# Quantifying Uncertainty in Bridge Condition Assessment Data

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The effectiveness of decisions in bridge management systems is based on the quality of data obtained regarding various processes in the systems. Hence, data play a crucial role in bridge management. In general, data collected has some amount of associated uncertainty. In order to assess the impact of this uncertainty on decisions, the uncertainty in the data should be quantified. In other words, by determining the level of uncertainty, we are judging the quality of the data, which is important for making effective decisions in bridge management systems. This paper describes a procedure for measuring the level of uncertainty in bridge condition assessment data. First, a bridge deterioration model was applied to historical data to estimate the current condition of a bridge and compared to current data. Next, reliability theory was applied to estimate the structural reliability of the bridge, again based on both historical and present data. Finally, the reliability of the bridge was compared to the results obtained from the deterioration model, using a coefficient of correlation. Because the deterioration model used Markov chains, which are probabilistic, and the reliability results are also reported as probabilities, the results can be compared.

## INTRODUCTION

Bridge management is a rational and systematic approach to organizing and carrying out the activities related to planning, design, construction, maintenance, rehabilitation, and replacement of bridges (*1*). A bridge management system (BMS) should assist decision-makers in selecting the optimal alternative needed to achieve desired levels of service within the allocated funds and to identify future funding requirements. The most basic requirement for bridge management is a bridge inventory, which includes bridge location, type, functional classification, importance within the network, condition, and maintenance history. Therefore, because the decisions made in a bridge management system are based on information obtained from these data (*2*), the quality of the data impacts the effectiveness of the decisions. To determine the extent of these impacts, some quantitative methods are required.

This research examines the uncertainty associated with condition assessment data and quantifies it based on mathematical and statistical principles. The authors have developed a procedure using deterioration models and reliability models to compare predicted condition with actual condition. This paper describes the application of the procedure to a small database of three bridges.

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## BACKGROUND

### Bridge Management

State and local agencies use a number of different bridge management systems. Most of these BMS employ probabilistic deterioration models to predict the future condition of a bridge. Pontis, one of the most widely used BMS, uses Markovian models, as do several other BMS. Markovian models are often considered to represent the bridge deterioration process most effectively (see, for example, *3*).

### Uncertainty

A significant body of research exists on data uncertainty, and much of the work has classified types of uncertainties and proposed models to express the level of uncertainty. However, these techniques have not been widely explored for bridge condition data. Much of the research to date, including the development of guidelines for identifying and reducing specific types of uncertainties, relates to industrial engineering. The accuracy and precision in manufacturing testing and maintenance is higher in industrial settings than in construction or other field-work, and the extensive use of computers and machines in industry reduces the potential for human error, as does the ability to control the environment.

A number of methods for classifying uncertainty have been proposed. For example, Ayyub attributes uncertainties in engineering systems to ambiguity and vagueness in defining the architecture, parameters, and governing prediction models for the systems (*4*). The ambiguity component is generally ascribed to noncognitive sources, such as physical randomness, statistical uncertainty due to limited information, and model uncertainties due to simplifying assumptions, simplified methods, and idealized representations of real performances. Vagueness typically is due to cognitive sources, such as qualitatively defined variables (e.g. "performance"), human factors, and interrelationships among variables of a problem, especially for a complex system. Similarly, Kikuchi and Parsula also classify uncertainty as cognitive (subjective and not be easily quantified) or noncognitive (typically associated with prediction) (*5*).

The National Institute of Standards and Technology (NIST) has developed guidelines for evaluating and expressing uncertainty in industrial engineering data. According to Taylor, uncertainty can be divided into two components – random uncertainty and systematic uncertainty (*6*). Random uncertainties are generally determined by applying reliability theory. The NIST work has addressed standards for and accuracy of data rather than any particular type of data.

## METHODOLOGY

This research addresses noncognitive, random uncertainty in bridge condition data. The methodology combines a comparison of predicted with actual data for both component condition and reliability of a bridge. A correlation coefficient is then used to quantify the level of agreement between the two, which is subsequently used to obtain an overall estimation of "accuracy."

### Condition Assessment

According to Aktan et al., condition assessment is a process, which can be summarized in the following steps (*7*):

1. Measure the extent of damage/deterioration.
2. Determine the effect of that damage/deterioration on the condition of facility.
3. Set the scale of parameters that describe the condition of the facility as a whole.
4. Compare the existing damage/deterioration with previous records of condition assessment.

