Multi-hazard Evacuation Route and Shelter Planning for Buildings

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16. Abstract

A bi-level, two-stage, binary stochastic program with equilibrium constraints, and three variants, are presented that support the planning and design of shelters and exits, along with hallway fortification strategies and associated evacuation paths in buildings. At the upper level, decisions are made regarding exit design, hallway fortification, and the location of shelters, their size and level of protection, with the objective of minimizing the expected maximum endured risk over all scenarios. At the lower level, the choice of evacuation routes by the users, following the upper-level design decisions, is modeled as a user equilibrium problem, where each individual seeks to minimize his/her risk exposure. Model variants involve both stochastic programming and robust optimization concepts under both user equilibrium (selfish) and system optimal (altruistic) conditions. Piecewise linearization of travel time functions and a disjunctive constraints transformation method that converts the single-level equivalent math program with complementarity constraints to a mixed integer program are employed to eliminate model nonlinearities. Integer L-shaped decomposition is adopted for solution of all four variants.

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Multi-hazard Evacuation Route and Shelter Planning for Buildings

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Executive Summary

A bi-level, two-stage, binary stochastic program with equilibrium constraints, and three variants, are presented that support the planning and design of shelters and exits, along with hallway fortification strategies and associated evacuation paths in buildings. At the upper level of this model, decisions are made regarding exit design, hallway fortification, and the location of shelters, their size and level of protection, with the objective of minimizing the expected maximum endured risk over all scenarios. At the lower level, the choice of evacuation routes by the users, following the upper-level design decisions, is modeled as a user equilibrium problem, where each individual seeks to minimize his/her risk exposure. Variants of the model involve both stochastic programming and robust optimization concepts under both user equilibrium (selfish) and system optimal (altruistic) conditions. A multi-hazard approach is utilized in which the performance of a plan is tested given various possible future emergency scenarios. Piecewise linearization of travel time functions and a disjunctive constraints transformation method that converts the single-level equivalent math program with complementarity constraints to a mixed integer program are employed to eliminate nonlinearities in the model. Integer L-shaped decomposition is adopted for solution of all four variants. These approaches are compared on a case study involving a single-story building.

Table of Contents

List of Figures

List of Tables

1. Introduction and Motivation

Regional evacuation studies have previously dealt with the problem of determining the optimal location and size of public shelters to which people can be evacuated in case of events such as floods and hurricanes. Studies on building evacuation, on the other hand, have mainly dealt with the question of how users can be evacuated as fast as possible to predefined building exits during an emergency. In practice, it might not be possible for all users to vacate a large or tall building *in time. This may be true in particular in the case of disabled or elderly users. In other cases, it might be possible for the users to reach an exit, but this will not be the safest option because of the presence of internal hazards such as fire or smoke on the path of evacuation inside the building, or because of external hazards that originate outside the building.

A possible alternative is to evacuate building users to shelters inside buildings, which offer a certain level of protection. This policy is already being implemented in some countries, such as Singapore and Israel, where buildings are required to contain air-raid shelters in every dwelling or on every floor. As is standard in some countries, shelters have a protective envelope of 20- to 30-cm-thick reinforced concrete walls and ceilings, as well as blast-proof doors and windows and an air filtration system. They usually contain a single room that serves an additional purpose, such as a bedroom in an apartment or a conference room in an office building. In high-rise buildings, they are built one on top of another, sometimes with trap doors and ladders that internally connect the shelters and can serve as an alternative evacuation route if staircases have become unusable. This creates a stable tower of shelters that will remain intact even if the rest of the building is heavily damaged. Such spaces have replaced the underground communal shelters that were originally built for this purpose in basements or even in public parks – serving several surrounding buildings. External communal shelters became less useful as buildings became higher, and the required time for evacuation decreased due to changing threats. This required shelters to be brought inside buildings and elevated to higher stories, so that they could be reached in time by evacuees. While the main purpose of existing shelters in buildings is to protect building users from missile attacks, they also offer protection during earthquakes. The possibility of using such shelters to protect users from additional hazards, such as fire or storms, is also considered herein.

While most shelters inside buildings are designed to house no more than a few dozen evacuees, local shelters, which serve an entire neighborhood, may house hundreds of evacuees. Such shelters are often located in public facilities, like schools or subway stations, and can serve the residents of buildings that do not contain internal shelters. The choice of where to locate these facilities depends on the type of hazard from which they are designed to protect. Regional evacuation may include even larger shelters, such as stadia that can house thousands of evacuees. The goal of this project was to develop mathematical models that support the planning of shelters and evacuation paths in buildings designed to accommodate a limited number of people. The objective of these models is to ensure that evacuees are optimally protected during emergencies, both during the evacuation as well as after reaching their destinations. The objective function is therefore defined to minimize the risk to which evacuees may be exposed, rather than minimize evacuation time. The models support identification of the shelters to which a population should evacuate in various emergency scenarios, in light of possible hazards on the evacuation paths. Moreover, the models can aid in investigating if it is preferable for building users to evacuate to shelters inside the building, rather than to building exits.

A network representation is used in the model to represent the layout of a building's circulation systems (i.e., the passageways along which building users can travel). A set of nodes may represent spaces inside buildings, such as rooms and corridors. A set of links represents connections between these spaces. The movement of evacuees toward shelters is modeled as link flows. The capacity of links and the risk exposure endured in traversing them may vary during emergencies as a result of structural failures or the spread of fire and smoke inside the building.

Different types of hazards may endanger a population's safety and require its evacuation. These may be natural (e.g., earthquakes), human-made (e.g., terror attacks), internal (e.g., fire), or external (e.g., hurricanes). Restricted construction budgets, and the difficulty to prepare evacuees for more than one evacuation procedure, imply the need to accommodate different hazards in a single solution. A multi-hazard approach was therefore adopted in which the performance of a plan was tested under various possible future emergency scenarios. This report presents a solution for the problem of designing a single building so that its users can minimize their exposure to risk in an emergency situation involving building egress or sheltering. This problem is referred to as the Building Evacuation Design Problem (BEDP). To solve the BEDP, a bi-level, two-stage stochastic program was defined. The program falls under the class of Stochastic Mathematical Programs with Equilibrium Constraints (SMPECs).

At the upper level of the proposed SMPEC, decisions are made regarding the location of shelters in the building, their size and level of protection, as well as the location of building exits, with the objective of minimizing the exposure of evacuees to risk over all scenarios. The uncertainty in the scenarios that will be realized is taken into account. It is assumed that construction costs are limited to a certain budget. This budget can be used for the planning of shelters that offer a high level of protection. Alternatively, the budget can be allocated for a partial fortification of sections of the hallways and staircases through which users evacuate to increase the level of protection that they offer, for widening hallways to increase their capacity, or for the construction of additional or redesign of existing building exits. The advantages of allocating the available budget for the construction of shelters can thus be weighed against the benefits of using it to add or redesign exits or to reduce the risks for evacuees on certain sections of the evacuation paths by fortifying or widening them.

At the lower level of the program, the choice of evacuation paths by the users, following the upper-level decisions on the location of safe locations (shelters, fortified hallways) and exits, is modeled as a User Equilibrium (UE) problem, while alternative, single-level system optimal (SO) formulations are posed as well. When modeled as a UE problem, it is assumed that users are homogenous, that they are perfectly informed of the conditions in the building or region, and each selfishly chooses a path to minimize his/her own risk. Evacuees will choose between evacuating to a specific shelter, evacuating to an exit, or staying in a partially fortified hallway. The UE approach ensures that no evacuee can do better by taking an alternative decision, but requires that evacuees be familiar with the building and with the risks imposed by the hazard, in order to have full information about all alternatives. On the other hand, when the choice of evacuation paths is modeled as a SO problem, it is assumed that evacuees are assigned to an exit or shelter and told which path to use to reach that location. The SO approach uses the available system resources optimally to ensure a social optimum, but requires compliance. That is, evacuees must act altruistically, following paths or taking cover in shelters that do not necessarily minimize their individual disutilities. Alternatively, command and control by a trained staff will be needed to direct the evacuees.

In the literature, inefficiencies created from selfish behavior are measured by the price of anarchy, which is computed as the average system performance cost (usually related to traffic) under a Wardrop equilibrium divided by the minimum possible average obtained from the system optimum over all origin-destination pairs and multiple networks. This concept was originally termed price of anarchy by Koutsoupias and Papadimitriuou (1999). Worst-case bounds on this price have been derived for several simpler objective functions. An overview of these findings is given by Roughgarden (2006).

