

**EVALUATION OF VARIABILITY
IN RESILIENT MODULUS TEST
RESULTS (ASTM D 41 23)**

by

**E. R. Brown
Kee Y. Foo**

October 1989

EVALUATION OF VARIABILITY IN RESILIENT MODULUS TEST RESULTS (ASTM D 41 23)

by

**E. R. Brown
Kee Y. Foo**

National Center for Asphalt Technology

NCAT Report No. 91-6

“The contents of this report reflect the views of the authors who are solely responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views and policies of the National Center for Asphalt Technology of Auburn University. This report does not constitute a standard, specification, or regulation.”

EVALUATION OF VARIABILITY IN RESILIENT MODULUS

TEST RESULT (ASTM D 4123)

ABSTRACT

Samples of **asphalt** mixture were evaluated in the laboratory under various conditions to evaluate the repeatability of the resilient modulus test and to evaluate the effect of stress on the measured resilient modulus. Some of the samples were prepared in the laboratory and others were obtained from in-place pavements that had been subjected to traffic. The independent variables investigated included stress, test temperature, and maximum aggregate size.

Tests were repeated a number of times and the data was analyzed by SAS to investigate its repeatability. This study quantified the repeatability of the ASTM D 4123 resilient modulus test as function of stiffness. The repeatability of resilient modulus test (ASTM D 4123) is low. A significant increase in the number of samples or number of measurements is required to improve the repeatability making it unfeasible. Tests conducted at different stresses showed resilient modulus to be stress sensitive. This indicated that stress should be specified in the test procedure. A correction factor was established for stresses differing from the recommended stress (15% of tensile stress) for test temperature of 25 °C and 40 °C.

Keywords: Resilient modulus, asphalt mixes, repeatability, variance, standard error, coefficient of variation.

TABLE OF CONTENTS

List of Tables	ii
List of Figures	iii
Introduction	1
Background	1
Objectives	1
Scope	2
Literature Review	4
Stiffness Moduli	4
Review and Analysis of Resilient Modulus Test (ASTM D 4123)	4
Test Plan	9
Part One	10
Part Two	13
Part Three	15
Prediction of Tensile Strength	16
Sample Information	19
Lab Samples	19
Field Samples	21
Test Results	
Results from Part One of Test Plan	22
Results from Part Two of Test Plan	23
Results from Part Three of Test Plan	28
Conclusion and Recommendation	35
References	39

List of Tables

<u>Table</u>	<u>Page</u>
1 Experimental Design #1	12
2 Variability of Test for Part One of Test Plan.....0.00.0	12
3 Experimental Design#2	14
4 Experimental Design #3.	16
5 Density and Tensile Strength of Laboratory Samples.....00.....00.....	19
6 Maximum Aggregate Size, Density, and Tensile Strength of Field Samples	21
7 σ^2_{test} of laboratory mixes at 25 °C0.	22
8 $\sigma^2_1, \sigma^2_2, \sigma^2_3,$ and σ^2_{ASTM} of laboratory mixes at 25°C	23
9 Standard Error, CV, and Acceptable Range of Two Tests for laboratory mixes at 25°C .	24
10 Maximum Aggregate Size, Slope and Mean MR of Laboratory Mixes	27
11 Variances of Field Mixes	28
12 Maximum Aggregate Size and Slope of Field Mixes	32

List of Figures

<u>Figure</u>		<u>Page</u>
1	Resilient Modulus Test Equipment .00	3
2	Schematic diagram for determining σ_{ASTM}^2 . . .	6
3	Typical recorder output of a resilient modulus test "	9
4	Graphical view of method of deformation measurement	10
5	Indirect Tensile Test (ASTM D 4123)	18
6	Asphalt Concrete Modulus-Temperature Relationship000**	18
7	Aggregate gradation of laboratory mixes . . .	20
8	Gyratory Calibration Graph . 0	20
9	Effect of Stress on Resilient Modulus of Laboratory Mixes at 25 °C	27
10	Sources of Variation in Resilient Modulus (ASTM D 4123) "	30
11	Variation in Resilient Modulus	30
12	CV and Acceptable Range of Two Test Results000	31
13	Effect of stress on MR for 25.4 mm (1 in) Aggregate Field Mixes	32
14	Effect of Stress on MR for 19.0 mm (3/4 in) Aggregate Field Mixes	33
15	Effect of Stress on MR for 12.7 mm (1/2 in) Aggregate Field Mixes	33
16	Effect of Stress on MR for Field Mixes at 25 °C000	35
17	Effect of Stress on MR for Field Mixes at 40 °C	35

INTRODUCTION

Background

In recent years, there has been a change in philosophy in flexible pavement design from the more empirical approach to the mechanistic approach based on elastic theory (1, 2, 3) . Proposed by AASHTO (1) in 1986, this mechanistic approach in the form of layered elastic theory is being used by increasing numbers of highway agencies. Elastic theory based design methods require as input the elastic properties of pavement materials. Resilient modulus of asphalt mixtures, measured in the indirect tensile mode (ASTM D 4123), is the most popular form of stress-strain measurement used to evaluate elastic properties. The resilient modulus along with other information is then used as input to the elastic theories model to generate an optimum thickness design. Therefore, the effectiveness of the thickness design procedure is directly related to the accuracy and precision in measuring the resilient modulus of the asphalt mixture. The accuracy and precision are also important in areas where resilient modulus is used as an index for evaluating stripping, fatigue, and low temperature cracking of asphalt mixtures. Items that affect the accuracy and precision of ASTM D 4123 are not well understood; thus research is needed.

Objectives

The principle objective of this paper was to evaluate the

repeatability of the ASTM D 4123 procedure using the resilient modulus test equipment shown in Figure 1. The repeatability measured in this study is for one operator using one type of test equipment in one laboratory . Repeatability evaluation involving comparison of test results from different operators using different pieces of equipment in different laboratories were not study here.

Another objective was to evaluate the effect of stress on resilient modulus. The effect of stress can then be accounted in **measured** resilient modulus values to standardize test results.

Scope

The test procedures used in this study were those outlined in ASTM D 4123. The machine used was an H & V resilient modulus device (Figure 1) which is a pneumatic device generating load pulses. The device was set to apply repeated 1 Hz repeated haversquare load waveform with load duration of 0.1 sec and rest period of 0.9 sec on test samples. LVDTs were used to measure deformation. Test transducers (load cell and **LVDTs**) were connected through A/C carrier preamplifiers to a two-channel **Oscillographic** strip-chart recorder.

Three mixes, **Mix** A, Mix B, and Mix C, each having maximum aggregate size of 25.4 mm (1 in), 19.0 mm (3/4 in), and 12.7 mm (1/ 2 in) respectively were used in this study. Five specimens were fabricated from each mix at optimum asphalt content established by Marshall mix design criteria using a gyratory compactive effort (set at 1° rotation angle, 30 revolutions, and 1380 **kN/m²**)

equivalent to 75 blows of Marshall procedure. Fourteen field mixes were obtained from cores taken from four pavements which contained several layers of asphalt concrete. Each core was separated into the various pavement layers and each layer was identified as one field mix. Three cores were obtained from each pavement giving three specimens for each field mix.

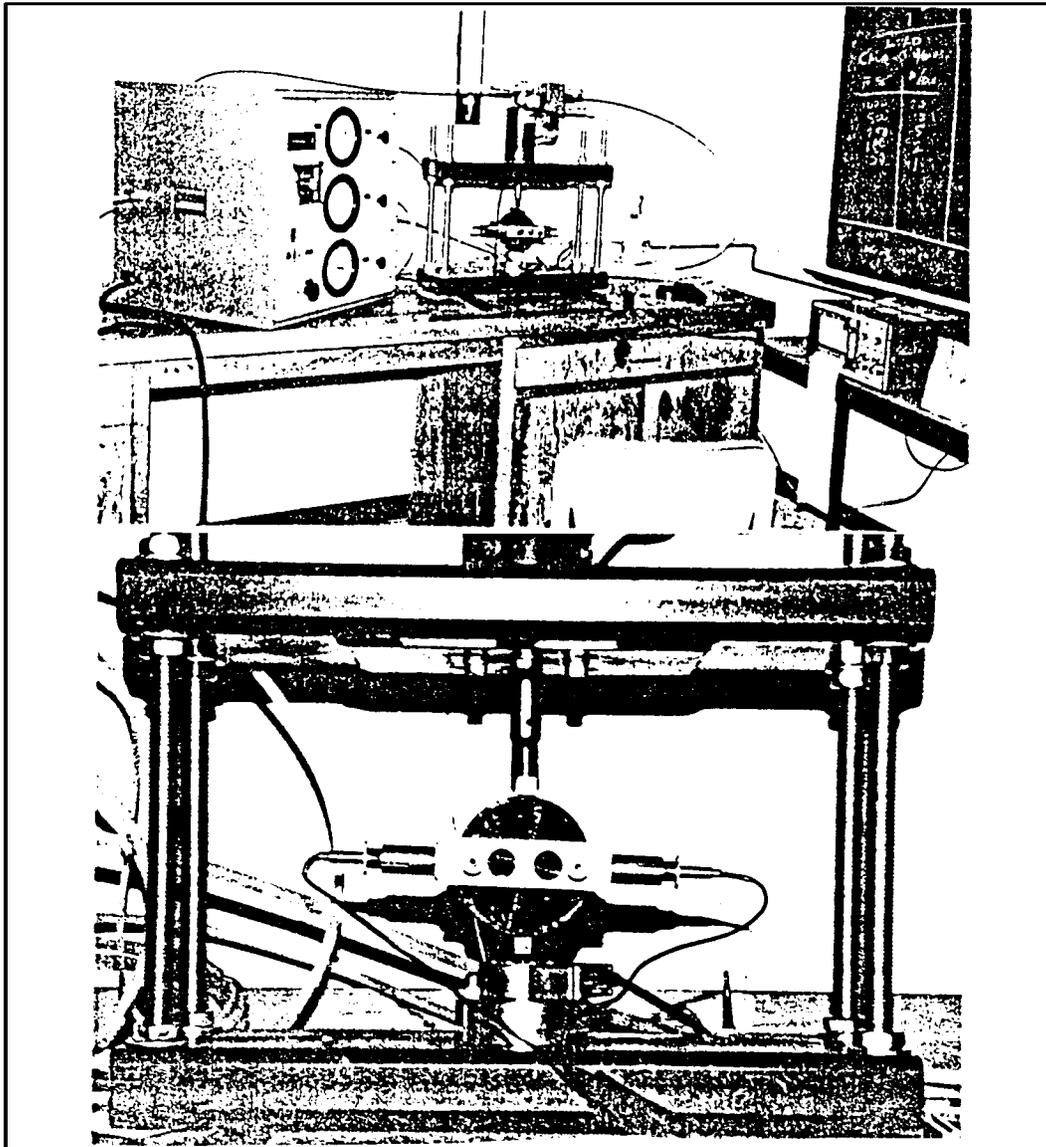


Figure 1. Resilient modulus test equipment.

