let $N_i = k_i N$

where k_i is the proportion of the total population in the i^{th} State, we have:

$$R_1 = \frac{Ns}{[N^{1/2} \sum_i k_i^{1/2}]^2} = \frac{s}{[\sum_i k_i^{1/2}]^2} \tag{18}$$

Using the 1970 census population figures for the 50 States and the District of Columbia, we have:

$$R_1 = \frac{51}{6.3207^2} = 1.277$$

so that the expected value of θ, the sum of the squared differences when criterion one is used exceeds its minimum value by 27.7 percent.

On the other hand, if we compare $E\Delta_i^2$ for the smallest and largest States when criterion two is used, we have:

$$R_2 = \frac{A(1-\Pi)/n_s \Pi}{A(1-\Pi)/n_L \Pi} = \frac{n_L}{n_s} = \frac{N_L^{1/2}}{N_s^{1/2}} = \frac{k_L^{1/2}}{k_s^{1/2}} \tag{19}$$

Using the same data as before, we have:

$$R_2 = \left[\frac{.098188}{.001476}\right]^{1/2} = 8.156$$

i.e., for the eligible person in Alaska, the expected value of the squared difference between the correct allocation and the sample-based allocation is slightly over eight times as great as it is for the eligible person in California.

If we drop the assumption of equal Π_i's, we find that—

$$R_1 = \frac{\sum_i^s N_i \Pi_i \sum_i^s \frac{(1-\Pi_i)}{\Pi_i}}{\left\{\sum_i^s [N_i(1-\Pi_i)]^{1/2}\right\}^2} \tag{20}$$

Suppose, for example, that we were planning to use a sample to estimate the number of persons currently living in poverty in each of the 50 States and the District of Columbia. Again using the 1970 census population figures for the N_i's, and taking the census figures on percent of persons below low income level in 1969 for the Π_i's, we find $R_1 = 1.373$.

To determine the States with the smallest and largest values of $E\Delta_i^2$ when criterion two is used, we must identify the States which have the smallest and largest values of the quantity.

$$\phi = \frac{(1-\Pi_i)}{\Pi_i [N_i(1-\Pi_i)]^{1/2}} \tag{21}$$

If we use the subscripts "S" and "L" to denote these two States, we have—

$$R_2 = \frac{\phi_L}{\phi_S} \tag{22}$$

In the present illustration, we find that the two States are Nevada and Texas, and that $R_2 = 10.459$.

Raking as a Statistical Adjustment Procedure

Prepared by

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In many situations, the statistics produced may not be the best. For example, in estimating an unknown parameter P (or more generally, a set of parameters *P*), the statistician may have neglected using available prior information about the parameters(s) in the estimation procedure. In a special case, this prior information may be in the nature of one or more constraints which may be imposed on the estimate of P. One technique for estimating the parameter(s) is called "raking". This technique may also find applications for the purpose of adjusting for effects from nonresponse and other deviations from a survey design.

In this appendix, we will consider the special case where prior information constraints have been imposed on the estimate of P. Thus, there is a population of N objects of some kind. With each object we associate a value of a characteristic Y. This value may be a measured quantity such as income or tax, or it may simply be a count.

The population is divided into RxC (mutually exclusive) categories or cells on the basis of two classification characteristics A and B:

B \ A	A_1 ····· A_j ····· A_C	$\sum_j$
B_1	Y_{11} ··· Y_{1j} ··· Y_{1C}	$Y_{1.}$
.	. . .	.
.	. . .	.
.	. . .	.
B_i	Y_{i1} ··· Y_{ij} ··· Y_{iC}	$Y_{i.}$
.	. . .	.
.	. . .	.
.	. . .	.
B_R	Y_{R1} ··· Y_{Rj} ··· Y_{RC}	$Y_{R.}$
$\sum_i$	$Y_{.1}$ ··· $Y_{.j}$ ··· $Y_{.C}$	$Y_{..}$

(In what follows, we will use the dot convention to denote summation over the suppressed index:

$$Y_{i.} = \sum_j Y_{ij} \quad \text{and so on.)}$$

In the case of a measured quantity, such as income, Y_{ij} is the total amount of income for the objects which belong to cell ij. In the case of a count, Y_{ij} is the number of objects in that cell; an alternative symbol for Y_{ij} is then N_{ij}.