Four variants of the BEDP were formulated using concepts of stochastic programming and robust optimization, each under UE and SO conditions. UE models involve the bi-level formulation described previously. By recognizing that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality, these models are reduced to equivalent single-level, two-stage stochastic integer programs. All variants are nonlinear. Using a disjunctive constraints transformation method and piecewise linearization, the models were linearized, and an integer L-shaped decomposition is proposed for solution of each of these mathematical programs. The capabilities of the modeling and solution techniques are illustrated on an office building using the original architectural plans. Similar to considering the price of anarchy, trade-offs between system optimal and UE solutions and their implications in terms of their application were investigated. Additionally, differences noted in performance between solutions from stochastic programming using expectation versus robust optimization were studied.

2. Literature Review

To the best of our knowledge, there have been no prior studies in the literature that address optimal shelter and exit location in buildings. However, models with relevance to the BEDP have been developed in the literature for locating shelters in the context of regional evacuation problems. These are reviewed next.

It appears that Sherali et al. (1991) were the first to study the shelter location problem for regional evacuation planning. They proposed a nonlinear, mixed-integer program to determine the shelter locations, resource allocations, and assignment of evacuees to minimize evacuation time. They suggest an SO approach, which assumes that a central authority controls the flow of evacuees. The model uses a single given hazard scenario. A deterministic, multi-objective p-median problem formulation is proposed by Alcada-Almeida et al. (2009) for locating *p* shelters in a given area so as to minimize demand-weighted distance traveled, incurred risk, and travel time associated with an evacuation. Similar deterministic and system-optimal assumptions are made. Congestion is not considered.

Kongsomaksakul et al. (2005) proposed a bi-level programming model for determining locations and sizes of shelters that can be used by evacuees to minimize evacuation time in the event of a flood. The model is intended for pre-disaster planning. The upper-level problem determines the number and locations of shelters among a given set of potential locations, and the lower-level problem is a combined trip distribution and assignment problem. The inclusion of the lower-level problem allows evacuees to freely select their preferred shelters and choose the shortest route to their chosen shelters. Shelter selection behavior is modeled with a logit model, and a Wardrop equilibrium is assumed to be reached. A genetic algorithm is employed to solve the problem. It is tested through a simulated flood scenario. Ng et al. (2010) also propose a bi-level programming model for regional shelter location, but optimize the shelter assignment in the upper-level problem, instead of assuming that evacuees themselves choose the shelters to which they will evacuate, as in Kongsomaksakul et al. (2005). A simulated annealing heuristic is proposed.

These earlier models all use a single given hazard scenario for locating shelters. Therefore, the identified solution may not be optimal for a wider range of hazard scenarios. Further, these models disregard the uncertain nature of disaster events. Kulshrestha et al. (2011) take into account uncertainty in demand for shelter capacity in a robust, bi-level program to determine the locations and sizes of shelters. As in Kongsomaksakul et al. (2005), it is assumed that the number of shelters, their locations and capacities are determined by a central authority, while the evacuees choose shelters and routes to access them. Although a set of possible demand scenarios is considered, other uncertainties regarding the type of hazard and the level of its severity are disregarded. An exact cutting plane algorithm is presented.

Li et al. (2011) study sheltering network planning and operations for natural disaster preparedness and response with a two-stage stochastic program. In their study, the number of evacuees present at each origin at the start of the evacuation period (i.e., the evacuation demand) and transportation costs are assumed to be known only with uncertainty. In the first stage, the locations, capacities, and resources required to supply the shelters are determined. In the second stage, the evacuees and resources are distributed to shelters under various disaster scenarios. With only continuous variables in the second stage, the L-shaped method can be employed. The proposed model and solution method were applied on a case study involving the Louisiana Gulf Coast. Another paper that explicitly addresses the uncertainties inherent in disaster situations is by Li et al. (2012). They developed a scenario-based, bi-level stochastic program for optimal shelter location that considers a range of possible hurricane scenarios. The program seeks to minimize expected total travel time and unmet shelter demand under one of a host of possible disaster scenarios. Such scenarios differ in the area of impact. A dynamic user equilibrium is sought in the lower level. Unlike earlier works, this paper considers the possibility that evacuees will exit the area and will not necessarily use the shelters. While this work is the most relevant to the current study, it considers only a single type of hazard. Moreover, the problem is solved using a heuristic rather than exact solution methodology.

This literature is summarized in Table 1.

Table 1 Synthesis of the related literature

Reference	SO vs. UE	What problem elements are stochastic	Optimization approach	Solution method	Hazard type	Application
Sherali et al. (1991)	SO	n/a	NLMIP	Generalized Benders & heuristic	Hurricane, flood	Geographic
Alcada-Almeida et al. (2009)	SO	n/a	Multi-objective p-Median program	Heuristic algorithm (nondominated solutions)	Generic	Geographic
Kongsomsaksakul (2005)	UE	n/a	Bi-level program	Genetic algorithm	Flood	Geographic
Ng & Park (2010)	UE	n/a	Bi-level program	Simulated annealing	Generic	Geographic
Kulshrestha et al. (2011)	UE	Number of evacuees	Bi-level RO	Cutting plane algorithm	Generic	Geographic
Li et al. (2011)	SO	Evacuation cost, number of evacuees	Two-stage SP	L-Shaped algorithm	Hurricane	Geographic
Li et al. (2012)	Dynamic UE	Evacuation capacity	Two-stage SP	Heuristic	Hurricane	Geographic
This study	Both	Evacuation risk exposure	Bi-level two-stage SP /RO	Integer L-shaped	Multi-hazard	Geographic & Building

The contributions of the current study are, in light of existing relevant works: (1) a mathematical formulation to address shelter and exit design and location, possible fortification of hallways with reduced risk exposure, and selection of evacuation routes for buildings; (2) a multi-hazard approach with applicability to not only a multitude of disaster types, but simultaneous consideration of special and competing needs arising from these hazard types; (3) explicit consideration of risk exposure and its relation to the effects of user route choice on travel congestion; (4) simultaneous consideration of shelter and exit use; (5) a comparison of stochastic programming and robust optimization modeling; (6) an evaluation of the role of cooperative behavior and related need for command and control through a comparison of user equilibrium and system optimum formulation applications; and (7) an exact solution methodology that addresses problem nonlinearities for a set of complicated SMPECs and Stochastic Nonlinear Programs (SNLPs). This innovative application of modeling and solution concepts from operations research to building evacuation and sheltering design can aid emergency planners and architects in improving safety in life-threatening circumstances. The development of models to replicate both selfish and coordinated behaviors enables evaluation of evacuation and sheltering designs over a spectrum of implementations. To this end, the value of command and control required to ensure that building users behave altruistically to optimize a social objective can be evaluated against a comparable laissez faire implementation, allowing benefit-cost evaluation.

3. Problem Definition

3.1. Notation

In modeling the BEDP, a network representation $G = (N, A)$ of the building circulation system layout is used. A set of nodes N corresponds with locations inside the building, such as evacuation points of origin, transition points, candidate shelter locations, existing exits and candidate exit locations, as well as a supersink d. A set of links $A = A_1 \cup A_2 \cup A_3$ connects these locations. A_1 is a subset of the links representing hallways, staircases, doorways and other passageways. A_2 is a subset of the links connecting existing and candidate shelters and fortified hallways (i.e., safe locations) to supersink d. Similar links from existing and candidate emergency exits to d are included in subset A_3. This network representation is illustrated in Fig. 1. The movement of evacuees in the circulation system is represented as flows along the links. The introduction of a supersink reduces the related network flow problem to that of a multi-source, single-sink problem.

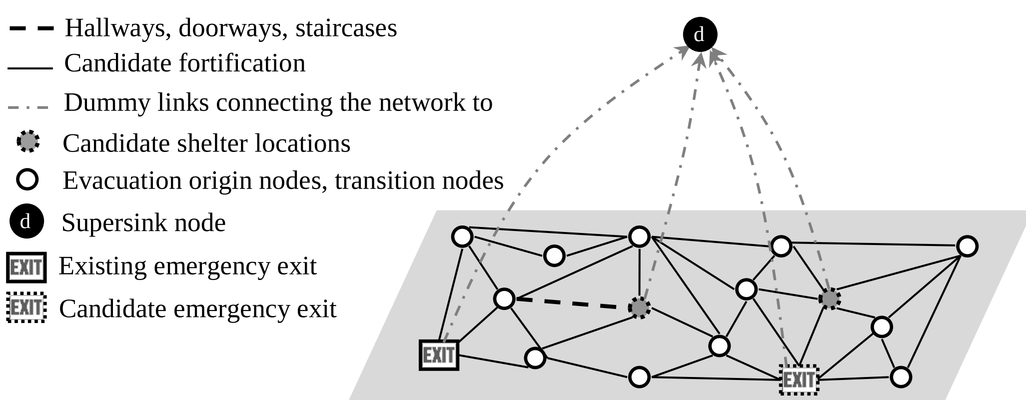


Fig. 1. Building network representation scheme

The network is considered under a host of potential states (or scenarios) that might arise for a building under no-notice disaster events. Unlike disaster events with notice, such as a hurricane with two to three days' advance warning, notification of such a no-notice event in the context of buildings, perhaps provided by an alarm system, may entail only minutes. In this context, it is assumed that such notification provides information to the evacuees and building managers on the disaster type and possibly the location within the building (e.g., fire on a particular floor). This information may be imperfect, but can permit assessment of risk exposure associated with evacuee options, both in terms of safe locations and exits, as well as the paths that lead to these locations.