LITERATURE REVIEW

Stiffness Moduli

Flexible pavement design methods based on elastic theories require that the elastic properties of the pavement materials be known (1, 2, 3). **Mamlouk** and Sarofim (4) concluded from their work that among the common methods of measurement of elastic properties of asphalt mixes (which are Young's, shear, bulk, complex, dynamic, double punch, resilient, and Shell nomograph **moduli**), the resilient modulus is more appropriate for use in **multilayer** elastic theories. Different test methods and equipment have been developed and employed to measure these different **moduli**. Some of the tests employed are **triaxial** tests (constant and repeated cyclic loads), cyclic flexural test, indirect tensile tests (constant and repeated cyclic load), and creep test. **Baladi** and Harichandran (5) indicated that resilient modulus measurement by indirect tensile test is the most promising in terms of repeatability. Resilient modulus measured in the indirect tensile mode (ASTM D 4123) has been selected by most engineers as the way to measure the resilient modulus of asphalt mixes. There is limited information on the precision of this test as presented in the ASTM standard or as published in other literature.

Review and **Analysis** of Resilient Modulus Test (ASTM D 4123)

ASTM D 4123 recommends a total of three laboratory fabricated

specimens or three cores be tested in order to determine the resilient modulus of that asphalt mix. Each of the specimens or cores is tested twice (the orientation of the specimen of the second test is 90° from the first test) producing a total of six measured resilient modulus values. The average of these six resilient modulus values is reported as the resilient modulus of the asphalt mix at that particular test temperature. Since ASTM D 4123 averages resilient modulus values measured from three specimens and at two orientations, it introduces three sources of error or variation, σ_1^2 , σ_2^2 and σ_3^2 . Experimental error (σ_1^2) is associated with random error that occurs in measurement of resilient modulus. Orientation variation (σ_2^2) is associated with the variation of resilient modulus values at different orientations in a specimen. Sample variation (σ_3^2) is associated with the variation of resilient modulus values of different samples. The combined effect of these three sources of variation produce the variation in resilient modulus, σ_{ASTM}^2 . If the resilient modulus at different orientations of a specimen remains constant ($\sigma_2^2 = 0$) and specimens from one mix are identical ($\sigma_3^2 = 0$), then the variation in resilient modulus (ASTM D 4123) equals to the experimental error ($\sigma_{ASTM}^2 = \sigma_1^2$). For materials such as rubber, fiberglass, and other homogeneous materials σ_2^2 and σ_3^2 would approach zero. However for asphalt mixtures which are not homogeneous the σ_2^2 and σ_3^2 error are likely to be relatively large.

Statistical analysis of data developed in this study will provide information needed to estimate the variation in resilient

modulus. The process on how the variation in resilient modulus was estimated through the three sources of variation is shown schematically in Figure 2.

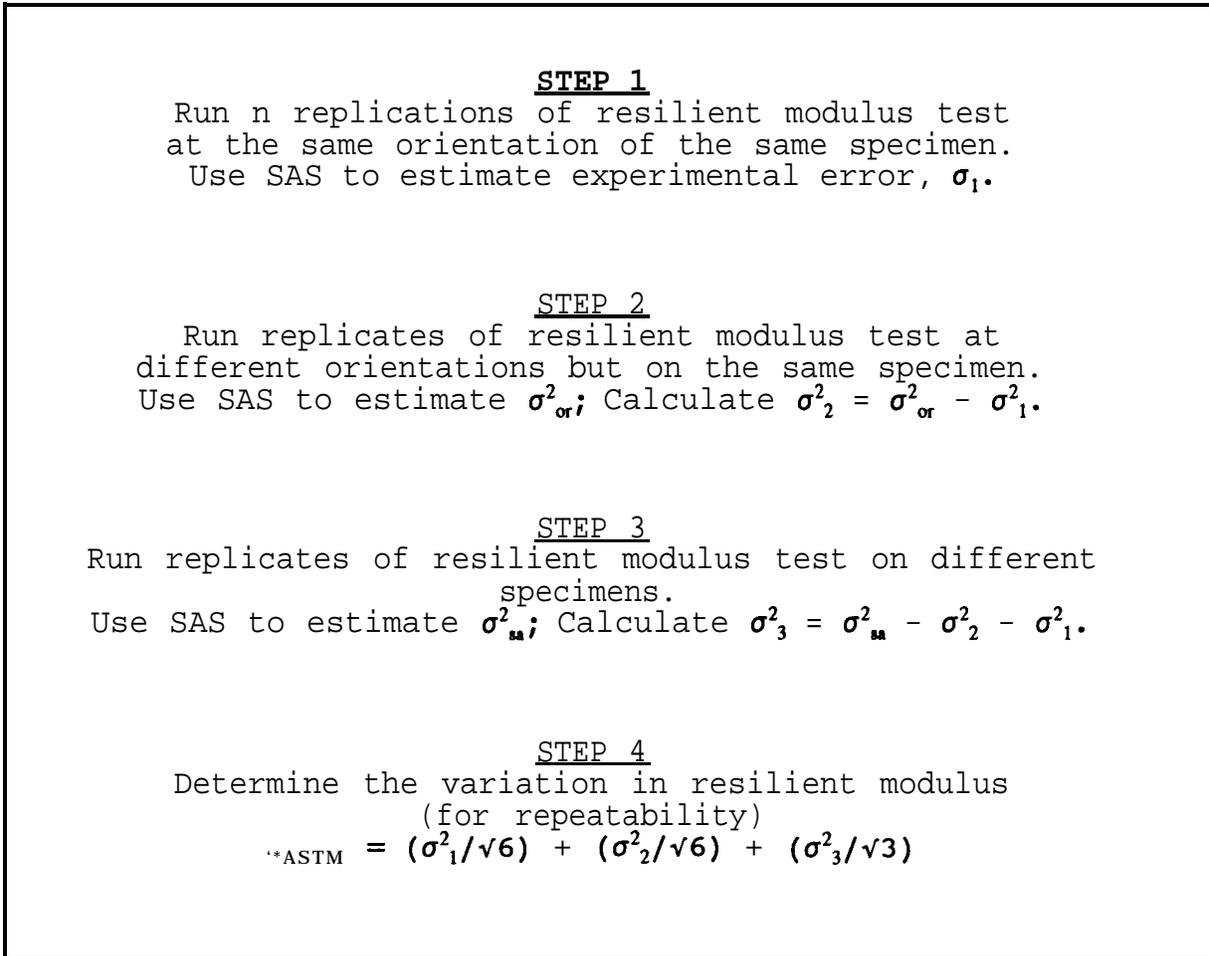


Figure 2. Schematic diagram for determining σ_{ASTM}^2 .

Experimental error (σ_1) is primarily a function of the resilient modulus equipment and operator. σ_1^2 was estimated by analyzing a number of repetitions of resilient modulus values measured at the same orientation of the same specimen. The

variation in the measured resilient modulus values was attributed to σ_1^2 , since the measurements were taken at the same orientation of the same specimen (σ_2^2 and σ_3^2 , equals 0). Next, resilient modulus was measured at different orientations of the same specimen, and the variation in the measured resilient modulus values was calculated. The calculated variation, σ_{or}^2 , was attributed to the combined effect of σ_1^2 and σ_2^2 , since the measured values were taken from the same sample (σ_3^2 , equals 0). Orientation variation (σ_2^2) was estimated by $\sigma_{or}^2 - \sigma_1^2$. Finally, resilient modulus was measured for different specimens at different orientations, and the variation, σ_{sa}^2 , in the measured resilient modulus values was calculated. σ_{sa}^2 was attributed to the combined effect of the three sources of variations. Sample variation, σ_3^2 , was estimated by $\sigma_{sa}^2 - \sigma_2^2 - \sigma_1^2$.

The variation in resilient modulus (σ_{ASTM}^2) can be estimated from the three sources of variation. If only one resilient modulus measurement at one orientation of one sample was recommended, then the formula for variation in resilient modulus is given by

$$\sigma_{ASTM}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$$

Since ASTM D 4123 averages six measured resilient modulus values (three specimens, each tested at two orientations), the variation of the mean should be used instead of individual variation. The variation of the mean for the averaged values of two orientations of the same specimen = $\sigma_{or}^2/\sqrt{2} = \sigma_1^2/\sqrt{2} + \sigma_2^2/\sqrt{2}$, and the variation of the mean for the averaged values of 3 specimens of the same mix = $\sigma_{sa}^2/\sqrt{3} = \sigma_3^2/\sqrt{3} + \sigma_{or}^2/\sqrt{3} = \sigma_3^2/\sqrt{3} + \sigma_2^2/\sqrt{6} + \sigma_1^2/\sqrt{6}$. As a result, the variation in resilient modulus is given by

$$\sigma^2_{\text{TEST}} = \sigma^2_3/\sqrt{N_s} + \sigma^2_2/\sqrt{(N_s N_o)} + \sigma^2_1/\sqrt{(N_s N_o)} \dots (1)$$

where NO = number of orientations
 N_s = number of samples

or

$$\sigma^2_{\text{ASTM}} = \sigma^2_3/\sqrt{3} + \sigma^2_2/\sqrt{6} + \sigma^2_1/\sqrt{6} \dots (2)$$

The analysis of variance (ANOVA) statistical technique was used to estimate the different variations (σ^2_1 , σ^2_2 , and σ^2_3) involved in ASTM D 4123 as described above. This technique is available in the SAS program (6).

TEST PLAN

The test procedures used to measured resilient modulus were outlined in ASTM D 4123. The setup was shown in Figure 1. An H & V resilient modulus device which is a pneumatic loading system generating load pulses was used as the loading device. The device was set to apply repeated 1 Hz repeated haversquare load waveform with load duration of 0.1 sec and rest period of 0.9 sec on test samples. Only horizontal deformation were measured using two spring loaded LVDTs placed in a diametrical yoke. Load and deformation were recorded with a two-channel **Oscillographic strip-chart** recorder. Figure 3 is a typical recorder output from a resilient modulus test. From the recorder output, the total resilient modulus of elasticity was determined. Since vertical deformation is not measured, Poisson's ratio was assumed to be 0.35 for all test temperature.

