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PB2002-105935

MEMORANDUM NO. 1580

Cyclic machine scheduling with tool transportation - additional calculations

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APRIL 2001

ISSN 0169-2690

REPRODUCED BY: NTIS
U.S. Department of Commerce
National Technical Information Service
Springfield, Virginia 22161

Cyclic Machine Scheduling with Tool Transportation – Additional Calculations

C.M.H. Kuijpers

Abstract

In the PhD Thesis of Kuijpers a cyclic machine scheduling problem with tool transportation is considered. For the problem with two machines, it is shown that there always exists an optimal schedule with a certain structure. This is done by means of an elaborate case study. For a number of cases some calculations are only sketched in the thesis, but not explicitly given. This paper contains a complete elaboration of the calculations that are not explicitly given in the thesis.

Keywords: scheduling, cyclic, tools
Mathematics Subject Classification: 90B35

1 Introduction

In Kuijpers[1] a certain machine scheduling problem is examined. Most results concern the problem for two machines. For this two-machine problem, it turns out that there always exists an optimal schedule with certain structure properties. The proof of this result can be found in Chapter 5 and Appendix A of [1]. On the last four pages of Appendix A some calculations are described, but not explicitly given. A complete elaboration of these calculations is given below.

(For the definitions of the used variables, see Kuijpers[1].)

We prove that for every pair $j^*, j \in \bigcup_{a=1}^{3}(P_a \cup Q_a)$ we have that $I(j^*) + II(j) < c$ in all of the following cases:

$$\textbf{Case 2(b)} \quad I(j^*) > p_{j^*} + 3d \text{ and } II(j) \leq p_j + 3d + \delta(j), \tag{1}$$

$$\textbf{Case 2(c)} \quad I(j^*) \leq p_{j^*} + 3d \text{ and } II(j) > p_j + 3d + \delta(j), \tag{2}$$

$$\textbf{Case 2(d)} \quad I(j^*) > p_{j^*} + 3d \text{ and } II(j) > p_j + 3d + \delta(j). \tag{3}$$

For proving this we have the following formulas at our disposal (see [1]).

General formulas

$$I(j^*) \in [p_{j^*} + 2kd + d, p_{j^*} + 2(k+1)d + d) \tag{4}$$

$$II(j) \in [p_j + 2l_j d + d + \delta(j), p_j + 2(l_j + 1)d + d + \delta(j)). \tag{5}$$

$$c = \sum_{a=1}^{3} l(P_a) + \sum_{a=1}^{3} l(Q_a). \tag{6}$$

$$\delta(j) = \begin{cases} \delta_2 & \text{if } j \in Q_1 \text{ and } T_j^2(4) \geq b(P_2)^1 + d, \\ \delta_3 & \text{if } j \in Q_2 \text{ and } T_j^2(4) \geq b(P_3)^1 + d, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

$$\delta(j) < 2d. \tag{8}$$

$$l(P_a) \geq 2d \qquad (a = 1, 2, 3). \tag{9}$$

$$l(Q_a) \geq 2d \qquad (a = 1, 2, 3). \tag{10}$$

$$2d(\lfloor \frac{p_j}{2d} \rfloor + 1) > p_j. \tag{11}$$

If j^* is the first task of Q_1 or Q_2:

$$I(j^*) = p_{j^*} + 2kd + d. \tag{12}$$

If $I(j^*) > p_{j^*} + 3d$:

$$k > 0 \tag{13}$$

$$p_{j^*} \geq 2d. \tag{14}$$

If $I(j^*) > p_{j^*} + 3d$ and $j^* \in P_a$ $(a = 1, 2, 3)$:

$$l(P_a| < j^*) \geq 2kd + \delta_a. \tag{15}$$

$$l(P_a| > j^*) + l(Q_a) + l(P_{a \oplus_3 1}) \geq 2d(k + \lfloor \frac{p_{j^*}}{2d} \rfloor). \tag{16}$$

$$p_{j^*} < \frac{1}{2} l(P_a) + \frac{1}{2} l(Q_a) + \frac{1}{2} l(P_{a \oplus_3 1}) - 2kd + d - \frac{1}{2}\delta_a. \tag{17}$$

If $I(j^*) > p_{j^*} + 3d$, $j^* \in P_a$ and j^* is not the last task of P_a $(a = 1, 2, 3)$:

$$l(P_a| > j^*) \geq 2d(k + \lfloor \frac{p_{j^*}}{2d} \rfloor) \tag{18}$$

If $I(j^*) > p_{j^*} + 3d$ and $j^* \in Q_a$ $(a = 1, 2, 3)$:

$$l(P_a) + l(Q_a| < j^*) \geq 2kd + \delta_a. \tag{19}$$

$$l(Q_a| > j^*) + l(P_{a \oplus_3 1}) \geq 2d(k + \lfloor \frac{p_{j^*}}{2d} \rfloor). \tag{20}$$

$$p_{j^*} < \frac{1}{2} l(P_a) + \frac{1}{2} l(Q_a) + \frac{1}{2} l(P_{a \oplus_3 1}) - 2kd + d - \frac{1}{2}\delta_a. \tag{21}$$