In the network representation, a particular state is given by the realization of parameters of link risk exposure functions. Risk exposure associated with a link consists of the likelihood of exposure while using the link and potential consequences. The longer the time spent en route to a safe location, the greater the likelihood of exposure. Thus, risk exposure is a function of travel time, which will depend not only on the link's length, but also on the number of people using it. It is assumed that the evacuees can assess risk exposure perfectly from the information they receive, and that all evacuees perceive risk identically. Risk associated with each safe location or an exit is also incorporated in the risk exposure functions. In the problem formulations proposed herein, evacuees choose or are guided to a safe location or exit with the goal of minimizing total risk exposure.

With this in mind, risk exposure associated with a link a is defined as a linear function of the link's flow-dependent travel time: $r_a[x_a(\xi)] = \alpha(\xi)t_a[x_a(\xi)] + \beta(\xi)$, where parameter $\alpha(\xi)$ converts the time it takes to evacuate through the hallways, staircases, and doorways to risk exposure, and parameter $\beta(\xi)$ is a measure of the risk associated with staying in a shelter or hallway, or exiting the building. Both parameters are a function of the scenario. Different emergency scenarios, ξ, may induce different behaviors or decisions to reduce risk exposure. For example, when an internal hazard occurs (e.g., a fire event), exiting from the building will be of the highest priority, whereas in the case of an external hazard (e.g., a storm), taking refuge within the building will provide protection. This is captured by parameter $\beta(\xi)$.

The BPR travel time function, originally used to estimate travel time on road networks, is adapted in the following form to estimate the evacuation travel time in a link $a \in A_1$, $t_a[x_a(\xi)]$, as a nonlinear function of link flow, $x_a(\xi)$ (see Schomborg et al. 2011). The travel time along link $a \in A_2 \cup A_3$ is also set to zero:

$$t_a[x_a(\xi)] = \begin{cases} t_a^0(\xi) + 0.15[\frac{x_a(\xi)}{c_a(\xi)}]^2, & \forall a \in A_1 \\ 0, & \forall a \in A_2 \cup A_3 \end{cases} \quad (1)$$

where t_a^0 and $c_a(\xi)$ are the freeflow travel time and capacity of link $a \in A_1$ under scenario ξ, respectively. The BPR function is generally formulated based on the velocity-density fundamental diagram for vehicle movement in road networks. Schomborg et al. (2011) argue that, in the context of macroscopic modeling, this function can also be utilized to estimate the pedestrian travel time using the parameter values adopted in Equation (1). That is, the velocity-density fundamental diagram in pedestrian and vehicular movements is similar. Thus, similarly structured mathematical models can be used. Coinciding findings were obtained from empirical observations and developed regression equations (Chattaraj et al., 2009; Seyfried et al., 2005).

Nomenclature used in the remainder of this report is provided next.

S	=	set of shelter/hallway fortification types
E	=	set of exit types/sizes
g_a^s	=	cost of fortification of type $s \in S$ in link $a \in A_2$
$\acute{g}_a^e$	=	cost of construction of exit type $e \in E$ in link $a \in A_3$
B	=	total budget for exit design and shelter/hallway fortification

p_a^s = capacity of shelter type $s \in S$, $a \in A_2$

q^o = number of evacuees originating at node $o \in N$

K_o = set of paths containing no cycles originating from node $o \in N$

$\delta_{a,k}^o$ = link-path incidence matrix (=1 if link a belongs to path k originated from node o, and =0 otherwise)

Ξ = set of possible scenarios $\xi \in \Xi$

Pre-event variables:

y_a^s = binary variable indicating if fortification of type $s \in S$ is selected for application to link $a \in A_2$ (=1 if selected, and =0 otherwise)

$\acute{y}_a^e$ = binary variable indicating if exit type $e \in E$ is selected for construction in link $a \in A_3$ (=1 if selected, and =0 otherwise)

Post-event variables:

$f_k^o(\xi)$ = flow along path $k \in K_o$ from demand node o under scenario ξ

$x_a(\xi)$ = flow along link $a \in A$ under scenario ξ

$t_a[x_a(\xi)]$ = travel time along link $a \in A$ under scenario ξ

$r_a[x_a(\xi)]$ = risk exposure associated with link $a \in A$ under scenario ξ; assumed to be a linear function of link travel time: $r_a[x_a(\xi)] = \alpha(\xi)t_a[x_a(\xi)] + \beta(\xi)$

$c_k^o(\xi)$ = risk exposure on path k, for $\forall k \in K_o, o \in N$

$u^o(\xi)$ = minimum risk exposure incurred by evacuees originating from node $o \in N$ under scenario ξ (under UE condition)

$w^o(\xi)$ = the worst (highest) evacuation risk exposure from node o (under SO condition)

3.2. Problem Formulations

Four BEDP formulations are presented. The programs use either Stochastic Programming (SP), which takes into account the expectation in performance over all future scenarios, or Robust Optimization (RO) with emphasis on the worst-case scenario imposing the highest evacuation risk exposure. The latter is a conservative approach, which may require a more expensive solution to attain the same level of risk exposure. Two of the models adopt a bi-level structure, where the evacuees choose their own routes to minimize their own risk exposure (taking a UE perspective). The remaining two models are single-level and assume the evacuees will follow system-optimal instructions (taking an SO perspective). This latter perspective requires altruistic user behavior or, more realistically, command and control for implementation. That is, users are commanded toward safe locations or exits that meet social goals and control is in place to ensure compliance (Feng and Miller-Hooks, 2012). These four programs are referred to by their acronyms: BEDP-SP-UE, BEDP-SP-SO, BEDP-RO-UE, and BEDP-RO-SO. The modeling specifications of these problems are summarized in Table 2.

Table 2 Modeling specifications for the proposed problems

Problem	Optimization approach	User behavior modeling	Modeling structure	Objective
BEDP-SP-UE	SP	UE	• Bi-level ○ UL:1st stage decision on design/fortification options ○ LL: user response to UL decisions	*min* E[*max* evacuation risk]
BEDP-RO-UE	RO	UE		
BEDP-SP-SO	SP	SO	• Single-level (command and control)	*minmax* [evacuation risk]
BEDP-RO-SO	RO	SO		

Objectives that minimize the maximum or expected maximum risk exposure are proposed herein, because they indirectly address issues of equity and consider the protection of each individual. This differs from other network design formulations in the literature. For both emergency and nonemergency applications, it is common to minimize total travel time or other disutility measures.

3.2.1. BEDP-SP-UE

This BEDP-SP-UE problem is formulated as a bi-level, two-stage stochastic program with equilibrium constraints, a type of stochastic MPEC.

$$(BEDP - SP - UE)$$

Upper-level: $\min\limits_{y} E_{\xi \in \Xi}[Z^{U}_{SP-UE}(\xi)]$ (2)

s.t.

$\sum_{a \in A_2} \sum_{s \in S} g^s_a y^s_a + \sum_{a \in A_3} \sum_{e \in E} \acute{g}^e_a \acute{y}^e_a \leq B$ (3)

$\sum_{s \in S} y^s_a \leq 1, \quad \forall a \in A_2$ (4)

$\sum_{e \in E} \acute{y}^e_a \leq 1, \quad \forall a \in A_3$ (5)

$y^s_a, \acute{y}^e_a \in \{0,1\}, \quad \forall a \in A_2 \cup A_3, s \in S, e \in E$ (6)

where

$Z^{U}_{SP-UE}(\xi) = \min\limits_{x} \max\limits_{o \in O} u^o(\xi)$ (7)

Lower-level: $Z^{L}_{SP-UE}(\xi) = \min \sum_a \int_0^{x_a(\xi)} r_a(w)\, dw$ (8)

s.t.

$$\sum_{k\in K_o} f_k^o(\xi) = q^o, \quad \forall o \in O \tag{9}$$

$$x_a(\xi) = \sum_{o\in O} \delta_{a,k}^o f_k^o(\xi), \quad \forall a \in A \tag{10}$$

$$x_a(\xi) \leq \sum_{s\in S} p_a^s y_a^s, \quad \forall a \in A_2 \tag{11}$$

$$x_a(\xi) \geq 0, \quad \forall a \in A \tag{12}$$

$$f_k^o(\xi) \geq 0, \quad \forall k \in K_o, o \in O \tag{13}$$

At the upper level, the problem is to determine the optimal location of exits, location and size of shelters to be constructed, and hallways to be fortified, as well as corresponding level of protection, aiming at minimizing the expectation of the worst-case (highest) risk exposure experienced by the evacuees over all origins, i.e., $\max_{o\in O} u^o(\xi)$. Construction costs are limited to an available budget in constraint (3). Constraints (4)-(6) ensure that only one type of fortification is constructed at any candidate location.

