FAA-72-31

REPORT NO. FAA-RD-73-30



# A SYSTEM OF SIXTEEN SYNCHRONOUS SATELLITES FOR WORLDWIDE NAVIGATION AND SURVEILLANCE

JOHN J. MORRISON

MARCH 1973
INTERIM REPORT

Prepared for
DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
Systems Research and Development Service
Washington, D.C. 20591

TECHNICAL REPORT STANDARD TITLE PAGE

| | | |
|---|---|---|
| 1. Report No.<br>FAA-RD-73-30 | 2. Government Accession No. | 3. Recipient's Catalog No. |
| 4. Title and Subtitle<br>A SYSTEM OF SIXTEEN SYNCHRONOUS SATELLITES FOR WORLDWIDE NAVIGATION AND SURVEILLANCE | | 5. Report Date<br>March 1973<br>6. Performing Organization Code |
| 7. Author(s)<br>John J. Morrison | | 8. Performing Organization Report No.<br>DOT-TSC-FAA-72-31 |
| 9. Performing Organization Name and Address<br>Department of Transportation<br>Transportation Systems Center<br>Kendall Square<br>Cambridge Ma. 02142 | | 10. Work Unit No.<br>R3112<br>11. Contract or Grant No.<br>FA311 |
| 12. Sponsoring Agency Name and Address<br>Department, of Transportation<br>Federal Aviation Administration<br>Systems Research and Development Service<br>Washington D.C. 20591 | | 13. Type of Report and Period Covered<br>July 1970 - June 1971<br>Interim Report<br>14. Sponsoring Agency Code |
| 15. Supplementary Notes | | |

16. Abstract

This report considers the orbital mechanics aspects of a system of satellites to be used for position determination of any point on or near the surface of the earth. Only satellites having a period of twenty-four hours are examined. No perturbing forces are taken into account. Three and four satellites are required to be visible at twenty and ten degrees elevation angles respectively. A system of sixteen satellites is described which has the required properties.

| | |
|---|---|
| 17. Key Words Communication<br>Navigation<br>Surveillance<br>Satellite coverage<br>Satellite constellations<br>Global coverage, Icosahedron | 18. Distribution Statement<br>DOCUMENT IS AVAILABLE TO THE PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22151. |

| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of Pages | 22. Price |
|---|---|---|---|
| Unclassified | Unclassified | 62 | |

Form DOT F 1700.7 (8-69)

# PREFACE

The work described in this report was performed in the context of an overall program at the Transportation Systems Center, supporting system design and development, avionics design, and test evaluation programs that influence the overall design of an aeronautical satellite system for an enroute-over-ocean-traffic, advanced air traffic control system. This program is sponsored by the Department of Transportation through the Federal Aviation Administration, Systems Research and Development Service. The program supports government activities designed to promote air traffic safety through improving the integrity of the air traffic control system.

In 1971 an orbital constellation was identified which would allow with a minimum of satellites, full worldwide coverage with either three satellites above 20° elevation angle or four satellites above 10°. A preliminary review of inclined elliptical orbits for regional coverage was completed.

## TABLE OF CONTENTS

## LIST OF ILLUSTRATIONS

## LIST OF ILLUSTRATIONS (CONT'D)

## LIST OF ILLUSTRATIONS (CONT'D)

## LIST OF TABLES

# 1.0 INTRODUCTION

There is a continuing interest in the utilization of a system of synchronous satellites to serve the communication, surveillance and/or navigation functions. Limited systems have been in operation for a number of years, and prospects appear to be good for expanded systems with commercial applications. As far as extent of coverage is concerned, a final goal is a global system facilitating the identification and location, by a ground-based, centralized network, of all mobile elements of the marine and aviation world community.

The problems involved in implementing such a system are numerous, encompassing technical, economic, political and even ecological considerations. One aspect of the total problem that is susceptible to a certain degree of isolation is what may be called the orbital mechanics problem. Stated briefly, the problem is to determine how many satellites are required and what their orbital elements must be in order for the system to be capable of performing a given mission in communication, surveillance or navigation. This problem has received considerable attention in the case where coverage of the earth excludes high northern and southern latitudes [1,2]. Information on systems that provide global coverage, however, is rather scarce.

It is of great interest to solve the problem of minimum number of satellites if certain constraints are imposed. One constraint is the visibility from any point on the surface of the earth of a certain number of satellites at all times, as is required for navigation purposes. Another constraint concerns the lower limit of antenna elevation above the local horizontal.

In this report a beginning has been made toward the minimum problem by deriving a lower bound on the number of synchronous satellites required for global coverage if three satellites are required to be visible at an elevation angle greater than 20 degrees. (Section 3).

In Section 4 the effects of variations of orbital elements on an orbit and its ground tracks are discussed.

In Section 5 a system of sixteen synchronous satellites is developed that yields visibility of three satellites for 20° elevation angle and of four satellites for 10° elevation angle.

The study is based on the model of Keplerian motion, only. It is essential that the system characteristics be understood first on this simplified model so that the effects of perturbing forces may be properly interpreted and evaluated. These effects on the proposed system will be the subject of a continuing investigation.

# 2.0 SPECIFICATION OF THE PROBLEM

## 2.1 DEFINITION

A satellite will be said to be visible from a point on the surface of the earth if there exists a straight line segment connecting that point with the satellite such that the line segment does not intersect the earth and makes an angle greater than some given angle with the local tangent plane.

In this report, the given angle, called the elevation angle, will be either 10 or 20 degrees. Local features of the terrain, e.g. mountains, are neglected, and the earth is assumed to be spherical. If a satellite is visible, it will be assumed that electromagnetic transmission is possible. The problems connected with this transmission, such as antenna design, power requirements, receiver and transmitter design, etc., will not be considered.

## 2.2 STATEMENT OF THE PROBLEM

The problem is to find the minimum number of satellites, each having a 24-hour period, and each in Keplerian orbit about the earth, such that from any point on the surface of the earth and at any time it is possible to have simultaneous visibility of (1) at least three satellites with an elevation angle of 20 degrees and (2) at least four satellites with an elevation angle of 10 degrees.

The choice of elevation angle relates to transmission difficulties. The requirement on the number of visible satellites is principally concerned with the technique used for purposes of navigation, surveillance or communication. The 24-hour period limitation appears to be a little more arbitrary, but seems to be a reasonable choice considering the number of satellites involved, the launch capabilities and cost, the station-keeping, the transmission problems, etc.

To avoid any possible misconceptions, it is pointed out, first, that the condition, "any point on the surface of the earth", includes both polar regions; and, second, that the phrase, "global coverage", as used in the present context means coverage of the total surface of the earth. From a practical point of view, to require global coverage is a luxury. However, it permits the statement of a well-defined problem. To except regions from coverage introduces an element of arbitrariness and makes the comparison of competing systems difficult. It is hoped that the system to be described in this report may serve as a benchmark against which other similar systems may be measured.

## 3.0 SOME THEORETICAL CONSIDERATIONS REGARDING A LOWER BOUND

Regarding the minimum number of satellites required to solve the problem, there appears to be very little that one can say from an analytical point of view. However, there are some general observations that may aid in grasping the complexity of the problem.

One satellite at an infinite distance from the earth is theoretically visible on a hemisphere of the earth if a zero-elevation angle is permitted. Now, suppose the satellite is positioned above the North-Pole. Then, as the satellite's distance from the earth is reduced, the visibility area on the earth is outlined by latitude circles. Increasing the elevation angle from zero to 20 degrees causes the latitude of these bounding circles to increase. With the satellite at a distance from the center of the earth as required for a synchronous, circular orbit (approximately 6.623 earth radii), let a point, P, be selected on the latitude circle bounding the area of visibility determined by a 20-degree minimum elevation angle. The three points: center of the earth, the point P, and the satellite, determine a triangle (See Fig. 1). The known quantities in the triangle are: the distance from the earth to the observation point, P (one earth radius), and the angle (GAMMA) at P: 90 + 20 degrees. The central angle at the earth (ALPHA) and the angle at the satellite (BETA) can then be found.

Elevation Angle 20°: GAMMA = 110°: ALPHA = 61.8° BETA = 8.2°
Elevation Angle 10°: GAMMA = 100°: ALPHA = 71.4° BETA = 8.6°

The angle BETA plays a role in determining the beam width of an antenna on the satellite.

A useful and interesting lower-bound on the problem can be obtained by positioning satellites at the 12 vertices of an icosahedron enclosed in an imaginary sphere having a radius of 6.623 earth radii. Let such an icosahedron (a twenty-sided solid with each side an equilateral triangle) be oriented so that one

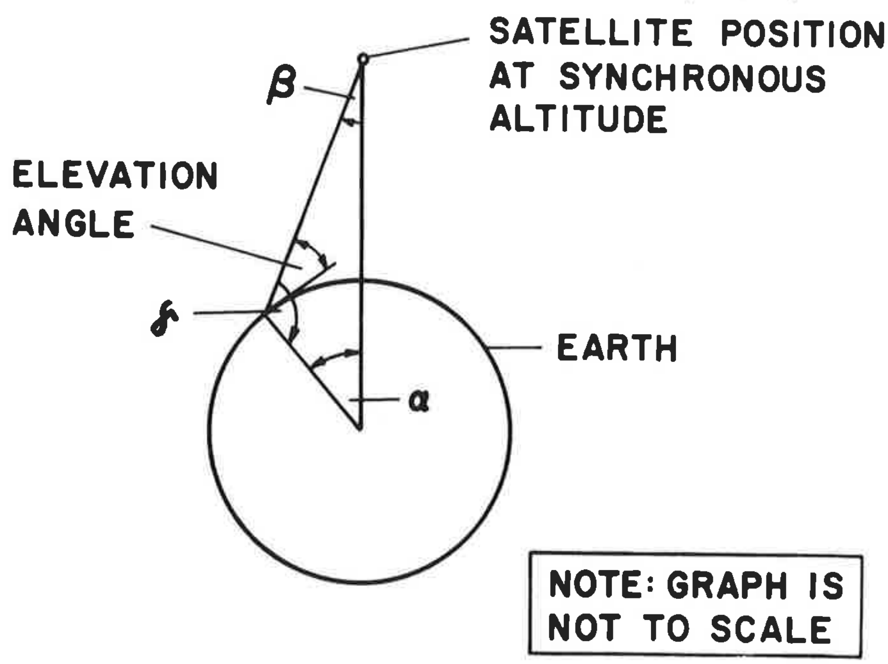


Figure 1. Angular Relationships

satellite is positioned directly above the North-Pole. Then, that satellite-point is common to five triangles with five satellites at the remaining five points of the five triangles. One may now ask the question: Can an observer standing on the North-Pole see these five satellites at an elevation angle greater than 20 degrees? He can, of course, see the one directly above him. The central angle between the two lines connecting the center of the icosahedron to any two adjacent vertices can be found and is analogous to ALPHA in the figure above. The central angle turns out to be greater than 63 degrees. Consequently, at the North-Pole, only one satellite is visible with an elevation angle greater than 20 degrees. There is, therefore, by analogy, a small circle about each subsatellite-point from which only one satellite can be seen. Since an icosahedron provides a uniform distribution of its vertices, there can be no improvement in the coverage by any other arrangement of 12 satellites, in circular, synchronous orbits. Therefore, one may conclude that, in order for three satellites to be simultaneously visible at an elevation angle greater than 20 degrees, it is necessary that there be at least 13 satellites.

It is obvious that it is impossible to choose orbital elements so that 12 satellites will maintain the type of spacing required by an icosahedron. Some effort has been expended in attempts to approximate such a distribution (using eccentric orbits), but with no success.

We do, however, have a useful lower bound on the problem: At least 13 satellites are required if each is to move in a circular synchronous orbit. Further, it will be shown that 16 satellites is an upper bound on the minimum. In one case, a 15 satellite system has come extremely close to satisfying the conditions. Perhaps, with some slight modification, it might become a candidate. On the other hand, it is obviously marginal and therefore would appear to be of no practical significance.

# 4.0 DISCUSSION OF THE EFFECTS OF VARIATIONS OF ORBITAL ELEMENTS ON AN ORBIT AND ITS GROUND-TRACK

Restricting satellite motion to orbits with 24-hour period fixes the value of the semi-major axis. The remaining elements i.e., inclination, eccentricity, argument of perigee, right ascension of the ascending node and the timing element, leave a wide range of choices. An examination of effects produced by varying these parameters, especially if viewed in relation to a rotating earth, should prove helpful.

In the remainder of the report the visibility of satellites on the surface of the earth is shown by a field of numbers, spread over a grid of longitude and latitude coordinates. The number at a gridpoint represents the number of satellites visible at that point at a reference time. This representation is called here a visibility map.

The fact that such representation utilizes finite intervals in space and time entails certain uncertainties as to continuity. In most cases, however, intervals are sufficiently small that the assumption of continuity is reasonably justified. The uncertainty remains for the extent to which numbers at the border of an area are representative for a region beyond its very point of computation.

## 4.1 STATIONARY SATELLITES

The first type orbit that comes to mind is the one in which a satellite is stationary above a given point on the equator, i.e., an orbit with zero-eccentricity (circular) and zero-inclination (equatorial). The visibility maps of a stationary satellite are shown in Figure 2 representing the area for an elevation angle of 20 degrees and Figure 3 representing that for 10 degrees. The satellite is assumed to be stationed at longitude zero. Since a stationary satellite does not alter its position relative to the earth in time, there is no need to specify the time at which the visibility maps are valid.