Assume now that the cell values Y_{ij} are *unknown*, but that the marginal values:

$$Y_{i.}; i = 1, \ldots, R$$

$$Y_{.j}; j = 1, \ldots, C$$

are *known*. Such a situation may occur in the context of a population census, where certain kinds of data are collected on a sample basis as a means of reducing the public's response burden.

As a basis for estimating the Y_{ij} values, the statistician has access to observations, say y_{ij}, for a sample of n objects. We shall assume here, without loss of generality, that the sample is selected by a scheme giving each object in the population the same probability of being selected. The statistician may now consider two different ways of estimating the Y_{ij} values. First, he may inflate the y_{ij} values by the reciprocal of the sampling fraction, that is use the estimates:

$$y'_{ij} = \frac{N}{n} y_{ij}$$

These estimates have the property of being unbiased. However, in general they are not consistent with the prior information available; more specifically, as a rule:

$$\sum_j y'_{ij} \neq Y_{i.}$$

$$\sum_i y'_{ij} \neq Y_{.j}$$

Second, the statistician may try to derive estimates Y'_{ij} which are consistent with the prior information. Such estimates may be more appealing to the users of the estimates, and may hopefully have some desirable technical properties, for example in terms of accuracy.

Estimates of the kind just referred to may be derived by first computing estimates y'_{ij} and then subjecting

these estimates to statistical adjustment. A commonly used term for such a procedure is "raking".

Raking has an old standing in statistical practice. The following procedure, known as "iterative proportional fitting", was discussed in considerable detail in Deming and Stephan (1940). The raking is carried out in one or more cycles, each of which comprises two steps, as illustrated below:

Cycle 1, Step 1: This calls for computing:

$$Y_{ij(1.1)} = y_{ij} \frac{Y_{i.}}{y_{i.}}$$

for each cell in row i, i = 1, . . ., R. Clearly,

$$\sum_j Y_{ij(1.1)} = Y_{i.(1.1)} = Y_{i.}$$

that is, the values computed are row-wise consistent with the prior information; in general, however:

$$\sum_i Y_{ij(1.1)} = Y_{.j(1.1)} \neq Y_{.j}$$

that is, these values are *not* column-wise consistent with the prior information.

Cycle 1, Step 2: This calls for computing:

$$Y_{ij(1.2)} = Y_{ij(1.1)} \frac{Y_{.j}}{Y_{.j(1.1)}}$$

These new values are column-wise consistent with the prior information, that is:

$$\sum_i Y_{ij(1.2)} = Y_{.j(1.2)} = Y_{.j}$$

but in general *not* row-wise consistent:

$$\sum_j Y_{ij(1.2)} = Y_{i.(1.2)} \neq Y_{i.}$$

The computations illustrated above are repeated for k cycles, until the resulting cell values $Y_{ij(k.2)}$ are (to a satisfactory degree of approximation) both row-wise and column-wise consistent with the prior information.

Raking is eminently well suited to the use of large-scale computers. The advances with the respect to computing capability in the last few years have served to stimulate important research and development with the respect to both the theory and the methods for raking, as evidenced by the recent references given below.

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An Agenda for Basic and Applied Research on Allocation Formula Problems[1]

Prepared by

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In the course of its work the Subcommittee adopted a useful paradigm for analyzing the formula allocation problem. This paradigm was expressed in terms of three entities—Need, Capability, and Effort —assumed to be measurable at the State and local level. The Subcommittee recognized that there were many definitional and other problems embedded in this apparently simple formulation. Such problems need to be addressed within a framework of a program of research on allocation formula problems. Although a variety of formula problems deserve to be analyzed in detail, it seems advisable to concentrate the modest resources which are likely to be available on a few selected problems. We are therefore selecting just three problem areas for immediate research. These problem areas are:

1. The nature of the interaction effects arising from the choice of allocation formula and the choice of estimates of Need, Capability, and Effort.

   It has been observed in the past that the use of different estimates may yield strikingly different allocations under a given formula. At the beginning of work in this research area a comprehensive review needs to be made of these previous experiences. The availability of more than one set of estimates of Need, Capability, and Effort presents the program designer with the practical problem of choosing the best allocation formula to be used with a given set of estimates, where one formula may be preferable with one set of estimates, and another formula may be preferable with another set of estimates. Research in this problem area should focus on the nature of the interaction mechanisms between estimates from specific statistical series and allocation formulas with specific structural differences.