The upper- and lower-level problems are linked through $u^o(\xi)$. This variable appears in the upper-level objective function $Z_{SP-UE}^U(\xi)$ and its value is determined through solution of the lower-level problem, given the decision on the network design made in the upper level. The lower-level problem is a path-based, capacitated user equilibrium problem with side constraints adapted from Larsson and Patriksson (1995). Objective function (8) is a standard traffic UE function, originally introduced by Beckmann et al. (1956). Beckmann et al. showed that a Wardrop equilibrium is reached when the link flows are chosen to minimize this function. Evacuees rationally seek to minimize their risk exposure, assuming that they have perfect information on the risks associated with the evacuation path choices under a given scenario ξ and the building design options (including the shelter capacities) determined at the upper level.

Evacuees are assigned to paths through constraints (9). Link flows are defined in constraints (10) as the total flow in terms of evacuees traveling from any origin along any path containing that link. In constraints (11), flow is allowed through a link $a \in A_2$ if a shelter of any type $s \in S$ is constructed along that link. The flow is limited to the shelter's capacity, p_a^s. An infinite capacity is presumed for all exit doors $a \in A_3$. Non-negativity requirements for link and path flows are captured through constraints (12)-(13).

The formulation can be readily extended to permit shelter capacities as a function of hazard type. This is important in real applications, because the amount of space required per evacuee while sheltered depends on the amount of time the evacuee will remain in the shelter. The longer the required time, the greater the required space. Because it is morally difficult to restrict the number of evacuees to enter a shelter when it appears that there is more space, constructing shelters for the worst-case as is supported by the proposed objective functions is desirable.

BEDP-SP-SO

As an alternative modeling approach, safe locations, exits and evacuation routes are designed to support a system optimal flow of evacuees under the assumption that evacuees are directed in emergency situations by trained staff or through commands given electronically. Thus, it is presumed that the evacuees will follow the instructions they are provided. This problem is formulated as a single-level, nonlinear two-stage stochastic program.

$$(BEDP - SP - SO)$$

$$\min_{y} E_{\xi \in \Xi}[Z_{SP-SO}(\xi)] \text{ s.t. (3-6)} \quad (14)$$

where

$$Z_{SP-SO}(\xi) = \min_{x} \max_{o \in O} w^o(\xi) \quad (15)$$

s.t. (9-13)

$$f_k^o(\xi) \cdot [c_k^o(\xi) - w^o(\xi)] \leq 0, \quad \forall k \in K_o, o \in N \tag{16}$$

As in the BEDP-SP-UE, the objective function is to minimize the expectation of the maximum evacuation risk exposure evacuees experience over all scenarios. $w^o(\xi)$ is defined as the worst (highest) evacuation risk exposure from node o. Through additional constraints (16), only the risk exposure of active paths from node o is used to determine $w^o(\xi)$. That is, the inequality $c_k^o(\xi) \leq w^o(\xi)$ is imposed if $f_k^o(\xi) > 0$.

3.2.2. BEDP-RO-UE and BEDP-RO-SO

By focusing on the worst evacuation risk exposure under the worst-case scenario rather than on the expectation of worst risk exposure over all scenarios, this robust optimization model is even more conservative than the BEDP models that use stochastic programming (BEDP-SP-UE and BEDP-SP-SO). Scenario probabilities are not included in robust optimization. Two problems, BEDP-RO-UE and BEDP-RO-SO, are formulated using the UE and SO principles, respectively:

$$(BEDP - RO - UE)$$

Upper-level: $\min\limits_{y} \max\limits_{\xi \in \Xi} [Z_{RO-UE}^U(\xi)]$ s.t. (3-6) (17)

where

$$Z_{RO-UE}^U(\xi) = \min_{x} \max_{o \in O} u^o(\xi) \tag{18}$$

and the lower-level problem as given in (8-13).

$$(BEDP - RO - SO)$$

$$\min_{y} \max_{\xi\in\Xi}[Z_{RO-SO}(\xi)] \text{ s.t. (3-6)} \tag{19}$$

where

$$Z_{RO-SO}(\xi) = min_x \max_{o\in O} w^o(\xi) \quad \text{s.t. (9-13), (16)} \tag{20}$$

Both formulations seek to minimize the maximum evacuation risk exposure over all scenarios.

4. Solving the BEDP Variants

4.1. Complementarity Constraints

4.1.1. Solving BEDP-SP-UE and BEDP-RO-UE programs

A common approach to solving bi-level programs is, when possible, to eliminate the lower-level problem by incorporating the original lower-level constraints along with related KKT conditions (first-order optimality conditions) within the upper level. This creates an equivalent single-level program. In the context of the BEDP-UE-SP and BEDP-UE-RO formulations, this includes constraints (9)-(13) and (21)-(24):

$$f_k^o(\xi) \cdot [\hat{c}_k^o(\xi) - u^o(\xi)] = 0, \quad \forall k \in K_o, o \in N \tag{21}$$

$$\hat{c}_k^o(\xi) - u^o(\xi) \geq 0, \quad \forall k \in K_o, o \in N \tag{22}$$

$$\lambda_a(\xi) \cdot [\textstyle\sum_{s \in S} p_a^s\, y_a^s - x_a(\xi)] = 0, \quad \forall a \in A_2 \tag{23}$$

$$\lambda_a(\xi) \geq 0, \quad \forall a \in A_2 \tag{24}$$

Building on the work of Larsson and Patriksson (1995), who considered the capacitated assignment problem in which users selfishly seek to minimize their experienced disutilities, it is assumed that a generalized Wardrop equilibrium can be reached. In such an equilibrium, no evacuee can unilaterally switch routes and improve his/her disutility (risk exposure in the context of this study).

In constraints (21)-(24), $\hat{c}_k^o(\xi)$ is the generalized path risk exposure adapted from Larsson and Patriksson (1995):

$$\hat{c}_k^o(\xi) = c_k^o(\xi) + \textstyle\sum_{a \in A_3} \delta_{a,k}^o\, \lambda_a(\xi), \quad \forall k \in K_o, o \in N. \tag{25}$$

where $c_k^o(\xi) = \sum_{a \in A} \delta_{a,k}^o\, r_a[x_a(\xi)]$ is the risk exposure on path k, for $\forall k \in K_o, o \in N$, and

$\lambda_a(\xi)$ is the Lagrange multiplier for link $a \in A_3$ associated with complementarity constraints (23). $\lambda_a(\xi)$ can be interpreted as the additional risk exposure that users passing through a saturated link are willing to endure to use the link (i.e., the link's shadow price). Constraints (21) imply that the equality $\hat{c}_k^o(\xi) = u^o(\xi)$ is achieved only if $f_k^o(\xi) > 0$ for each scenario ξ, origin o and path k. That is, a path originating from node $o \in N$ can take flow only if its generalized risk exposure equals the minimum risk exposure $u^o(\xi)$ under scenario ξ.

In their compatible formulation, Larsson and Patriksson showed that the KKT conditions are both necessary and sufficient for optimality. Constraints (21) and (23) for the KKT conditions fall under the class of complementarity constraints, and thus are nonlinear. A transformation methodology, specifically a disjunctive constraints approach, initially introduced by Fortuny-Amat and McCarl (1981), is employed in which the introduction of binary variables converts these constraints into equivalent linear mixed-integer constraints.