If $I(j^*) > p_{j^*} + 3d$, $j^* \in Q_a$ $(a = 1,2,3)$ and j^* is not the first task of Q_a $(a = 1,2,3)$:

$$p_{j^*} < \frac{1}{2}l(P_a) + \frac{1}{2}l(Q_a) + \frac{1}{2}l(P_{a\oplus_3 1}) - 2kd - \frac{1}{2}\delta_a. \tag{22}$$

If $II(j) > p_j + 3d + \delta(j)$:

$$l_j > 0 \tag{23}$$

$$j \text{ is not the last task of } P_a \ (a = 1,2,3) \tag{24}$$

If $II(j) > p_j + 3d + \delta(j)$ and $j \in P_a$ $(a = 1,2,3)$:

$$l(P_a| > j) > 2l_j d. \tag{25}$$

$$l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a| < j) \geq 2d(l_j + \lfloor\frac{p_j}{2d}\rfloor + 1). \tag{26}$$

$$p_j < \frac{1}{2}l(P_{a\oplus_3 2}) + \frac{1}{2}l(Q_{a\oplus_3 2}) + \frac{1}{2}l(P_a) - 2l_j d. \tag{27}$$

If $II(j) > p_j + 3d + \delta(j)$ and $j \in Q_a$ $(a = 1,2)$:

$$l(Q_a| > j) + l(P_{a\oplus_3 1}) > 2l_j d + \delta(j). \tag{28}$$

$$l(P_a) + l(Q_a| < j) \geq 2d(l_j + \lfloor\frac{p_j}{2d}\rfloor + 1). \tag{29}$$

$$p_j < \frac{1}{2}l(P_a) + \frac{1}{2}l(Q_a) + \frac{1}{2}l(P_{a\oplus_3 1}) - 2l_j d - \frac{1}{2}\delta(j). \tag{30}$$

If $II(j) > p_j + 3d + \delta(j)$ and $j \in Q_3$:

$$l(Q_3| > j) + l(P_1) \geq 2l_j d + 2d + \delta(j). \tag{31}$$

$$l(P_3) + l(Q_3| < j) \geq 2d(l_j + \lfloor\frac{p_j}{2d}\rfloor). \tag{32}$$

$$p_j < \frac{1}{2}l(P_3) + \frac{1}{2}l(Q_3) + \frac{1}{2}l(P_1) - 2l_j d - \frac{1}{2}\delta(j). \tag{33}$$

2 Calculations

2.1 $\mathbf{j^* \in P_a, j \in P_a\ (a = 1,2,3)}$

2.1.1 Case 2(b)

$\mathrm{j^* = j}$

$$\begin{aligned} I(j^*) + II(j) &\overset{(1)(4)}{<} p_{j^*} + p_j + 2kd + 6d + \delta(j) \\ &\overset{(7)}{=} p_{j^*} + p_j + 2kd + 6d \\ &\overset{j^*=j}{=} 2p_{j^*} + 2kd + 6d \end{aligned}$$

$$
\begin{aligned}
&\overset{(17),\delta_a\geq 0}{<} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})-2kd+8d\\
&\overset{(13)}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})+6d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 2})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+6d\\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

$\mathbf{j^* < j}$

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(1)(4)}{<} && p_{j^*}+p_j+2kd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2kd+6d\\
&\overset{(15),\delta_a\geq 0}{\leq} && p_{j^*}+p_j+l(P_a|<j^*)+6d\\
&\overset{j^*,j\in P_a;j^*<j}{\leq} && l(P_a)+6d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 1})-l(P_{a\oplus_3 2})-\sum_{a=1}^{3}l(Q_a)+6d\\
&\overset{(9)(10)}{\leq} && c-4d.
\end{aligned}
$$

$\mathbf{j^* > j}$

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(1)(4)}{<} && p_{j^*}+p_j+2kd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2kd+6d\\
&\overset{(16)}{\leq} && p_{j^*}+p_j+l(P_a|>j^*)+l(Q_a)+l(P_{a\oplus_3 1})-\\
& && 2d\lfloor\frac{p_{j^*}}{2d}\rfloor+6d\\
&\overset{j^*,j\in P_a;j^*>j}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+6d\\
&\overset{(14)}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})+4d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 2})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+4d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.1.2 Case 2(c)

$\mathbf{j^* = j}$

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(2)(5)}{<} && p_{j^*}+p_j+2l_jd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2l_jd+6d\\
&\overset{j^*=j}{=} && 2p_j+2l_jd+6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{(27)}{<} && l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a) - 2l_j d + 6d \\
&\overset{(23)}{\leq} && l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a) + 4d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 1}) - l(Q_a) - l(Q_{a\oplus_3 1}) + 4d \\
&\overset{(9)(10)}{\leq} && c - 2d.
\end{aligned}
$$