2. The deterioration over time of various estimates of Need, Capability, and Effort.

   To some degree, any estimate based on statistical data collected at a point in time will, between periodic recollections of such data, deteriorate with the passage of time. One estimate, say, of Need, may quickly become very inaccurate, while another estimate, say, of Capability, may remain reasonably accurate for a longer period of time.

   Research in this area should aim at analyzing how information about the behavior of such estimates over time can be taken into account in the design and administration of allocation programs so that the actual allocation will remain close to the ideal allocation as time progresses between updates.

3. The issue of possible adjustment of future allocations to compensate for past inaccurate allocations.

   As a consequence of the inherent delays in the production of statistics, it is always necessary to use old statistics in an allocation formula. When the new statistics become available it is possible to compare the actual allocations in past years with recomputed allocations based on the new statistics. The policy issue is whether or not this kind of information should be taken into account in future allocations: Should States which received too little or too much in the past have their future allocations adjusted upward or downward to compensate in full or in part for underpayments or overpayments?

   The research in this issue area should focus on measuring the distributional impact of using old statistics and on developing criteria for the use of accounting correction versus equity compensation. In our context, accounting correction is meant to include the actual adjustments required to make a stream of benefits conform

[1] Based in part on an invited paper by W. Smith presented at the Annual Meeting of the American Statistical Association, Boston, Massachusetts, August 1976.

as closely as possible to a known entitlement. On the other hand, equity compensation entails additional considerations in terms of the legislative intent of the program.

Taken together, the three research areas discussed above constitute a manageable agenda for studies of allocation formula problems. As part of an implementation of this agenda, each research area would be reviewed in terms of:

a. the level of precision achieved or achievable in translating legislative goals into fund allocations under existing programs;
b. the implicit and explicit performance criteria for the particular Federal program;
c. the related modeling questions such as: "How well does a particular formula reflect the real-world dynamics it is designed to address?";
d. the structural aspects of typical allocation formulas, including additive, multiplicative, iterative, and mixed structures;
e. the effects of the presence or absence of constraints such as floors or ceilings (e.g., hold harmless provisions); and
f. the data required to compute the allocations by means of the chosen formula.

Furthermore, attention would be paid to the special problems involved in various allocation programs. For example, Need may, appropriately, have different components in two differing geographic areas; that taxable real estate and indicators of personal income may not provide an adequate basis for Capability; and that local tax revenue Effort would be most appropriately analyzed in terms of the purpose to which the revenues are applied.

The proposed research would be primarily applied in nature. It would, to a large extent, make use of existing substantive theories and methods. While it would be highly speculative to try to make an exhaustive list of potentially useful tools, we will briefly mention those disciplines which should prove to be of instrumental value in this research. These disciplines are:

1. *Statistics and Stochastic Modeling*
The relevance of these areas is exemplified in a paper by Savage stating a "working hypothesis . . . that the collection of the major statistical series and some of their important uses can be formulated as a statistical decision problem" (Savage, I. R. (1975), "Cost-benefit analysis of demographic data", *Supplement to Advances in Applied Probability, 7,* 62-71) and in a paper by Singer and Spilerman in which they discuss "how to select the specific structure . . . which should be associated with the empirical process" and also the problems of modeling with fragmentary data and observations containing noise and other sources of error (Singer B, and Spilerman, S. (1976), "The representation of social processes by Markov models", *American Journal of Sociology, 82,* 1-54). Both of these papers emphasize the need to improve basic data.
2. *Applied Mathematics and Control Theory*
We want to emphasize the potential usefulness of techniques from linear and nonlinear programming, as well as other optimization techniques (e.g., generalized Lagrange multipliers), since some allocation formulas may be viewed as generating constrained optimization problems. Extensions of existing raking and other adjustment techniques may require the application of advanced topological concepts and the development of new computer software. Incorporation of equity considerations may require tools from both control theory and system simulation.
3. *Economics and Utility Theory*
The meanings of such terms as "best allocation" and "equitable allocation" are by no means self-evident. The crucial question here is how the intent of the Congress is to be translated adequately into objectives within the framework of formula allocation techniques.
The appropriate choice of a utility or loss function for overshooting or undershooting a desired allocation requires consideration of the issues discussed recently in R. N. Waud (1976), "Asymmetric policymaker utility functions and optimal policy under uncertainty," *Econometrica,* 44, 53-66.

We must be mindful in our proposed research that there is a lack of extant theory which is fully suitable to our allocation problems and while we seek new theory we must continue to explore and improve our approximation techniques.

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