The implementation of this methodology given by Wang and Lo (2010) is followed herein. Thus, constraints (13) are replaced by constraints (26)-(28):

$$L \cdot \varphi_k^o(\xi) + \varepsilon \leq f_k^o(\xi) \leq U \cdot [1 - \varphi_k^o(\xi)], \quad \forall k \in K_o, o \in N \tag{26}$$

$$L \cdot \varphi_k^o(\xi) \leq \hat{c}_k^o(\xi) - u^o(\xi) \leq U \cdot \varphi_k^o(\xi), \quad \forall k \in K_o, o \in N \tag{27}$$

$$\varphi_k^o(\xi) \in \{0,1\}, \quad \forall k \in K_o, o \in N \tag{28}$$

where L and U are very large negative and positive numbers, respectively, and ε is a very small positive number. Binary variable $\varphi_k^o(\xi)$ indicates whether or not path k from origin node o receives a flow, i.e. $\varphi_k^o(\xi) = 0$ resulting in $\hat{c}_k^o(\xi) = u^o(\xi)$ if $f_k^o(\xi) > 0$; $\varphi_k^o(\xi)$=1, otherwise.

Similarly, constraints (23) are replaced by constraints (29-31):

$$L \cdot \phi_a(\xi) + \varepsilon \leq \lambda_a(\xi) \leq U \cdot [1 - \phi_a(\xi)], \quad \forall a \in A_2 \tag{29}$$

$$L \cdot \phi_a(\xi) \leq \textstyle\sum_s p_a^s y_a^s - x_a(\xi) \leq U \cdot \phi_a(\xi), \quad \forall a \in A_2 \tag{30}$$

$$\phi_a(\xi) \in \{0,1\}, \quad \forall a \in A_2 \tag{31}$$

where binary variable $\phi_a(\xi)$ indicates whether or not flow along link a reaches the link capacity. When the flow along link a reaches the link's capacity limitation, $\phi_a(\xi) = 0$, resulting in $\lambda_a(\xi) > 0$; and $\phi_a(\xi) = 1$, otherwise.

4.1.2. Solving BEDP-SP-SO and BEDP-RO-SO programs

BEDP-SO-SP and BEDP-SO-RO do not involve UE constraints, and thus the need for the complementarity constraints described in the prior section is eliminated; they are, thus, single-level problems. However, complementarity constraints (16) are required to ensure that risk exposure is considered within the objective only for active paths. Thus, the programs are nonlinear. Again, a disjunctive constraints transformation approach is applied wherein constraints (32)-(34) replace constraints (16).

$$L \cdot \sigma_k^o(\xi) + \varepsilon \le f_k^o(\xi) \le U \cdot [1 - \sigma_k^o(\xi)], \quad \forall k \in K_o, o \in N \tag{32}$$

$$c_k^o(\xi) - w^o(\xi) \le U \cdot \sigma_k^o(\xi), \quad \forall k \in K_o, o \in N \tag{33}$$

$$\sigma_k^o(\xi) \in \{0,1\}, \quad \forall k \in K_o, o \in N \tag{34}$$

where $\sigma_k^o(\xi)$ is a binary variable indicating whether a path is active or not: $\sigma_k^o(\xi) = 0$ if $f_k^o(\xi) > 0$; and $\sigma_k^o(\xi) = 1$, otherwise.

4.2. Piecewise Linearization of the Travel Time Function

For each link $a \in A_1$, the nonlinear travel time function is replaced by a piecewise linear function using a method presented by Sherali (2001) and also applied by Farvaresh and Sepehri (2011). The first step of this technique is to bound link flow $x_a(\xi)$ by lower and upper bounds. One simple approach to setting these bounds is to use zero and total evacuation demand from all origin nodes, i.e. $0 \leq x_a(\xi) \leq \sum_{o \in O} q^o$, $\forall\, a \in A_1$. Next, this range is partitioned into I_a non-overlapping segments. Let the link flow $x_a(\xi)$ be represented as follows:

$$x_a(\xi) = \sum_{i=1}^{I_a} \dot{x}_{a,i-1}\pi_{a,i}^L + \dot{x}_{a,i}\pi_{a,i}^R, \quad \forall a \in A_1 \tag{35}$$

where $\dot{x}_{a,i-1}$ and $\dot{x}_{a,i}$ are link flow values at endpoints of segment i, and $\pi_{a,i}^L$ and $\pi_{a,i}^R$ are convex-combination weights of that segment such that equations (36) and (37) hold.

$$\pi_{a,i}^L + \pi_{a,i}^R = \theta_{a,i}, \quad \forall a \in A_1, i = 1,2,..,I_a \tag{36}$$

$$\sum_{i=1}^{I_a} \theta_{a,i} = 1, \quad \forall a \in A_1 \tag{37}$$

where

$$\pi_{a,i}^L, \pi_{a,i}^R \geq 0, \quad \forall a \in A_1, i = 1,2,..,I_a \tag{38}$$

$$\theta_{a,i} \in \{0,1\}, \quad \forall a \in A_1, i = 1,2,..,I_a \tag{39}$$

Then, the link travel time function can be replaced by the piecewise linear function given in (40).

$$t_a[x_a(\xi)] = t_a^0 + b_a.[\sum_{i=1}^{I_a} \dot{x}_{a,i-1}^2\pi_{a,i}^L + \dot{x}_{a,i}^2\pi_{a,i}^R], \quad \forall a \in A_1 \tag{40}$$

An advantage of this linearization method is that the matrix of coefficients in these added constraints (constraints (36)-(39)) is totally unimodular, making it possible to relax integrality constraints (39) (see Sherali (2001) for more details).

Given the above mathematical replacements, the nonlinear BEDPs are reformulated as SMIPs presented in Table 3.

Table 3 BEDPs reformulated as two-stage SMIPs

Problem	Objective function	Constraints					
		1st stage	2nd stage				
		Design decisions	Link/path flow assignment	UE CCs*	Capacitated link CCs*	Active path CCs *	Link travel time function linearization
		(3)-(6)	(9)-(13)	(26)-(28)	(29)-(31)	(32)-(34)	(35)-(40)
BEDP-SP-UE	$\min_y E_{\xi\in\Xi}[\min_x \max_{o\in O} u^o(\xi)]$	✓	✓	✓	✓	-	✓
BEDP-RO-UE	$\min_y \max_{\xi\in\Xi}[\min_x \max_{o\in O} u^o(\xi)]$	✓	✓	✓	✓	-	✓
BEDP-SP-SO	$\min_y E_{\xi\in\Xi}[\min_x \max_{o\in O} w^o(\xi)]$	✓	✓	-	-	✓	✓
BEDP-RO-SO	$\min_y \max_{\xi\in\Xi}[\min_x \max_{o\in O} w^o(\xi)]$	✓	✓	-	-	✓	✓

* CC: Complementarity Constraints

5. Solution Methodology

The integer L-shaped method, introduced by Laporte and Louveaux (1993), is adopted to solve the four variants of the BEDP, each having only binary decision variables in the first stage as required by the procedure. This method is exact. It decomposes the original program into a master problem and set of subproblems representing second-stage problems $Z(\xi)$ for each scenario. Let $y \in Y = \{y_a^s, \acute{y}_a^e\}_{(a)\in A_2 \cup A_3, s\in S, s\in E}$ represent all first-stage variables. The master problem is generally formulated as follows.

$$min\ \theta \tag{41}$$

s.t.

(3-5)

$$0 \le y \le 1 \tag{42}$$

$$f(y, \theta) \le 0 \tag{43}$$

where the objective is to minimize θ, an approximation of the expectation (maximum) of the second-stage objective functions $Z(\xi)$ over all scenarios $\xi \in \Xi$ for a general stochastic program or in robust optimization. Constraints (42) are relaxations of integrality constraints (6) for first-stage variables.

To solve the master problem, branch-and-bound steps are integrated within the procedure to obtain binary solutions at each iteration. The binary variables of these solutions are fixed in the subproblems. Optimality cuts (43) are iteratively generated and added to the master problem based on solution of the subproblems, creating a tighter feasible region. No feasibility cut is required, since the master problem solution is always feasible for the subproblems.

The number of feasible first-stage solutions, each of which is indexed by ϵ, is finite, as all first-stage variables are binary. The binary solution corresponding to the ϵth feasible solution set is represented by y_r^ϵ, where r is the index of first-stage variables in the ϵth feasible solution set. Let R_1^ϵ be the set of indices with corresponding binary solutions equal to 1, i.e. $R_1^\epsilon = \{r | y_r^\epsilon = 1\}$. Then valid optimality cuts can be generated by (44).

$$f(y,\theta) = \{\theta^\epsilon - LB\}\left[\sum_{r \in R_1^\epsilon} y_r^\epsilon - \sum_{r \notin R_1^\epsilon} y_r^\epsilon\right] - \{\theta^\epsilon - LB\}(|R_1^\epsilon| - 1) + LB - \theta \leq 0, \qquad (44)$$

where $|R_1^\epsilon|$ is the cardinality of the set R_1^ϵ, and LB is a finite lower bound, which can be set to zero in this problem. However, a tighter lower bound could significantly improve the solution time. A tighter lower bound can be obtained by relaxing the budget constraint and solving the subproblems assuming best-quality shelters are constructed in all candidate locations.