$\mathbf{j^* < j}$:

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(2)(5)}{<} && p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2l_j d + 6d \\
&\overset{(25)}{<} && p_{j^*} + p_j + l(P_a| > j) + 6d \\
&\overset{j^*, j \in P_a; j^* < j}{\leq} && l(P_a) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 1}) - l(P_{a\oplus_3 2}) - \sum_{a=1}^{3} l(Q_a) + 6d \\
&\overset{(9)(10)}{\leq} && c - 4d.
\end{aligned}
$$

$\mathbf{j^* > j}$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(2)(5)}{<} && p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2l_j d + 6d \\
&\overset{(26)}{\leq} && p_{j^*} + p_j + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a| < j) - \\
& && 2d(\lfloor \frac{p_j}{2d} \rfloor + 1) + 6d \\
&\overset{j^*, j \in P_a; j^* > j}{\leq} && l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a) - 2d\lfloor \frac{p_j}{2d} \rfloor + 4d \\
&\overset{p_j > 0}{\leq} && l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a) + 4d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 1}) - l(Q_a) - l(Q_{a\oplus_3 1}) + 4d \\
&\overset{(9)(10)}{\leq} && c - 2d.
\end{aligned}
$$

2.1.3 Case 2(d)

$\mathbf{j^* = j}$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(4)(5)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2kd + 2l_j d + 6d \\
&\overset{j^* = j}{=} && 2p_{j^*} + 2kd + 2l_j d + 6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{(24)(18)}{\leq} && 2p_{j^*} + l(P_a| > j^*) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 2l_j d + 6d \\
&\overset{(26)}{\leq} && 2p_{j^*} + l(P_a| > j^*) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + l(P_{a\oplus_3 2}) + l(Q_{a\ominus_3 2}) + \\
& && l(P_a| < j) - 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 6d \\
&\overset{j^*,j\in P_a;j^*=j}{=} && p_{j^*} + l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) - 4d\lfloor\frac{p_{j^*}}{2d}\rfloor + 4d \\
&\overset{(11)}{<} && l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 6d \\
&\overset{(14)}{\leq} && l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + 4d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 1}) - l(Q_a) - l(Q_{a\oplus_3 1}) + 4d \\
&\overset{(9)(10)}{\leq} && c - 2d.
\end{aligned}
$$

$j^* < j$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(4)(5)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2kd + 2l_j d + 6d \\
&\overset{(15),\delta_a\geq 0}{\leq} && p_{j^*} + p_j + l(P_a| < j^*) + 2l_j d + 6d \\
&\overset{(25)}{<} && p_{j^*} + p_j + l(P_a| < j^*) + l(P_a| > j) + 6d \\
&\overset{j^*,j\in P_a;j^*<j}{\leq} && l(P_a) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 1}) - l(P_{a\oplus_3 2}) - \sum_{a=1}^{3} l(Q_a) - 6d \\
&\overset{(9)(10)}{\leq} && c - 4d.
\end{aligned}
$$

$j^* > j$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(4)(5)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2kd + 2l_j d + 6d \\
&\overset{(16)}{\leq} && p_{j^*} + p_j + l(P_a| > j^*) + l(Q_a) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + \\
& && 2l_j d + 6d \\
&\overset{(26)}{\leq} && p_{j^*} + p_j + l(P_a| > j^*) + l(Q_a) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + \\
& && l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + l(P_a| < j) - 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 6d \\
&\overset{j^*,j\in P_a;j^*>j}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2})
\end{aligned}
$$

$$
\begin{aligned}
& -2d\lfloor\frac{p_{j^*}}{2d}\rfloor - 2d\lfloor\frac{p_j}{2d}\rfloor + 4d \\
\overset{(14),p_j>0}{\leq} \quad & l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + 2d \\
\overset{(6)}{=} \quad & c - l(Q_{a\oplus_3 1}) + 2d \\
\overset{(10)}{\leq} \quad & c.
\end{aligned}
$$

2.2 $\mathbf{j^* \in P_a, j \in P_b\ (a, b \in \{1, 2, 3\}, a \neq b)}$

2.2.1 Case 2(b)

$$
\begin{aligned}
I(j^*) + II(j) \quad \overset{(1)(4)}{<} \quad & p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
\overset{(7)}{=} \quad & p_{j^*} + p_j + 2kd + 6d \\
\overset{(15),\delta_a \geq 0}{\leq} \quad & p_{j^*} + p_j + l(P_a| < j^*) + 6d \\
\overset{j^* \in P_a, j \in P_b}{\leq} \quad & l(P_a) + l(P_b) + 6d \\
\overset{(6),a\neq b}{=} \quad & c - \sum_{i\neq a,b} l(P_i) - \sum_{a=1}^{3} l(Q_a) + 6d \\
\overset{(9)(10)}{\leq} \quad & c - 2d.
\end{aligned}
$$