MAP 1: ELEVATION ANGLE = 20 DEGREES

| LAT | LONG | -90 | | 0 | | 90 | | 180 | |
|---|---|---|---|---|---|---|---|---|---|
| 90 | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | 90 |
| 80 | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | 80 |
| 70 | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | 70 |
| 60 | ØØØØØØØØ | Ø | ØØØØØØØ1 | 1 | 1ØØØØØØØ | Ø | ØØØØØØØØ | Ø | 60 |
| 50 | ØØØØØØØØ | Ø | ØØØØ1111 | 1 | 1111ØØØØ | Ø | ØØØØØØØØ | Ø | 50 |
| 40 | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | 40 |
| 30 | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | 30 |
| 20 | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | 20 |
| 10 | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | 10 |
| Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | Ø |
| -10 | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | -10 |
| -20 | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | -20 |
| -30 | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | -30 |
| -40 | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | -40 |
| -50 | ØØØØØØØØ | Ø | ØØØØ1111 | 1 | 1111ØØØØ | Ø | ØØØØØØØØ | Ø | -50 |
| -60 | ØØØØØØØØ | Ø | ØØØØØØØ1 | 1 | 1ØØØØØØØ | Ø | ØØØØØØØØ | Ø | -60 |
| -70 | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | -70 |
| -80 | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | -80 |
| -90 | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | -90 |

Figure 2. Visibility Map for a Stationary Satellite

MAP 2: ELEVATION ANGLE = 1Ø DEGREES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 9Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | 9Ø |
| 8Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | 8Ø |
| 7Ø | ØØØØØØØØ | Ø | ØØØØØØ11 | 1 | 11ØØØØØØ | Ø | ØØØØØØØØ | Ø | 7Ø |
| 6Ø | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | 6Ø |
| 5Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | 5Ø |
| 4Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | 4Ø |
| 3Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | 3Ø |
| 2Ø | ØØØØØØØØ | Ø | Ø1111111 | 1 | 1111111Ø | Ø | ØØØØØØØØ | Ø | 2Ø |
| 1Ø | ØØØØØØØØ | Ø | Ø1111111 | 1 | 1111111Ø | Ø | ØØØØØØØØ | Ø | 1Ø |
| Ø | ØØØØØØØØ | Ø | Ø1111111 | 1 | 1111111Ø | Ø | ØØØØØØØØ | Ø | Ø |
| -1Ø | ØØØØØØØØ | Ø | Ø1111111 | 1 | 1111111Ø | Ø | ØØØØØØØØ | Ø | -1Ø |
| -2Ø | ØØØØØØØØ | Ø | Ø1111111 | 1 | 1111111Ø | Ø | ØØØØØØØØ | Ø | -2Ø |
| -3Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | -3Ø |
| -4Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | -4Ø |
| -5Ø | ØØØØØØØØ | Ø | ØØ111111 | 1 | 111111ØØ | Ø | ØØØØØØØØ | Ø | -5Ø |
| -6Ø | ØØØØØØØØ | Ø | ØØØ11111 | 1 | 11111ØØØ | Ø | ØØØØØØØØ | Ø | -6Ø |
| -7Ø | ØØØØØØØØ | Ø | ØØØØØØ11 | 1 | 11ØØØØØØ | Ø | ØØØØØØØØ | Ø | -7Ø |
| -8Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | -8Ø |
| -9Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | ØØØØØØØØ | Ø | -9Ø |

Figure 3. Visibility Map for a Stationary Satellite

A stationary satellite offers many simplifications from the point of view of ground stations, station-keeping, and prediction of the motion of the satellite. However, stationary satellites alone cannot give global coverage because such satellites are not visible above 71.4 degrees latitude for 10 degree elevation angles or 61.8 degrees for 20 degrees elevation angles. Stationary orbits, however, in combination with other types of orbits, must be kept in mind as possible candidates.

4.2 CIRCULAR, INCLINED ORBITS

Due to the rotation of the earth, circular, inclined orbits produce ground tracks (i.e., the track of the subsatellite point laid down on the surface of the earth by the satellite) that are in the shape of a figure eight. Increasing the inclination produces figure eights with larger and larger loops. (Typical examples are presented Figure 4). As the loops of the figure eight attain higher latitudes, they become wider in longitude at higher latitudes.

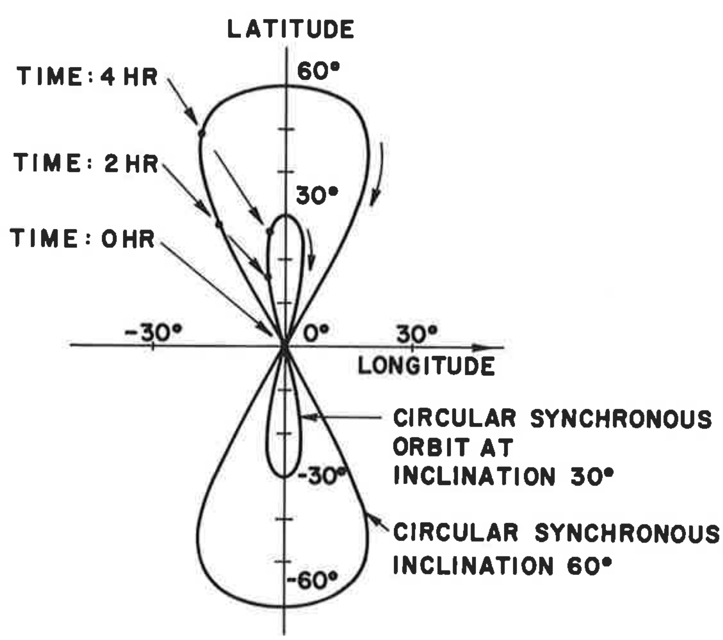


Figure 4. Ground Tracks of Two Inclined Circular Synchronous Orbits

Inclinations of satellite orbits affect the visibility problem in two ways: first, when a satellite, in an inclined orbit, is at its most northerly excursion, its area of visibility from the ground in the Southern Hemisphere is decreased. Thus, in designing a system for global coverage, such reductions in coverage must be compensated for by introducing other satellites. Second, under the same conditions, coverage at the equator is reduced. Consequently, satellites must be more closely spaced around the equator to ensure coverage in the lower latitudes than would be required by stationary satellites.

## 4.3 ECCENTRIC ORBITS

Introduction of positive eccentricities produces a perigee and apogee in the orbit and, therefore, a well-defined argument of perigee. This latter element locates the perigee in the plane of the motion with respect to the ascending node. Positive eccentricity has two interesting effects. A satellite in an eccentric orbit is closer to the earth at perigee (and further away at apogee) than in a corresponding circular orbit. Secondly, the amount of time a satellite spends in the neighborhood of perigee is less than the amount of time it spends in a corresponding neighborhood of apogee. As a result, at apogee the area of visibility is greater and the satellite stays for a longer time in this region than is the case for the satellite in circular orbit. One can then manipulate the argument of perigee so that the satellite is at perigee in some area where coverage is not so important.

From an engineering point of view, there are two kind of disadvantages that must be taken into account in designing the operational capabilities of a satellite. Non-uniformity in the rate of traverse of the orbit complicates the antenna-pointing problems and variable distance from the earth implies antenna beamwidth modifications.

## 4.4 COMBINING VARIATIONS OF INCLINATION AND ECCENTRICITY

This produces some interesting results. Introducing positive eccentricity in an orbit with zero inclination brings about a longitudinal oscillation in position of the satellite along with variation in distance from the earth. These effects do not appear to be particularly useful for coverage purposes. For positive inclinations, however, variations in eccentricity lead to modifications of the figure eight ground-track. Permitting the eccentricity to take on values from 0 to 1 and locating the perigee at the southern extremity of the orbit, causes the intersection point of the figure eight to move northward along a meridian, increasing the size of the lower loop and decreasing the size of the upper loop (See Figure 5). At some value of the eccentricity, the upper loop disappears and the remaining loop tends to spread out along the equator. For another value of the eccentricity, the remaining loop is nearly circular. Different measures of approximation to circularity can be devised. One such definition is the following. In this report, a ground-track will be called nearly circular if the angle from the meridian, on which the subsatellite

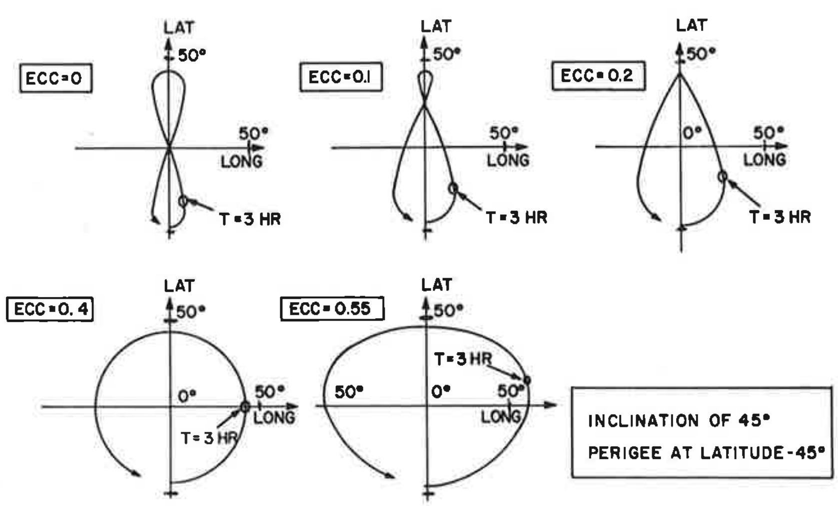


Figure 5. Ground Tracks of Synchronous Orbits of Varied Eccentricity

point reaches its maximum latitude, to the meridian, at which the equatorial crossing occurs, equals the inclination. Table 1 lists the eccentricity for each degree of inclination from zero to 89 which yields a nearly circular ground track.

If there are three of four satellites equally spaced in time, traversing such a nearly circular ground-track, and if there is a stationary satellite at the center of the circle, one has the so called Y-configuration or X-configuration. Such satellite systems appear to be especially suitable for deriving accurate results from the navigation equations. It may also be noted that the satellites move from east to west in the northern hemisphere, whereas the reverse is the case for orbits with zero-eccentricity.

It has been our experience, based on numerical studies, that while good coverage can be obtained using eccentric orbits, the solutions are valid for only very narrow ranges of inclination and eccentricity. As long as other solutions, less sensitive to these parameters, can be found, eccentric orbits are not recommended. In addition to the reasons already mentioned, the perigee of an eccentric orbit is subject to comparatively large variations due to the triaxiality of the earth. (Acceptable coverage was not found at 63.4 degrees inclination for a selection of values of eccentricity). However, for special cases, such as coverage restricted to a specific area or for communication links via a "hand-over" link, e.g., over the North-Pole, eccentric orbits may still prove useful.

TABLE 1. INCLINATION AND ECCENTRICITY FOR NEARLY CIRCULAR GROUND TRACKS

| INCLINATION | ECCENTRICITY | INCLINATION | ECCENTRICITY |
|---|---|---|---|
| 0.00 | 0.00000000 | 45.00 | 0.40397276 |
| 1.00 | 0.00872677 | 46.00 | 0.41353475 |
| 2.00 | 0.01745421 | 47.00 | 0.42314255 |
| 3.00 | 0.02618291 | 48.00 | 0.43279820 |
| 4.00 | 0.03491367 | 49.00 | 0.44250376 |
| 5.00 | 0.04364727 | 50.00 | 0.45226143 |
| 6.00 | 0.05238385 | 51.00 | 0.46207353 |
| 7.00 | 0.06112461 | 52.00 | 0.47194255 |
| 8.00 | 0.06987007 | 53.00 | 0.48187107 |
| 9.00 | 0.07862099 | 54.00 | 0.49186184 |
| 10.00 | 0.08737779 | 55.00 | 0.50191782 |
| 11.00 | 0.09614143 | 56.00 | 0.51204211 |
| 12.00 | 0.10491254 | 57.00 | 0.52223805 |
| 13.00 | 0.11369181 | 58.00 | 0.53250921 |
| 14.00 | 0.12247997 | 59.00 | 0.54285942 |
| 15.00 | 0.13127773 | 60.00 | 0.55329272 |
| 16.00 | 0.14008589 | 61.00 | 0.56381359 |
| 17.00 | 0.14890512 | 62.00 | 0.57442678 |
| 18.00 | 0.15773619 | 63.00 | 0.58513743 |
| 19.00 | 0.16657992 | 64.00 | 0.59595117 |
| 20.00 | 0.17543708 | 65.00 | 0.60687411 |
| 21.00 | 0.18430844 | 66.00 | 0.61791289 |
| 22.00 | 0.19319486 | 67.00 | 0.62907491 |
| 23.00 | 0.20209715 | 68.00 | 0.64036820 |
| 24.00 | 0.21101615 | 69.00 | 0.65180169 |
| 25.00 | 0.21995278 | 70.00 | 0.66338532 |
| 26.00 | 0.22890791 | 71.00 | 0.67513020 |
| 27.00 | 0.23788244 | 72.00 | 0.68704885 |
| 28.00 | 0.24687733 | 73.00 | 0.69915536 |
| 29.00 | 0.25589354 | 74.00 | 0.71146580 |
| 30.00 | 0.26493208 | 75.00 | 0.72399867 |
| 31.00 | 0.27399399 | 76.00 | 0.73677538 |
| 32.00 | 0.28308023 | 77.00 | 0.74982092 |
| 33.00 | 0.29219198 | 78.00 | 0.76316495 |
| 34.00 | 0.30133035 | 79.00 | 0.77684287 |
| 35.00 | 0.31049643 | 80.00 | 0.79089792 |
| 36.00 | 0.31969151 | 81.00 | 0.80538365 |
| 37.00 | 0.32891681 | 82.00 | 0.82036785 |
| 38.00 | 0.33817359 | 83.00 | 0.83593878 |
| 39.00 | 0.34746321 | 84.00 | 0.85221528 |
| 40.00 | 0.35678706 | 85.00 | 0.86936443 |
| 41.00 | 0.36614659 | 86.00 | 0.88763575 |
| 42.00 | 0.37554331 | 87.00 | 0.90743524 |
| 43.00 | 0.38497880 | 88.00 | 0.92951999 |
| 44.00 | 0.39445473 | 89.00 | 0.95571879 |

## 4.5 LONGITUDE OF THE ASCENDING NODE AND EPOCH

The principal effect on the ground-track of varying the inclination is to cause a variation in the maximum latitude. To position the ground-track of an inclined orbit longitudinally, as over a specified region of the earth, use is made of the orbital element called the longitude of the node.

The longitude of a node is one of the two angles that exist in an inertial coordinate system between the intersection of the orbital plane with the equatorial plane and an inertial reference axis in the equatorial plane (X-axis of an inertial rectangular coordinate system). The longitude of the ascending node (here called "RA") then is the angle between the positive X-axis and that half of the intersection at which the satellite crosses from the southern into the northern hemisphere. By varying this angle, it is possible to rotate the plane of the motion through 360 degrees about the earth.

In order to be able to coordinate, time-wise, the positions of two or more satellites, it is necessary to know where the satellites are in their respective orbits at some given instant of time, called epoch. Then, on the basis of this information, along with a knowledge of the other orbital elements, it is just a matter of calculation to determine where the satellites will be at any other instant of time. It is clear that, in order to complete the description of the motion of a satellite, we need two more quantities, namely epoch and a parameter that locates the satellite in its orbit.

It is customary in celestial mechanics to identify the position of a satellite in its orbit by referring to its angular advance from its perigee point or an equivalent time interval. Since in this chapter only circular orbits with positive inclination are considered, the perigee reference will be substituted for by the ascending node.

The angle between the position of the satellite and the ascending node of its orbit will be measured then in terms of hours with the understanding that in synchronous circular orbits one hour corresponds to 15 degrees.

At "epoch", i.e., at a fixed initial time, this angle will be designated "PTO".

For synchronous circular orbits of given inclination it is then obvious that the two quantities RA and PTO define the position of a satellite at epoch. The position at any other time is then a matter of computation.