Different structural types of bridges, such as reinforced concrete slab, steel stringer, prestressed concrete, and box-reinforced concrete slab, have similar response and loading mechanisms. However, no two bridges are similar in all respects, especially in their deterioration and aging characteristics. Therefore, it is difficult to assess all types of bridges using the same condition analysis framework. This research examines condition assessment data for concrete bridge decks, slabs, girders (or beams), columns, railings, and abutments.

Once the current condition of a bridge has been assessed, future condition can be predicted using deterioration models. Deterioration is a long-term, gradual degradation leading to a reduction in the performance of a member, a structure, and ultimately the entire facility. Considering bridges specifically, deterioration can be defined as a decline in bridge element condition (*1*). Most deterioration models are based on basic theories of mathematics – statistical regression and/or stochastic modeling.

A Markovian process, which is used in most BMS, is a stochastic process that takes the uncertainties involved in the bridge deterioration process into consideration. Current models do not, however, account for uncertainties in the original data. In a Markov process, the state probabilities (the percentage of the inventory predicted to be in a particular condition state) and the transition probabilities (the probability that the condition of a component will deteriorate from one state to a lower state) are used to predict the future condition of the bridge or bridge component.

The framework proposed here requires the use of deterioration models, and for the test cases presented, the deterioration models in Pontis (*8*) were used to determine the transition probabilities and to predict future condition. Visual inspection data provide the type and severity of element deterioration, which are recorded as the condition state for that element. Pontis uses a Markovian deterioration model to predict the probability of transition among condition states each year (*8*).

### Reliability Model

Random uncertainty can be mathematically modeled using reliability theory (*6*). According to Gertsbakh, "The word 'reliability' refers to the ability of a system to perform its stated purposes adequately for a specified period of time under the operational conditions encountered" (*9*). The reliability of a system (in this case, a bridge) is based on the probability of failure of the system. The reliability of the entire system is a combination of the reliabilities of the components that comprise the system.

A bridge is a complex system in itself. In order to calculate the reliability of a bridge or a bridge component, Newton's law (to every action there is equal and opposite reaction) is used. For all complex structures, two components – resistance and capacity – are used in the calculation of reliability. If the resistance (or "demand") is greater than the capacity (or "supply"), the system will fail (*10*). The calculated probability of failure depends on the reasonableness of the underlying assumptions. It is based on empirical models and relies on observational data, as formulated below (*10*).

If the reliability of a structural system is

$$\text{Reliability} = 1 - P_f \qquad (1)$$

where $P_f$ is the probability of failure,

then, for discrete variables,

$$P_f = P(A<B) = \Sigma\, P(A<B/B=b)P(B=b) \qquad (2)$$

where A is the Capacity (Supply),
B is the Resistance (Demand), and
b is the Resistance at a given instance.

Generally, the variables whose functions are discussed above are normal random variables, and their distribution is normal. For calculating the reliability of a particular variable, two moments, the mean and the variance, are estimated. Only two moments of the random variables are considered practical, as large amounts of data are required to evaluate for further moments.

For a complex system, the capacity (supply) and resistance (demand) may each be functions of several other variables. Hence, the problem of calculating the reliability becomes complex, as the selected variable depends on various other random variables. Further, the complexity of the problem increases when the correlation between the variables is considered. To simplify the problem for the given bridges, only the two moments mentioned above are considered.

The total load effect (S) is

$$S = D + L + I \qquad (3)$$

where D is the dead load,
L is the live load, and
I is the impact load.

All three loads are considered random variables, as the loads at any particular time are not constant. Failure for a particular component will occur when S, the total load, exceeds the strength or resistance, R. Thus,

$$P_f = P[R<S] \qquad (4)$$

For this model, the mean (g) is the difference between resistance and capacity; that is,

$$g = R - S \qquad (5)$$

And the variance ($\sigma_g$) is

$$(\sigma_g)^2 = (\sigma_R)^2 - (\sigma_S)^2 \qquad (6)$$

where $\sigma_g$ is the difference between the variance of resistance and capacity,
$\sigma_R$ is the variance of resistance, and
$\sigma_S$ is the variance of load.

The failure probability is the region where g<0, and, in discrete form,

$$P_f = \Sigma\, P[g = g_i] \quad \text{for all values where } g<0 \qquad (7)$$

The safety index (β) is defined as the ratio of the difference between the resistance and capacity (the mean) and the variance:

$$\beta = g/\sigma_g \quad (8)$$

Thus, the failure probability is the sum of probabilities over the range where the safety index obtained is negative (*10*), and the probability of failure can be expressed as a function ($\Phi$) of the ratio of difference between the loads and the difference between variances.

$$P_f = \Phi(g/\sigma_g) \quad (9)$$

Substituting,

$$P_f = \Phi(\beta) \quad (10)$$

The quantitative relation between the safety index and the probability of failure is shown in Table 1 (*10*).