2.2.2 Case 2(c)

$$
\begin{aligned}
I(j^*) + II(j) \quad \overset{(2)(5)}{<} \quad & p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
\overset{(7)}{=} \quad & p_{j^*} + p_j + 2l_j d + 6d \\
\overset{(25)}{<} \quad & p_{j^*} + p_j + l(P_b| > j) + 6d \\
\overset{j^* \in P_a, j \in P_b}{\leq} \quad & l(P_a) + l(P_b) + 6d \\
\overset{(6),a\neq b}{=} \quad & c - \sum_{i\neq a,b} l(P_i) - \sum_{a=1}^{3} l(Q_a) + 6d \\
\overset{(9)(10)}{\leq} \quad & c - 2d.
\end{aligned}
$$

2.2.3 Case 2(d)

$$
\begin{aligned}
I(j^*) + II(j) \quad \overset{(4)(5)}{<} \quad & p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
\overset{(7)}{=} \quad & p_{j^*} + p_j + 2kd + 2l_j d + 6d \\
\overset{(15),\delta_a \geq 0}{\leq} \quad & p_{j^*} + p_j + l(P_a| < j^*) + 2l_j d + 6d \\
\overset{(25)}{<} \quad & p_{j^*} + p_j + l(P_a| < j^*) + l(P_b| > j) + 6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{j^*\in P_a, j\in P_b}{\leq} && l(P_a)+l(P_b)+6d\\
&\overset{(6),a\neq b}{=} && c-\sum_{i\neq a,b} l(P_i)-\sum_{a=1}^{3} l(Q_a)+6d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.3 $\mathbf{j^* \in P_a, j \in Q_b (a \in \{1,2,3\}, b \in \{a, a \oplus_3 1\}}$

2.3.1 Case 2(b)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(1)(4)}{<} && p_{j^*}+p_j+2kd+6d+\delta(j)\\
&\overset{(8)}{<} && p_{j^*}+p_j+2kd+8d\\
&\overset{(15),\delta_a\geq 0}{\leq} && p_{j^*}+p_j+l(P_a|<j^*)+8d\\
&\overset{j^*\in P_a, j\in Q_b}{\leq} && l(P_a)+l(Q_b)+8d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 1})-l(P_{a\oplus_3 2})-l(Q_{b\oplus_3 1})-l(Q_{b\oplus_3 2})+8d\\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.3.2 Case 2(c)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(2)(5)}{<} && p_{j^*}+p_j+2l_jd+6d+\delta(j)\\
&\overset{(28)/(31)}{<} && p_{j^*}+p_j+l(Q_b|>j)+l(P_{b\oplus_3 1})+6d\\
&\overset{j^*\in P_a, j\in Q_b}{\leq} && l(P_a)+l(Q_b)+l(P_{b\oplus_3 1})+6d\\
&\overset{(6),b\in\{a,a\oplus_3 1\}}{=} && c-\sum_{i\neq a,b\oplus_3 1} l(P_i)-l(Q_{b\oplus_3 1})-l(Q_{b\oplus_3 2})+6d\\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.3.3 Case 2(d)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(4)(5)}{<} && p_{j^*}+p_j+2kd+2l_jd+6d+\delta(j)\\
&\overset{(15),\delta_a\geq 0}{\leq} && p_{j^*}+p_j+l(P_a|<j^*)+2l_jd+6d+\delta(j)\\
&\overset{(28)/(31)}{<} && p_{j^*}+p_j+l(P_a|<j^*)+l(Q_b|>j)+l(P_{b\oplus_3 1})+6d\\
&\overset{j^*\in P_a, j\in Q_b}{\leq} && l(P_a)+l(Q_b)+l(P_{b\oplus_3 1})+6d\\
&\overset{(6),b\in\{a,a\oplus_3 1\}}{=} && c-\sum_{i\neq a,b\oplus_3 1} l(P_i)-l(Q_{b\oplus_3 1})-l(Q_{b\oplus_3 2})+6d\\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.4 $\mathbf{j^* \in P_a, j \in Q_{a\oplus_3 2}\ (a = 1, 2, 3)}$

2.4.1 Case 2(b)

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(1)(4)}{<} p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2kd + 8d \\
&\overset{(15),\delta_a \geq 0}{\leq} p_{j^*} + p_j + l(P_a| < j^*) + 8d \\
&\overset{j^* \in P_a, j \in Q_{a\oplus_3 2}}{\leq} l(P_a) + l(Q_{a\oplus_3 2}) + 8d \\
&\overset{(6)}{=} c - l(P_{a\oplus_3 1}) - l(P_{a\oplus_3 2}) - l(Q_a) - l(Q_{a\oplus_3 1}) + 8d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.4.2 Case 2(c)

$a = 1$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} p_{j^*} + p_j + 2l_j d + 6d \\
&\overset{(32)}{\leq} p_{j^*} + p_j + l(P_3) + l(Q_3| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{j^* \in P_1, j \in Q_3}{\leq} l(P_1) + l(P_3) + l(Q_3) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{p_j > 0}{\leq} l(P_1) + l(P_3) + l(Q_3) + 6d \\
&\overset{(6)}{=} c - l(P_2) - l(Q_1) - l(Q_2) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