Let $Z(Y^\epsilon, \xi)$ be the second-stage problem under scenario ξ with first-stage variables fixed at the ϵth set of first-stage values, Y^ϵ. Laporte and Louveaux (1993) proved that cuts given by (44), where $\theta^\epsilon = \theta_{SP}^\epsilon = E_{\xi \in \Xi}[Z(Y^\epsilon, \xi)]$ (i.e., the expectation over second-stage objective functions corresponding to first-stage feasible solutions Y^ϵ), are valid for stochastic programs. Therefore, cuts (44) can be directly applied to solve both the BEDP-SP-UE and BEDP-SP-SO. In this paper, these cuts are further modified for solving robust optimization versions: BEDP-RO-UE and BEDP-RO-SO.

Proposition 1. Let $\theta^\epsilon = \theta_{RO}^\epsilon = max_{\xi \in \Xi}[Z(Y^\epsilon, \xi)]$ be the maximum second-stage objective function over all scenarios $\xi \in \Xi$ corresponding to first-stage feasible solutions Y^ϵ. Modified optimality cuts (45) are valid cuts for BEDP-RO-UE and BEDP-RO-SO.

$$f(y,\theta) = \{\theta_{RO}^\epsilon - LB\}\left[\sum_{r \in R_1^\epsilon} y_r^\epsilon - \sum_{r \notin R_1^\epsilon} y_r^\epsilon\right] - \{\theta_{RO}^\epsilon - LB\}(|R_1^\epsilon| - 1) + LB - \theta \leq 0. \qquad (45)$$

Proof. The inequality $\sum_{r \in R_1^\epsilon} y_r^\epsilon - \sum_{r \notin R_1^\epsilon} y_r^\epsilon \leq |R_1^\epsilon|$ always holds; thus, the right-hand side of (45) takes a value less than or equal to LB. In the extreme case where $\sum_{r \in R_1^\epsilon} y_r^\epsilon - \sum_{r \notin R_1^\epsilon} y_r^\epsilon = |R_1^\epsilon|$, the right-hand side will be equal to θ_{RO}^ϵ. Therefore, the cuts (45) will never eliminate the globally optimal solution, and it is valid to impose them on first-stage solutions. □

Note that in numerical experiments described in Section 6, to improve the implementation time of the UE-based problems, the corresponding SO-based problems were solved first and their objective function values were used as the LB in optimality cuts (44) and (45).

The general algorithm of the integer L-shaped method (Laporte and Louveaux 1993) to solve the BEDPs is presented in the following. Let $\bar{Z}$ be the upper bound of the desired stochastic program or robust optimization model Z, and μ be the algorithm iteration number:

Step 0: Set $\mu = 0$, upper bound $\bar{Z} = \infty$. The value of θ is set to $-\infty$ or other absolute lower bound. A pendant node list is created that contains only a single pendant node corresponding to the initial subproblem.

Step 1: Select a pendant node in the list. Stop if the pendant node list is empty.

Step 2: Set $\mu = \mu + 1$ and solve the current problem. If the problem is infeasible, fathom the current node and go to *Step 1*. Otherwise, let (y^{μ}, θ^{μ}) be an optimal solution.

Step 3: Check for integrality. If violated, create two new branches in which the most fractional variable is set to 0 or 1. Append the two nodes to the pendant node list and go to *Step 1*.

Step 4: Given the first-stage solutions y^{μ}, solve the sub-problems $Z(y^{\mu}, \xi)$ for each scenario ξ. If the model is a stochastic program, calculate the expectation value over all scenarios, $Z(y^{\mu}) = \underset{\xi \in \Xi}{E}[Z(y^{\mu}, \xi)]$. Otherwise, if the model is of robust optimization models, calculate the corresponding maximum value over all scenarios, $Z(y^{\mu}) = \max_{\xi \in \Xi}[Z(y^{\mu}, \xi)]$. If $Z(y^{\mu}) < \bar{Z}$, update upper bound $\bar{Z} = Z(y^{\mu})$.

Step 5: If $\theta^{\mu} \geq Z(y^{\mu})$, then fathom the current node and go to *Step 1*; otherwise, impose an optimality cut to the master problem, and return to *Step 2*.

6. Numerical Example

6.1. Network Representation

Numerical experiments were conducted using the design of an actual office building. The building has a reinforced concrete structure and consists of two connected wings that surround an inner courtyard. In the original design of the building, each wing has a core containing a shelter. The layout of the building is illustrated in Fig. 2.

Two exits (E1 and E2) were already included in the initial building design. One additional emergency exit (E3) was also considered for incorporation in the design, and is represented by dashed lines. Seven locations were taken as candidates to fortify as shelters represented by dashed ovals (S1-S7). Four hallways (H1-H4) were already included in the building evacuation plan as relatively safe locations for evacuees in case of a hazard. One additional hallway, H5, was also considered in this example as a candidate for fortification. The network representation includes 75 links, as well as 15 dummy links that connect the locations of shelters, exits, and fortified hallways to the supersink node.

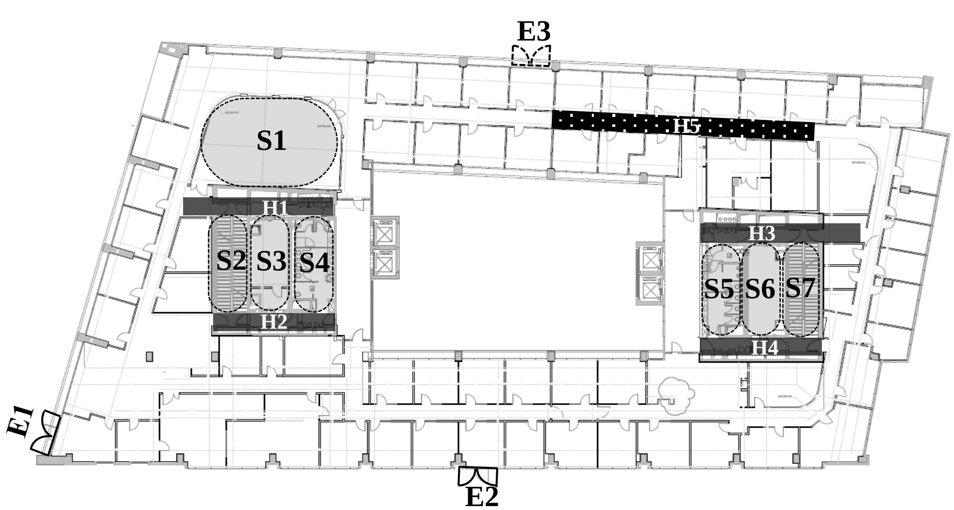


Fig. 2. Office building layout

Forty rooms in the building were considered evacuation origin nodes. The number of evacuees in these rooms was estimated based on their maximum occupancies from the National Fire Protection Association (NFPA) Life Safety Code (2009), and given in Table 4.

Table 4 Maximum occupancy of rooms in building

Room #	Max occ.	Room #	Max occ.	Room #	Max occ.	Room #	Max occ.
1	4	12	6	22	4	39	4
2	4	13	2	23	5	40	4
3	2	14	2	24	1	41	4
4	2	15	4	25	2	42	4
5	3	16	4	26	4	43	4
6	5	17	4	27	5	44	4
7	1	18	4	28	2	45	4
8	2	19	4	32	5	49	6
10	4	20	4	33	5	50	6
Total building occupancy =150 people							

6.2. Modeling Parameters

In this example, only one fortification or construction type was considered for each location in terms of level of protection, cost, and capacity. However, the general formulation of the optimization model allows different design options to be considered for any single location out of which one option can then be selected through the optimization. The costs and capacities (in terms of number of evacuees) of the design options are given in Table 5. These were estimated based on current average construction costs.

Table 5 Costs and capacities of design options

Design option	ID	Design cost ($)	Capacity
Shelter	S1	6,700	35
	S2	4,100	15
	S3	5,600	25
	S4	5,000	25
	S5	3,700	15
	S6	3,900	25
	S7	4,100	15
Unfortified hallway	H1	-	30
	H2	-	30
	H3	-	30
	H4	-	30
Hallway fortification	H5	3,600	40
Emergency exit	E3	2,200	-

Five disaster scenarios were generated, assuming 20% occurrence probability of each: one scenario for an external malicious act which is likely to affect the whole building equally, and four scenarios for an internal fire in different parts of the building (north, south, west, and east). The stochastic nature of these scenarios is captured through parameters $\alpha(\xi)$ and $\beta(\xi)$ in the risk exposure function; $\alpha(\xi)$ represents the slope of the risk function line converting the evacuation time through passageways to a risk exposure value, and $\beta(\xi)$ represents the risk imposed by exiting the building or staying in a safe location.