**TABLE 1 Relationship Between the Probability of Failure ($P_f$) and the Safety Index ($\beta$)**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_f$ | 0.5 | 0.25 | 0.16 | 0.10 | 0.05 | 0.01 | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ |
| $\beta$ | 0 | 0.67 | 1.00 | 1.28 | 1.65 | 2.33 | 3.10 | 3.72 | 4.25 | 4.75 |

### Coefficient of Correlation

Correlation techniques are used to study relationships (associations) between variables. Correlation is calculated as the level and direction of a relationship between two variables X and Y. The range of values of a correlation coefficient is from "-1" to "+1". The closer the value is to "+1," the stronger the positive correlation, and the closer the value is to "-1," the stronger the negative correlation (*11*).

The Pearson product moment correlation (r) is the most common "Correlation Coefficient." A number of assumptions must be made for the Pearson r (*11*):

1. Data for both X and Y must be measured at regular time intervals (e.g. data are collected each year).
2. Both X and Y must be normally distributed.
3. The sample must be representative of the population.
4. The relationship between X and Y should be linear.

It is assumed that he data used in the research satisfies the above requirements.

The Correlation Coefficient (r) is calculated as:

$$r = \frac{\sum XY - \frac{(\sum X)(\sum Y)}{n}}{\sqrt{\left(\sum X^2 - \frac{(\sum X)^2}{n}\right)\left(\sum Y^2 - \frac{(\sum Y)^2}{n}\right)}} \quad (11)$$

where n is the sample size.

To summarize the procedure:

- Estimate the transition probabilities for the components of a bridge based on historical condition assessment data (for *j* years).
- Determine the transition probabilities for the same components including condition assessment data from year *j+1*.
- Compare the transition probabilities based on the historical data (for *j* years) and current data (for *j+1* years) using a coefficient of correlation.
- Similarly, use a reliability model to calculate the current probability of failure based on the condition assessment data at time T = *j* years and time T = *j+1* years.
- Compare the probabilities of failure by calculating the coefficient of correlation.
- Calculate a final coefficient of correlation. The uncertainty is quantified in terms of this correlation coefficient.

## CASE STUDY

Three bridges in the state of Missouri form a case study for the methodology described. The condition data for these bridges were obtained from the Missouri Department of Transportation (MoDOT). For this work, concrete bridges were selected based on the age, type, and availability of the required data. The raw data are in the form of condition ratings for bridge components and for the bridge as a whole. These data are used as input to determine the transition probabilities for the deterioration models in Pontis and for calculating the reliability of the components. The transition probability for each component is calculated using deterioration models in Pontis. Table 2 shows the transition probabilities calculated for one of the three bridges.

**TABLE 2 Transition Probabilities for Bridge H198**

| | Transition Probabilities | |
|---|---|---|
| Element | based on past data | based on present data |
| Deck | 93.28 | 92.42 |
| Slab | 95.86 | 93.71 |
| Girder | 94.77 | 92.18 |
| Columns | 96.82 | 95.67 |
| Railings | 97.52 | 97.81 |
| Abutment | 95.47 | 93.25 |

Next, the reliability of each component is calculated using past and present data. Tables 3 and 4 show the reliability of components for the same bridge, H198.

The correlation coefficients for the transition probabilities (between past and present data) are calculated for each bridge, as are the coefficients of correlation for the reliabilities. Table 5 shows the coefficients of correlation for each of the three bridges and the final coefficient of correlation for the whole data set.

## CONCLUSIONS

Based on the coefficient of correlation, the uncertainty in condition assessment data can be quantified. The coefficient of correlation varies from 0 to 1, and the closer the value of the coefficient to 1, the higher the correlation between the predicted and present probabilities. These values can be attached to the bridge data, and weights can be assigned to different data elements used in bridge management based on the coefficients of correlation, which would enhance decision making in bridge management.

In this research, the value assigned to uncertainty associated with the data is in the form of a coefficient of correlation. From the coefficients of correlation obtained for these bridges (Table 5), it is clear that the uncertainty is very low. In other words, the results obtained from two different data sets for the same bridge are very close. However, the data used in this example are for only three bridges. If data for a whole network of bridges are used, the procedure will be more efficient and effective, as uncertainty in any problem cannot be eliminated, but only can be reduced. Thus, as the number of iterations increases, the uncertainty in the result decreases.