$a = 2, 3$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2l_j d + 8d \\
&\overset{(29)}{\leq} p_{j^*} + p_j + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}| < j) - 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 8d \\
&\overset{j^* \in P_a, j \in Q_{a\oplus_3 2}}{\leq} l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{p_j > 0}{\leq} l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(6)}{=} c - l(P_{a\oplus_3 1}) - l(Q_a) - l(Q_{a\oplus_3 1}) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.4.3 Case 2(d)

$a = 1$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_jd + 6d + \delta(j) \\
&\overset{(7)}{=} p_{j^*} + p_j + 2kd + 2l_jd + 6d \\
&\overset{(15),\delta_a \geq 0}{\leq} p_{j^*} + p_j + l(P_1| < j^*) + 2l_jd + 6d \\
&\overset{(32)}{\leq} p_{j^*} + p_j + l(P_1| < j^*) + l(P_3) + l(Q_3| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{j^* \in P_1, j \in Q_3}{\leq} l(P_1) + l(P_3) + l(Q_3) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{p_j > 0}{\leq} l(P_1) + l(P_3) + l(Q_3) + 6d \\
&\overset{(6)}{=} c - l(P_2) - l(Q_1) - l(Q_2) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

$a = 2, 3$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_jd + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2kd + 2l_jd + 8d \\
&\overset{(15),\delta_a \geq 0}{\leq} p_{j^*} + p_j + l(P_a| < j^*) + 2l_jd + 8d \\
&\overset{(29)}{\leq} p_{j^*} + p_j + l(P_a| < j^*) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}| < j) - \\
&\qquad 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 8d \\
&\overset{j^* \in P_a, j \in Q_{a\oplus_3 2}}{\leq} l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{p_j > 0}{\leq} l(P_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(6)}{=} c - l(P_{a\oplus_3 1}) - l(Q_a) - l(Q_{a\oplus_3 1}) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.5 $\mathbf{j^* \in Q_a, j \in P_a\ (a = 1, 2, 3)}$

2.5.1 Case 2(b)

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(1)(4)}{<} p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(7)}{=} p_{j^*} + p_j + 2kd + 6d \\
&\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{j^*\in Q_a, j\in P_a}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+6d\\
&\overset{(14)}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})+4d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 2})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+4d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.5.2 Case 2(c)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(2)(5)}{<} && p_{j^*}+p_j+2l_jd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2l_jd+6d\\
&\overset{(25)}{<} && p_{j^*}+p_j+l(P_a|>j)+6d\\
&\overset{j^*\in Q_a, j\in P_a}{\leq} && l(P_a)+l(Q_a)+6d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 1})-l(P_{a\oplus_3 2})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+6d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.5.3 Case 2(d)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(4)(5)}{<} && p_{j^*}+p_j+2kd+2l_jd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2kd+2l_jd+6d\\
&\overset{(20)}{\leq} && p_{j^*}+p_j+l(Q_a|>j^*)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+\\
& && 2l_jd+6d\\
&\overset{(25)}{<} && p_{j^*}+p_j+l(Q_a|>j^*)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+\\
& && l(P_a|>j)+6d\\
&\overset{j^*\in Q_a, j\in P_a}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+6d\\
&\overset{(14)}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 1})+4d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 2})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+4d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.6 $\mathbf{j^*\in Q_a, j\in P_{a\oplus_3 1}\ (a=1,2,3)}$

2.6.1 Case 2(b)

$$
I(j^*)+II(j) \quad \overset{(1)(4)}{<} \quad p_{j^*}+p_j+2kd+6d+\delta(j)
$$

$$
\begin{aligned}
&\overset{(7)}{=} && p_{j^*} + p_j + 2kd + 6d \\
&\overset{(19),\delta_a \geq 0}{\leq} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + 6d \\
&\overset{j^* \in Q_a, j \in P_{a\oplus_3 1}}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.6.2 Case 2(c)

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(2)(5)}{<} && p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2l_j d + 6d \\
&\overset{(25)}{<} && p_{j^*} + p_j + l(P_{a\oplus_3 1}| > j) + 6d \\
&\overset{j^* \in Q_a, j \in P_{a\oplus_3 1}}{\leq} && l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_a) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c - 2d.
\end{aligned}
$$

2.6.3 Case 2(d)

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(4)(5)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2kd + 2l_j d + 6d \\
&\overset{(19),\delta_a \geq 0}{\leq} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + 2l_j d + 6d \\
&\overset{(25)}{<} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + l(P_{a\oplus_3 1}| > j) + 6d \\
&\overset{j^* \in Q_a, j \in P_{a\oplus_3 1}}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.7 $\mathbf{j^* \in Q_a, j \in P_{a\oplus_3 2}\ (a = 1, 2, 3)}$