To quantify the risk to which evacuees are exposed, a range of 0-100 points was considered, where 0 indicates no risk exposure and 100 indicates a maximum risk exposure (which can be interpreted as a high risk of death). To find risk equivalency of evacuation time, it was assumed that the maximum tolerable evacuation time is equal to a risk exposure of 100 points and occurs at 120 seconds for an external malicious act and at 180 seconds for an internal fire. This results in $\alpha(\xi)$ values of 0.83 (=100/120) and 0.55 (=100/180), respectively. Moreover, given the range of 0-100, the risk exposure of using each individual evacuation option under different hazard types was estimated and is given in Table 6.

Table 6 Scenario-dependent values of parameter $\beta(\xi)$ in risk exposure function

Scenario	Evacuation option			
	Exit	Shelter	Unfortified hallway	Fortified hallway
External malicious act	100	5	30	10
Internal fire	0	20	100	40

The travel time function is divided into 20 linear segments with respect to link flow, and the function parameters for passageways $a \in A_1$, t_a^0 and c_a, are estimated from the Society of Fire Protection Engineers' (SFPE) Handbook (2002) based on passageway lengths, widths, and average speed of evacuees. These are presented in Table 7. Finally, four budget levels of $0, $7500, $15,000, and $42,000 (a sufficient budget for the construction of all the design options) are considered for experimental runs.

Table 7 Values of passageway travel time function parameters

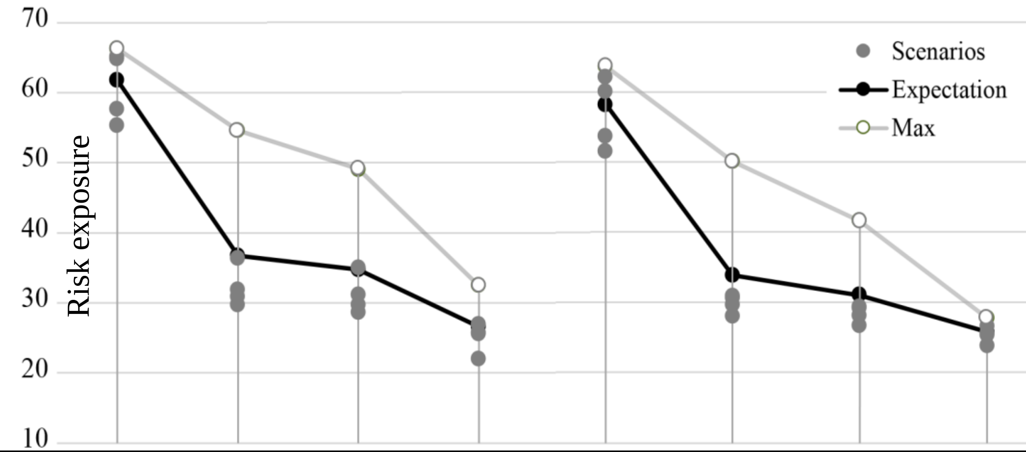
Link ID	Link type*	t_a^0 (s)	c_a (evac./s)	Link ID	Link type*	t_a^0 (s)	c_a (evac./s)	Link ID	Link type*	t_a^0 (s)	c_a (evac./s)
1	C	2.5	2	26	C	5.6	2	51	C	4.5	2
2	C	3.0	2	27	C	2.7	2	52	C	4.3	2
3	C	2.1	2	28	D	3.1	1	53	C	4.1	2
4	D	3.1	1	29	C	4.0	3	54	D	4.9	1
5	C	2.3	2	30	D	9.5	1	55	C	3.6	2
6	C	2.6	2	31	D	4.1	1	56	C	3.5	2
7	C	1.7	2	32	D	8.2	1	57	C	3.1	2
8	C	2.1	2	33	D	6.6	1	58	C	4.7	2
9	C	2.5	2	34	C	2.3	3	59	D	10.8	1
10	C	3.2	2	35	D	2.8	1	60	C	0.8	3
11	C	4.0	2	36	S	4.3	1	61	D	8.5	1
12	C	4.3	2	37	C	4.6	2	62	D	1.7	1
13	C	4.4	2	38	C	3.5	2	63	D	3.7	1
14	C	3.6	2	39	C	3.4	2	64	S	2.4	1
15	D	5.2	1	40	D	5.6	1	65	D	7.6	1
16	D	5.6	1	41	S	2.8	1	66	C	3.0	3
17	C	4.1	2	42	D	3.9	1	67	D	7.0	1
18	C	4.3	2	43	D	2.1	1	68	D	7.9	1
19	D	3.7	1	44	D	8.6	1	69	D	3.3	1
20	C	3.8	2	45	D	2.0	1	70	D	3.9	1
21	C	2.2	2	46	D	9.8	1	71	S	4.8	1
22	C	2.4	2	47	D	9.7	1	72	D	3.2	1
23	C	3.5	2	48	C	20.8	2	73	C	4.2	2
24	C	3.4	2	49	D	6.4	1	74	C	3.8	2
25	C	3.5	2	50	C	2.2	2	75	C	3.6	2

*D=Door, C=Corridor, S=Stairs

6.3. Experimental Results

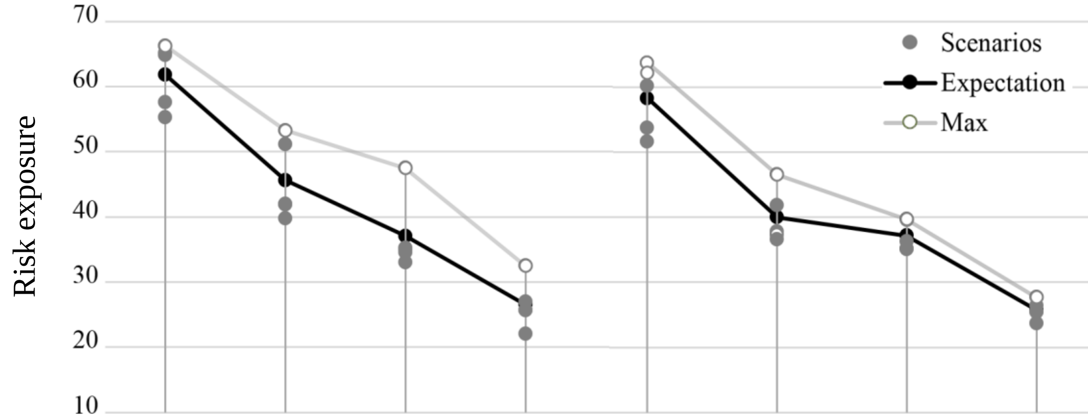
The SP (BEDP-SP-UE, BEDP-SP-SO) and RO (BEDP-RO-UE and BEDP-RO-SO) model results are reported in Tables 8 and 9, respectively. The RO and SP approaches led to different design solutions. Scenarios with external hazards frequently give the worst results in terms of evacuation risk exposure. Under these scenarios, the RO design solutions are best, because they target these worst-case situations.