Strengths of this methodology include the following:

1. Two different models are used for every bridge to calculate data uncertainty. By doing so, the possibility of simply validating the models is decreased.

**TABLE 3 Reliability of Components for Bridge H198 Based on Historical Data**

| Element | Dead Load 1000 lbs | Live Load 1000 lbs | Impact Load 1000 lbs | Impact Load Factor | Total Load carrying capacity 1000 lbs | Capacity Reduction Factor | Capacity (R) at time t=x 1000 lbs | Load (S) at time t=x 1000 lbs | $\sigma_R$ lbs | $\sigma_S$ 1000 lbs | Safety Index β | Reliability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Deck | 202.75 | 70.00 | 16.10 | 0.23 | 288.85 | 0.94 | 271.52 | 252.65 | 0 | 14.66 | 1.287 | 90.1 |
| Slab | 202.75 | 70.00 | 16.10 | 0.23 | 288.85 | 0.94 | 271.52 | 252.65 | 0 | 14.66 | 1.287 | 90.1 |
| Girder | 659.05 | 70.00 | 16.10 | 0.23 | 745.15 | 0.84 | 625.93 | 565.42 | 0 | 40.69 | 1.4872 | 92.8 |
| Column | 683.21 | 70.00 | 16.10 | 0.23 | 769.31 | 0.84 | 646.22 | 595.25 | 0 | 33.60 | 1.5168 | 93.2 |
| Railings | | | | | 10.00 | 0.94 | 9.40 | 7.93 | 0 | 1.02 | 1.45 | 92.3 |
| Abutment | | | | | 16.54 | 0.84 | 13.89 | 12.87 | 0 | 0.67 | 1.5316 | 93.4 |

**TABLE 4 Reliability of Components for Bridge H198 Based on Current Data**

| Element | Dead Load 1000 lbs | Live Load 1000 lbs | Impact Load 1000 lbs | Impact Load Factor | Total Load Carrying Capacity 1000 lbs | Capacity Reduction Factor | Capacity (R) at time t=x 1000 lbs | Load (S) at time t=x 1000 lbs | $\sigma_R$ lbs | $\sigma_S$ 1000 lbs | Safety index β | Reliability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Deck | 202.75 | 70.00 | 16.10 | 0.23 | 288.85 | 0.84 | 242.63 | 223.07 | 0 | 15.53 | 1.26 | 88.27 |
| Slab | 202.75 | 70.00 | 16.10 | 0.23 | 288.85 | 0.84 | 242.63 | 223.07 | 0 | 15.53 | 1.26 | 88.27 |
| Girder | 659.05 | 70.00 | 16.10 | 0.23 | 745.15 | 0.76 | 566.31 | 504.55 | 0 | 48.14 | 1.283 | 90.42 |
| Column | 683.21 | 70.00 | 16.10 | 0.23 | 769.31 | 0.76 | 584.67 | 522.24 | 0 | 44.44 | 1.405 | 91.78 |
| Railings | | | | | 10.00 | 0.84 | 8.40 | 7.35 | 0 | 0.79 | 1.33 | 90.89 |
| Abutment | | | | | 16.54 | 0.76 | 12.57 | 11.32 | 0 | 0.90 | 1.38 | 91.21 |

**TABLE 5 Coefficients of Correlation**

| | Coefficient of correlation | |
|---|---|---|
| Bridge | Reliability theory | Deterioration models |
| A814 | 0.92746438 | 0.95069567 |
| H198 | 0.96602507 | 0.88902714 |
| H199 | 0.98538182 | 0.89800344 |
| | Final coefficient of correlation | |
| All | 0.891760796 | |

2. The coefficients of correlation have no units and can be compared to obtain a numerical value for the uncertainty of the condition assessment data.
3. As the amount of data increases, the uncertainty in the procedure decreases (repetitive analysis). Hence, the procedure has great potential for network level bridge management.

While the methodology shows promise, weaknesses which must be addressed in future research include:

1. Additional methods, which are based on the data themselves and collection methods rather than on models, should be explored.
2. Uncertainty is present in the procedure itself, due to the assumptions made in simplifying the models and in calculating the correlation coefficients.
3. The methodology requires large amounts of historical data for bridges in the network to give good results.

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