Next, the relationship between the inertial coordinates and the ground-track on the rotating earth may be discussed. As previously mentioned, a circular inclined orbit produces a figure eight ground-track on the earth. To place a satellite in such an orbit so that its ground-track lies on a specified area of the earth requires specific values for both RA and PTO. A few examples should make this clear.

Suppose that the zero-meridian of the earth at time TO intersects the positive X-axis of an earth-centered inertial coordinate system and that the equatorial plane and the X, Y inertial plane coincide.

A satellite with PTO=0 (i.e., its nodal passage occurs at epoch) and RA=0 (i.e., the intersection of the orbital and equatorial planes coincides with the X-axis) will produce a figure eight ground-track centered at zero-longitude on the earth. An identical ground-track results if PTO=TO+1 hours and RA=15 degs. In this case, the second satellite at epoch is already one hour past its nodal crossing. Therefore, to find the subsatellite-point at the time of the nodal crossing - the center of the figure eight - the earth must be rotated backwards through 15 degrees. We then find that zero-longitude on the earth lines up with the longitude of the node. This second satellite is one hour ahead of the first with both generating the same ground-track.

In general, if C is the longitude of the center of a figure eight ground-track at TO, a satellite with orbital elements satisfying the condition

RA=C-15*PTO

will have a figure eight ground-track centered at C.

If, on the other hand, a satellite has the conditions; PTO= 0 hours and RA=-15 degrees, a ground-track centered at 15 degrees west longitude results. Again, a satellite with condition: PTO= TO+1 hours and RA-0 degrees, has a ground-track centered at 15 degrees east longitude.

Thus, by the manipulation of these two parameters, one is able to locate a figure eight ground-track with center at any desired longitude on the earth and with the satellite above any given point on the ground-track at a given time.

Finally, it must be kept in mind that the satellite traces out its ground-track in time. This implies that, while a given configuration of satellites may produce good coverage at one time, it may be unsatisfactory at another. Therefore, it is not only necessary to select values for the six orbital elements, but also to investigate the time-history of a satellite system over an entire 24-hour period. It is sometimes possible to shorten this time-interval due to a repetition of the initial satellite configuration in less than 24 hours.

## 5.0 A SYSTEM OF SIXTEEN SATELLITES

We shall present in this section the orbital elements and visibility maps of the satellite system that we have found best to meet the conditions of the problem.

The search for an acceptable solution was carried out over the following ranges. Fourteen-, fifteen- and sixteen-satellite systems were considered with principal emphasis on the last. Inclination was allowed to vary from zero to 180 degrees, at 5-degrees intervals in the less interesting regions, to 0.1 degree intervals in some cases. Inclinations greater than ninety degrees produce retrograde orbits with the accompanying increase in fuel requirements for launching and, perhaps, should be ruled out for that reason. Systems based on orbits with 120 degrees inclination, however, do have good coverage properties, but none was found producing complete coverage.

In the cases where variation of eccentricity from zero was tested, the limits were zero to 0.6. Except for special configurations, no clear-cut rationale was established for testing positive eccentricities.

All reasonable combinations of RA and PTO were tried in the interesting regions of inclination and zero eccentricity. The criterion of "reasonable" here means a distribution of orbital planes and initial positions that would not lead to some portion of the globe or time-interval being favored over another. Thus, for example, one would not consider cases of the type: all satellites in the same plane or all satellites initially at their equatorial crossings.

The filter has yielded only one region where satisfactory solutions exist. The system presented below is, more or less, at the center of this region.

The proposed system consists of 16 satellites. All satellites have the following orbital elements in common: 24-hour period, zero-eccentricity and 60-degree inclination. Table 2 lists the remaining data.

The satellites are numbered, from 1 to 16. The center of the figure eight ground-track for each is given in degrees of meridianal longitude (earth fixed). Finally, the longitude of the ascending node (degrees of longitude in an inertial coordinate system) and position (in terms of hours since last crossing of the ascending node) at epoch are listed.

TABLE 2: SIXTEEN SATELLITE SYSTEM

| Longitude of Center (Deg) | No. | RA (Deg) | PTO (Hr) | No. | RA (Deg) | PTO (Hr) |
|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 9 | 180 | 12 |
| 135 | 2 | 112.5 | 1.5 | 10 | 292.5 | 13.5 |
| 270 | 3 | 225 | 3 | 11 | 45 | 15 |
| 45 | 4 | 337.5 | 4.5 | 12 | 157.5 | 16.5 |
| 180 | 5 | 90 | 6 | 13 | 270 | 18 |
| 315 | 6 | 202.5 | 7.5 | 14 | 22.5 | 19.5 |
| 90 | 7 | 315 | 9 | 15 | 135 | 21 |
| 225 | 8 | 67.5 | 10.5 | 16 | 247.5 | 22.5 |

In this table the columns labeled "RA" show that no two satellite planes are identical. Also, one may observe that each two satellites, shown side by side in the table, differ in right ascension by 180 degrees and in their positions by twelve hours. This implies that both members of a pair produce identical ground-tracks. There are eight such ground-tracks, spaced at 45-degree intervals about the equator.

An examination of Table 2 will make it evident that 1.5 hours after epoch the satellite system returns to its initial configuration. To clarify this point, one may note that the "center" column of Table 2 indicates that the centers of the ground-tracks make six cycles around the earth, by steps of 135 degrees. Figure 6 is a plot of the 16 subsatellite points, spread over the six cycles.

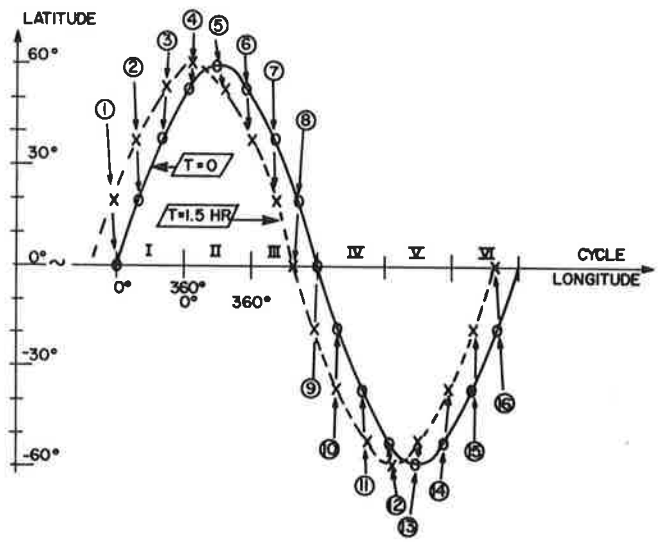


Figure 6. Sixteen Satellites Plotted Over Repeated Longitude Scale for Times T=0 Hr. and T=1.5 Hr.

These points are connected, producing a sinusoidal curve. In a simular manner the 16 positions are plotted 1.5 hours after epoch and connected. Except for the numbering of the individual satellites and a shift to the left of 135 degrees, the two sinusoidal curves are identical. The similarities in the shape of the curves and in the distribution of points on each curve indicate the identity of the two configurations. The fact that the two curves do not coincide is a result of a difference in the relation of the two configurations to the earth. This is of no importance, however, as far as global coverage is concerned, that is, if one configuration gives satisfactory coverage, so does the other.

Figure 7 is a diagram of the eight figure eights with the sixteen positions at epoch, two in each ground-track, indicated in their respective positions. Figure 8 is another version of Figure 7. In this instance, however, the figure eights are not drawn in. Instead, the position-points are connected by solid lines, producing an ellipse-like pattern superimposed to an X-type

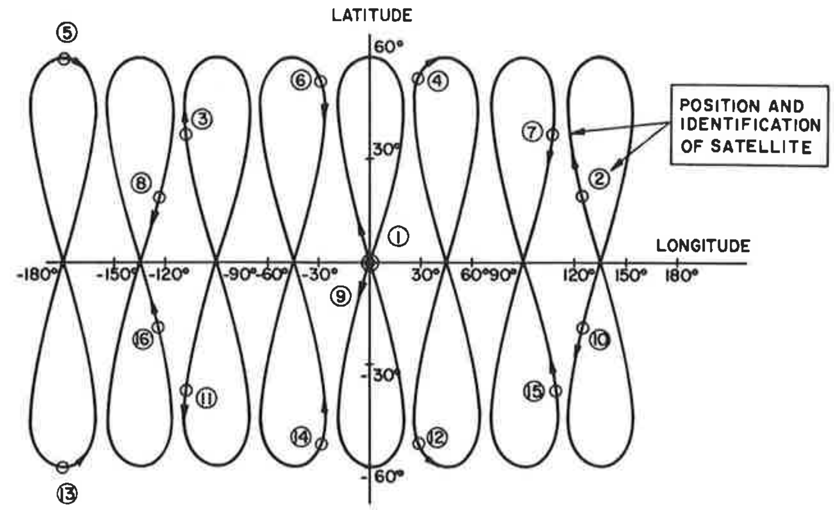


Figure 7. Groundtracks and Zero-Time Positions of the Sixteen Satellites

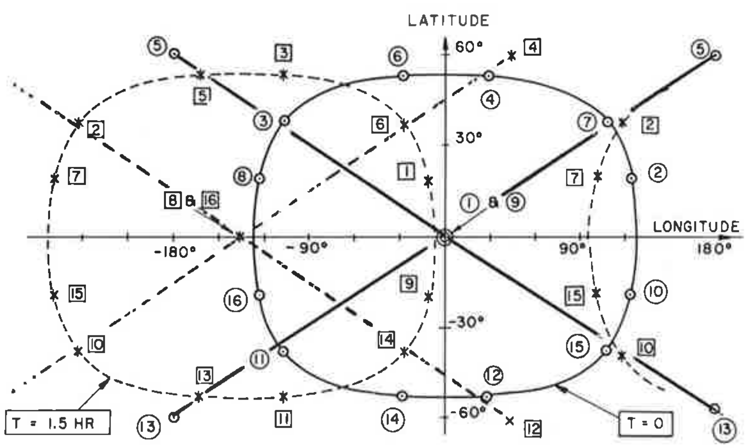


Figure 8. Positions of the Sixteen Satellites at T=0 and T=1.5 Hr.

pattern. The position-points at 1.5 hours are also plotted and similarly connected, by dashed lines. The dashed and solid lines serve merely as an aid to the reader in giving structure to the two sets of points. Their similarity helps to point out the identity between the two configurations. The data for these are contained in Tables 3 to 5 and in Figures 9-20.

The visibility maps at epoch and at 1.5 hours (Figs. 9-14 and 15-20) do not precisely bear out this conclusion. The two maps should be identical, but they are not. The reason is that the grid of observation points is based on a 10-degree increment, while the rotation of 135 degrees is not an exact multiple of 10 degrees. However, the maps are close enough to substantiate the assertion.

Tables 3 and 4 list some pertinent data of a typical satellite. Table 3 lists the rectangular coordinates in an inertial coordinate system. Table 4 gives the ground-track for the same orbital path. Both tables list data for only one quarter of a period.

Table 5 contains a list of the earth-fixed longitude and latitude of each of the 16 satellites over a time-interval of 1.5 hours. The individual satellite is identified by number (No.); the time is listed at increments of 0.30 hours.

The visibility maps (Figures 9-20) then show the number of satellites visible from each of the 614 gridpoints, Figures 9 to 14 referring to the minimum of 20 degrees elevation angle, Figures 15 to 20 referring to the 10 degrees minimum elevation.

The primary points of interest regarding these maps are the following two facts. First, all digits in Figures 9 to 14 are greater than 2, signifying that at least three satellites are simultaneously visible from all the grid-points at all the time-points. Second, all digits in Figures 15 to 20 are greater than 3, proving that at least four satellites are visible from all the grid-points at all the time-points for an antenna elevation angle lowered to 10 degrees. The inference is that the conditions of the problem appear to have been met.

TABLE 3. POSITION VECTOR IN RECTANGULAR COORDINATES
(DIMENSIONS: HOURS AND EARTH RADII)

| TIME | X | Y | Z |
|---|---|---|---|
| Ø,Ø | 6,6227812 | Ø,2ØØØØØØ | Ø,ØØØØØØØ |
| Ø,5 | 6,5661224 | Ø,4322232 | Ø,7486326 |
| 1,Ø | 6,3971154 | Ø,857Ø51Ø | 1,4844559 |
| 1,5 | 6,1186521 | 1,2672143 | 2,1948796 |
| 2,Ø | 5,7354968 | 1,6556953 | 2,8677484 |
| 2,5 | 5,2542Ø57 | 2,Ø158469 | 3,4915492 |
| 3,Ø | 4,683Ø135 | 2,3415Ø67 | 4,Ø556Ø86 |
| 3,5 | 4,Ø316939 | 2,6271Ø28 | 4,55Ø2754 |
| 4,Ø | 3,31139Ø6 | 2,8677484 | 4,967Ø858 |
| 4,5 | 2,5344287 | 3,Ø59326Ø | 5,2989Ø81 |
| 5,Ø | 1,7141Ø18 | 3,1985577 | 5,54ØØ644 |
| 5,5 | Ø,8644465 | 3,283Ø612 | 5,6864288 |
| 6,Ø | Ø,ØØØØØØØ | 3,31139Ø6 | 5,7354967 |

TABLE 4. COORDINATES OF THE GROUND-TRACK
(DIMENSIONS: HOURS AND DEGREES)

| TIME | LONGITUDE | LATITUDE |
|---|---|---|
| Ø,Ø | Ø,ØØØØØØØ | Ø,ØØØØØØØ |
| Ø,5 | -3,7338671 | 6,49Ø53Ø1 |
| 1,Ø | -7,3692596 | 12,952539Ø |
| 1,5 | -1Ø,799Ø8ØØ | 19,354595Ø |
| 2,Ø | -13,897886Ø | 25,65897Ø6Ø |
| 2,5 | -16,51ØØ14Ø | 31,816657Ø |
| 3,Ø | -18,434948Ø | 37,761243Ø |
| 3,5 | -19,411223Ø | 43,398155Ø |
| 4,Ø | -19,1Ø66Ø5Ø | 48,59Ø376Ø |
| 4,5 | -17,139272Ø | 53,139953Ø |
| 5,Ø | -13,186783Ø | 56,774Ø57Ø |
| 5,5 | -7,2514457 | 59,161589Ø |
| 6,Ø | Ø,ØØØØØØØ | 59,999999Ø |