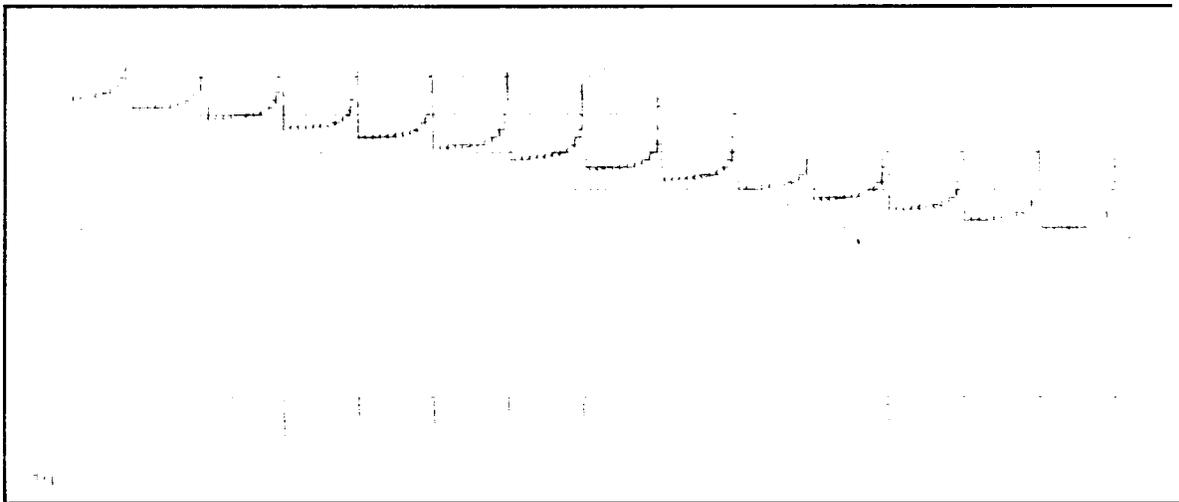


Figure 3. Typical recorder output of a resilient modulus test.

Part One

It is believed that experimental error (σ^2_1) is sensitive to the method of measuring deformation. It is thus important to insure that the deformation measurement by ASTM D 4123 produces the lowest experimental error (σ^2_1). The **ASTM's** method of placing spring loaded LVDTs in direct contact with the sample surface was studied against two other methods which use a thin membrane placed between the spring loaded LVDTs and sample surface. Figure 4 is a graphical view of the methods of deformation measurement.

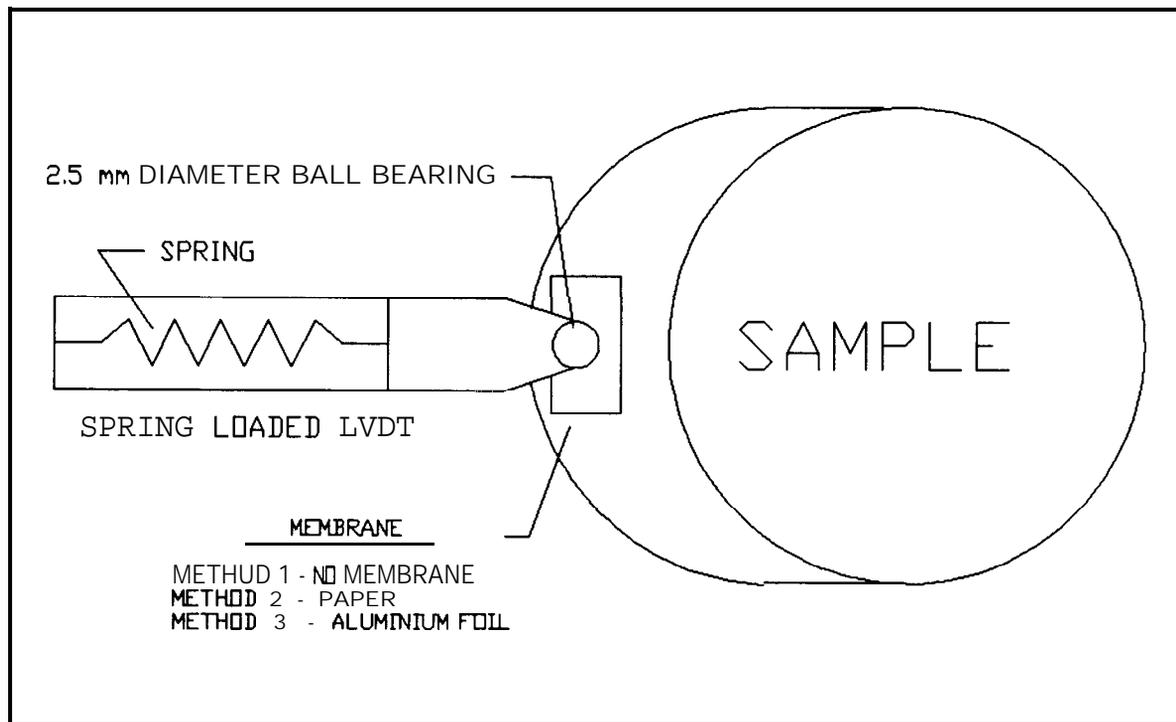


Figure 4. Graphical view of method of deformation measurement.

A thin membrane was used because it was thought that LVDTs may be placed on small depressions or on small aggregates on the sample surface which may increase the variation in the measured resilient modulus causing a higher experimental error, σ^2_1 . The use of a thin

membrane placed between the sample and LVDTs to bridge over these depressions or small aggregates may lower σ^2_1 . The method with the lowest value of σ^2_1 will be selected as the standard method of deformation measurement in this study. A lower value of σ^2_1 will result in a more repeatable test procedure by decreasing the variation in resilient modulus (ASTM D 4123). The three methods of deformation measurement studied were:

- Method 1 - Direct contact between spring loaded LVDTs and sample surface (ASTM D 4123).
- Method 2 - A piece of thin paper was placed between spring loaded LVDTs and the sample surface
- Method 3 - A piece of aluminum foil was placed between LVDTs and the sample surface.

Methods 2 and 3 are somewhat crude; however, the results from these tests should provide some indication of the effect of a membrane between the LVDTs and the sample.

The effect of the three methods of deformation measurement on three laboratory mixes (Mix A, Mix B, and Mix C) at 25 °C were studied. Each mix was represented by five laboratory fabricated specimens. For each mix and method of deformation measurement, experimental design #1 (Table 1) was conducted. Using the test results, σ^2_{test} was estimated using SAS. The variation in resilient modulus due to different stresses was factored out by SAS. The estimated variation in test result ($\sigma^2_{\text{test}} = \sigma^2_1 + \sigma^2_2 + \sigma^2_3$) was recorded (Table 2) .

A comparison of σ^2_{test} among the three methods of deformation in each mix revealed the best way to measure deformation (lowest σ^2_1) .

The method producing the lowest σ_{test}^2 ($\sigma_{test}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$) will have the lowest σ_1^2 , since σ_2^2 and σ_3^2 remained constant for each mix.

Table 1. Experimental design 1.

Sample	Stress 1					Stress 2					Stress 3				
	Orientation					Orientation					Orientation				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1															
2															
3															
4															
5															

Stress 1 = 10% tensile stress
 Stress 2 = 15% tensile stress
 Stress 3 = 20% tensile stress
 Orientation 1 = 1st random orientation
 Orientation 2 = 2nd random orientation
 Orientation 3 = 3rd random orientation
 Orientation 4 = 4th random orientation
 Orientation 5 = 5th random orientation

Table 2. Variability of Test for Part One of Test Plan

Test Data From	SAS Estimates	Choose
Mix A using Method 1	σ_{test}^2	Minimum σ_1^2
Mix A using Method 2	σ_{test}^2	
Mix A using Method 3	σ_{test}^2	
Mix B using Method 1	σ_{test}^2	Minimum σ_1^2
Mix B using Method 2	σ_{test}^2	
Mix B using Method 3	σ_{test}^2	
Mix C using Method 1	σ_{test}^2	Minimum σ_1^2
Mix C using Method 2	σ_{test}^2	
Mix C using Method 3	σ_{test}^2	

Part Two

The method of deformation measurement which produced the minimum σ^2_1 (determined in Part One) was used as the standard method of deformation measurement for the remaining part of this study. The purpose of Part Two of the test plan was to estimate the variation in resilient modulus (ASTM D 4123) of laboratory fabricated mixes at 25 °C. Another purpose was to determine the effect of stress on resilient modulus of laboratory mixes at this temperature.

Three laboratory mixes (Mix A, Mix B, Mix C), with each mix represented by five laboratory fabricated specimens, were studied. For each laboratory specimen, experimental design #2 (Table 3) was conducted. Therefore for this study, three laboratory mixes were evaluated and each mix was represented by five specimens. The tests were conducted at 25 °C, two sample orientations, three stresses, and five repetitions resulting in a total of 450 tests. Each repetition was represented by removing and remounting the LVDTs on the same sample location before the test was repeated.

ANOVA in SAS was used to factor out the variation due to different stresses. Experimental error (σ^2_1) was estimated with SAS from data measured at five repetitions at the same orientation and specimen in each mix. Next, the compounded orientation variation and experimental error (σ^2_{or}) was estimated from data measured at different orientations of the same specimen. Orientation variation (σ^2_2) was then calculated using the equation $\sigma^2_2 = \sigma^2_{or} - \sigma^2_1$. Finally, the compounded effect of sample variation, orientation

variation and experimental error (σ_{sa}^2) was estimated from data measured from different specimens of each mix. Sample variation (σ_3^2) were calculated from the equations $\sigma_3^2 = \sigma_{sa}^2 - \sigma_2^2 - \sigma_1^2$. The variation in resilient modulus is given by

$$s_{ASTM} = \sigma_3^2/\sqrt{3} + \sigma_2^2/\sqrt{6} + \sigma_1^2/\sqrt{6} \dots\dots\dots(2)$$

Table 3. Experimental design #2.

Repet- ition	Stress 1		Stress 2		Stress 3	
	Orie 1	Orie 2	Orie 1	Orie 2	Orie 1	Orie 2
1						
2						
3						
4						
5						

Stress 1 = 10% of tensile stress
 Stress 2 = 15% of tensile stress
 Stress 3 = 20% of tensile stress
 Orie 1 = 1st randomly selected orientation
 Orie 2 = 2nd randomly selected orientation

To analyze the effect of stress on resilient modulus, the differences in measured resilient modulus values due to orientations and specimens was factored out before the data were used to analyze the effect of stress. A regression analysis was performed with resilient modulus as Y, the dependent variable. The independent class variables were sample and orientation and the independent continuous variable was stress (% of tensile stress) . Equations were developed from these regression to predict resilient modulus at a stress of 15% tensile stress for each mix evaluated.

Each measured resilient modulus value for a given mix type was divided by the predicted resilient modulus at a stress of 15% of tensile stress. This resulting ratio (MR @ X% / MR @ 15%) will show the expected difference between measured resilient modulus values at various stresses and that measured at 15% of tensile stress for typical asphalt mixes. The ratio (MR @ X% / MR @ 15%) for each sample tested was plotted against stress in percent of tensile stress to evaluate the effect of stress on MR for Mix A, Mix B, Mix C, and for a combination of all mixes at the test temperature.

Part Three

The purpose of Part Three of the test plan was to estimate the variation in resilient modulus (σ^2_{ASTM}) of field mixes. Three test temperatures (4, 25, and 40 °C) were used in this part instead of one test temperature (25 °C) used in part two. The effect of stress on resilient modulus of field mixes **was also** analyzed.

Fourteen different field mixes (each mix represented by three samples) were studied. For each sample and test temperature, experimental design #3 (Table 4) was conducted. Therefore for this study, 14 field mixes were evaluated. Each field mix was represented by three samples. The tests were conducted at three temperatures (4 °C, 25 °C, and 40 °C), four sample orientations, three stresses, and 2 repetitions. This resulted in a **total of** 3024 tests.