2.7.1 Case 2(b)

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(1)(4)}{<} && p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(7)}{=} && p_{j^*} + p_j + 2kd + 6d \\
&\overset{(19),\delta_a \geq 0}{\leq} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + 6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{j^*\in Q_a, j\in P_{a\oplus_3 2}}{\leq} && l(P_a)+l(Q_a)+l(P_{a\oplus_3 2})+6d\\
&\overset{(6)}{=} && c-l(P_{a\oplus_3 1})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+6d\\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.7.2 Case 2(c)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(2)(5)}{<} && p_{j^*}+p_j+2l_jd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2l_jd+6d\\
&\overset{(25)}{<} && p_{j^*}+p_j+l(P_{a\oplus_3 2}|>j)+6d\\
&\overset{j^*\in Q_a, j\in P_{a\oplus_3 2}}{\leq} && l(Q_a)+l(P_{a\oplus_3 2})+6d\\
&\overset{(6)}{=} && c-l(P_a)-l(P_{a\oplus_3 1})-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+6d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.7.3 Case 2(d)

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(4)(5)}{<} && p_{j^*}+p_j+2kd+2l_jd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2kd+2l_jd+6d\\
&\overset{(20)}{\leq} && p_{j^*}+p_j+l(Q_a|>j^*)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor\\
& && +2l_jd+6d\\
&\overset{(25)}{<} && p_{j^*}+p_j+l(Q_a|>j^*)+l(P_{a\oplus_3 1})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+\\
& && l(P_{a\oplus_3 2}|>j)+6d\\
&\overset{j^*\in Q_a, j\in P_{a\oplus_3 2}}{\leq} && l(Q_a)+l(P_{a\oplus_3 1})+l(P_{a\oplus_3 2})-2d\lfloor\frac{p_{j^*}}{2d}\rfloor+6d\\
&\overset{(14)}{\leq} && l(Q_a)+l(P_{a\oplus_3 1})+l(P_{a\oplus_3 2})+4d\\
&\overset{(6)}{=} && c-l(P_a)-l(Q_{a\oplus_3 1})-l(Q_{a\oplus_3 2})+4d\\
&\overset{(9)(10)}{\leq} && c-2d.
\end{aligned}
$$

2.8 $\mathbf{j^*\in Q_a, j\in Q_a\ (a=1,2,3)}$

2.8.1 Case 2(b)

$\mathbf{j^*=j}$ If j^* is not a task of Q_1 or Q_2:

$$
\begin{aligned}
I(j^*)+II(j) \quad &\overset{(1)(4)}{<} && p_{j^*}+p_j+2kd+6d+\delta(j)\\
&\overset{(7)}{=} && p_{j^*}+p_j+2kd+6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{j^*=j}{=} && 2p_{j^*} + 2kd + 6d \\
&\overset{(21),\delta_a\geq 0}{<} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) - 2kd + 8d \\
&\overset{(13)}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

If j^* is the first task of Q_1 or Q_2 :

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(1)(12)}{\leq} && p_{j^*} + p_j + 2kd + 4d + \delta(j) \\
&\overset{(8)}{<} && p_{j^*} + p_j + 2kd + 6d \\
&\overset{j^*=j}{=} && 2p_{j^*} + 2kd + 6d \\
&\overset{(21),\delta_a\geq 0}{<} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) - 2kd + 8d \\
&\overset{(13)}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

If j^* is a task of Q_1 or Q_2, but not the first task of Q_1 or Q_2:

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(1)(4)}{<} && p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(8)}{<} && p_{j^*} + p_j + 2kd + 8d \\
&\overset{j^*=j}{=} && 2p_{j^*} + 2kd + 8d \\
&\overset{(22),\delta_a\geq 0}{<} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) - 2kd + 8d \\
&\overset{(13)}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

$\mathbf{j^* < j}$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(1)(4)}{<} && p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(8)}{<} && p_{j^*} + p_j + 2kd + 8d \\
&\overset{(19),\delta_a\geq 0}{\leq} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + 8d \\
&\overset{j^*,j\in Q_a;j^*<j}{\leq} && l(P_a) + l(Q_a) + 8d
\end{aligned}
$$

$$\overset{(6)}{=} \quad c - l(P_{a\oplus_3 1}) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 8d$$
$$\overset{(9)(10)}{\leq} \quad c.$$

$j^* > j$

$$I(j^*) + II(j) \overset{(1)(4)}{<} p_{j^*} + p_j + 2kd + 6d + \delta(j)$$
$$\overset{(8)}{<} p_{j^*} + p_j + 2kd + 8d$$
$$\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 8d$$
$$\overset{j^*,j\in Q_a;j^*>j}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 8d$$
$$\overset{(14)}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + 6d$$
$$\overset{(6)}{=} c - l(P_a) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d$$
$$\overset{(9)(10)}{\leq} c - 2d.$$