TABLE 5. EARTH-FIXED COORDINATES OF THE 16 SATELLITES

| TIME | NO. | LONGITUDE | LATITUDE | NO. | LONGITUDE | LATITUDE |
|---|---|---|---|---|---|---|
| 0.00 | 1 | 0.00000 | 0.00000 | 9 | 0.00000 | 0.00000 |
| | 2 | 124.20092 | 19.35460 | 10 | 124.20091 | -19.35460 |
| | 3 | -108.43495 | 37.76124 | 11 | -108.43495 | -37.76124 |
| | 4 | 27.86073 | 53.13995 | 12 | 27.86073 | -53.13995 |
| | 5 | 180.00000 | 60.00000 | 13 | 180.00000 | -60.00000 |
| | 6 | -27.86073 | 53.13995 | 14 | -27.86073 | -53.13995 |
| | 7 | 108.43495 | 37.76124 | 15 | 108.43494 | -37.76124 |
| | 8 | -124.20092 | 19.35460 | 16 | -124.20092 | -19.35460 |
| 0.30 | 1 | -2.24652 | 3.89611 | 9 | -2.24653 | -3.89611 |
| | 2 | 122.29279 | 23.15172 | 10 | 122.29277 | -23.15172 |
| | 3 | -109.15424 | 41.18792 | 11 | -109.15424 | -41.18792 |
| | 4 | 29.98260 | 55.45076 | 12 | 29.98261 | -55.45076 |
| | 5 | -175.55484 | 59.69548 | 13 | -175.55485 | -59.69548 |
| | 6 | -26.45940 | 50.50087 | 14 | -26.45940 | -50.50087 |
| | 7 | 107.37556 | 34.22461 | 15 | 107.37556 | -34.22461 |
| | 8 | -126.22764 | 15.52249 | 16 | -126.22764 | -15.52249 |
| 0.60 | 1 | -4.47207 | 7.78616 | 9 | -4.47207 | -7.78616 |
| | 2 | 120.53508 | 26.90400 | 10 | 120.53507 | -26.90400 |
| | 3 | -109.46460 | 44.47751 | 11 | -109.46460 | -44.47751 |
| | 4 | 32.85166 | 57.36223 | 12 | 32.85166 | -57.36223 |
| | 5 | -171.42342 | 58.79988 | 13 | -171.42342 | -58.79988 |
| | 6 | -25.71189 | 47.59598 | 14 | -25.71190 | -47.59598 |
| | 7 | 106.03537 | 30.59970 | 15 | 106.03537 | -30.59971 |
| | 8 | -128.34500 | 11.66386 | 16 | -128.34500 | -11.66386 |
| 0.90 | 1 | -6.65500 | 11.66386 | 9 | -6.65500 | -11.66386 |
| | 2 | 118.96463 | 30.59970 | 10 | 118.96462 | -30.59970 |
| | 3 | -109.28810 | 47.59598 | 11 | -109.28811 | -47.59598 |
| | 4 | 36.42342 | 58.79988 | 12 | 36.42342 | -58.79988 |
| | 5 | -167.85166 | 57.36223 | 13 | -167.85166 | -57.36223 |
| | 6 | -25.53539 | 44.47751 | 14 | -25.53540 | -44.47751 |
| | 7 | 104.46491 | 26.90400 | 15 | 104.46491 | -26.90400 |
| | 8 | -130.52793 | 7.78616 | 16 | -130.52793 | -7.78616 |
| 1.20 | 1 | -8.77236 | 15.52249 | 9 | -8.77236 | -15.52249 |
| | 2 | 117.62444 | 34.22460 | 10 | 117.62443 | -34.22460 |
| | 3 | -108.54060 | 50.50087 | 11 | -108.54060 | -50.50087 |
| | 4 | 40.55484 | 59.69548 | 12 | 40.55484 | -59.69548 |
| | 5 | -164.98260 | 55.45076 | 13 | -164.98260 | -55.45076 |
| | 6 | -25.84576 | 41.18792 | 14 | -25.84576 | -41.18793 |
| | 7 | 102.70721 | 23.15172 | 15 | 102.70721 | -23.15172 |
| | 8 | -132.75347 | 3.89611 | 16 | -132.75347 | -3.89611 |
| 1.50 | 1 | -10.79908 | 19.35460 | 9 | -10.79908 | -19.35460 |
| | 2 | 116.56505 | 37.76124 | 10 | 116.56504 | -37.76124 |
| | 3 | -107.13927 | 53.13995 | 11 | -107.13928 | -53.13995 |
| | 4 | 45.00000 | 60.00000 | 12 | 45.00001 | -60.00000 |
| | 5 | -162.86073 | 53.13995 | 13 | -162.86073 | -53.13995 |
| | 6 | -26.56505 | 37.76124 | 14 | -26.56505 | -37.76124 |
| | 7 | 100.79908 | 19.35460 | 15 | 100.79908 | -19.35460 |
| | 8 | -135.00000 | 0.00000 | 16 | -135.00000 | 0.00000 |

```
                         MAP  3:  TIME =  Ø.ØØ HOURS

          LONG     -9Ø             Ø             9Ø           18Ø
  LAT
  9Ø                               5                                  9Ø
  8Ø  55666666 5 44455555 5 55555444 5 66666655 5   8Ø
  7Ø  66665555 5 55444444 5 44444455 5 55556666 7   7Ø
  6Ø  55444445 5 55544446 5 64444555 5 54444455 5   6Ø
  5Ø  53334444 4 55535554 4 45553555 4 44443335 5   5Ø
  4Ø  53344554 4 43455544 4 44555434 4 45544335 5   4Ø
  3Ø  44444445 4 33355444 4 44455333 4 54444444 5   3Ø
  2Ø  54445544 5 43345444 4 44454334 5 44554445 3   2Ø
  1Ø  44554444 4 55443344 4 44334455 4 44445544 3   1Ø

   Ø  44444444 4 44644446 6 64444644 4 44444444 6

 -1Ø  44554444 4 55443344 4 44334455 4 44445544 3  -1Ø
 -2Ø  54445544 5 43345444 4 44454334 5 44554445 3  -2Ø
 -3Ø  44444445 4 33355444 4 44455333 4 54444444 5  -3Ø
 -4Ø  53344554 4 43455544 4 44555434 4 45544335 5  -4Ø
 -5Ø  53334444 4 55535554 4 45553555 4 44443335 5  -5Ø
 -6Ø  55444445 5 55544446 5 64444555 5 54444455 5  -6Ø
 -7Ø  66665555 5 55444444 5 44444455 5 55556666 7  -7Ø
 -8Ø  55666666 5 44455555 5 55555444 5 66666655 5  -8Ø
 -9Ø                               5                                 -9Ø
```

Figure 9. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs.

```
                         MAP  4:  TIME =  Ø.3Ø HOURS

  9Ø                               5                                  9Ø
  8Ø  55555444 4 44444445 5 55555566 6 66666666 6   8Ø
  7Ø  76555555 5 54444444 4 54444555 5 55566666 6   7Ø
  6Ø  53444455 5 55444555 4 45545555 5 54444555 5   6Ø
  5Ø  53334444 4 55454554 4 44544555 4 44444345 5   5Ø
  4Ø  43444455 4 43345544 4 44554433 4 44433335 5   4Ø
  3Ø  44444445 4 33355444 4 44455333 4 55444434 5   3Ø
  2Ø  44444454 4 43444444 4 44454335 5 44554444 3   2Ø
  1Ø  44455444 4 54444454 4 44434355 4 44445544 4   1Ø

   Ø  46444444 4 44644444 6 64444444 4 44444444 6    Ø

 -1Ø  44455444 4 54444454 4 44434355 4 44445544 4  -1Ø
 -2Ø  44444454 4 43444444 4 44454335 5 44554444 3  -2Ø
 -3Ø  44444445 4 33355444 4 44455333 4 55444434 5  -3Ø
 -4Ø  43444455 4 43345544 4 44554433 4 44433335 5  -4Ø
 -5Ø  53334444 4 55454554 4 44544555 4 44444345 5  -5Ø
 -6Ø  53444455 5 55444555 4 45545555 5 54444555 5  -6Ø
 -7Ø  76555555 5 54444444 4 54444555 5 55566666 6  -7Ø
 -8Ø  55555444 4 44444445 5 55555566 6 66666666 6  -8Ø
 -9Ø                               5                                 -9Ø
```

Figure 10. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs.

```
                MAP  5:  TIME =  Ø,6Ø HOURS

     LONG     -9Ø            Ø             9Ø            18Ø
LAT
 9Ø                          5                                 9Ø
 8Ø  66444444 4 44444444 4 55556666 6 66666666 6   8Ø
 7Ø  66555555 4 44444444 4 45555555 5 55665555 6   7Ø
 6Ø  64344555 5 54445555 5 44545555 5 44445555 6   6Ø
 5Ø  43334444 4 55454454 4 44445454 4 44444444 5   5Ø
 4Ø  44444455 4 43445544 4 44545433 4 44443335 5   4Ø
 3Ø  54444445 5 33355444 4 44455333 3 56444334 4   3Ø
 2Ø  44444443 4 43444444 4 44454345 5 54455444 4   2Ø
 1Ø  34455444 4 53454445 4 44434455 4 44445544 4   1Ø

  Ø  44544444 4 46444444 4 64446444 4 44444444 4    Ø

-1Ø  34455444 4 53454445 4 44434455 4 44445544 4  -1Ø
-2Ø  44444443 4 43444444 4 44454345 5 54455444 4  -2Ø
-3Ø  54444445 5 33355444 4 44455333 3 56444334 4  -3Ø
-4Ø  44444455 4 43445544 4 44545433 4 44443335 5  -4Ø
-5Ø  43334444 4 55454454 4 44445454 4 44444444 5  -5Ø
-6Ø  64344555 5 54445555 5 44545555 5 44445555 6  -6Ø
-7Ø  66555555 4 44444444 4 45555555 5 55665555 6  -7Ø
-8Ø  66444444 4 44444444 4 55556666 6 66666666 6  -8Ø
-9Ø                          5                    -9Ø
```

Figure 11. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs.

```
                MAP  6:  TIME =  Ø,9Ø HOURS

 9Ø                          5                     9Ø
 8Ø  55554444 4 44444444 4 45666666 6 66666666 6   8Ø
 7Ø  45544444 4 44444555 5 55666555 6 66555555 5   7Ø
 6Ø  55445555 5 44455555 4 43565555 4 44445555 5   6Ø
 5Ø  44444554 4 54554445 3 33455444 4 44444544 5   5Ø
 4Ø  54444455 5 43445554 4 44545433 3 44443335 4   4Ø
 3Ø  54444445 5 44345544 4 44455334 4 46643334 5   3Ø
 2Ø  44444444 4 44444444 4 44453344 5 54455434 4   2Ø
 1Ø  34445544 4 43444445 5 44434445 4 44445544 4   1Ø

  Ø  44664444 4 46444444 4 46446444 4 44444446 4    Ø

-1Ø  34445544 4 43444445 5 44434445 4 44445544 4  -1Ø
-2Ø  44444444 4 44444444 4 44453344 5 54455434 4  -2Ø
-3Ø  54444445 5 44345544 4 44455334 4 46643334 5  -3Ø
-4Ø  54444455 5 43445554 4 44545433 3 44443335 4  -4Ø
-5Ø  44444554 4 54554445 3 33455444 4 44444544 5  -5Ø
-6Ø  55445555 5 44455555 4 43565555 4 44445555 5  -6Ø
-7Ø  45544444 4 44444555 5 55666555 6 66555555 5  -7Ø
-8Ø  55554444 4 44444444 4 45666666 6 66666666 6  -8Ø
-9Ø                          5                    -9Ø
```

Figure 12. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs.

```
                     MAP  7:  TIME =  1.20 HOURS

        LONG     -90            0            90             180
  LAT
   90                           5                                  90
   80 55555544 4 44444444 5 55556666 6 66666665 5   80
   70 44544444 4 44455555 5 56666666 6 55555555 4   70
   60 45445555 4 44555554 4 43556554 4 44455555 4   60
   50 45444455 4 45554444 3 33455434 4 44444554 4   50
   40 54444455 5 44444554 4 44355333 3 44444344 5   40
   30 54444445 5 43345544 4 44445344 4 44543334 5   30
   20 44444444 4 43444454 4 44454444 5 54455334 4   20
   10 34444544 3 43544444 5 54444445 5 44444544 4   10

    0 44664444 4 44444444 4 46446444 4 44444446 4    0

  -10 34444544 3 43544444 5 54444445 5 44444544 4  -10
  -20 44444444 4 43444454 4 44454444 5 54455334 4  -20
  -30 54444445 5 43345544 4 44445344 4 44543334 5  -30
  -40 54444455 5 44444554 4 44355333 3 44444344 5  -40
  -50 45444455 4 45554444 3 33455434 4 44444554 4  -50
  -60 45445555 4 44555554 4 43556554 4 44455555 4  -60
  -70 44544444 4 44455555 5 56666666 6 55555555 4  -70
  -80 55555544 4 44444444 5 55556666 6 66666665 5  -80
  -90                           5                  -90
```

Figure 13. Visibility Map for a 16 Satellite System
Elevation = 20 Degs.

```
                     MAP  8:  TIME =  1.50 HOURS

   90                           5                                  90
   80 55555555 5 44445666 6 65555556 6 66654444 5   80
   70 44444444 4 45555555 5 66666666 5 55555554 4   70
   60 44655644 4 45555544 4 45555554 4 44555554 4   60
   50 55444455 3 45544444 4 33455433 4 44444554 3   50
   40 54444445 5 34344454 4 33355333 4 45444343 5   40
   30 54444445 5 43334544 4 44344344 4 44543334 5   30
   20 44444444 4 43355455 4 44444444 4 55455334 4   20
   10 34444443 3 43544444 5 54444445 5 44444534 3   10

    0 44466444 4 46444444 4 44444444 4 44444464 4    0

  -10 34444443 3 43544444 5 54444445 5 44444534 3  -10
  -20 44444444 4 43355455 4 44444444 4 55455334 4  -20
  -30 54444445 5 43334544 4 44344344 4 44543334 5  -30
  -40 54444445 5 34344454 4 33355333 4 45444343 5  -40
  -50 55444455 3 45544444 4 33455433 4 44444554 3  -50
  -60 44655644 4 45555544 4 45555554 4 44555554 4  -60
  -70 44444444 4 45555555 5 66666666 5 55555554 4  -70
  -80 55555555 5 44445666 6 65555556 6 66654444 5  -80
  -90                           5                  -90
```

