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Value of Information for Optimal Adaptive Routing in Stochastic Time-Dependent Traffic Networks: Algorithms and Computational Tools

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Abstract Real-time information is important for travelers' routing decisions in uncertain networks by enabling online adaptation to revealed traffic conditions. Usually there are spatial and/or temporal limitations in traveler information. In this research, a generic description of online information is provided based on which three types of partial online information and one no online information schemes are derived. A theoretical analysis shows that more error-free information is always better (or at least not worse) for optimal adaptive routing in flow-independent networks. For the empirical evaluation of information benefit in a general network, a heuristic algorithm is designed for the optimal adaptive routing problem with the three partial and no online information schemes, based on a set of necessary conditions for optimality. The effectiveness of the heuristic is shown to be satisfactory over the tested random networks. The work is potentially of interest for traveler information system evaluation and design.

Keywords: Traveler Information; Stochastic Time-Dependent Network; Adaptive Routing; Value of Information; Routing Policy

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Real-Time Traveler Information for Optimal Adaptive Routing in Stochastic Time-Dependent Networks

1. Introduction

An advanced traveler information system (ATIS) aims to provide travelers with real-time traffic conditions information, in the hope that better informed travelers could make better decisions. In order to assess the effects of an ATIS, a comprehensive model is needed to take into account travelers' decision making and the demand-supply interaction under the influence of ATIS. This research deals with the demand side of the problem, which describes the optimal within-day routing decisions a traveler can make with the help of real-time information on realized travel times and how much benefit can be obtained from traveler information. No demand-supply interaction is modeled in this research, i.e., travel times are not affected by travelers' choices. No day-to-day learning is modeled – the traveler is assumed to have already formed a steady perception of the random travel times and the problem is how he/she uses the real-time information to update conditional distributions of travel times on a given day and make good decisions accordingly.

The benefit of information depends on how decision makers make use of the information. For two information schemes to be comparable, we must ensure decision makers' behavior under the two schemes are consistent. For example, an in-vehicle GPS unit with the most up-to-date traffic information does not provide any benefit if the driver chooses to ignore it, while a radio message delayed for 20 minutes can help travelers divert if taken into account in a correct way. In this research, we assume decision makers optimize an objective function (e.g., minimal travel time, minimal variability) and make a series of optimal routing decisions based on time-of-day and realized link travel times during the trip. The value of traveler information is therefore defined as the difference between optimal routing outcomes (e.g., minimal expected travel time) a traveler could obtain with and without the information.

This definition of value of information is akin to that in decision analysis and information economics. A general definition from Marschak and Miyasawa (1968) is as follows:

"An information system is a set of potential messages to be received by the decision maker. It is characterized by the statistical relation of the messages to the payoff-relevant events, and also by the message cost. Neglecting this cost, the (gross) value of an information system for a given user is the (gross) payoff that he would obtain, on the average, if he would respond to each message by the most appropriate decision."

Examples of studies of the value of traveler information consistent with the above definition include Arnott et al. (1996, 1999), Levinson (2003), Denant-Boemont and Petiot (2003), Chorus et al. (2006) and de Palma and Picard (2006).

Traveler information can be characterized in a number of ways, e.g., quantitative or qualitative, historical, prevailing (realized) or predictive, and noise levels of the information. In this research, quantitative error-free information is studied that reveals realized link travel times without error. The focus is on the scope of information in time and space. In Gao and Chabini (2006), perfect online information is studied that approximates an ideal in-vehicle system providing information on all links at all time periods up to the decision time. However realistic information situations are generally limited in scope temporally and/or spatially. For example, a variable message sign (VMS) is usually fixed in location providing information on a small number of routes, and only travelers passing it can obtain the information. Radio can provide information to travelers anywhere in the radio coverage, however the scope is usually limited to major highways and arterials. Limitations on the temporal side also prevail. For example, radio traffic reports could be delayed for 15 minutes, such that at 8:00am travelers only know the traffic conditions up to 7:45am. Internet can provide travelers with access to traffic information, but might not be unavailable *en route*. It can be treated as pre-trip information, available up to the departure time.