2.8.2 Case 2(c)

$j^* = j$

$$I(j^*) + II(j) \overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j)$$
$$\overset{j^*=j}{=} 2p_j + 2l_j d + 6d + \delta(j)$$
$$\overset{(30)/(33)}{<} l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) - 2l_j d + 6d$$
$$\overset{(23)}{\leq} l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 4d$$
$$\overset{(6)}{=} c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 4d$$
$$\overset{(9)(10)}{\leq} c - 2d.$$

$j^* < j$

$$I(j^*) + II(j) \overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j)$$
$$\overset{(28)/(31)}{<} p_{j^*} + p_j + l(Q_a| > j) + l(P_{a\oplus_3 1}) + 6d$$
$$\overset{j^*,j\in Q_a;j^*<j}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + 6d$$
$$\overset{(6)}{=} c - l(P_a) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d$$
$$\overset{(9)(10)}{\leq} c - 2d.$$

$\mathbf{j^* > j}$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2l_j d + 8d \\
&\overset{(29)/(32)}{\leq} p_{j^*} + p_j + l(P_a) + l(Q_a| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 8d \\
&\overset{j^*, j \in Q_a; j^* > j}{\leq} l(P_a) + l(Q_a) - 2d\lfloor\frac{p_j}{2d}\rfloor + 8d \\
&\overset{p_j > 0}{\leq} l(P_a) + l(Q_a) + 8d \\
&\overset{(6)}{=} c - l(P_{a\oplus_3 1}) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 8d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.8.3 Case 2(d)

$\mathbf{j^* = j}$

$a = 1, 2$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2kd + 2l_j d + 8d \\
&\overset{j^* = j}{=} 2p_{j^*} + 2kd + 2l_j d + 8d \\
&\overset{(20)}{\leq} 2p_{j^*} + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 2l_j d + 8d \\
&\overset{(29)}{\leq} 2p_{j^*} + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + l(P_a) + \\
&\qquad l(Q_a| < j) - 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 8d \\
&\overset{j^*, j \in Q_a; j^* = j}{\leq} l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + p_{j^*} - 4d\lfloor\frac{p_{j^*}}{2d}\rfloor + 6d \\
&\overset{(11)(14)}{<} l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

$a = 3$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} p_{j^*} + p_j + 2kd + 2l_j d + 6d \\
&\overset{j^* = j}{=} 2p_{j^*} + 2kd + 2l_j d + 6d
\end{aligned}
$$

$$
\begin{aligned}
&\overset{(20)}{\leq} && 2p_{j^*} + l(Q_3| > j^*) + l(P_1) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 2l_j d + 6d \\
&\overset{(32)}{\leq} && 2p_{j^*} + l(Q_3| > j^*) + l(P_1) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + l(P_3) \\
& && +l(Q_3| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{j^*,j\in Q_3;j^*=j}{\leq} && l(P_3) + l(Q_3) + l(P_1) + p_{j^*} - 4d\lfloor\frac{p_{j^*}}{2d}\rfloor + 6d \\
&\overset{(11)(14)}{<} && l(P_3) + l(Q_3) + l(P_1) + 6d \\
&\overset{(6)}{=} && c - l(P_2) - l(Q_1) - l(Q_2) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

$j^* < j$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(4)(5)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(19),\delta_a\geq 0}{\leq} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + 2l_j d + 6d + \delta(j) \\
&\overset{(28)/(31)}{<} && p_{j^*} + p_j + l(P_a) + l(Q_a| < j^*) + l(Q_a| > j) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{j^*,j\in Q_a;j^*<j}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

$j^* > j$

$$
\begin{aligned}
I(j^*) + II(j) \quad &\overset{(4)(5)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(8)}{<} && p_{j^*} + p_j + 2kd + 2l_j d + 8d \\
&\overset{(20)}{\leq} && p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 2l_j d + 8d \\
&\overset{(29)/(32)}{\leq} && p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + l(P_a) \\
& && +l(Q_a| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 8d \\
&\overset{j^*,j\in Q_a;j^*>j}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor - 2d\lfloor\frac{p_j}{2d}\rfloor + 8d \\
&\overset{(14),p_j>0}{\leq} && l(P_a) + l(Q_a) + l(P_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} && c - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} && c.
\end{aligned}
$$

2.9 $\mathbf{j^* \in Q_a, j \in Q_{a\oplus_3 1}\ (a = 1, 2, 3)}$

2.9.1 Case 2(b)

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(1)(4)}{<} p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2kd + 8d \\
&\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 8d \\
&\overset{j^*\in Q_a, j\in Q_{a\oplus_3 1}}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(Q_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 8d \\
&\overset{(14)}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(Q_{a\oplus_3 1}) + 6d \\
&\overset{(6)}{=} c - l(P_a) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.9.2 Case 2(c)