Figure 14. Visibility Map for a 16 Satellite System
Elevation = 20 Degs.

```
                     MAP  9Ø  TIME =  Ø.ØØ HOURS

      LONG      -9Ø             Ø            9Ø           18Ø
LAT
 9Ø                             7                              9Ø
 8Ø  77766666 6 66666555 5 55566666 6 66666777 7    8Ø
 7Ø  77766666 6 55566577 7 77566555 6 66666777 7    7Ø
 6Ø  77766555 5 55577667 7 76677555 5 55566777 7    6Ø
 5Ø  56555565 5 55776666 5 66667755 5 56555565 5    5Ø
 4Ø  56455555 5 55765554 4 45556755 5 55555465 5    4Ø
 3Ø  65445666 6 54665554 4 45556645 6 66654456 5    3Ø
 2Ø  65555566 5 57565544 4 44556575 5 66555556 7    2Ø
 1Ø  75555555 5 57856556 6 65565875 5 55555557 8    1Ø

  Ø  86666444 4 68864666 6 66646886 4 44466668 6     Ø

-1Ø  75555555 5 57856556 6 65565875 5 55555557 8   -1Ø
-2Ø  65555566 5 57565544 4 44556575 5 66555556 7   -2Ø
-3Ø  65445666 6 54665554 4 45556645 6 66654456 5   -3Ø
-4Ø  56455555 5 55765554 4 45556755 5 55555465 5   -4Ø
-5Ø  56555565 5 55776666 5 66667755 5 56555565 5   -5Ø
-6Ø  77766555 5 55577667 7 76677555 5 55566777 7   -6Ø
-7Ø  77766666 6 55566577 7 77566555 6 66666777 7   -7Ø
-8Ø  77766666 6 66666555 5 55566666 6 66666777 7   -8Ø
-9Ø                             7                  -9Ø
```

Figure 15. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs.

```
                     MAP 1Ø:  TIME =  Ø.3Ø HOURS

 9Ø                             6                              9Ø
 8Ø  77777666 6 66655555 5 66666666 6 66666667 7    8Ø
 7Ø  77776655 5 55555666 6 66766666 6 66666677 7    7Ø
 6Ø  77765555 5 55666666 7 77766555 5 55666677 7    6Ø
 5Ø  55655566 5 55676666 6 66677655 5 54445566 5    5Ø
 4Ø  66445555 5 66775555 4 45566655 5 55555455 5    4Ø
 3Ø  65444565 5 56665554 4 44556646 6 66665466 6    3Ø
 2Ø  55455566 6 57555654 4 44556655 5 66555556 7    2Ø
 1Ø  66555554 5 58856555 6 65567855 5 54555557 8    1Ø

  Ø  86666444 4 68864466 6 66646866 4 44446668 6     Ø

-1Ø  66555554 5 58856555 6 65567855 5 54555557 8   -1Ø
-2Ø  55455566 6 57555654 4 44556655 5 66555556 7   -2Ø
-3Ø  65444565 5 56665554 4 44556646 6 66665466 6   -3Ø
-4Ø  66445555 5 66775555 4 45566655 5 55555455 5   -4Ø
-5Ø  55655566 5 55676666 6 66677655 5 54445566 5   -5Ø
-6Ø  77765555 5 55666666 7 77766555 5 55666677 7   -6Ø
-7Ø  77776655 5 55555666 6 66766666 6 66666677 7   -7Ø
-8Ø  77777666 6 66655555 5 66666666 6 66666667 7   -8Ø
-9Ø                             6                  -9Ø
```

Figure 16. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs.

```
                    MAP 11:  TIME =  Ø.6Ø HOURS

         LONG      -9Ø            Ø             9Ø            18Ø
  LAT
   9Ø                             6                                  9Ø
   8Ø  77777777 6 55555566 6 66666666 6 66666666 6   8Ø
   7Ø  77776555 5 55555556 6 77777666 6 66666667 7   7Ø
   6Ø  77765555 5 56655666 7 77766655 5 56665566 7   6Ø
   5Ø  66655666 6 56666666 6 66776655 5 44445566 6   5Ø
   4Ø  65445555 5 66765555 4 45566555 4 65555555 6   4Ø
   3Ø  65444555 5 56655554 4 44566545 6 66665465 6   3Ø
   2Ø  54455556 6 56566655 5 44556655 5 56555557 6   2Ø
   1Ø  66655554 5 58855555 5 66567855 5 54555567 8   1Ø

    Ø  66666644 4 68864466 6 66666866 4 44446668 6    Ø

  -1Ø  66655554 5 58855555 5 66567855 5 54555567 8  -1Ø
  -2Ø  54455556 6 56566655 5 44556655 5 56555557 6  -2Ø
  -3Ø  65444555 5 56655554 4 44566545 6 66665465 6  -3Ø
  -4Ø  65445555 5 66765555 4 45566555 4 65555555 6  -4Ø
  -5Ø  66655666 6 56666666 6 66776655 5 44445566 6  -5Ø
  -6Ø  77765555 5 56655666 7 77766655 5 56665566 7  -6Ø
  -7Ø  77776555 5 55555556 6 77777666 6 66666667 7  -7Ø
  -8Ø  77777777 6 55555566 6 66666666 6 66666666 6  -8Ø
  -9Ø                      6                        -9Ø
```

Figure 17. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs.

```
                    MAP 12:  TIME =  Ø.9Ø HOURS

   9Ø                      6                         9Ø
   8Ø  66666655 5 55567777 7 77766666 6 66666666 6   8Ø
   7Ø  77766655 5 55555555 7 77777766 6 66666666 7   7Ø
   6Ø  67776665 5 56555555 5 67777655 6 66555556 6   6Ø
   5Ø  76666666 6 66556666 5 55666655 5 44455566 6   5Ø
   4Ø  65444555 5 76665555 4 45665555 5 56664565 6   4Ø
   3Ø  55444455 5 66655554 4 44566645 6 66666456 6   3Ø
   2Ø  54445556 6 56566655 5 44567555 5 56655567 6   2Ø
   1Ø  65665555 5 58655555 5 56567855 5 54455577 7   1Ø

    Ø  66666644 4 68864446 6 66666866 4 44446688 6    Ø

  -1Ø  65665555 5 58655555 5 56567855 5 54455577 7  -1Ø
  -2Ø  54445556 6 56566655 5 44567555 5 56655567 6  -2Ø
  -3Ø  55444455 5 66655554 4 44566645 6 66666456 6  -3Ø
  -4Ø  65444555 5 76665555 4 45665555 5 56664565 6  -4Ø
  -5Ø  76666666 6 66556666 5 55666655 5 44455566 6  -5Ø
  -6Ø  67776665 5 56555555 5 67777655 6 66555556 6  -6Ø
  -7Ø  77766655 5 55555555 7 77777766 6 66666666 7  -7Ø
  -8Ø  66666655 5 55567777 7 77766666 6 66666666 6  -8Ø
  -9Ø                      6                        -9Ø
```

Figure 18. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs.

```
                 MAP 13:  TIME =  1.20 HOURS

        LONG     -90             0              90            180
LAT
 90                              6                              90
 80 66655555 5 56666667 7 77777766 6 66666666 6   80
 70 77666665 4 55555566 6 77777766 6 66666666 6   70
 60 76777666 5 65555555 5 77777776 6 65555556 6   60
 50 76656666 6 76556665 5 57556665 4 44555556 7   50
 40 55544555 5 76565555 5 45655555 5 55554566 6   40
 30 55444455 5 66555664 4 44565644 5 66666556 6   30
 20 54444556 5 66556655 5 54567555 5 56655576 6   20
 10 65665555 5 78555455 5 55677755 5 55455577 6   10

  0 46666664 6 68664446 6 66666866 6 44446688 6    0

-10 65665555 5 78555455 5 55677755 5 55455577 6  -10
-20 54444556 5 66556655 5 54567555 5 56655576 6  -20
-30 55444455 5 66555664 4 44565644 5 66666556 6  -30
-40 55544555 5 76565555 5 45655555 5 55554566 6  -40
-50 76656666 6 76556665 5 57556665 4 44555556 7  -50
-60 76777666 5 65555555 5 77777776 6 65555556 6  -60
-70 77666665 4 55555566 6 77777766 6 66666666 6  -70
-80 66655555 5 56666667 7 77777766 6 66666666 6  -80
-90                       6                      -90
```

Figure 19. Visibility Map for a 16 Satellite System
Elevation = 10 Degs.

```
                 MAP 14:  TIME =  1.50 HOURS

 90                       7                       90
 80 65555556 6 66666666 6 77777777 6 66666666 6   80
 70 55777755 6 55556666 6 67777776 6 66665555 6   70
 60 66677666 7 55555555 6 67777776 6 55555555 7   60
 50 66666666 7 75555655 5 56655665 5 55655557 7   50
 40 55544555 6 77565555 5 46655664 5 55556577 6   40
 30 55444455 6 66566666 4 44666644 4 66666566 6   30
 20 54444445 5 67556655 5 54577545 5 55665576 5   20
 10 65566556 5 78555455 5 55777755 5 55455587 5   10

  0 46666664 6 88664444 6 66666666 6 44446688 6    0

-10 65566556 5 78555455 5 55777755 5 55455587 5  -10
-20 54444445 5 67556655 5 54577545 5 55665576 5  -20
-30 55444455 6 66566666 4 44666644 4 66666566 6  -30
-40 55544555 6 77565555 5 46655664 5 55556577 6  -40
-50 66666666 7 75555655 5 56655665 5 55655557 7  -50
-60 66677666 7 55555555 6 67777776 6 55555555 7  -60
-70 55777755 6 55556666 6 67777776 6 66665555 6  -70
-80 65555556 6 66666666 6 77777777 6 66666666 6  -80
-90                       7                      -90
```

Figure 20 Visibility Map for a 16 Satellite System
Elevation = 10 Degs.

We must not fail to note, however, that, since two satellites are tracing out each figure eight and since the members of each pair differ by 12 hours in time, collisions (doublets) have to occur at the equatorial crossings. In practical applications, collisions must be avoided. Further, leaving aside the disastrous consequences of such catastrophes, the conditions of the problem are not realistically fulfilled. A doublet is counted in the maps as two distinct satellites, and for navigation purposes, a doublet has to be counted as a singlet. Thus, for example, in Figure 9 , the pairs of 3's, located at ±20 and ±30 degrees longitude, ±10 degrees latitude, count the doublet that, at epoch, is at zero-longitude, zero-latitude, as two distinct satellites. Therefore, these particular 3's should be replaced by 2's, and the conditions of the problem are not satisfied.

However, this shortcoming can be removed by modifying the proposed system in small measure. Among the various ways that this can be done, four modifications will be described and discussed in the next section of the report.

# 6.0 SYSTEM MODIFICATIONS

Among the various small modifications that might be introduced for avoiding collisions, the following have been considered. The first splits each ground-track into two closely adjacent figure eight's. From Table 4, it is easily verified that the maximum excursion in longitude of the satellite is approximately 19 degrees. Since the centers of neighboring ground-tracks have a separation of 45 degrees, there is a minimum separation between ground-track of about 6 degrees. We may, therefore, without introducing other possible collisions, introduce a split of 6 degrees.

A second alternative consists in retaining the original spacing of the ground-tracks and introducing time-shift for one satellite in each pair. An obvious third possibility is a combination of a split and a time-shift.

All three modifications have been examined. The first and third have been tried employing a split of from one to five degrees and a time-shift up to 0.3 hours. While coverage remains complete for an elevation angle of 20 degrees, there are one or two failures (that is, one or two observation-time-points at which only three satellites are visible) over a 12-hour interval with time-increments of 0.3 hours for an elevation angle of 10 degrees. (It is interesting to note that the second condition, in this instance, is more restrictive. It cannot be concluded that this is the case in general, since other examples have been found in which the reverse holds).

The second alternative, a time-shift, appears to permit the largest deviation from the basic model while maintaining the visibility requirements. The maximum allowable time-shift lies between 6 and 12 minutes. (If the second condition, 10-degree elevation angle, were relaxed to 9 degrees, both the second and the third modifications result in complete coverage with a split of at least 4 degrees and a time-shift of 0.3 hours). Table 6 and Figures 21 to 32 are based on a time-shift of 0.1 hours. Refering to Table 2,