In this research, three types of partial online information are introduced: delayed global information, global pre-trip information and radio information on a subset of links without delay. Compared with perfect online information, the first two are limited temporally and the last spatially. The contributions of the research are threefold: 1) a theoretical proof that for optimal adaptive routing in a flow-independent stochastic time-dependent (STD) network, more error-free information is always better (or at least not worse); 2) an analysis of the optimal adaptive routing problem with partial and no online information indicating that Bellman's principle of optimality does not apply, and the proposal of a set of necessary conditions for optimality; and 3) a heuristic algorithm based on the necessary conditions with polynomial running time and satisfactory effectiveness tested computationally.

The report is organized as follows. In Section 2, a literature review is given in two areas: value of traveler information and optimal routing policy problems. In Section 3, the optimal routing policy problem in an STD network is defined for partial online information situations. Section 4 presents a theoretical proof of the non-negative value of error-free traveler information. In Section 5, Bellman's principle of optimality is shown to be invalid for the problem with partial and no online information. A set of necessary conditions for optimality is then proposed and proved. A heuristic algorithm is designed based on the necessary condition and computational test results are presented. Section 6 gives conclusions and future research directions.

2. Literature Review

There are a large number of studies on traveler information since two decades ago. One critical problem is how to represent various types of information situations in a network. Under a traffic equilibrium framework, some (e.g., Hall, 1996; Yang, 1998; Levinson, 2003) assume full information for travelers with access to ATIS, which is sometimes too ideal. In Mahmassani and Jayakrishnan (1991), Hall (1996) and Engelson (2003), travelers are assumed to switch routes based on instantaneous path travel times, rather than those that they will actually experience. This assumption circumvents the need to retrieve future link travel times. In Yin and Yang (2003) and Lo and Szeto (2004), the imperfection of various ATIS is represented through random errors added to the true path travel times, and different degrees of errors suggest different information systems. Under a dynamic process framework, information could be included in travelers' learning process to represent traffic conditions from the previous day or time period (e.g., Ben-Akiva et al., 1991; Friesz et al., 1994; Emmerink et al., 1995; Jha et al., 1998; Mahmassani and Liu, 1999). A common shortcoming of these studies is that the information representation cannot be directly related to real life situations, e.g., the spatially or temporally limited information systems discussed in Section 1.

There is another school of information theoretic studies on simplified networks. Arnott et al. (1999) study effects of online information in a two-link network with random capacities under equilibrium in both departure time and route, using the bottleneck model to calculate congested travel times. Rigorous studies of zero information, full information, and imperfect information are carried out. Other studies in this school include Arnott et al. (1991, 1996), Emmerink et al.

(1998), de Palma and Picard (2006) and Chorus et al. (2006). Denant-Boemont and Petiot (2003) evaluate travel information value using human subjects' willingness to pay in an experimental setting with limited mode and route choices.

It is difficult to generalize the results in a highly simplified network to a general network. While the optimal choice problem can be solved by observation in a simplified network, algorithms are needed in a general one. Two possible types of routing problems exist in stochastic networks: non-adaptive and adaptive. Non-adaptive routing ignores information available during a trip, and thus a fixed path is determined at the origin and followed regardless of the realizations of the stochastic network. On the contrary, adaptive routing considers intermediate decision nodes, and a next link (or sub-path) is chosen based on collected information at each decision node. Adaptive routing is no worse than non-adaptive routing, since the latter can be viewed as a constrained version of the former. In this review, routing policy is used to denote the adaptive routing process. The review focuses on problems in time-dependent (as opposed to static) networks, as summarized in Table 1 with various assumptions on link stochastic dependencies and information access.

Table 1. Taxonomy of the optimal routing policy problems

Information / Network	Perfect online information	Partial online information	No online information (time-adaptive)
No time-wise or link-wise dependency		Opasanon and Miller-Hooks (2006)	**See the note below***
Complete dependency	**Gao and Chabini** (2002, 2006)	**This research**	This research
Partial dependency		**Psaraftis and Tsitsiklis (1993), Kim et al. (2005), Boyles (2006)**	

*: Hall (1986), Miller-Hooks and Mahmassani (2000), Chabini (2000), Pretolani (2000), Miller-Hooks (2001), Bander and White (2002), Nielson et al. (2003), Yang and Miller-Hooks (2004), Fan et al. (2005), Fan and Nie (2006), Pretolani et al. (2009).