Using the procedure identical to Part Two, ANOVA in SAS was

used to estimate σ_1^2 , σ_2^2 , and σ_3^2 of each field mix after factoring out the effect of different stresses.

Table 4. Experimental design #3.

Repetition	Stress 1				Stress 2				Stress 3			
	Orientation				Orientation				Orientation			
	1	2	3	4	1	2	3	4	1	2	3	4
1												
2												

Stress 1 = 10% of tensile stress
 Stress 2 = 15% of tensile stress
 Stress 3 = 20% of tensile stress
 Orientation 1 = 1st randomly selected orientation
 Orientation 2 = 2nd randomly selected orientation
 Orientation 3 = 3rd randomly selected orientation
 Orientation 4 = 4th randomly selected orientation

At each test temperature, a procedure identical to that discussed in Part Two of the test plan was used to factor out the differences in measured resilient modulus values due to orientation and sample. The factored out data were then analyzed for the effect of stress on resilient modulus. The analysis of the effect of stress on resilient modulus was conducted at three temperatures: 4, 25 and 40 °C.

Prediction of Tensile Strength

It was necessary to estimate the tensile stress of asphalt mixes in order to estimate the applied stress as a percent of tensile stress.

The indirect tensile stress of laboratory mixes was estimated

from Marshall stability values obtained during mix design. Indirect tensile stress was assumed to be Marshall stability divided by 20 (7). Based on this estimated tensile stress, the corresponding load was applied during resilient modulus testing. After resilient modulus tests were completed, actual indirect tensile stress of each sample was obtained according to ASTM D 4123 with load rate of 50.8 mm per minute and temperature of 25 °C (Figure 5) . Therefore, the stress applied during modulus testing at 25 °C was divided by the sample actual indirect tensile stress of the sample to determine stress as percent of tensile stress.

Tensile stress of field samples at 25 °C were first estimated from indirect tensile strength test results of cores taken adjacent to the field samples. Figure 6 was used to predict the indirect tensile stress at 4 and 40 °C from the estimated tensile stress at 25 °C (8). Figure 6 shows that the indirect tensile stress at 4 °C was approximately 3 times greater than the tensile stress at 25 °C approximately 7.5 times greater than at 40 °C. Based on the predicted tensile stress, the desired stress (10%, 15%, or 20% of tensile stress) was applied during each resilient modulus test. When all resilient modulus tests were completed, indirect tensile strength tests were conducted on the actual test samples to obtain the actual tensile stress of samples at 25 °C. The tensile stress at 4 °C and 40 °C were calculated using the measured strength at 25 °C and Figure 6.

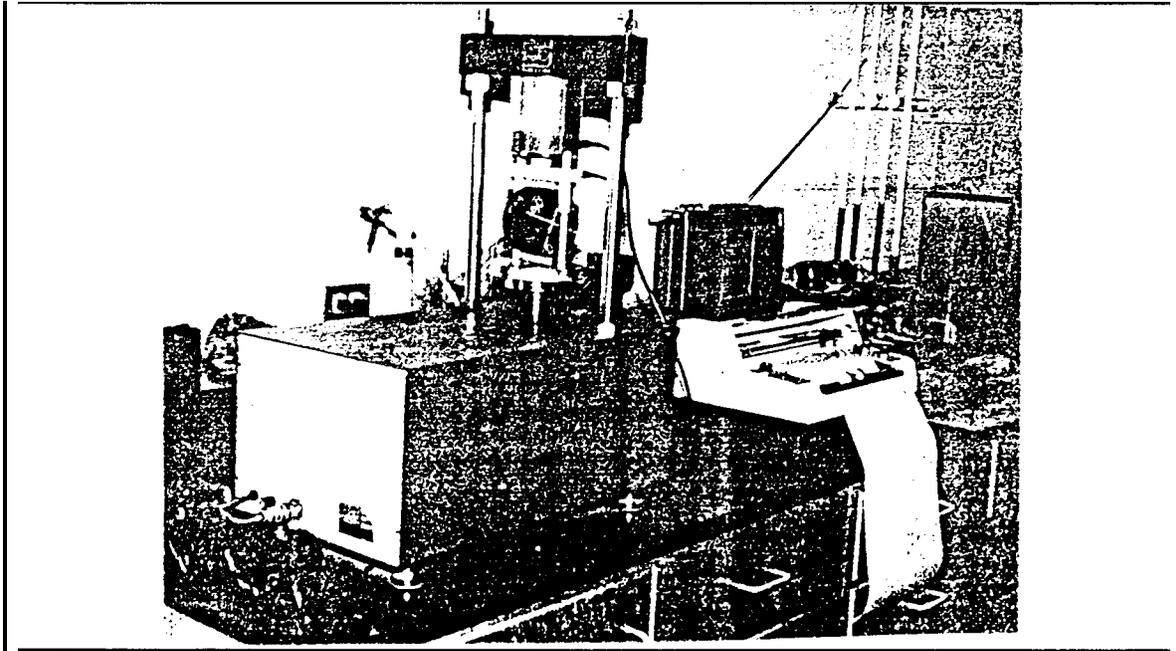


Figure 5. Indirect tensile test (ASTM D 4123).

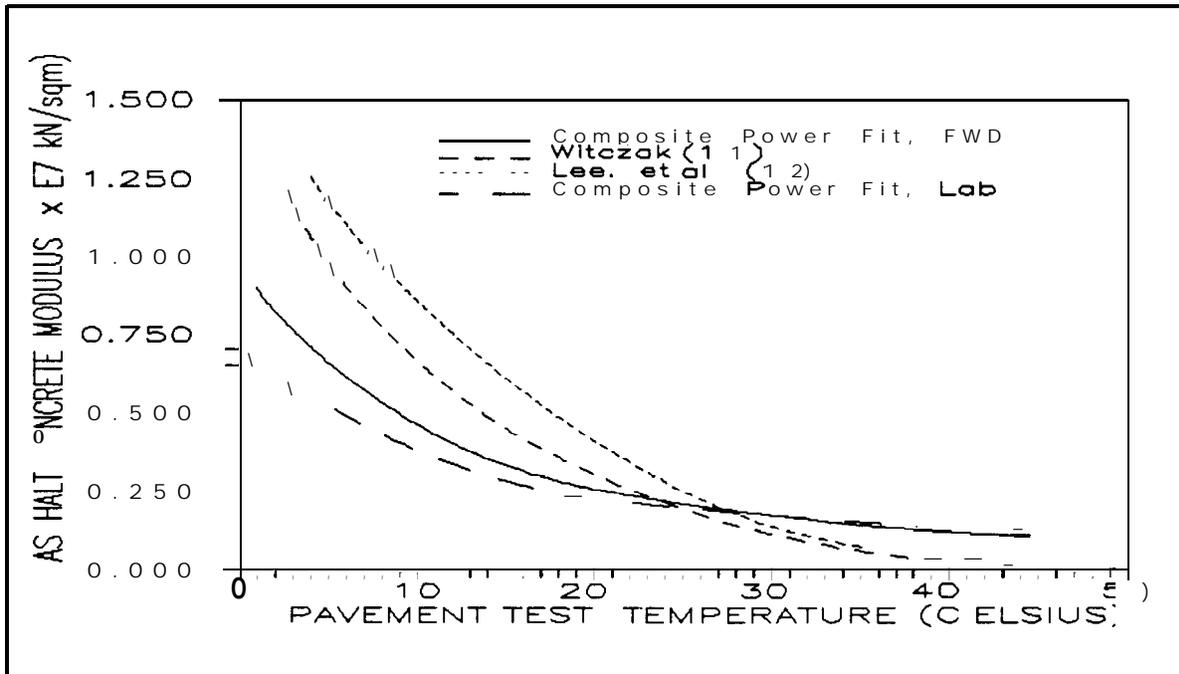


Figure 6. Asphalt concrete modulus-temperature relationship (8).

SAMPLE INFORMATION

Lab Samples

The aggregate gradations for the three mixes (Mix A, Mix B, Mix C) of laboratory samples are shown in Figure 7. The optimum asphalt content of each mix established by Marshall mix design criteria using a gyratory compactor (set at 1 degree angle, 30 revolutions, and 1380 kN/m²) was 4.2% for Mix A, 4.8% for Mix B, and 5.8% for Mix C. This gyratory setting produces a density equivalent to that with 75 blows of the Marshall hand hammer (Figure 8). It appeared that much of the larger aggregate in Mix A was broken when compacted with the gyratory compactor. This problem is more severe with the Marshall hammer and is primarily caused by compacting large aggregate in a small mold (9).

Five samples were prepared from each mix. The density test results (ASTM D 1188) and indirect tensile strength test results (ASTM D 4123) of all the samples are shown in Table 5.

Table 5. Density and tensile strength of laboratory samples.

Sample	Mix A		Mix B		Mix C	
	Density (g/cm ³)	Ten Str (kN/m [*])	Density (g/cm ³)	Ten Str (kN/m [*])	Density (g/cm ³)	Ten Str (kN/m [*])
1	2.521	614.72	2.505	815.51	2.473	1016.44
2	2.536	633.14	2.525	1044.80	2.476	1156.30
3	2.546	683.86	2.518	955.10	2.480	1019.68
4	2.543		2.541	926.33	2.463	1041.76
5	2.558	746.44	2.500	1069.91	2.471	1133.05

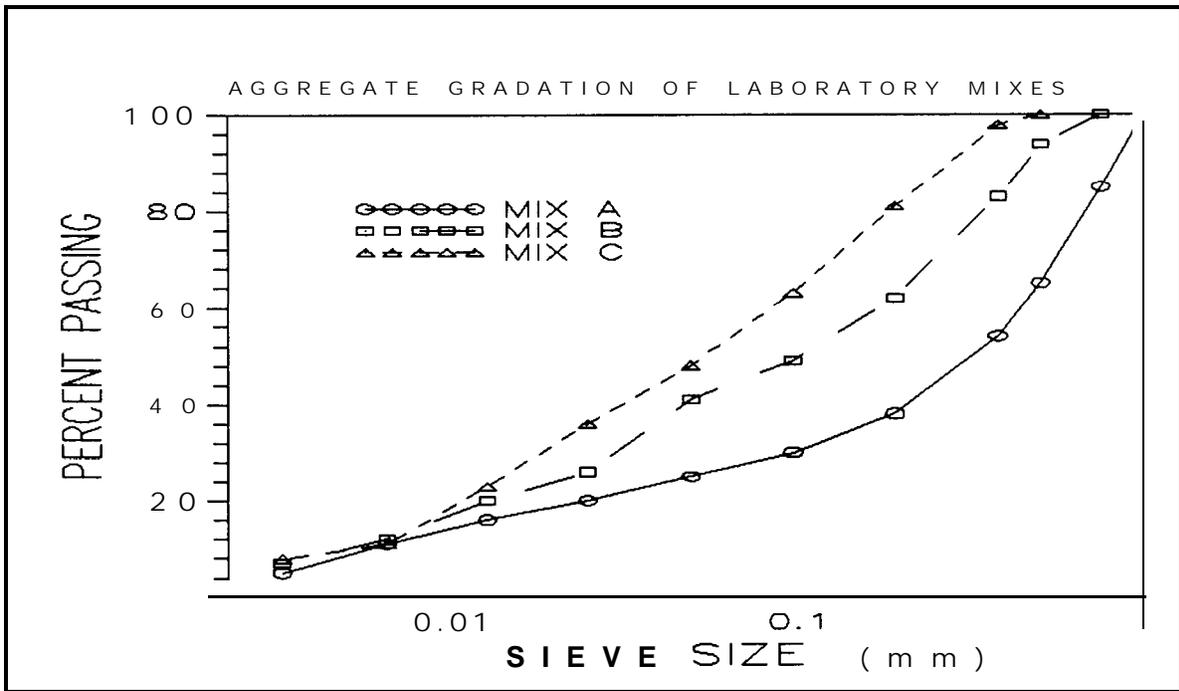


Figure 7. Aggregate gradation of laboratory mixes.