Table 8 SP run results



Problem	BEDP-SP-UE				BEDP-SP-SO			
Budget ($×1000)	0	7.5	15	42	0	7.5	15	42
Selected design options	-	S7, H5	S4, S7, H5, E3	All	-	S7, H5	S2, S7, H5, E3	All
Expected risk	61.8	36.7	34.7	26.6	58.3	33.8	31.0	25.8
Max. risk	66.3	54.6	49.1	32.5	63.7	50.1	41.6	27.7
Standard deviation	4.5	9.2	7.5	3.4	4.8	8.2	5.4	1.3

Table 9 RO run results

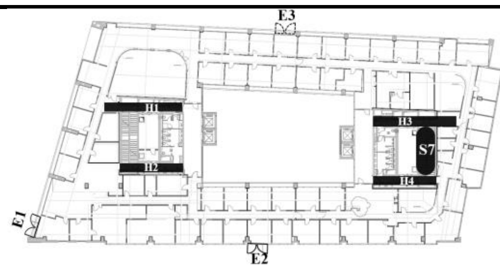
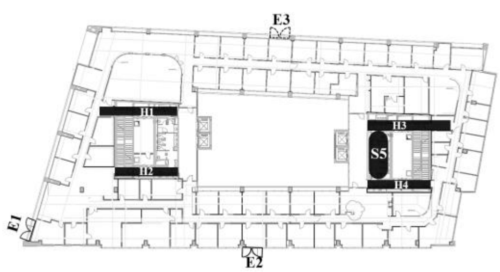
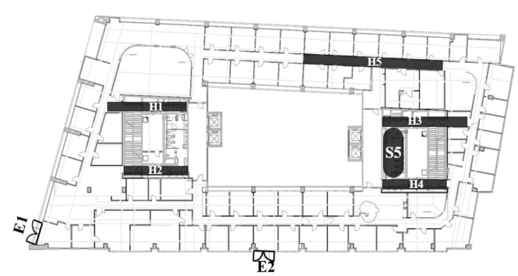
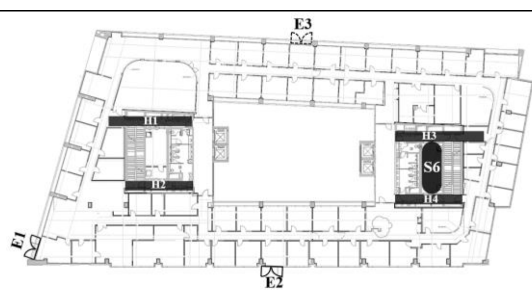
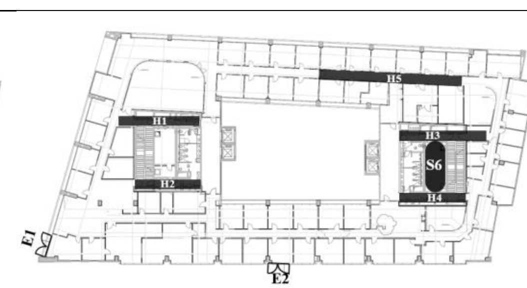


Problem	BEDP-RO-UE				BEDP-RO-SO			
Budget ($×1000)	0	7.5	15	42	0	7.5	15	42
Selected design options	-	S6, E3	S4, S5, H5, E3	All	-	S6, E3	S4, S6, H5	All
Expected risk	61.8	45.6	37.1	26.6	58.3	40.0	37.2	25.8
Max. risk	66.3	53.3	47.5	32.5	63.7	46.5	39.7	27.7
Standard deviation	4.5	5.5	5.3	3.4	4.8	3.7	2.1	1.3

SO solutions have only a slightly lower evacuation risk exposure compared to modeling under the UE condition for the same level of budget. This is also true in those cases in which the same optimal design solution was identified under SO or UE conditions. The difference in objective function values quantifies the benefits to the system of enforcing SO-derived routes and shelter/exit assignments. With a budget of $15,000, for example, the reduction in expected risk exposure achieved by enforcing the SO solution over allowing individuals the freedom to choose their own paths is approximately 12%. Thus, for this specific application, the price of anarchy or inefficiency created by allowing users to behave selfishly is moderate. By comparison, estimates of such inefficiencies were obtained by Youn et al. (2008) for several real-world traffic networks. Assuming that delays increase very steeply with large traffic volumes, under significant congestion, the traffic networks in New York, London, and Boston were found to operate with inefficiencies of between 24 and 30%.

Moreover, the maximum as well as the dispersion of risk data points over all scenarios (measured by standard deviation) diminishes through an RO approach. That is, RO modeling results in better solutions. Similar reduction in standard deviation is noted when comparing implementations with SO and UE conditions. That is, as expected, the SO solutions outperform the UE solutions. Of course, their practical implementation requires some level of support to ensure that evacuees adhere to directives.

Table 10 Optimal design solutions under internal-only scenarios vs. internal and external scenarios (budget= $7,500)

<table>
<tr><th rowspan="2">Problem</th><th colspan="2">Hazard type</th></tr>
<tr><th>Internal</th><th>Internal & external</th></tr>
<tr><td>BEDP-SP-UE</td><td colspan="2" rowspan="2">
</td></tr>
<tr><td>BEDP-SP-SO</td></tr>
<tr><td>BEDP-RO-UE</td><td>
</td><td>
</td></tr>
<tr><td>BEDP-RO-SO</td><td>
</td><td>
</td></tr>
</table>

The optimal design solutions were also determined under only internal fire scenarios given a budget of $7,500. The corresponding results are reported and compared with the design solutions under both internal and external scenarios in Table 8, and resulting designs are depicted in Table 10. Identical solutions are found for SPs under UE and SO conditions. However, a design shift is made from fortification of hallway 5 to construction of exit 3 for internal only scenarios. Evacuating out of the building through an emergency exit is the least desired option under the external malicious act scenario. When only an internally produced hazard is considered, evacuation from the building will produce best results. The presence of such diametrically opposed optimal design solutions highlights the importance of pursuing a multi-hazard approach.

7. Conclusions and Extensions

The mathematical program presented in this study allows the identification of building design solutions that ensure the safety of evacuees during emergencies. The program can be used to investigate different alternatives for the design of shelters, fortified hallways and exits in buildings, and permits exact solution that minimizes the exposure of evacuees to risks under various hazard scenarios. This solution requires a novel approach that differs from previous studies on building evacuation, which deal mainly with the analysis of a predefined building design, as well as previous studies on regional evacuation problems, which have focused on the minimization of evacuation time for a single type of hazard. The explicit consideration of risk exposure includes not only the time evacuees will spend in different locations in the building (which in turn depends on the length of the path traveled as well as on the number of people using that path), but also the level of protection from hazards that these locations provide.

This study follows a multi-hazard approach, in which different types of hazards are simultaneously taken into account when searching for an optimal solution. This can be crucial, since for each type of hazard a different solution may produce the best results, but eventually a single design solution must be chosen. All other relevant works in the literature consider only a single hazard class. Furthermore, the program allows the use of an objective function based on expectation, which gives weight to a range of hazard scenarios, or a more conservative RO approach, which focuses on the worst-case scenario in terms of evacuation risk exposure.

Finally, model variants allow different types of user responses to be considered by embedding either SO or UE conditions. The SO approach assumes that evacuees will be guided by a trained staff person who is fully informed of the conditions in the building. This may be appropriate in certain types of buildings (e.g., train stations), in certain circumstances in which a building may be used (e.g., a concert or sporting event), and for certain types of events for which such information can be provided (e.g., an internal fire). The UE approach assumes that fully informed evacuees will themselves choose their evacuation paths and destinations, and that the evacuees have full information about their options. This may be appropriate in buildings with which the evacuees are highly familiar (e.g., their home or workplace), and for certain types of events for which they have been repeatedly trained or which they have repeatedly experienced.

The actual behavior of evacuees during emergencies will vary depending on factors including their familiarity with this type of event, the building layout, the complexity of this layout, their relationships with other evacuees, and the type of guidance and information they receive in real time. Thus, aspects of evacuee behavior, such as the degree to which they will be well-informed on the actual risks at hand and whether they will behave selfishly, are difficult to precisely predict in advance. A model which would seek to accurately reflect actual evacuee behavior might combine to some degree the UE and SO approaches, depending on the particular context of the building, its occupants and the event. Since this paper study sought to support the design of buildings, rather than the management of an actual evacuation, it was useful to model the evacuation under both UE and SO assumptions and compare the obtained results, while taking into account that the reality will likely lie somewhere in between those two extremes. Such an approach is in line with the general practice in building design, in which systems are subjected to extreme loads of different types to ensure their robustness to varying conditions.

Though the present set of programs is appropriate for supporting the design of buildings, an implementation of the UE approach for the actual management of evacuation events would require the development of a dynamic model in which link travel times are continuously reassessed, and of a sensor-based system that can capture in detail the movements of evacuees and provide in real-time information to each evacuee.

An implementation of the program in a case study of a geographical evacuation problem is planned as well. The use of a program that minimizes the exposure of evacuees to risk, through an explicit consideration of the level of protection that different evacuation routes and shelters provide, may constitute an improvement on previous geographical evacuation models that did not address such an objective. For example, in a flooding scenario, the risks of using different evacuation routes, depending on their location and elevation, can be considered when planning the location of emergency shelters.

Additional extensions may be desirable. For example, shelter capacities may be uncertain due to their multi-purpose use. That is, a shelter may be used for a community activity and thus filled to capacity at the time it is needed. Heterogeneity in the evacuee population is ignored herein. However, some evacuees may move more quickly than others. Some evacuees may put more weight on risk exposure from traveling in the corridors versus waiting for help in a shelter than other evacuees. Moreover, risk perception may vary by evacuee and may be imperfect. Thus, alternative models for handling risk may be appropriate. Individualized risk functions may be warranted, and a stochastic UE may be beneficial.

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