In the studies of no time-wise or link-wise dependency and no online information, marginal distributions of link travel times are used and the routing is only adaptive to arrival times at decision nodes (hence the name time-adaptive). Hall (1986) studies for the first time the time-dependent version of the optimal routing policy problem, showing that in an STD network, routing policies are more effective than paths. Based on the concept of decreasing order of time, Chabini (2000) gives a dynamic programming algorithm, which is optimal in the sense that no algorithms with better time complexity exist. Miller-Hooks and Mahmassani (2000) develop a label-correcting algorithm, which Miller-Hooks (2001) compares with the dynamic programming algorithm (Chabini, 2000) computationally. Yang and Miller-Hooks (2004) extend the study to a signalized network.

Pretolani (2000) uses a hyper-path formulation of the adaptive routing problem based on arrival times. Bander and White (2002) design a heuristic approach with a promising feature: it will terminate with an optimal solution if one exists, given that the heuristic function underestimates the true cost-to-go. Fan et al. (2005) maximize the probability of arriving on time with continuous probability density functions on link travel times. Later in Fan and Nie (2006), algorithmic issues are explored for the same problem. Nielson et al. (2003) study the bicriterion time-adaptive problem.

In the case of partial online information, Opasanon and Miller-Hooks (2006) study the multicriterion adaptive routing problem with information on traversed link travel times in a statistically independent network. Later on Pretolani et al. (2009) distinguishes between time-adaptive and history-adaptive routing in a multicriteron optimization context.

Psaraftis and Tsitsiklis (1993) study networks where link costs evolve as Markov processes and travelers learn the current state of the Markovian chain at any time. The network is assumed to be acyclic to enable the design of a polynomial-time algorithm. Kim et al. (2005) study the problem in a general Markovian network with a wider information range. In Boyles (2006), conditional probabilities of adjacent link travel costs are utilized and travelers are assumed to remember only the travel time on the last link they traverse. The objective function is a general piece-wise polynomial function of arrival time at the destination.

Gao and Chabini (2002, 2006) study the problem in a general STD network with both time-wise and link-wise dependency with perfect online information. This research fills in the blank by a study of the optimal routing policy problem in the same general STD network with partial and no online information. Note that a heuristic rather than an exact algorithm is designed.

3. Problem Definition

3.1. The Network

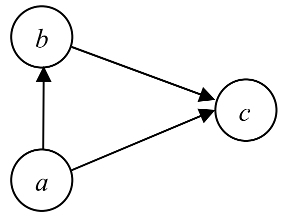

Fig. 1. An illustrative small network

Table 2. Support points for the small network ($p_1 = p_2 = p_3 = 1/3$)

Time	Link	C^1	C^2	C^3
0	(*a, b*)	1	1	1
	(*b, c*)	2	2	1
	(*a, c*)	3	3	2
1	(*a, b*)	1	1	2
	(*b, c*)	1	2	1
	(*a, c*)	3	2	2

Let $G=(N,A,T,\tilde{C})$ denote an STD network. N is the set of nodes and A the set of links, with $|N|=n$ and $|A|=m$. Assume there is at most one directional link from node j to k, denoted as (j,k). T is the set of time periods $\{0,1,\ldots,K\text{-}1\}$. A support point is defined as a distinct value (vector of values) a discrete random variable (vector) can take. A probability mass function (PMF) of a random variable (vector) is a combination of support points and the associated probabilities. In this research, a symbol with a ~ over it is a random variable (vector), while the same symbol without the ~ is its support point. The travel time on each link (j,k) at each time t is a random variable $\tilde{C}_{jk,t}$ with a finite number of discrete, positive and integral support points. The time period from 0 to K-1 is denoted as dynamic, while that beyond K-1 static. The peak hour period is generally modeled as dynamic, while off-peak as static when traffic is more stable. $\{C^1,\ldots,C^R\}$ is the universal set of support points for the joint probability distribution of all link travel times at all times, where C^r is a vector of time-dependent link travel times with a dimension of $K\times m$, $r=1,2,\ldots,R$. $C^r_{jk,t}$ is the travel time of link (j,k) at time t in the r-th support point, with probability p_r, and $\sum_{r=1}^{R} p_r = 1$. The travel time on a given link (j,k) at any time $t>K$-1 is equal to that at time K-1 for any support point: $C^r_{jk,t} = C^r_{jk,K-1}, \forall (j,k), \forall t > K-1, \forall r$.