TABLE 6. EARTH-FIXED COORDINATES OF THE 16 SATELLITES

| TIME | NO. | LONGITUDE | LATITUDE | NO. | LONGITUDE | LATITUDE |
|---|---|---|---|---|---|---|
| 0,00 | 1 | 0,00000 | 0,00000 | 9 | -7,82483 | -1,42889 |
| | 2 | 124,20092 | 19,35460 | 10 | 123,48591 | -22,75142 |
| | 3 | -108,43495 | 37,76124 | 11 | -108,74161 | -39,03176 |
| | 4 | 27,86073 | 53,13995 | 12 | 28,55251 | -54,02897 |
| | 5 | 180,00000 | 60,00000 | 13 | -178,35273 | -59,95888 |
| | 6 | -27,86073 | 53,13995 | 14 | -27,26620 | -52,20689 |
| | 7 | 108,43495 | 37,76124 | 15 | 108,08266 | -36,47599 |
| | 8 | -124,20092 | 19,35460 | 16 | -124,93183 | -17,95378 |
| 0,30 | 1 | -2,24652 | 3,89611 | 9 | -3,26612 | -5,32349 |
| | 2 | 122,29279 | 23,15172 | 10 | 121,62902 | -24,53334 |
| | 3 | -109,15424 | 41,18792 | 11 | -109,31946 | -42,41166 |
| | 4 | 29,98260 | 55,45076 | 12 | 30,94832 | -56,27194 |
| | 5 | -175,55484 | 59,69548 | 13 | -173,98853 | -59,43370 |
| | 6 | -26,45940 | 50,50087 | 14 | -26,11362 | -49,46362 |
| | 7 | 107,37556 | 34,22461 | 15 | 106,91391 | -32,93472 |
| | 8 | -126,22764 | 15,52249 | 16 | -126,99486 | -14,11033 |
| 0,60 | 1 | -4,47207 | 7,78616 | 9 | -5,27861 | -9,22976 |
| | 2 | 120,53508 | 26,90400 | 10 | 119,93538 | -28,26635 |
| | 3 | -109,46460 | 44,47751 | 11 | -109,46078 | -45,64267 |
| | 4 | 32,85166 | 57,36223 | 12 | 34,08495 | -57,94825 |
| | 5 | -171,42342 | 58,79988 | 13 | -170,03989 | -58,33235 |
| | 6 | -25,71189 | 47,59598 | 14 | -25,58540 | -46,47489 |
| | 7 | 106,03537 | 30,59970 | 15 | 105,48385 | -29,25202 |
| | 8 | -128,34500 | 11,66386 | 16 | -129,13904 | -10,24388 |
| 0,90 | 1 | -6,65500 | 11,66386 | 9 | -7,44025 | -13,08128 |
| | 2 | 118,96463 | 30,59970 | 10 | 118,44399 | -31,93790 |
| | 3 | -109,28810 | 47,59598 | 11 | -109,08484 | -48,68845 |
| | 4 | 36,42342 | 58,79988 | 12 | 37,88424 | -59,19435 |
| | 5 | -167,85166 | 57,36223 | 13 | -166,71382 | -56,71245 |
| | 6 | -25,53539 | 44,47751 | 14 | -25,59697 | -43,28923 |
| | 7 | 104,46491 | 26,90400 | 15 | 103,84010 | -25,53407 |
| | 8 | -130,52793 | 7,78616 | 16 | -131,34018 | -6,36090 |
| 1,20 | 1 | -8,77236 | 15,52249 | 9 | -9,52742 | -16,93109 |
| | 2 | 117,62444 | 34,22460 | 10 | 117,20045 | -35,53266 |
| | 3 | -108,54060 | 50,50087 | 11 | -108,10678 | -51,50249 |
| | 4 | 40,55484 | 59,69548 | 12 | 42,16403 | -59,87748 |
| | 5 | -164,98260 | 55,45076 | 13 | -164,11803 | -54,64592 |
| | 6 | -25,84576 | 41,18792 | 14 | -26,06587 | -39,94582 |
| | 7 | 102,70721 | 23,15172 | 15 | 102,02325 | -21,76409 |
| | 8 | -132,75347 | 3,89611 | 16 | -133,57588 | -2,46792 |
| 1,50 | 1 | -10,79908 | 19,35460 | 9 | -11,51409 | -20,75142 |
| | 2 | 116,56505 | 37,76124 | 10 | 116,25839 | -39,03176 |
| | 3 | -107,13927 | 53,13995 | 11 | -106,44749 | -54,02897 |
| | 4 | 45,00000 | 60,00000 | 12 | 46,64727 | -59,95888 |
| | 5 | -162,86073 | 53,13995 | 13 | -162,26619 | -52,20689 |
| | 6 | -26,56505 | 37,76124 | 14 | -26,91734 | -36,47599 |
| | 7 | 100,79908 | 19,35460 | 15 | 100,06816 | -17,95308 |
| | 8 | -135,00000 | 0,00000 | 16 | -135,82483 | 1,42889 |

```
                    MAP 15!  TIME =  Ø.ØØ HOURS

     LONG     -9Ø            Ø            9Ø            18Ø
LAT
 9Ø                          5                                  9Ø
 8Ø  55666666 5 44455555 5 55555444 5 66666655 5   8Ø
 7Ø  66665555 5 55444444 5 44444455 5 55556666 7   7Ø
 6Ø  55444445 5 55544445 5 54444555 5 54444455 5   6Ø
 5Ø  53334444 4 55535554 4 45543555 4 44443335 5   5Ø
 4Ø  53344554 4 43455544 4 44554434 4 44543335 5   4Ø
 3Ø  44444445 4 33355444 4 44455333 4 54444444 5   3Ø
 2Ø  54444544 5 43345444 4 44454334 5 44554445 3   2Ø
 1Ø  44454444 4 55443344 4 44334455 4 44445544 4   1Ø

  Ø  45444444 4 44644445 6 64444644 4 44444444 6    Ø

-1Ø  44555444 4 55443344 4 44334455 4 44445544 3  -1Ø
-2Ø  54445544 5 43445444 4 44454334 5 44554445 3  -2Ø
-3Ø  44444445 4 33355444 4 44455333 4 55444444 5  -3Ø
-4Ø  43344554 4 43455544 4 44555434 4 45544335 5  -4Ø
-5Ø  53334444 4 55535554 4 44553555 4 44444335 5  -5Ø
-6Ø  55444455 5 55444456 5 55444555 5 54444454 5  -6Ø
-7Ø  66655555 5 55444444 5 44444555 5 55556666 7  -7Ø
-8Ø  55555554 4 44444555 5 55555555 6 66666665 5  -8Ø
-9Ø                          5                     -9Ø
```

Figure 21. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                    MAP 16!  TIME =  Ø.3Ø HOURS

 9Ø                          5                                  9Ø
 8Ø  55555444 4 44444445 5 55555566 6 66666666 6   8Ø
 7Ø  76555555 5 54444444 4 54444555 5 55566666 6   7Ø
 6Ø  53444455 5 55444555 4 45545555 5 54444555 5   6Ø
 5Ø  53334444 4 55454554 4 44444555 4 44444345 5   5Ø
 4Ø  43444455 4 43345544 4 44554433 4 44433335 5   4Ø
 3Ø  54444445 4 33355444 4 44455333 3 55444434 5   3Ø
 2Ø  44444443 4 43444444 4 44454335 5 44555444 3   2Ø
 1Ø  44455444 4 54444455 4 44434355 4 44445544 4   1Ø

  Ø  45444444 4 44644444 6 64444444 4 44444444 6    Ø

-1Ø  34455444 4 54444454 4 44434355 4 44445544 4  -1Ø
-2Ø  44444454 4 43444444 4 44454335 5 54554444 3  -2Ø
-3Ø  44444445 5 33355444 4 44455333 4 45444434 5  -3Ø
-4Ø  43444455 4 43445544 4 44555433 4 44433335 5  -4Ø
-5Ø  53334444 4 55454554 4 44544454 4 44444344 5  -5Ø
-6Ø  53444555 5 55445555 4 45545555 5 44444555 6  -6Ø
-7Ø  66555555 5 44444444 4 55445555 5 55566666 6  -7Ø
-8Ø  55554444 4 44444444 5 55555666 6 66666666 6  -8Ø
-9Ø                          5                     -9Ø
```

Figure 22. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
MAP 17:  TIME =  Ø.6Ø HOURS

      LONG    -9Ø           Ø            9Ø            18Ø
LAT
 9Ø                         5                                9Ø
 8Ø  66444444 4 44444444 4 55556666 6 66666666 6   8Ø
 7Ø  66555555 4 44444444 4 45555555 5 55665555 6   7Ø
 6Ø  64344555 5 54445555 5 44545555 5 44445555 6   6Ø
 5Ø  43344544 4 55454454 4 43445454 4 44444444 5   5Ø
 4Ø  44444455 4 43445544 4 44545433 4 44443335 5   4Ø
 3Ø  54444445 5 33355444 4 44455333 4 56544334 4   3Ø
 2Ø  44444443 4 43444444 4 44454344 5 54455444 4   2Ø
 1Ø  34445444 4 53454445 5 44434455 4 44445544 4   1Ø

  Ø  44644444 4 46444444 4 65446444 4 44444445 4    Ø

-1Ø  34455544 4 53454445 4 44434455 4 44445544 4  -1Ø
-2Ø  44444443 4 43444444 4 44454345 5 54455444 4  -2Ø
-3Ø  54444445 5 33345444 4 44455333 3 56444334 4  -3Ø
-4Ø  44444455 4 44445544 4 44545433 4 44443335 5  -4Ø
-5Ø  43334444 4 54554455 4 44445454 4 44444444 5  -5Ø
-6Ø  55345555 5 54455555 5 44565555 5 44445555 5  -6Ø
-7Ø  65555554 4 44444455 5 55555555 5 56655555 5  -7Ø
-8Ø  65444444 4 44444444 4 45566666 6 66666666 6  -8Ø
-9Ø                        6                      -9Ø
```

Figure 23. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
MAP 18:  TIME =  Ø.9Ø HOURS

 9Ø                         5                                9Ø
 8Ø  55554444 4 44444444 4 45666666 6 66666666 6   8Ø
 7Ø  45544444 4 44444555 5 55666555 6 66555555 5   7Ø
 6Ø  55445555 5 44455555 4 43565555 4 44445555 5   6Ø
 5Ø  45444554 4 54554444 3 33455444 4 44444544 5   5Ø
 4Ø  54444455 5 43445554 4 44445433 3 44443335 4   4Ø
 3Ø  54444445 5 44345544 4 44445334 4 45543334 5   3Ø
 2Ø  44444444 4 44444444 4 44453344 5 54455434 4   2Ø
 1Ø  34445544 4 43444445 5 44434445 4 44445544 4   1Ø

  Ø  44664444 4 46444444 4 46446444 4 44444446 4    Ø

-1Ø  34445544 4 44444445 5 44434445 4 44444544 4  -1Ø
-2Ø  44444444 4 44444444 4 44454344 5 54455434 4  -2Ø
-3Ø  54444445 5 43345544 4 44455334 4 46643334 5  -3Ø
-4Ø  54444455 5 44445554 4 44445433 3 44443344 4  -4Ø
-5Ø  44444554 4 54554445 3 33455444 4 44444554 4  -5Ø
-6Ø  55445555 4 44555555 4 43465555 4 44455555 5  -6Ø
-7Ø  44544444 4 44445555 5 55766666 6 65555555 5  -7Ø
-8Ø  55554444 4 44444444 4 55566666 6 66666666 5  -8Ø
-9Ø                        5                      -9Ø
```

Figure 24. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                  MAP 19:  TIME =  1.2Ø HOURS

       LONG    -9Ø            Ø             9Ø            18Ø
 LAT
  9Ø                          5                                9Ø
  8Ø 55555544 4 44444444 5 55556666 6 66666665 5   8Ø
  7Ø 44544444 4 44455555 5 56666666 6 55555555 4   7Ø
  6Ø 45445555 4 44555554 4 43556554 4 44455555 4   6Ø
  5Ø 55444455 4 45554444 3 33455434 4 44444554 4   5Ø
  4Ø 54444455 5 44444554 4 43355333 3 54444344 5   4Ø
  3Ø 54444445 5 43345544 4 44445344 4 44543334 5   3Ø
  2Ø 44444444 4 43345455 4 44454444 5 54455334 4   2Ø
  1Ø 34444443 3 43544444 5 54444445 5 44444544 4   1Ø

   Ø 44565444 4 44444444 4 45446444 4 44444446 4    Ø

 -1Ø 34444554 3 43544444 5 54444445 5 44444544 3  -1Ø
 -2Ø 44444444 4 43444454 4 44454444 5 55455334 4  -2Ø
 -3Ø 54444445 5 43345544 4 44445344 4 44543334 5  -3Ø
 -4Ø 54444455 5 44444554 4 44355333 3 44444344 5  -4Ø
 -5Ø 45444455 4 45554444 4 33355433 4 44444554 4  -5Ø
 -6Ø 45544554 4 45555554 4 44455554 4 44455555 4  -6Ø
 -7Ø 44454444 4 44555555 5 56676666 6 55555554 4  -7Ø
 -8Ø 55555554 4 44444445 5 55555666 6 66666655 5  -8Ø
 -9Ø                      5                       -9Ø
```

Figure 25. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                  MAP 2Ø:  TIME =  1.5Ø HOURS

  9Ø                      5                        9Ø
  8Ø 55555555 5 44445666 6 65555556 6 66654444 5   8Ø
  7Ø 44444444 4 45555555 5 66666666 5 55555554 4   7Ø
  6Ø 45655654 4 45555544 4 45555554 4 44555554 4   6Ø
  5Ø 55444455 3 45544444 4 33455433 4 44444554 4   5Ø
  4Ø 54444445 5 34344454 4 33355334 4 45544343 5   4Ø
  3Ø 54444445 5 43334544 4 44344444 4 44544334 5   3Ø
  2Ø 44444444 4 43355455 4 44444444 4 55454334 4   2Ø
  1Ø 34444443 3 44544444 5 54444445 5 44444534 3   1Ø

   Ø 44466444 4 56444444 4 44454444 4 44444464 4    Ø

 -1Ø 34444443 3 43544444 5 54444445 5 44444534 3  -1Ø
 -2Ø 44444444 4 43355455 4 44444444 4 55455334 4  -2Ø
 -3Ø 54444445 5 53334544 4 44354344 4 44553334 5  -3Ø
 -4Ø 54444445 5 44344454 4 33355333 4 45444343 5  -4Ø
 -5Ø 55444455 3 45544444 4 33455433 4 44444554 3  -5Ø
 -6Ø 44555544 4 55555544 4 45555444 4 44555554 4  -6Ø
 -7Ø 44445444 4 45555555 6 66667666 5 55555544 4  -7Ø
 -8Ø 55555555 5 55566666 6 66555555 5 55444444 4  -8Ø
 -9Ø                      5                       -9Ø
```

Figure 26. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                MAP 21:  TIME =  0.00 HOURS

      LONG    -90                0               90              180
LAT
 90                              7                                     90
 80  77766666 6 66666555 5 55566666 6 66666777 7   80
 70  77766666 6 55566566 7 66566555 6 66666777 7   70
 60  77766555 5 55567667 7 76676555 5 55566777 7   60
 50  56555566 5 55676666 5 66667655 5 55554565 5   50
 40  56455555 5 65765554 4 45556755 5 55555465 5   40
 30  65444666 6 55665554 4 45556645 6 66654456 5   30
 20  65555566 5 57565544 4 44556565 5 66555556 7   20
 10  65555555 5 57856555 6 65565875 5 55555557 8   10

  0  86666444 4 68864566 6 66646886 4 44456668 6    0

-10  75555554 5 57856556 6 65566875 5 54555557 8  -10
-20  55555566 5 57565544 4 44556575 5 66555556 7  -20
-30  65445666 6 54665554 4 45556645 6 66654456 5  -30
-40  56455555 5 55775554 4 45556755 5 55555465 5  -40
-50  56555565 5 55776666 6 66677755 5 55555566 5  -50
-60  77765555 5 55576667 7 76677555 5 55566677 7  -60
-70  77766655 5 55556677 7 77566666 6 66656677 7  -70
-80  77776666 6 66666555 5 55666666 6 66666677 7  -80
-90                      6                        -90
```

Figure 27. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                MAP 22:  TIME =  0.30 HOURS

 90                      6                         90
 80  77777666 6 66655555 5 66666666 6 66666667 7   80
 70  77776655 5 55555666 6 66766666 6 66666677 7   70
 60  77765555 5 55665666 7 77766555 5 55666677 7   60
 50  56655566 5 55676666 6 66676655 5 54445566 5   50
 40  66445555 5 66775555 4 45566655 5 55555455 5   40
 30  65444555 5 56665554 4 44556646 6 66665465 6   30
 20  55455566 6 56556655 4 44556655 5 66555557 6   20
 10  66555554 5 58856555 6 65567855 5 54555557 8   10