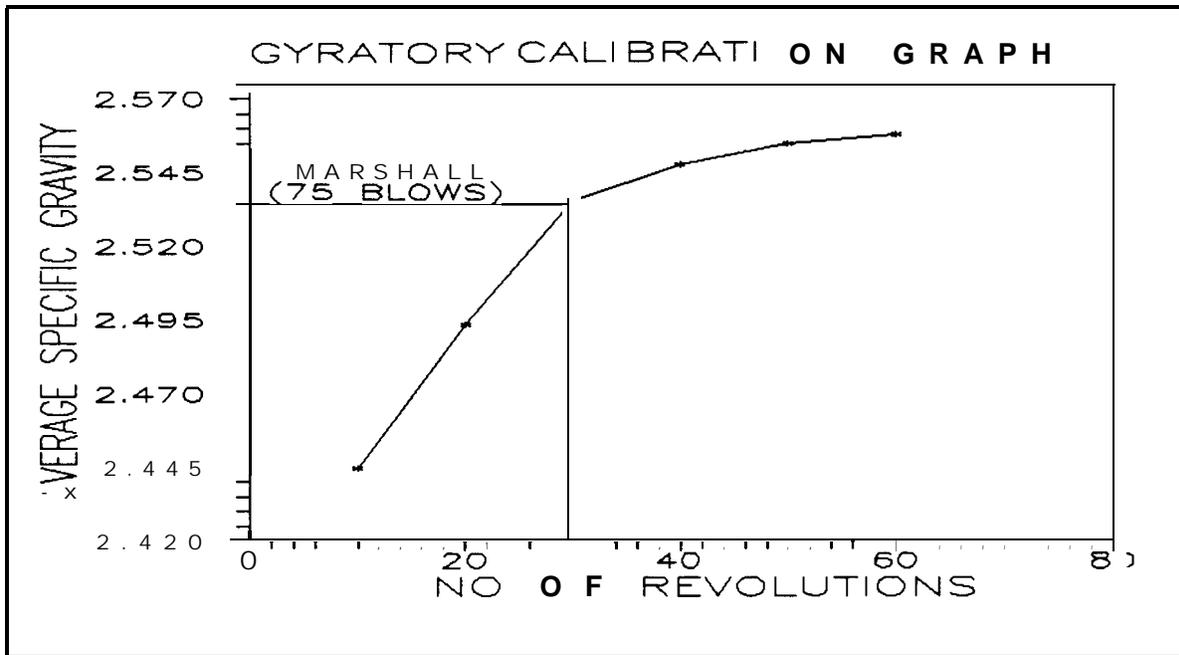


Figure 8. Gyrotory calibration graph.

Field Samples

The maximum aggregate size, density and indirect tensile strength measured from field cores are shown in Table 6.

The field mixes are identified by a letter of the alphabet, D, followed by two numbers for identification purpose (mixes A, B, and C are laboratory mixes). The first number indicates the pavement site number, and second number indicates the pavement layer. Therefore, Mix D42, was identified as a field mix obtained from the second layer of pavement number 4.

Table 6. Maximum aggregate size, density and tensile strength of field samples.

Mix	Max Agg		Core 4		Core 5		Core 8	
	Size (mm)	Density (g/cm ³)	Ten Str (kN/m ²)	Density (g/cm ³)	Ten Str (kN/m ²)	Density (g/cm ³)	Ten Str (kN/m ²)	
D23	19.0	2.338		2.337	400.5	2.348	586.6	
D24	19.0	2.356	560.2	2.321	-	2.322	818.1	
D25	25.4	1.724	184.9	1.712	179.5	1.700	-	
D32	12.7	2.261		2.261	341.4	2.253	333.9	
D41	19.0	2.361		2.329	541.7	2.381	580.6	
D42	25.4	2.389	603.8	2.361	-	2.391	587.8	
D43	25.4	2.361		2.362	598.3	2.354	497.4	
D44	25.4	2.349	665.6	2.357	-	2.253	434.6	
D45	25.4	2.293	530.3	2.295	542.8	2.285	-	
D52	12.7	2.341			362.7	2.357	507.6	
D53	25.4	2.389	344.6			2.383	272.0	
D54	19.0	2.375	332.9	2.329	282.7	2.389	-	
D55	25.4	2.421		2.446	402.9	2.463	387.3	
D56	25.4	2.393	310.4	2.434	-	2.413	372.7	

mixes showed that Method 1 has the lowest value of σ_{test}^2 ; Method 1 has the lowest experimental error (σ_1^2). It was concluded that Method 1 (deformation measurement by ASTM) is the best method of deformation among the three methods studied.

Results from Part Two of Test Plan

Table 8 shows the experimental errors (σ_1^2), orientation variation (σ_2^2), sample variation (σ_3^2), and variation in resilient modulus (σ_{ASTM}^2) of the laboratory mixes at 25 °C.

Table 8. σ_1^2 , σ_2^2 , σ_3^2 , and σ_{ASTM}^2 of laboratory mixes at 25 °C.

	Mix A	Mix B	Mix C
Max. aggr. size (mm)	25.4	19.0	12.7
Mean MR (kN/m ²)	2078190	2687302	2086739
σ_1^2	3.4371 E10	6.8558 E10	2.6471 E10
σ_2^2	1.1872 E10	6.7916 E09	5.0151 E09
σ_3^2	3.7095 E10	1.5917 E11	2.8177 E10
σ_{ASTM}^2	2.0072 E10	6.5615 E10	1.4640 E10

Experimental error (σ_1^2) is a function of the test equipment and operators. For $\sigma_1^2 = 0$ (completely repeatable), all repeated resilient modulus values measured at any one orientation of a specimen must be identical. Orientation variation (σ_2^2) is the variation in resilient modulus values obtained by testing at different orientations of a specimen. Orientation variation (σ_2^2) is related to the specimen homogeneity. For a homogeneous specimen, resilient modulus measured at different orientations of

the specimen would be identical ($\sigma^2_2 = 0$) . The test results showed that mixes with larger maximum aggregate sizes have higher values of σ^2_2 . The data supports the obvious fact that homogeneity of specimens decrease with increasing maximum aggregate size. The variation in resilient modulus caused by different orientations is minimal and does not have a significant effect on the variation. It is the smallest variation among the three sources of variation. Sample variation (σ^2_3) is the variation in resilient modulus values obtained by testing different specimens of the same mix. Sample variation (σ^2_3) is related to reproducibility of identical test specimens. If it is possible to reproduce identical specimens from a mix, the resilient modulus of different specimens of the same mix would be identical ($\sigma^2_3 = 0$) . It was suspected that mixes with smaller maximum aggregate size would have a lower resilient modulus value and higher reproducibility (lower σ^2_3) . As suspected, test results showed that the mix with smallest maximum aggregate size (Mix C) had a higher reproducibility (minimum σ^2_3) and lower resilient modulus value. It is unclear why Mix A had lower mean MR and lower variability than Mix C. The breaking of the larger aggregate size (Mix A) during compaction may have something to do with it.

Table 9. Standard error, CV and acceptable range of two tests for laboratory mixes at 25 °C.

	Mix A	Mix B	Mix C
Standard error (kN/m ²)	141676	256154	120996
Coeff of variation (%)	6.82	9.53	5.80
Acceptable range (%)	19.29	26.98	16.41

Useful information can be extracted from the variation in resilient modulus (ASTM D 4123) , σ_{ASTM}^2 . Standard error (σ_{ASTM}) , coefficient of variation ($CV = \sigma_{ASTM}/\text{Mean } Mr$) , and acceptable **range** of two tests according to ASTM C 670 ($2.83 * CV$) were calculated and tabulated in Table 9.

If the same operator repeated the ASTM D 4123 test with specimens from the same batch at the same temperature (25 °C) using the same machine, the two results should not differ more than $2.83 * Cv$. It was concluded that resilient modulus measurement of asphalt mixes does not have a high degree of precision. The maximum expected difference between two test measurements from the same batch of materials by the same operator in the same laboratory using the same machine can be as high as 20% for Mix A, 27% for Mix B, and 16% for Mix C.

Of the three components of variation in resilient modulus, given as $\sigma_{ASTM}^2 = (\sigma_1^2/\sqrt{6}) + (\sigma_2^2/\sqrt{6}) + (\sigma_3^2/\sqrt{3})$, the last term ($\sigma_3^2/\sqrt{3}$) was the major contributing component. The most effective way to decrease the variation in resilient modulus or increase the precision is to minimize the last term ($\sigma_3^2/\sqrt{3}$) where 3 is the number of samples tested. The term (σ_3^2/\sqrt{n}) can be decreased by averaging the resilient modulus values of a larger number of test samples, n. Therefore, there is a tradeoff between precision of the test procedure and the number of specimens to be tested. The acceptable range of two test results can be calculated using the equations below: $AR = CV * 2.83 \dots \dots \dots (3)$

$$Cv = \sqrt{\sigma_{TEST}^2}/Mr * 100 \dots \dots \dots (4)$$

$$\sigma_{\text{TEST}}^2 = \sigma_3^2/N_s + \sigma_2^2/(N_s N_o) + \sigma_1^2/(N_s N_o) \dots \dots \dots (1)$$

Substituting equations (4) and (1) into (3)

$$A R = 283/MR * [\sigma_3^2/N_s + \sigma_2^2/(N_s N_o) + \sigma_1^2/(N_s N_o)]^{1/2} \dots (5)$$

where NO = number of orientations
 N_s = number of samples
 AR = Acceptable range in %
 MR = mean resilient modulus

Equation (5) can be used to calculate the acceptable range of two test result when more samples or orientations were tested. For example, quadrupling the testing effort, an increase from 6 to 24 tests (from **ASTM's** 3 samples at 2 orientations to 6 samples at 4 orientations) , will improved the acceptable range from 19.29 to 12.26 for Mix A, 26.98% to 18.14% for Mix B, and 16.41 to 10.51 for Mix C. It was not be feasible to improve the ASTM D 4123 by using more samples or orientations.