An example network is shown in Figure 1 and Table 2 with 3 nodes, 3 links and 2 time periods. There are 3 support points, each with a probability of 1/3, for the joint distribution of 6 travel time random variables (links (a, b), (b, c) and (a, c) over time periods 0 and 1). A support point can be viewed as a distinctive day. Travel times beyond time 1 are the same as those at time 1 in each of the 3 support points.

The discrete distributions of link travel times are assumed for the convenience of defining routing policies (Section 3.4), which are based on realized travel times. Even if the underlying travel time distribution is continuous, in order to define a routing policy with finite number of states, one has to discretize the distribution. The extension of routing policy definition to continuous travel time distribution is a challenging task and will be included in the future work.

3.2. Online Information

Let H be a trajectory of (node, time) pairs a traveler could experience in the network to the current node j and current time t: $H=\{(j_0,t_0),\ldots,(j,t)\}$, where j_0 is the origin and t_0 the departure time. Denote the information coverage over links and time periods as $Q \subseteq A\times T$. Information is represented as travel time realizations on time-dependent links of Q. No predictive information is assumed, i.e., Q cannot contain elements beyond the current time t. It is assumed that there is no error in revealing the true travel times. An information scheme is defined as a mapping from trajectory H to information coverage Q, that is, information depends on traversed locations and times. Here are some examples of online information schemes with trajectory $H=\{(j_0,t_0),\ldots,(j,t)\}$:

- Perfect online information: $Q(H) = A\times\{0,1,\ldots,t\}$ (all links up to the current time t)
- Delayed global information with time lag Δ: $Q(H) = A\times\{0,1,\ldots,t\text{-}\Delta\}$ (all links up to Δ time units ago)
- Global pre-trip information with departure time t_0: $Q(H) = A\times\{0,1,\ldots,t_0\}$ (all links up to the departure time t_0)
- Radio information on $B\subseteq A$ with no time lag: $Q(H) = B\times\{0,1,\ldots,t\}$ (a subset of links up to the current time t)
- No online information: $Q(H)=\emptyset$ (no information on any link at any time)

The example in Figure 1 and Table 2 is used to illustrate different information schemes. At time 0, a traveler with perfect online information knows the travel time realizations of $\{\tilde{C}_{ab,0}, \tilde{C}_{bc,0}, \tilde{C}_{ac,0}\}$: either {1,2,3} or {1,1,2}; a traveler with global information with 1 unit time lag (LAG1) does not know any travel time realization yet; a traveler with global pre-trip information with departure time 0 has the same knowledge as with perfect online information; a traveler with radio information on link (*a, b*) with no time lag knows the travel time realization of $\tilde{C}_{ab,0}$ that is 1; and a traveler with no online information simply does not know any travel time realization. At time 1, a traveler with perfect online information knows the travel time realizations of $\{\tilde{C}_{ab,0}, \tilde{C}_{bc,0}, \tilde{C}_{ac,0}, \tilde{C}_{ab,1}, \tilde{C}_{bc,1}, \tilde{C}_{ac,1}\}$, which could be each of the 3 support points; a traveler with delayed information knows what happened at time 0 and gains the same information as with perfect online information at time 0; a traveler with pre-trip information does not gain any more information *en route* and thus his/her information remains unchanged; a traveler with radio information knows the travel time realization of $\{\tilde{C}_{ab,0}, \tilde{C}_{ab,1}\}$ that could be {1,1} or {1,2}; and a traveler with no online information still does not know any travel time realization. At time 2, only the traveler with delayed information will gain more useful information, as he/she now knows what happened in time 1. A traveler with perfect online, pre-trip or radio information does not gain any more useful information, because the information he/she had at time 1 is enough for any time periods beyond 1 due to the static period assumption. A traveler with no online information does not gain any more information by definition. Note that routing under no online information could still be adaptive to the arrival time at each decision node, which is random due to random travel times.