  0  86666444 4 68864466 6 66656866 4 44446668 6    0

-10  66555554 5 58856555 6 65567855 5 54555567 8  -10
-20  54455566 6 57555654 4 44556655 5 66555556 7  -20
-30  65444565 5 57655554 4 44566645 6 66665466 6  -30
-40  65445555 5 66775555 4 45566655 5 55555655 5  -40
-50  55655666 5 56676666 6 66677655 5 54445566 6  -50
-60  77755555 5 55666666 7 77766555 5 55666677 7  -60
-70  77776555 5 55555566 6 67776666 6 66666677 7  -70
-80  77777766 6 66555555 6 66666666 6 66666666 7  -80
-90                      6                        -90
```

Figure 28. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                MAP 23!  TIME =  Ø.6Ø HOURS

      LONG     -9Ø             Ø             9Ø            18Ø
LAT
 9Ø                            6                                 9Ø
 8Ø  77777777 6 55555566 6 66666666 6 66666666 6   8Ø
 7Ø  77776555 5 55555556 6 77777666 6 66666667 7   7Ø
 6Ø  77776655 5 56655566 7 76766655 5 56665566 7   6Ø
 5Ø  76655666 6 56666666 6 66776655 5 44445566 6   5Ø
 4Ø  65445555 5 66765555 4 45566555 5 66655555 6   4Ø
 3Ø  55444555 5 56655554 4 44566545 6 66665466 6   3Ø
 2Ø  54455556 6 56566655 5 44556655 5 56555557 6   2Ø
 1Ø  65655554 5 58755555 5 66567855 5 54555567 8   1Ø

  Ø  66666644 4 68864456 6 66666866 4 44446678 6    Ø

-1Ø  66655555 5 58855555 5 66567855 5 54555567 7  -1Ø
-2Ø  54455556 6 56566655 5 44556655 5 56655557 6  -2Ø
-3Ø  65444455 5 56655554 4 44566545 6 66665465 6  -3Ø
-4Ø  65445555 5 66765555 4 45566555 4 66555555 6  -4Ø
-5Ø  67556666 6 56666666 6 66776655 5 44455566 6  -5Ø
-6Ø  77665555 5 56655666 6 78776655 5 56655556 6  -6Ø
-7Ø  77776555 5 55555556 6 77777666 6 66666667 7  -7Ø
-8Ø  67777777 6 65556677 7 76666666 6 66666666 6  -8Ø
-9Ø                            6                      -9Ø
```

Figure 29. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                MAP 24!  TIME =  Ø.9Ø HOURS

 9Ø                            6                      9Ø
 8Ø  66666655 5 55567777 7 77766666 6 66666666 6   8Ø
 7Ø  77766655 5 55555555 7 77777766 6 66666666 7   7Ø
 6Ø  68776666 5 56555555 5 67777655 6 66555556 6   6Ø
 5Ø  76666666 6 66556666 5 55666655 5 44455566 7   5Ø
 4Ø  65444555 5 76665555 4 45665555 5 55554566 6   4Ø
 3Ø  55444455 5 66655554 4 44566644 6 66666556 6   3Ø
 2Ø  54445556 6 56666655 5 54567555 5 56655566 6   2Ø
 1Ø  65665555 5 58655555 5 56567855 5 54455577 6   1Ø

  Ø  66666644 4 68864446 6 66666866 4 44446688 6    Ø

-1Ø  65665555 5 68655555 5 56667855 5 54455577 7  -1Ø
-2Ø  54445556 6 56556655 5 44567555 5 56655567 6  -2Ø
-3Ø  55444455 5 66655554 4 44566645 5 66666456 6  -3Ø
-4Ø  65544555 5 76565555 5 45665545 5 56664565 6  -4Ø
-5Ø  76666666 6 66556665 5 56666655 5 44455556 6  -5Ø
-6Ø  67776665 5 66555555 5 77777766 6 66555556 6  -6Ø
-7Ø  77766655 5 45555555 7 77777766 6 66666666 7  -7Ø
-8Ø  66666555 5 55666677 7 77776666 6 66666666 6  -8Ø
-9Ø                            6                      -9Ø
```

Figure 30. Visibility Map for a 16 Satellite System
Elevation = 1Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                MAP 25:  TIME =  1.20 HOURS

       LONG      -90            0              90             180
LAT
 90                             6                                  90
 80 66555555 5 56666667 7 77777766 6 66666666 6    80
 70 77666665 4 55555566 6 77777766 6 66666666 6    70
 60 76777666 6 65555555 5 77777776 6 65555556 6    60
 50 76656666 6 76555665 5 56556665 4 44555557 7    50
 40 55544555 5 76565555 5 45655555 5 55555566 6    40
 30 55444455 5 66556664 4 44565644 5 66666466 6    30
 20 54444445 5 66556655 5 54567555 5 56655576 6    20
 10 65666555 5 78555455 5 55677755 5 55455587 6    10

  0 46666664 6 68664445 6 66666866 6 44446688 6     0

-10 65665556 5 78555455 5 55677755 5 55455577 6   -10
-20 54444556 5 66556655 5 54567555 5 55655576 5   -20
-30 55444455 5 66555664 4 44665644 5 66666556 6   -30
-40 55544555 5 76565555 5 45655565 5 55555567 6   -40
-50 66666666 7 76556665 5 57555665 4 44555556 7   -50
-60 76777666 6 65555555 5 67777776 6 65555555 6   -60
-70 66666666 5 55555566 6 77777776 6 66666666 6   -70
-80 66555555 5 66666666 7 77777766 6 66666666 6   -80
-90                     6                         -90
```

Figure 31. Visibility Map for a 16 Satellite System Elevation = 1Ø Degs. (Time-Shift of Ø.1Ø Hours)

```
                MAP 26:  TIME =  1.50 HOURS

 90                     7                          90
 80 65555556 6 66666666 6 77777777 6 66666666 6    80
 70 56777765 6 55556666 6 67777776 6 66665555 6    70
 60 66677666 7 55555555 6 67777776 6 55555555 7    60
 50 66666666 7 75555555 4 56655666 5 55655557 7    50
 40 55541555 6 77565555 5 46555664 5 55556577 6    40
 30 55444455 6 66566666 5 44666644 4 66665566 6    30
 20 54444445 5 67556655 5 54576545 5 55665566 5    20
 10 65566556 5 78555455 5 55777655 5 55455587 5    10

  0 46666654 6 88664444 6 66766666 6 44446688 6     0

-10 65566556 6 78555455 5 55777755 5 55455587 5   -10
-20 54444445 5 67556655 5 54577545 5 55665576 5   -20
-30 55444455 6 66466666 4 44666644 4 66666566 5   -30
-40 55544555 6 76565555 5 56655654 5 55556567 6   -40
-50 66666656 7 75555655 5 56655564 5 55655557 7   -50
-60 66677766 7 55555555 6 67777776 6 55555555 7   -60
-70 55666656 6 66666666 6 67777777 6 66655555 5   -70
-80 55555566 6 66666666 6 67777777 6 66666666 6   -80
-90                     6                         -90
```

Figure 32. Visibility Map for a 16 Satellite System Elevation = 1Ø Degs. (Time-Shift of Ø.1Ø Hours)

the "center" column and the data for satellites 1 to 8 remain unchanged. For satellites 9 to 16, a value of 1.5 degrees is subtracted from the values for RA and a value of 0.1 hour is added to the values of PTO. The effect of these changes is to move the respective satellites ahead by 0.1 hours in their original ground-tracks. The complete period of 24 hours is not required before the configuration is repeated. However, from symmetry considerations, an examination of a 12-hour cycle is sufficient. One may verify this observation by noting that, in Table 6, the block of data at time 0.00 also represents the positions of the 16 satellites 12 hours later if all latitudes are replaced by their negatives. A comparison of Figures 21 and 33, and 27 and 34 confirm this fact. These tables and figures are presented as a sample and for purposes of comparison with the basic model.

Ever present in the background throughout this investigation has been the perplexing question: Why don't systems based on eccentric orbits exist that satisfy the visibility conditions at least as well as those based on circular orbits? From the beginning it seemed intuitively clear that, if such systems exist, the most likely candidates should be systems that use positive eccentricity to stretch the orbits toward the poles in order to cover the higher latitudes. In other words, the system should be based on orbits with argument of perigee having values of ±90 degrees. Such systems were considered and are mentioned in the Appendix. The case that seemed least interesting to the author was the system in which the argument of perigee is zero for all orbits, i.e. eccentric orbits with perigee and apogee points in the equatorial plane. However, such systems were finally studied. Unexpectedly, the present sixteen satellite configuration, subjected to variations of eccentricity up to 0.5, in steps of 0.1, and variations of ±5 degrees in inclination about the sixty-degree mark, shows relatively good coverage. An eccentricity of 0.2 at sixty degrees inclination appears to give the best results. The ground-track produced by an eccentric orbit of this type is a distorted figure eight. The center of the figure eight remains fixed,

MAP 27! TIME = 12.ØØ HOURS

| LAT | LONG | -9Ø | | Ø | | 9Ø | | 18Ø | |
|---|---|---|---|---|---|---|---|---|---|
| 9Ø | | | | 5 | | | | | 9Ø |
| 8Ø | 55555554 | 4 | 44444555 | 5 | 55555555 | 6 | 66666665 | 5 | 8Ø |
| 7Ø | 66655555 | 5 | 55444444 | 5 | 44444555 | 5 | 55556666 | 7 | 7Ø |
| 6Ø | 55444455 | 5 | 55444456 | 5 | 55444555 | 5 | 54444454 | 5 | 6Ø |
| 5Ø | 53334444 | 4 | 55535554 | 4 | 44553555 | 4 | 44444335 | 5 | 5Ø |
| 4Ø | 43344554 | 4 | 43455544 | 4 | 44555434 | 4 | 45544335 | 5 | 4Ø |
| 3Ø | 44444445 | 4 | 33355444 | 4 | 44455333 | 4 | 55444444 | 5 | 3Ø |
| 2Ø | 54445544 | 5 | 43445444 | 4 | 44454334 | 5 | 44554445 | 3 | 2Ø |
| 1Ø | 44555444 | 4 | 55443344 | 4 | 44334455 | 4 | 44445544 | 3 | 1Ø |
| Ø | 45444444 | 4 | 44644445 | 6 | 64444644 | 4 | 44444444 | 6 | Ø |
| -1Ø | 44454444 | 4 | 55443344 | 4 | 44334455 | 4 | 44445544 | 4 | -1Ø |
| -2Ø | 54444544 | 5 | 43345444 | 4 | 44454334 | 5 | 44554445 | 3 | -2Ø |
| -3Ø | 44444445 | 4 | 33355444 | 4 | 44455333 | 4 | 54444444 | 5 | -3Ø |
| -4Ø | 53344554 | 4 | 43455544 | 4 | 44554434 | 4 | 44543335 | 5 | -4Ø |
| -5Ø | 53334444 | 4 | 55535554 | 4 | 45543555 | 4 | 44443335 | 5 | -5Ø |
| -6Ø | 55444445 | 5 | 55544445 | 5 | 54444555 | 5 | 54444455 | 5 | -6Ø |
| -7Ø | 66665555 | 5 | 55444444 | 5 | 44444455 | 5 | 55556666 | 7 | -7Ø |
| -8Ø | 55666666 | 5 | 44455555 | 5 | 55555444 | 5 | 66666655 | 5 | -8Ø |
| -9Ø | | | | 5 | | | | | -9Ø |

Figure 33. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

MAP 28! ELEVATION ANGLE = 2Ø DEGS TIME = 12.ØØ HOURS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 9Ø | | | | 6 | | | | | 9Ø |
| 8Ø | 77776666 | 6 | 66666555 | 5 | 55666666 | 6 | 66666677 | 7 | 8Ø |
| 7Ø | 77766665 | 5 | 55556677 | 7 | 77566666 | 6 | 66666677 | 7 | 7Ø |
| 6Ø | 77765555 | 5 | 55576667 | 7 | 76677555 | 5 | 55566677 | 7 | 6Ø |
| 5Ø | 56555565 | 5 | 55776666 | 6 | 66677755 | 5 | 55555566 | 5 | 5Ø |
| 4Ø | 56455555 | 5 | 55775554 | 4 | 45556755 | 5 | 55555465 | 5 | 4Ø |
| 3Ø | 65445666 | 6 | 54665554 | 4 | 45556645 | 6 | 66654456 | 5 | 3Ø |
| 2Ø | 55555566 | 5 | 57565544 | 4 | 44556575 | 5 | 66555556 | 7 | 2Ø |
| 1Ø | 75555554 | 5 | 57856556 | 6 | 65566875 | 5 | 54555557 | 8 | 1Ø |
| Ø | 86666444 | 4 | 68864566 | 6 | 66646886 | 4 | 44456668 | 6 | Ø |
| -1Ø | 65555555 | 5 | 57856555 | 6 | 65565875 | 5 | 55555557 | 8 | -1Ø |
| -2Ø | 65555566 | 5 | 57565544 | 4 | 44556565 | 5 | 66555556 | 7 | -2Ø |
| -3Ø | 65444666 | 6 | 55665554 | 4 | 45556645 | 6 | 66654456 | 5 | -3Ø |
| -4Ø | 56455555 | 5 | 65765554 | 4 | 45556755 | 5 | 55555465 | 5 | -4Ø |
| -5Ø | 56555566 | 5 | 55676666 | 5 | 66667655 | 5 | 55554565 | 5 | -5Ø |
| -6Ø | 77766555 | 5 | 55567667 | 7 | 76676555 | 5 | 55566777 | 7 | -6Ø |
| -7Ø | 77766666 | 6 | 55566566 | 7 | 66566555 | 6 | 66666777 | 7 | -7Ø |
| -8Ø | 77766666 | 6 | 66666555 | 5 | 55566666 | 6 | 66666777 | 7 | -8Ø |
| -9Ø | | | | 7 | | | | | -9Ø |

Figure 34. Visibility Map for a 16 Satellite System
Elevation = 2Ø Degs. (Time-Shift of Ø.1Ø Hours)

with the northern loop pulled to the east and the southern loop pulled to the west. None of these systems, however, resulted in complete coverage.

The investigation of these orbits has led to the consideration of a fourth possible small modification for avoiding collisions. It has been found that an eccentricity of 0.005 does not degrade the complete coverage capacity of the sixteen satellite system. With this eccentricity and zero argument of perigee, when the two satellites in each of the eight figure eight's make their simultaneous equatorial crossings, one is at perigee and the other at apogee. The distance between them is equal to 2*A*E which is approximately equal to 6.6*0.01 earth radii. This amounts to a separation of about 250 miles. The collision problem is avoided. The doublet problem remains, but only for the subsatellite point. This latter problem is ameliorated by the difference in the relative velocities and the difference in the directions of flight.

The data for this system is exactly the same as that given for the original system, except that the eccentricity must be incorporated in the system. The PTO parameter does not need to be redefined since perigee occurs at the ascending node crossing. The repetition interval remains exactly the same. This system was tested over a three-hour interval at time-increments of 0.15 hours.

# 7.0 CONCLUSION

In this report it has been shown that at least thirteen satellites are required at synchronous altitude for at least two satellites to be visible from any point on the surface of the earth at any time at an elevation angle of twenty degrees. A fortiori, the same statement holds for three, instead of two, satellites. The effects on the coverage properties of a satellite produced by changes in the various orbital parameters were examined in order to give some insight into where and how to search for those sets of parameters that would yield a system of satellites with certain properties.

Although it seems likely that a fifteen satellite system having the required visibility characteristics, does exist (See Appendix), none was found. The sixteen satellite system described in the report does have the required properties and, moreover, has sufficient redundancy to tolerate small positioning errors. The system is based only on Keplerian motion of the satellites. The redundancy will also permit some latitude in station-keeping.