Figure 9 shows the effect of stress on **MR** of the laboratory mixes at 25 °C. The Y axis is given by Y =MR @ X% / **MR @ 15%** as shown in part two of test plan. The X-axis is the stress in percent of tensile stress. The data shows that the equation for the best fit straight line through all data is Y = -0.02252X + 1.340.

The maximum aggregate size, slope, and mean **MR** of the three mixes were tabulated in Table 10. The table shows that Mix A is more sensitive to stress followed by Mix C, and Mix B is least sensitive to stress. It seems that the stiffer the mix, the less sensitive it is to stress. When all mixes were analyzed, the slope is -0.02252. Therefore, a change in stress from 15% of tensile

stress to 10% of tensile stress will increase the measured MR by 11.26% ($[10 - 15] * -0.02252$).

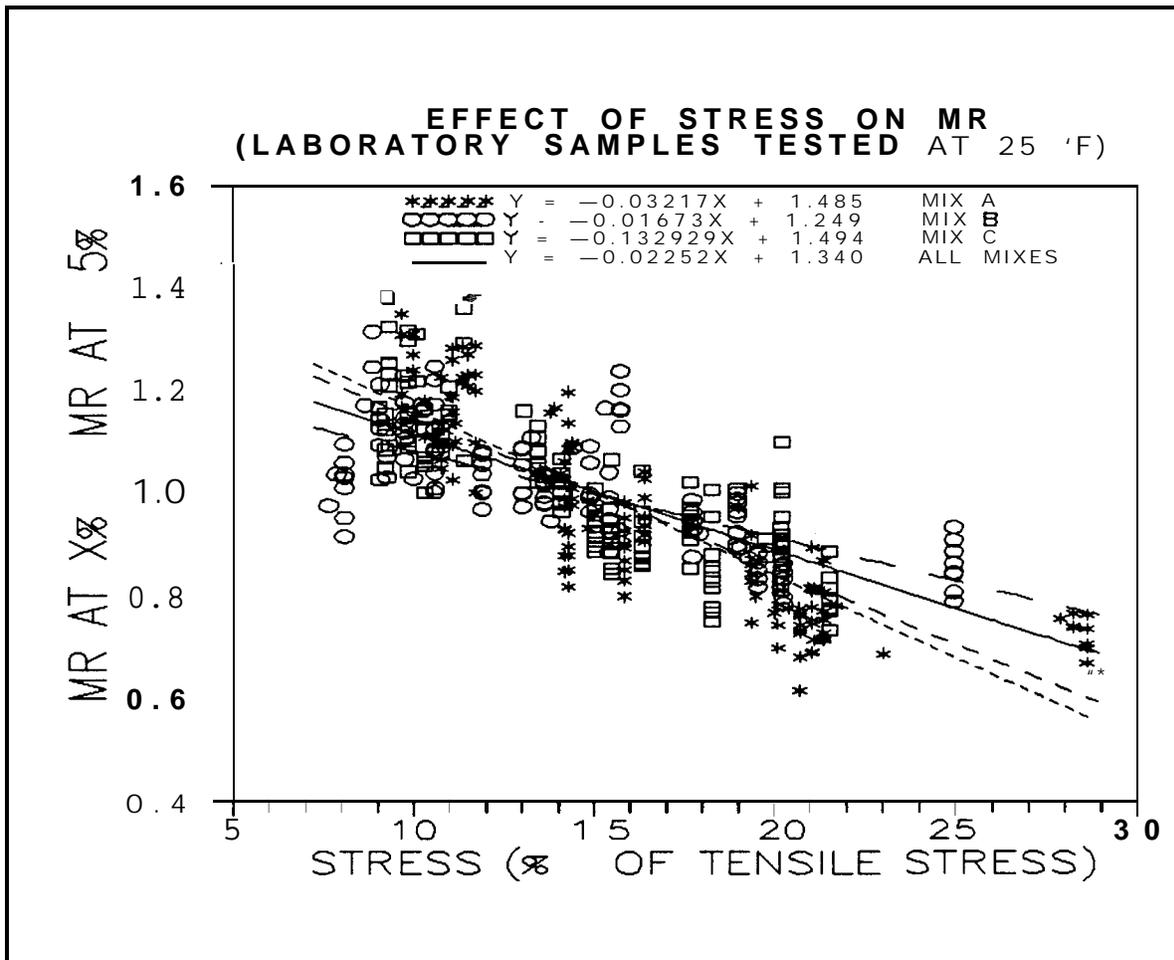


Figure 9. Effect of stress on resilient modulus of laboratory mixes at 25 °C.

Table 10. Maximum aggregate size, slope, and mean MR of laboratory mixes.

Mix	Max. Aggregate Size	Slope	Mean MR
Mix A	23.4 mm	-0.03217	2078190 kN/m ²
Mix B	19.0 mm	-0.01673	2687302 kN / m ²
Mix C	12.7 mm	-0.02929	2086739 kN / m ²
All Mixes		-0.02252	

Results from Part Three of Test Plan

Table 11 shows the experimental errors (σ^2_1), variation in resilient modulus caused by different orientations (σ^2_2), and variation in resilient modulus caused by different specimens (σ^2_3). There are a total of 42 points from 14 field mixes tested at 4 °C, 25 °C, and 40 °C with measured resilient modulus values ranging from 7×10^5 to 1.75×10^7 kN/m².

Table 11. Variances of field mixes.

Mix	Variances	40 °C	25 °C	4 °C
D23	σ^2_3	2.640 E12	3.790 E12	5.262 E11
	σ^2_2	1.432 E10	1.010 E10	1.650 E11
	σ^2_1	1.803 E11	6.021 E11	8.416 E12
D24	σ^2_3	7.932 E11	4.357 E12	1.029 E10
	σ^2_2	7.328 E09	1.739 E11	2.750 E09
	σ^2_1	3.664 E11	4.756 E11	4.302 E12
D25	σ^2_3	6.449 E11	3.950 E11	1.611 E12
	σ^2_2	9.404 E08	3.481 E08	1.619 E10
	σ^2_1	7.543 E09	9.774 E10	3.220 E11
D32	σ^2_3	2.291 E11	6.956 E11	
	σ^2_2	7.943 E07	6.921 E07	
	σ^2_1	1.538 E10	2.656 E11	
D41	σ^2_3	5.982 E07	4.211 E10	2.442 E11
	σ^2_2	5.765 E08	5.767 E09	4.093 E11
	σ^2_1	9.654 E09	6.581 E10	3.940 E12
D42	σ^2_3	2.287 E09	9.412 E10	7.732 E11
	σ^2_2	1.231 E08	3.004 E10	1.688 E11
	σ^2_1	1.426 E10	1.958 E11	5.853 E12
D43	σ^2_3	1.047 E11	8.758 E11	4.834 E12
	σ^2_2	5.708 E08	3.397 E11	8.592 E11
	σ^2_1	2.544 E10	1.425 E11	9.758 E12
D44	σ^2_3	1.155 E11	1.167 E12	3.159 E12
	σ^2_2	7.620 E09	2.738 E10	1.531 E11
	σ^2_1	2.033 E10	1.081 E11	4.836 E12

Table 11. Continued.

D45	σ^2_3	1.220 E10	4.321 E10	1.080 E12
	σ^2_2	5.379 E09	7.524 E10	1.139 E10
	σ^2_1	2.982 E10	1.904 E11	3.103 E12
D52	σ^2_3	4.562 E11	2.095 E12	4.447 E10
	σ^2_2	9.651 E08	3.796 E09	1.192 E10
	σ^2_1	1.547 E10	1.006 E11	1.712 E12
D53	σ^2_3	5.636 E11	2.409 E12	1.471 E13
	σ^2_2	3.060 E09	1.241 E10	1.522 E11
	σ^2_1	5.158 E10	1.265 E11	1.833 E12
D54	σ^2_3	1.566 E11	1.156 E12	6.970 E12
	σ^2_2	3.312 E08	5.707 E09	1.147 E11
	σ^2_1	6.865 E09	4.129 E10	9.596 E11
D55	σ^2_3	1.473 E10	6.431 E10	1.129 E12
	σ^2_2	2.285 E09	4.847 E10	3.164 E10
	σ^2_1	1.058 E10	8.850 E10	1.964 E12
D56	σ^2_3	2.524 E10	1.555 E11	4.493 E11
	σ^2_2	1.987 E08	5.204 E09	2.947 E10
	σ^2_1	7.070 E09	3.751 E10	2.561 E12

Figure 10 is a plot of sample variation (σ^2_3), orientation variation (σ^2_2), and experimental error (σ^2_1) versus mean MR. It showed at mean MR less than 6×10^6 kN/m², sample variation (σ^2_3) has the highest variation and at mean MR greater than 6×10^6 kN/m², experimental error has the highest variation. Orientation variation (σ^2_2) was significantly lower throughout the ranges of mean MR. Since the stress applied during resilient modulus testing remained practically the same, deformation is inversely proportional to the mean MR (mix stiffness). The amount of deformation in stiff mixes is therefore very small. The error of the test equipment in measuring deformation at this range increases. Therefore, as the mean MR increases, the influence of

Figure 11 is a plot of σ_{ASTM}^2 ($\sigma_{ASTM}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$) versus mean MR. The regression equation $\sigma_{ASTM}^2 = MR^{1.4158} * 97.3673$ was developed from data points in the plot. Figure 12, a plot of CV and acceptable range of two test results versus mean MR, were obtained using the equation $CV = \sigma_{ASTM}/MR * 100$ and the acceptable range of two test results according to ASTM C 670 = $2.83 * CV$.

Figure 11 showed σ_{ASTM}^2 increasing with increasing mean MR while Figure 12 showed CV decreasing with increasing mean MR. The variation (σ_{ASTM}^2) in the test result using the same operator and machine increased with stiffness of the mixes. When this variation was expressed in percent of mean MR ($CV = \sigma_{ASTM}/\text{mean MR} * 100$), it decreases with stiffness of the mix. Figure 12 also shows that the maximum difference between two repeated test results can be as high as 35% for mixes with stiffness of $3 * 10^6$ kN/m². As the stiffness increases to $1.7 * 10^7$ kN/m², the maximum difference of acceptable range decreased to 22%.