3.3. Event Collection

The concept of event collection is generalized from that in Gao and Chabini (2006) to the case of a general information scheme. Let $\tilde{C}_Q$ be the vector of random travel times of time-dependent links in *Q*. For a given support point C_Q, there exists one or more support points of the whole network that are expansions of C_Q. In other words, for any possible revealed link travel times of *Q*, a set of support points of the network can be identified as compatible with the information. Such a set is defined as an event collection, *EV*. It can be viewed as the conditional joint distribution of link travel times given realized link travel times in the coverage *Q*. With more information collected, information coverage Q grows and the size of EV decreases or remains unchanged. When EV becomes a singleton, a deterministic network (not necessarily static) is revealed to travelers. If a traveler has perfect online information, the network becomes deterministic no later than the start of static period K-1. If travelers have less than perfect online information, the network may remain stochastic beyond the dynamic period.

All the possible event collections with information coverage Q, denoted as $\boldsymbol{EV}(Q)$, can be generated by performing a partition of $\{C^1,\ldots,C^R\}$ based on $\tilde{C}_Q$. $\boldsymbol{EV}(Q)=\{EV_1,EV_2,\ldots\}$, where $C^r_{jk,t}$ is invariant over $r\in EV_i$, $\forall((j,k),t)\in Q$, $\forall i$, and $\exists((j,k),t)\in Q$ such that $C^r_{jk,t}\neq C^{r'}_{jk,t}$, for $r\in EV_i$, $r'\in EV_j$, $\forall j\neq i$. In other words, support points in an EV are undistinguishable in terms of revealed travel times of Q, but are distinctive from those in another EV. All the possible event collections for a given information scheme can be generated in preprocessing.

The generation of event collection can be carried out in an increasing order of time, as the information is error-free and later information will not contradict earlier one. An example from Figure 1 and Table 2 is shown here for a traveler with up-to-date radio information on link (a,b). Since the information coverage in questions depends only on the current time t and not the whole trajectory, $Q(H)$ is simplified as $Q(t)$ and $\boldsymbol{EV}(Q)$ as $\boldsymbol{EV}(t)$. At time 0, information coverage $Q(0)=\{(a,b)\}\times\{0\}$. The travel time on link (a,b) at time 0 is 1 for all 3 support points, so the partition yields only one event collection and $\boldsymbol{EV}(0)=\{\{C^1,C^2,C^3\}\}$. At time 1, information coverage $Q(1)=\{(a,b)\}\times\{0,1\}$ where the incremental information is on $\{(a,b)\}\times\{1\}$. The partition can be carried out on $\boldsymbol{EV}(0)$ based on travel time realizations of link (a,b) at time 1, which can be either 1 or 2. Therefore, $\boldsymbol{EV}(1)=\{\{C^1,C^2\},\{C^3\}\}$. In the static period, no more useful information is available, so $\boldsymbol{EV}(t)=\{\{C^1,C^2\},\{C^3\}\}$, for all $t>1$. The same logic can be applied to other information schemes.

3.4. The Decision and the Optimal Routing Policy Problem

It is assumed that travelers make decisions only at nodes. The decision is what node k to take next based on the state defined as a triplet $\{j, t, EV\}$, where j is the node, t is the time, and EV is the event collection.

Definition 1. (Routing Policy) A routing policy μ is a mapping from state to decision, for all possible states and all possible next nodes out of a given state, $\mu:\{j,t,EV\}\mapsto k$.