No system was studied that utilizes orbits which are not all equal either in eccentricity or inclination, except when stationary satellites were considered in combinations with non-stationary ones. In the latter cases, various combinations of stationary satellites with satellites in inclined, circular or eccentric orbits were tested, although not exhaustively. Further, the possibility of reducing the number of satellites required, if the polar regions, above 75 degrees latitude, were excepted from the coverage requirements, has not proved to be fruitful. No system was discarded for this reason alone.

Among the many other difficult and complex aspects of the problem that will have to be resolved before a global system can come to fruition, some impinge directly on the field of orbital mechanics and require further investigation.

Among these is the degradation of the global system subsequent to the failure of one or more satellites to function properly. It is a simple matter to calculate the effect of the failure of one satellite. Because of a certain amount of redundancy in the proposed system, indicating non-optimality with respect to the number of satellites, coverage falls to about ninety-eight percent. This means there are an average of about 12 out of the 614 observation points at which the required number of satellites cannot be seen at any one time. The lack of coverage grows rather rapidly with additional satellite failures. Four grouped failures (two adjacent figure eight's) bring about a twenty-five percent drop in coverage. Of course, one cannot predict the geographic location of the hiatus in coverage since one does not know in advance which satellite will fail. Further, the uncovered area is a function of time; it changes with the motion of the system during the basic time-interval.

From another point of view, the problem of failures is the problem of the initial deployment of the system. While the former can be managed in a predictable manner, the latter seems, in large part, not amenable to mathematical techniques. Regarding the system proposed here, it can be said that, because it is so interdependent in all its parts, the area of coverage, that is, the area in which the visibility conditions are satisfied, is insignificant until almost all sixteen satellites are on station. In view of this fact, the most effective procedure for initiating the system would appear to be the following. The feasibility of the total concept should be demonstrated with a few experimental satellites. Then, as soon as a sufficient number of users are properly equipped, all sixteen satellites should be deployed as quickly as possible. Any piecemeal, individually tailored, limited systems will cause skepticism and be more costly in the long run.

Besides these difficult problems, there remains the possibility of improvement of the satellite configuration itself. The many possible combinations of the orbital parameters have by no means been exhausted. In particular, it is strange that no global system has been proposed, based on eccentric orbits. The flexibility introduced by allowing positive eccentricities does permit design of special purpose, limited systems and suggests an idea for further study, particularly from the point of view of partial independence of subsystems in a global system.

Another point that cannot finally be avoided is the long-term stability of any proposed system. Assuming a five-year lifetime for a satellite, it is well-known that over such a time-interval a satellite left unattended will wander far from its original path due to the perturbing forces exerted by the triaxiality of the earth, the moon, sun, drag, etc. For any specific systems, these forces must be calculated and provision made to counteract them or incorporate their effect in the predicted evolution of the system.

## APPENDIX

The system described in this report was culled from many other candidates. Some of the rejected systems were sufficiently close to meeting the requirement that they merit some comment. Two systems reported on by other authors appear to be susceptible to improvement or more explicit formulation. The necessary data for four other systems are listed below.

As was mentioned in the report, a fifteen satellite system was found for which only one failure was detected. The system consists of fifteen satellites in circular synchronous orbits, inclined at 59 degrees. The ground-tracks are fifteen separate figure eights, equally spaced around the equator. There are five planes with three satellites equally spaced in each plane: a 5/3 system. The data for this system are listed in Table A-1.

TABLE A-1: A 5/3, FIFTEEN SATELLITE SYSTEM
(DIMENSIONS: DEGREES AND HOURS)

| RA | CENTER | PTO | CENTER | PTO | CENTER | PTO |
|---|---|---|---|---|---|---|
| 0 | 0 | 0.0 | 120 | 8.0 | 240 | 16.0 |
| 288 | 312 | 1.6 | 72 | 9.6 | 192 | 17.6 |
| 216 | 264 | 3.2 | 24 | 11.2 | 144 | 19.2 |
| 144 | 216 | 4.8 | 336 | 12.8 | 96 | 20.8 |
| 72 | 168 | 6.4 | 288 | 14.4 | 48 | 22.4 |

This system of satellites has a repetition interval of 1.6 hours. If the visibility maps were based on the adopted grid of observation points and calculated at time-increments of 0.2 hours over the time-interval from zero to 1.6 hours, no failures would be observed. The visibility conditions would be satisfied. However, the repetition of the configuration in the second time-interval involves a rotation about the Z-axis of 48 degrees, while the earth, during the first 1.6 hours, has rotated only 24 degrees. The difference of 24 degrees introduces a displacement of four degrees in the relation between the grid and the start of the new configuration. An examination of the time-history during this second time-interval shows one failure at 20 degrees elevation.

This means that the grid size during the first interval was not small enough to pick up the failure which happened to occur between observation points. Weaknesses of this type are common to numerical methods.

Another system investigated in some detail and described in an ERC report is the so-called 3/4+4 system. It consists of sixteen satellites: four stationary and four equally spaced satellites in each of three equally spaced planes. The orbits are all circular and synchronous. The characterization of 3/4+4 is not sufficient to specify the distribution of figure eights. The configuration we found to give the best results is the following. Three satellites are equally spaced in each of four figure eights which, in turn, are equally spaced around the equator. The four stationary satellites are at the midpoints between the figure eights. Table A-2 gives the relevant data.

TABLE A-2: A 3/4+4, SIXTEEN SATELLITE SYSTEM
(DIMENSIONS: DEGREES AND HOURS)

| RA | CENTER | PTO | CENTER | PTO | CENTER | PTO | CENTER | PTO |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 90 | 6 | 180 | 12 | 270 | 18 |
| 120 | 180 | 4 | 270 | 10 | 0 | 16 | 90 | 22 |
| 240 | 270 | 2 | 0 | 8 | 90 | 14 | 180 | 20 |
| | 45 | | 135 | | 225 | | 315 | |

This system shows complete coverage at an elevation angle of 20 degrees for inclinations ranging from 69 to 78 degrees. However, for an elevation angle of 10 degrees, no clean case was found for this interval of inclinations. There is a number of failures at the higher latitudes. This is not too surprising since four of the sixteen satellites are required to remain in the equatorial plane and cannot, therefore, be seen above 70 degrees latitude.

The motivation leading to consideration of systems of this type appears to be twofold. It is assumed that stationary satellites will be the first to be deployed, and it would be economical to incorporate them in any expanded system, secondly, if one considers the global distribution of air and marine transport, the population is thin in the neighborhood of the poles. The possibility arises that fewer satellites could be used if polar coverage, say, above 75 degrees latitude, were to be neglected. It turns out, however, that the polar caps are relatively small in area when compared to the regions between 50 and 75 degrees latitude. Any system that covers these latter regions must, a fortiori, cover the former. No system that we considered was rejected because of failure in the polar regions. This is further borne out by the systems just discussed; the failures occur below 75 degrees latitude.

Among the various configurations of eighteen satellites considered in the TRW report, the one which seems to be favored consists of eighteen separate figure eights equally spaced around the equator. All satellites are in circular, synchronous orbits at 60 degrees inclination. The system is designated as 3/6: three equally spaced planes with six satellites in each plane. This system does not yield one-hundred percent coverage.

The following eighteen satellite system more than meets the visibility conditions. It consists of eighteen satellites in circular, synchronous orbit at 60 degrees inclination. It is a 9/2 system: nine equally spaced planes with two satellites in each plane or six equally spaced figure eight's with three satellites in each figure eight. On the basis of a cursory examination, we find that four satellites are visible with an elevation angle up to 14 degrees, and three are visible with an elevation angle up to 24 degrees. Table A-3 contains the data for this system.

TABLE A-3: A 9/2, EIGHTEEN SATELLITE SYSTEM
(DIMENSIONS: DEGREES AND HOURS)

| CENTER | RA | PTO | RA | PTO | RA | PTO |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 240 | 24/3 | 120 | 48/3 |
| 300 | 280 | 4/3 | 160 | 28/3 | 40 | 12/3 |
| 240 | 200 | 8/3 | 80 | 32/3 | 320 | 56/3 |
| 180 | 120 | 12/3 | 0 | 36/3 | 240 | 30/3 |
| 120 | 40 | 16/3 | 280 | 40/3 | 160 | 64/3 |
| 60 | 320 | 20/3 | 200 | 44/3 | 80 | 68/3 |

A disappointing shortcoming of the sixteen satellite system described in the body of this report is that there is no simple solution to the initial phase problem. The use of eccentric orbits partially overcomes this difficulty, but requires the deployment of more than sixteen satellites, and entails the inherent complications introduced by eccentric orbits. However, the partial independence of the subsystems may be of sufficient interest to warrant serious consideration of a system of the following type. This twenty satellite system consists of four so-called X-configurations with the four centers equally spaced around the equator. Each X-configuration consists of four satellites equally spaced in time in the same nearly circular ground-track. A fifth, stationary satellite is located at the center of the ground-track. All non-stationary orbits are inclined at 63.4 degrees and have an eccentricity of 0.59. The satellites in one pair of opposite clusters have perigees in the southern hemisphere; those in the other pair have their perigees in the northern hemisphere. Table A-4 presents the data for this system. Since eccentric orbits are being considered, it is necessary to redefine PTO.

In this table, PTO means the position at epoch that a satellite would have reached in the time, given by PTO, since its last perigee passage. The first two groups of satellites, having centers at zero and 180 degrees longitude, have an argument of perigee of -90 degrees. The second two groups, at 270 and 90 degrees longitude, have argument of perigee of +90 degrees. In addition, there are four stationary satellites, one at each of the four centers.

TABLE A-4: A TWENTY SATELLITE SYSTEM
(DIMENSIONS: DEGREES AND HOURS)

| CENTER | RA | PTO | RA | PTO | RA | PTO | RA | PTO |
|---|---|---|---|---|---|---|---|---|
| 0 | 90 | 0.0 | 0 | 6.0 | 270 | 12.0 | 180 | 18.0 |
| 180 | 225 | 3.0 | 135 | 9.0 | 45 | 15.0 | 315 | 21.0 |
| 270 | 157.5 | 1.5 | 67.5 | 7.5 | 337.5 | 13.5 | 247.5 | 19.5 |
| 90 | 292.5 | 4.5 | 202.5 | 10.5 | 112.5 | 16.5 | 22.5 | 22.5 |

The complete system satisfies the visibility conditions. Two maps are appended that are representative of the extremes of coverage for one five-satellite configuration (Figures A-1 and A-2). Not only do these figures exhibit the coverage for one subsystem, but by translation and inversion one can derive a general indication of the geographic possibilities of deploying one subsystem at a time. It is conceivable, for instance, that the United States might initiate the program by deploying one cluster for domestic and/or North Atlantic coverage. Russia could follow suit with a similar subsystem on the opposite side of the earth for use over Russia and Siberia. Then, the USA might wish to fill in the gap in the South Pacific; and, finally, Western Europe and Africa could complete the system with the fourth cluster. There is the possibility of independent operation and maintenance of the individual subsystems, but a certain degree of cooperation will be required to ensure the compatibility of the subsystems and to fully realize the capabilities of the integrated system.

Finally, a system based on Y-configurations has not been given serious consideration because it has a number of failures. However, there may be some interest in such a system. A Y-configuration consists of three satellites, equally spaced in time, tracing out the same, nearly circular ground-track, with a stationary satellite at the center. Four such clusters, arranged in a manner similar to the system above, have a maximum of six failures for some time-points and for both elevation angles during a two-hour interval with time-increments of 0.5 hours.

```
                 MAP 291  TIME =  0.00 HOURS

     LONG     -90              0              90             180
LAT
 90                            3                                    90
 80 33333333 3 33333333 3 33333333 3 33333333 3  80
 70 33333333 3 33333333 3 33333333 3 33333333 3  70
 60 12222222 3 33333334 4 43333333 3 22222221 1  60
 50 00122222 2 23334444 4 44443332 2 22222100 0  50
 40 00000122 2 22244444 4 44444222 2 22100000 0  40
 30 00000011 2 22234444 4 44443222 2 11000000 0  30
 20 00000001 1 12233444 4 44433221 1 10000000 0  20
 10 00000000 1 11233344 4 44333211 1 00000000 0  10

  0 00000000 0 11222224 4 42222211 0 00000000 0   0

-10 00000000 0 00222222 1 22222220 0 00000000 0 -10
-20 00000000 0 00011222 2 22211000 0 00000000 0 -20
-30 00000000 0 00022222 2 22222000 0 00000000 0 -30
-40 00000000 0 01122222 2 22222110 0 00000000 0 -40
-50 00000000 1 11112222 2 22221111 1 00000000 0 -50
-60 00000111 1 11111112 2 21111111 1 11100000 0 -60
-70 11111111 1 11111111 1 11111111 1 11111111 1 -70
-80 11111111 1 11111111 1 11111111 1 11111111 1 -80
-90                            1                                   -90
```

Figure A-1. Visibility Map for a 5 Satellite System
Elevation = 20 Degs.

```
                 MAP 301  TIME =  3.00 HOURS

 90                            2                                    90
 80 22222223 3 33333333 2 33333333 3 32222222 2  80
 70 22222333 3 33333333 4 33333333 3 33322222 2  70
 60 22222333 3 33333335 5 53333333 3 33322222 2  60
 50 00111333 3 33334445 5 54443333 3 33311100 0  50
 40 00000222 3 33344445 5 54444333 3 22200000 0  40
 30 00000112 2 23344445 5 54444332 2 21100000 0  30
 20 00000111 1 22244444 5 44444222 1 11100000 0  20
 10 00000011 1 12334444 5 44443321 1 11000000 0  10

  0 00000011 1 11223334 5 43332211 1 11000000 0   0

-10 00000001 1 11222222 1 22222211 1 10000000 0 -10
-20 00000001 1 11122221 1 12222111 1 10000000 0 -20
-30 00000000 0 11122211 1 11222111 0 00000000 0 -30
-40 00000000 0 00011111 1 11111000 0 00000000 0 -40
-50 00000000 0 00001111 1 11110000 0 00000000 0 -50
-60 00000000 0 00000001 1 10000000 0 00000000 0 -60
-70 00000000 0 00000000 0 00000000 0 00000000 0 -70
-80 00000000 0 00000000 0 00000000 0 00000000 0 -80
-90                            0                                   -90
```

Figure A-2. Visibility Map for a 5 Satellite System
Elevation = 20 Degs.

## REFERENCES


1. ERC Phase A, Analytical Report, Navigation Traffic Control Satellite, May, 1970, prepared by Satellite Programs Office, ERC.
2. Navigation Satellite Constellation Study. TRW Report No. 09424-6001-T000, prepared for Navy Space Systems Activity, Contract No. 123-68-C0319, Nov., 1967.