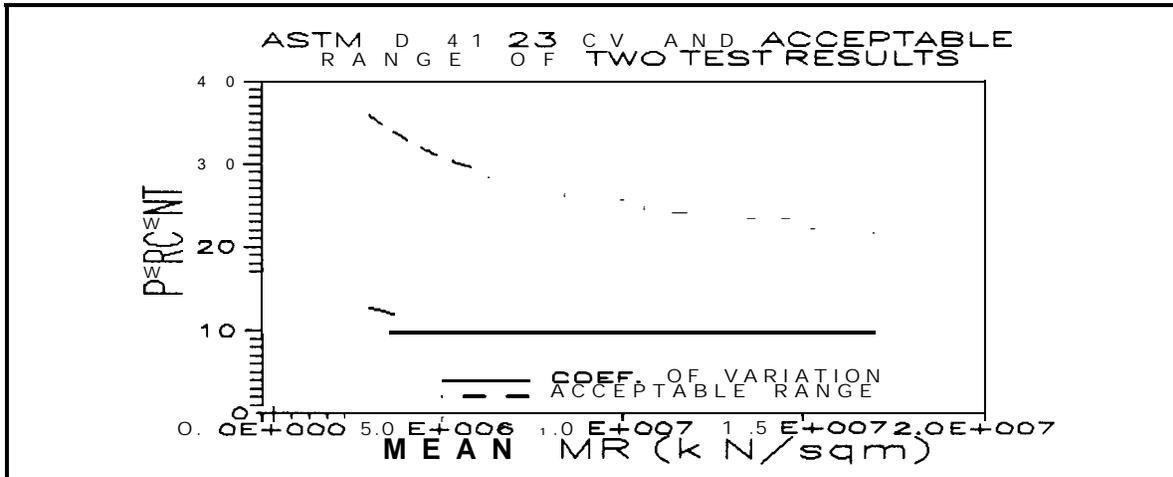


Figure 12. CV and acceptable range of two test results.

Figure 13, 14, and 15 are plots of resilient modulus ratio versus stress at 25 °C for field mixes with maximum aggregate size of 25.4, 19.0, 12.7 mm respectively. A straight line was fitted in each figure. The figures showed a decrease in **MR** with increasing load. However, and there does not seem to be any correlation between maximum aggregate size and the slope of the fitted line (Table 12) . The slope measured the sensitivity of **MR** to stress.

Table 12. Maximum aggregate size and slope of field mixes.

Maximum aggregate size	Slope
25.4 mm	-0.0243
19.0 mm	-0.0275
12.7 mm	-0.0228

Figure 16 is a plot of resilient modulus ratio versus stress of all field mixes at 25 °C. The slope of the equation is -0.025. Therefore, a change in stress from 15% of tensile stress to 10% of tensile stress will increase the measured **MR** at 77 degrees F by

12.53% ($[10 -15] * -0.025$). The slope selected for test results on field samples is very similar to that selected for laboratory samples (-0.0225). Figure 15 is a plot of resilient modulus versus stress of field mixes at 40 °C. The slope of the equation is -0.0423. A change in stress from 15% of tensile stress to 10% of tensile stress will increase the measured **MR** at 40 °C by 21.13%. At higher temperature, the effect of stress on **MR** is more pronounced.

The effect of stress at 4 °C was not analyzed because of the lack of air pressure. The maximum stress that could be applied by the test equipment was in the range of 5 to 10 % of tensile stress at 4 °C.

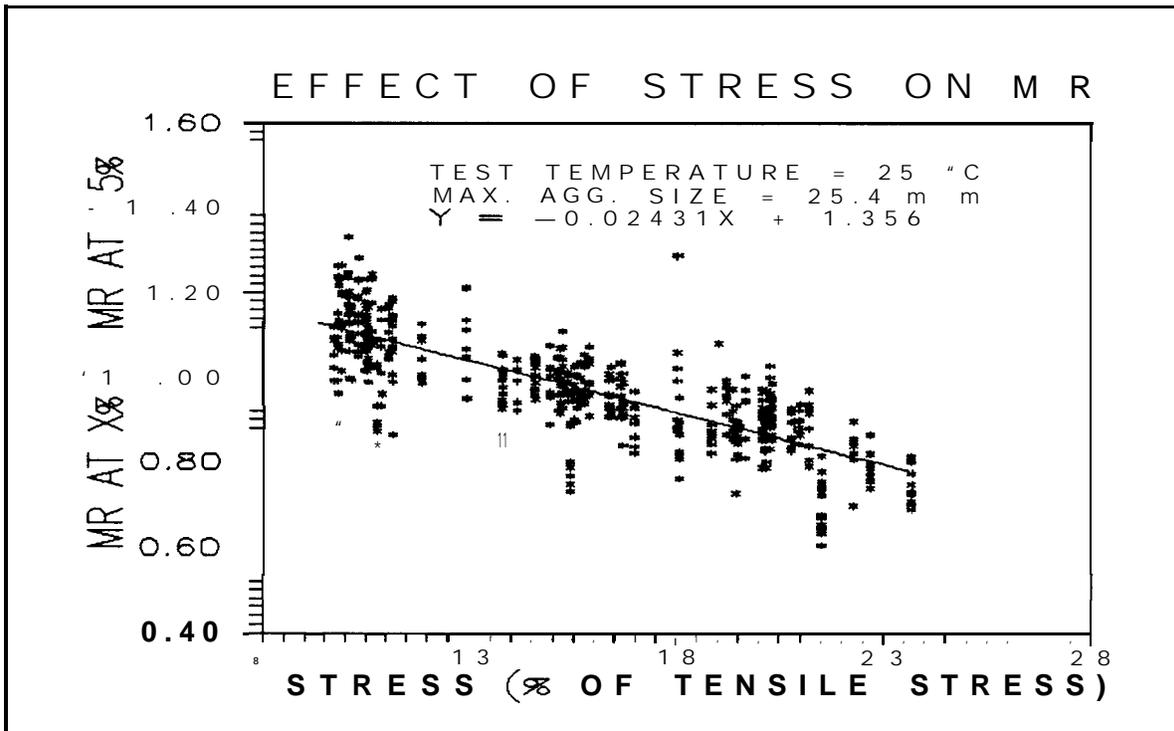


Figure 13. Effect of stress on **MR** for 25.4 mm aggregate field mixes.

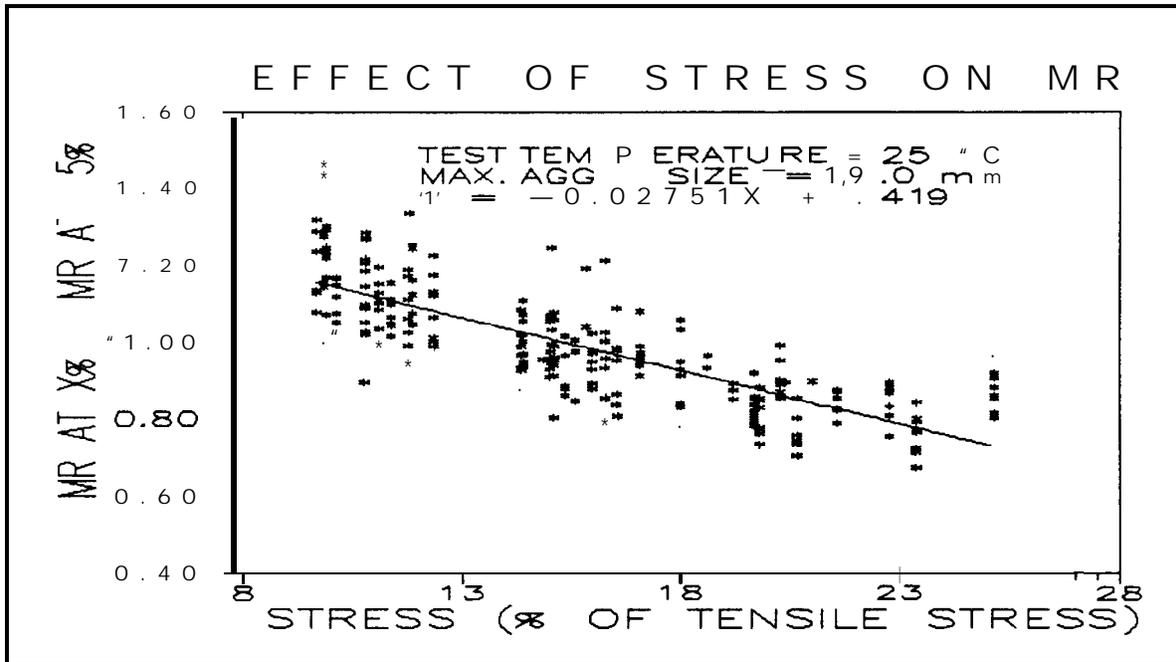


Figure 14. Effect of stress on MR for 19.0 mm aggregate field mixes.

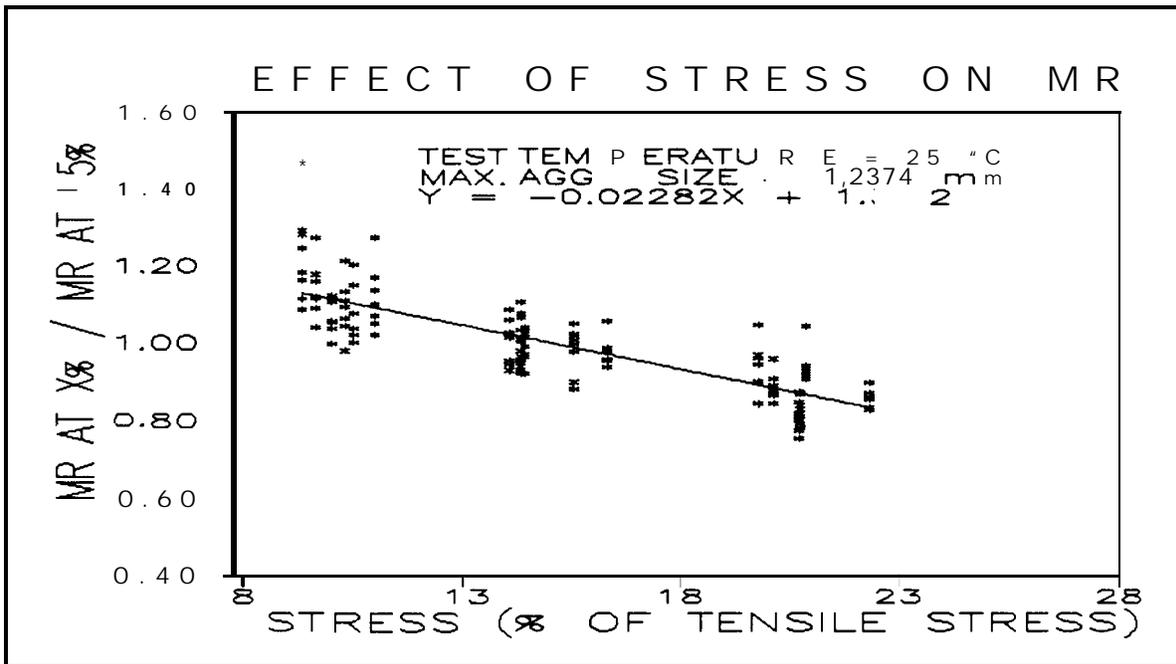


Figure 15. Effect of stress on MR for 12.7 mm aggregate field mixes.