A routing policy can be visualized as a contingence table with as many rows as the number of combinations of node, time and event collection, and for each combination, a next node is given. A path (e.g., as defined in Ahuja et al., 1993) is a purely topological concept and a special case of a routing policy, such that the same next node is given regardless of the time and event collection. The travel time by following a routing policy (sometimes terms routing policy travel time) from any origin and departure time to a destination is a random variable, with one realization in each support point. The routing policy travel time then can be represented as a list of travel times in all support points with the associated probabilities. The routing policy itself can also be viewed as a collection of paths with the associated probabilities.

Definition 2. (Optimal routing policy problem) The optimal routing policy problem in an STD network is to find the routing policy that optimizes an objective function of routing policy travel times over all support points to a given destination, from a given origin and departure time.

Note that an optimal routing policy is not necessarily *ex post* optimal for any given support point (day), but is optimal on average over all possible support points.

The objective function could be, e.g., expected travel time, travel time variance, expected travel time schedule delay, or a combination of a number of criteria. The discussions in Section 4 are not restricted to a particular objective functional form. It however does affect the algorithm design and as such only expected travel time is dealt with in Section 5.

Let $e_\mu(j,t)$ be the objective function (to be minimized) of following routing policy μ from origin node j at departure time t to a given destination. The optimal objective function value $e^*(j,t)=\min_\mu e_\mu(j,t)$.

Given an information scheme, a partition of the universal support point set $\{C^1,\ldots,C^R\}$ at (j, t) provides the initial set of event collections $\boldsymbol{EV}(Q(j,t))$. Note that generally the event collection will change during the trip with more information (one exception being pre-trip information), as described in Section 3.3. If the objective function is additive over support points, e.g., in the case of expected travel time or expected schedule delay, an optimal routing policy for the initial universal set of support points is also optimal for any of the initial event collections. In this case, finding an optimal routing policy for the universal set of support points is equivalent to finding an optimal routing policy for each of the initial event collection, and as such Section 5 deals with optimal routing policies with regard to initial event collections. However this is not necessarily true for a non-additive objective function, e.g., variance, and in such cases, solving an optimal routing policy problem cannot be broken down to solving a number of similar problems with initial event collections.

4. Theoretical Analysis of the Value of Information

We compare the optimal routing outcomes under two information schemes 1 and 2 in the same network with different coverage.

Assumption 1. (A1) For any trajectory H, information scheme 2 has a larger coverage Q_2 than that of information scheme 1, Q_1, that is, $Q_1(H) \subseteq Q_2(H)$.

Definition 3. (S_1 contains S_2). Let S_1 and S_2 be two partitions of S. S_1 is said to contain S_2 if for any $y \in S_2$, there exists $z \in S_1$, such that $y \subseteq z$. In other words, any element of S_2 is a subset of one and only one element of S_1, and any element of S_1 is the union of one or more elements of S_1. See Figure 2 for a schematic representation.

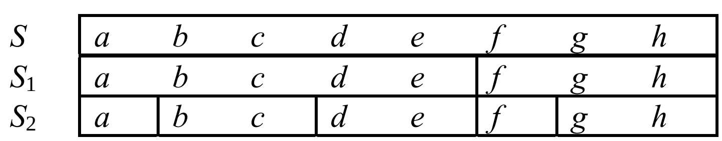

Fig. 2. A schematic view of S_1 containing S_2

Lemma 1. With assumption A1, $\boldsymbol{EV}(Q_1)$ contains $\boldsymbol{EV}(Q_2)$ for any trajectory H.

Proof. $\boldsymbol{EV}(Q_1)$ and $\boldsymbol{EV}(Q_2)$ are partitions of the set of support points $\{C^1, \ldots, C^R\}$. For any $EV_2 \in \boldsymbol{EV}(Q_2)$, travel times on time-dependent links of Q_2 are invariant across support points in EV_2. Since $Q_1 \subseteq Q_2$, travel times on time-dependent links of Q_1 are also invariant across support points in EV_2. Therefore there must exist $EV_1 \in \boldsymbol{EV}(Q_1)$ such that $EV_2 \subseteq EV_1$. **Q.E.D.**

With Lemma 1, we can proceed to compare the optimal objective function values under two different information schemes. Note that two travelers with different information schemes generally do not have the same starting information coverage and thus not the same initial set of event collections, even with the same origin and departure time. For example, assume the radio only reports travel times on the highway, while a pre-trip information source (e.g. a website) reports travel times on both the highway and arterial. There are two initial event collections under radio with the highway being normal or congested, and four initial event collections under pre-trip information, with the additional combination with the arterial being normal or congested.