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(28)/(31)}{<} p_{j^*} + p_j + l(Q_{a\oplus_3 1}| > j) + l(P_{a\oplus_3 2}) + 6d \\
&\overset{j^*\in Q_a, j\in Q_{a\oplus_3 1}}{\leq} l(Q_a) + l(Q_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) + 6d \\
&\overset{(6)}{=} c - l(P_a) - l(P_{a\oplus_3 1}) - l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.9.3 Case 2(d)

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_j d + 6d + \delta(j) \\
&\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 2l_j d + \\
&\qquad 6d + \delta(j) \\
&\overset{(28)/(31)}{<} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + \\
&\qquad l(Q_{a\oplus_3 1}| > j) + l(P_{a\oplus_3 2}) + 6d \\
&\overset{j^*\in Q_a, j\in Q_{a\oplus_3 1}}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(Q_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 6d \\
&\overset{(14)}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(Q_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) + 4d \\
&\overset{(6)}{=} c - l(P_a) - l(Q_{a\oplus_3 2}) + 4d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.10 $j^* \in Q_a, j \in Q_{a\oplus_3 2}$ $(a = 1, 2, 3)$

2.10.1 Case 2(b)

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(1)(4)}{<} p_{j^*} + p_j + 2kd + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2kd + 8d \\
&\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 8d \\
&\overset{j^*\in Q_a, j\in Q_{a\oplus_3 2}}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(Q_{a\oplus_3 2}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 8d \\
&\overset{(14)}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(6)}{=} c - l(P_a) - l(P_{a\oplus_3 2}) - l(Q_{a\oplus_3 1}) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

2.10.2 Case 2(c)

$a = 1$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(7)}{=} p_{j^*} + p_j + 2l_j d + 6d \\
&\overset{(32)}{\leq} p_{j^*} + p_j + l(P_3) + l(Q_3| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{j^*\in Q_1, j\in Q_3}{\leq} l(Q_1) + l(P_3) + l(Q_3) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{p_j>0}{\leq} l(Q_1) + l(P_3) + l(Q_3) + 6d \\
&\overset{(6)}{=} c - l(P_1) - l(P_2) - l(Q_2) + 6d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}
$$

$a = 2, 3$

$$
\begin{aligned}
I(j^*) + II(j) &\overset{(2)(5)}{<} p_{j^*} + p_j + 2l_j d + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2l_j d + 8d \\
&\overset{(29)}{\leq} p_{j^*} + p_j + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}| < j) - 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 8d \\
&\overset{j^*\in Q_a, j\in Q_{a\oplus_3 2}}{\leq} l(Q_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{p_j>0}{\leq} l(Q_a) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + 6d \\
&\overset{(6)}{=} c - l(P_a) - l(P_{a\oplus_3 1}) - l(Q_{a\oplus_3 1}) + 6d
\end{aligned}
$$

$$\overset{(9)(10)}{\leq} \quad c.$$

2.10.3 Case 2(d)

$a = 1$

$$\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_jd + 6d + \delta(j) \\
&\overset{(7)}{=} p_{j^*} + p_j + 2kd + 2l_jd + 6d \\
&\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_1| > j^*) + l(P_2) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + 2l_jd + 6d \\
&\overset{(32)}{\leq} p_{j^*} + p_j + l(Q_1| > j^*) + l(P_2) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + l(P_3) + \\
&\quad l(Q_3| < j) - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{j^*\in Q_1, j\in Q_3}{\leq} l(Q_1) + l(P_2) + l(P_3) + l(Q_3) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{(14), p_j>0}{\leq} l(Q_1) + l(P_2) + l(P_3) + l(Q_3) + 4d \\
&\overset{(6)}{=} c - l(P_1) - l(Q_2) + 4d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}$$

$a = 2, 3$

$$\begin{aligned}
I(j^*) + II(j) &\overset{(4)(5)}{<} p_{j^*} + p_j + 2kd + 2l_jd + 6d + \delta(j) \\
&\overset{(8)}{<} p_{j^*} + p_j + 2kd + 2l_jd + 8d \\
&\overset{(20)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor + \\
&\quad 2l_jd + 8d \\
&\overset{(29)}{\leq} p_{j^*} + p_j + l(Q_a| > j^*) + l(P_{a\oplus_3 1}) - 2d\lfloor\frac{p_{j^*}}{2d}\rfloor \\
&\quad + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}| < j) - 2d(\lfloor\frac{p_j}{2d}\rfloor + 1) + 8d \\
&\overset{j^*\in Q_a, j\in Q_{a\oplus_3 2}}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) - \\
&\quad 2d\lfloor\frac{p_{j^*}}{2d}\rfloor - 2d\lfloor\frac{p_j}{2d}\rfloor + 6d \\
&\overset{(14), p_j>0}{\leq} l(Q_a) + l(P_{a\oplus_3 1}) + l(P_{a\oplus_3 2}) + l(Q_{a\oplus_3 2}) + 4d \\
&\overset{(6)}{=} c - l(P_a) - l(Q_{a\oplus_3 1}) + 4d \\
&\overset{(9)(10)}{\leq} c.
\end{aligned}$$

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