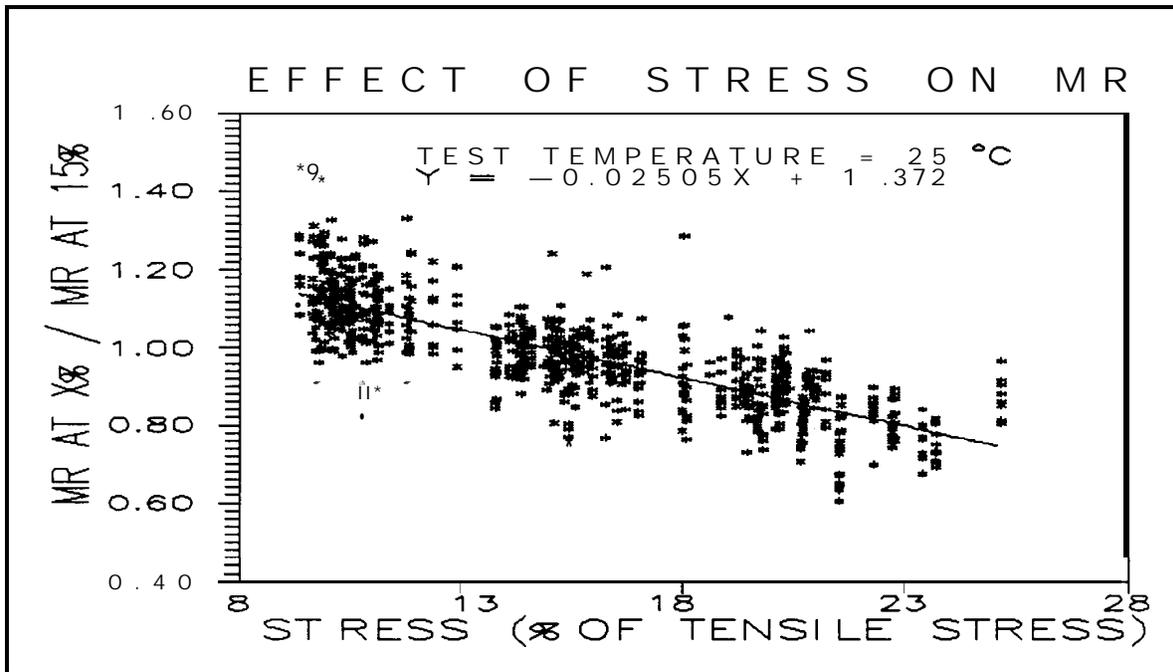


Figure 16. Effect of stress on MR for field mixes at 25 °C.

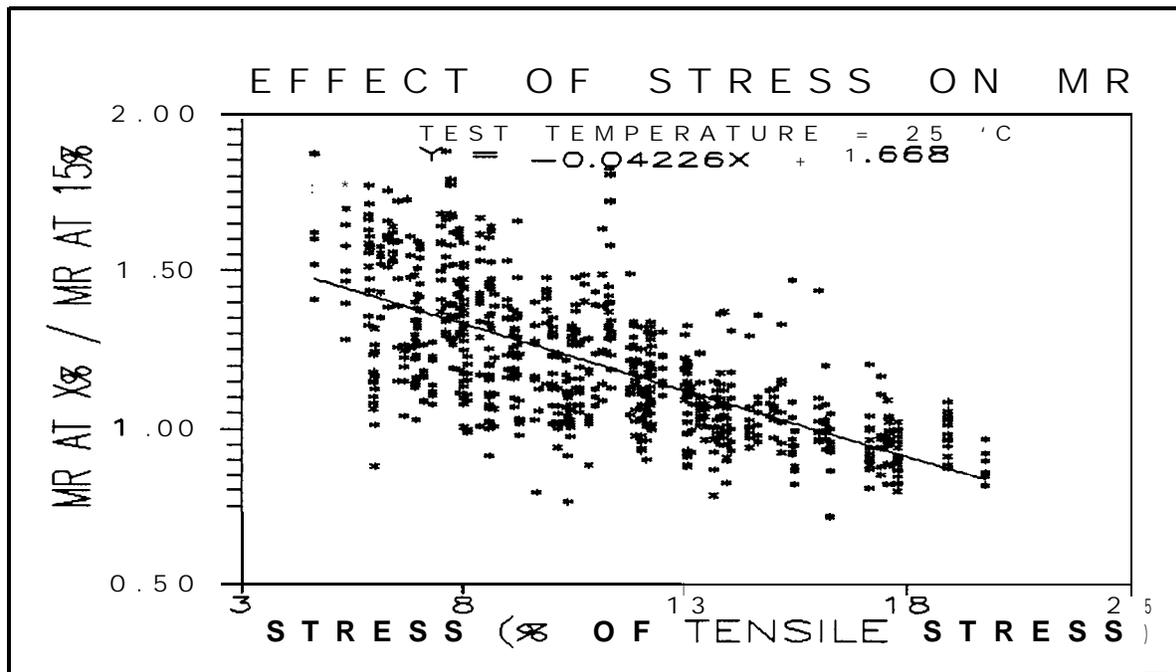


Figure 17. Effect of stress on MR for field mixes at 40 °C.

CONCLUSIONS AND RECOMMENDATIONS

One source of variation in resilient modulus (ASTM D 4123) is experimental error (σ^2_1). For the variation in resilient modulus (ASTM D 4123) to be minimal, the experimental error (σ^2_1) has to be minimal. It was found the ASTM D 4123 method of deformation (spring loaded LVDTs placed in contact with sample) has the lowest σ^2_1 compared with two other methods of deformation measurement (using membrane between the LVDTs and sample).

Other sources of variation in resilient modulus (ASTM D 4123) are σ^2_2 and σ^2_3 . It was found that sample variation (σ^2_3) is the most important factor influencing the variation in resilient modulus for mix with stiffness less than $6 \times 10^6 \text{ kN/m}^2$. Sample variation (σ^2_3) is a measure of within laboratory variability for specimens or cores taken from the same asphalt mix. Sample variation (σ^2_3) values obtained in this study were typically high, showing significant differences in resilient modulus among samples of the same mix. For stiffer mixes (M_r greater than $6 \times 10^6 \text{ kN/m}^2$) with small deformations, the capability of the test machine to accurately measure deformation becomes the major factor for the variation in resilient modulus (ASTM D 4123). This is reflected by the higher value of experimental error (σ^2_1) for mean **MR** values greater than $6 \times 10^6 \text{ kN/m}^2$.

The acceptable range of two test results ($2.83 * CV$) is another measure of the variation in resilient modulus. This study

shows that resilient modulus measurement of asphalt mixes by ASTM D 4123 does not have a high degree of precision. For field mixes, the acceptable range of two test results ranges from 35% for a mix stiffness of $3 \times 10^6 \text{ kN/m}^2$ and decreases to 22% at a mix stiffness of $1.7 \times 10^7 \text{ kN/m}^2$. For the three laboratory mixes whose averaged stiffness is $2.3 \times 10^6 \text{ kN/m}^2$ (2.1×10^6 , 2.7×10^6 , and $2.1 \times 10^6 \text{ kN/m}^2$), the average acceptable range of two test results is 20.89% (19.29%, 26.98%, and 16.41%). As expected the variation of field mixes is higher than laboratory mixes.

It is not feasible to improve the precision of ASTM D 4123 or acceptable range by using more samples and orientations. The effect of quadruple the testing effort (from ASTM D 4123 recommended 6 tests with 3 samples at 2 orientations to 24 tests with 6 samples at 4 orientations) were calculated using equation 5. The acceptable range of two test results were improved from 19.29% to 12.26% for Mix A, 26.98% to 18.14% for Mix B, and 16.41% to 10.51% for Mix C. The time and samples required for a significant amount reduction in variation of resilient modulus (ASTM D 4123) is too large.

The amount of stress applied to the sample during testing has a significant effect on the measured resilient modulus values. It is recommended to characterize asphalt mixes at a standard stress of 15% of tensile stress. Resilient modulus at other stresses can be converted to the standard stress using the relationship obtained in this study. The regression equations obtained for field and laboratory mixes tested at 25 °C are as shown:-

$$\text{Field mixes: } Y = -0.025X + 1.372$$

$$\text{Laboratory mixes: } Y = -0.0225X + 1.34$$

where $Y = \text{MR @ } X\% / \text{MR @ } 15\%$ and $X = \text{stress as \% of tensile stress}$. There is no significant difference in the effect of stress on field and laboratory mixes at 25 °C. The combined equations of field and laboratory mixes is $Y = -0.0238X + 1.36$. Therefore, a change in stress from 15% to 10% of tensile stress at 25 °C will increase the measured **MR** by 11.89% $[(10-15) * -0.023785]$. For field mixes tested at 40 °C, the regression obtained was $Y = -0.04226 + 1.668$. A change in stress from 15% to 20% of tensile stress will decrease the measure **MR** by 21.13% $[(20-15) * -0.4226]$.

This study is limited since only one machine and one operator was used. However, the information obtained is useful in establishing variation of resilient modulus values obtained within any one laboratory. Further work is needed to include round robin study using a number of laboratories, test machines, and operators.

REFERENCES

1. **"AASHTO** Interim Guide for Design of Pavement Structures,^t American Association of State Highway and Transportation Officials, 1972, chapter III revised, 1981.
2. **Baladi, G. Y.**, ^tCharacterization of Flexible Pavement: A Case Study,^{tt} American Society for Testing and Material, Special Technical Paper No. 807, 1983, p 164-171.
3. Kenis, W. J., ^IMaterial Characterizations for Rational Pavement Design,["] American Society for Testing and Material, Special Technical Paper No. 561, 1973, p 132-152.
4. **Mamlouk S. Micheal** and Sarofim T. Ramsis, **"The Modulus of Asphalt Mixtures - An Unresolved Dilemma,"** Transportation Research Board, 67th annual meeting, 1988.
5. Baladi, G. Y. and Harichandran S. Ronald, **"Asphalt Mix Design and The Indirect Test: A New Horizon,"**
6. SAS Guide, 1979, SAS Institute Inc., **Cary**, North Carolina.
7. Brown, Elton Ray, "Evaluation of Properties of Recycled Asphalt Concrete Hot **Mix**," Dissertation report. Dept. of Civil Engineering, Texas A&M, August 1983.
8. Parker, Frazier Jr and Elton, David J, "Methods for Evaluating Resilient Modulus of Paving **Materials**," Final Report - Vol 1, Project Number ST-2019-7, Auburn University Highway Research Center, June 1989.
9. Bassett, Charles. , Master thesis draft report, Dept. of Civil Engineering, Auburn University, May 1989.
10. **"AASHTO** Test and Material Specifications,^{oc} Parts I and II, 13th edition, American Association for State Highway and Transportation Official, 1982.
11. Witczak, M. W., ^tDesign of Full Depth Air Field Pavements,^j Proceedings, 3rd International Conference on the Structural Design of Asphalt Pavements, 1972.
12. Lee, S. W., J. P. Mahoney and N. C. Jackson, ^lVerification of **Backcalculation** of Pavement **Moduli**," Transportation Research Record 1196, Transportation Research Board, 1988.