The comparison of the two information schemes is based on all the possible initial event collections under each scheme.

Theorem 1. With assumption A1, the optimal objective function value under information scheme 2 is no worse than that under information scheme 1, for the same origin j_0 and departure time t_0.

$$e_2^*(j_0,t_0) \le e_1^*(j_0,t_0), \ \ \forall j_0 \in N, \forall t_0 \in T.$$

Proof. Given an optimal routing policy μ_1 under information scheme 1, an equivalent feasible routing policy μ_2 under information scheme 2 can be constructed as follows. At the original node j_0 and departure time t_0, partition the universal set of support points based on the two information schemes to obtain the initial event collection sets: $\boldsymbol{EV}(Q_1(j_0,t_0))$ and $\boldsymbol{EV}(Q_2(j_0,t_0))$. For any $EV_2 \in \boldsymbol{EV}(Q_2(j_0,t_0))$, according to Lemma 1 there must exists $EV_1 \in \boldsymbol{EV}(Q_1(j_0,t_0))$, such that $EV_2 \subseteq EV_1$. We can then set $\mu_2(j_0,t_0,EV_2)=\mu_1(j_0,t_0,EV_1)$. As μ_1 and μ_2 give exactly the same next node under any support point, they produce the same trajectory under any support point at the next decision node. Let the arrival at the next node j occur at time t, then the information coverage Q_1 is a subset of Q_2 from the same trajectory $\{(j_0, t_0), (j, t)\}$. By Lemma 1, $\boldsymbol{EV}(Q_1)$ contains $\boldsymbol{EV}(Q_2)$, therefore we can set $\mu_2(j_0,t_0,EV'_2)=\mu_1(j_0,t_0,EV'_1)$, $\forall EV'_2 \in \boldsymbol{EV}(Q_2)$, $EV'_2 \subseteq EV'_1$. The process continues and a routing policy μ_2 is constructed with exactly the same trajectory as μ_1 under any support point, and thus the same objective function value. The optimal objective function value under scheme 2 is at least as good as that from the feasible solution μ_2 by definition, and thus at least as good as the optimal objective function value under scheme 1, namely, $e_2^*(j_0,t_0) \le e_{\mu_2}(j_0,t_0) = e_{\mu_1}(j_0,t_0) = e_1^*(j_0,t_0)$. **Q.E.D.**

The intuition behind Theorem 1 is that with larger information coverage throughout the trip, one has more flexibility in every decision node based on a finer partition of the possible outcomes (support points). For example, instead of having to choose a next node based on whether the highway is congested, now one can make the decision based on whether both the highway and arterial are congested. One can always ignore the additional information on arterial and act as if only information on the highway was available, and this ensures that optimal actions under larger information coverage is at least as good.

Theorem 1 also applies when only a subset of the universal set of support points is used to evaluate routing policies. The proof is the same with the universal set replaced by the subset.

The theorem can be alternatively stated as follows: more error-free information is always better (or at least not worse) for adaptive routing in a flow-independent network. It is consistent with Marschak and Miyasawa (1968)'s Theorem 11.3 regarding noiseless information systems: if two information systems are noiseless and one is finer than (in this report's terminology, contained by) the other, then it is also more informative in the sense that "it can never have smaller value than the other for any payoff function defined on a given set of events". The decision problem in Marschak and Miyasawa (1968) is however single-staged, and Theorem 1 extends the result to a multi-staged routing decision situation in a network context.

5. Solutions to the Partial and No Online Information Cases

Theorem 1 provides a theoretical comparison between two information schemes, however it is applicable only when one coverage is larger or no smaller in both spatial and temporal dimensions. In reality an information scheme can have larger coverage in one dimension but smaller coverage in the other. In order to evaluate the value of traveler information empirically for more complicated situations, computer algorithms to solve the optimal routing policy problem with partial and no online information are needed.

Since a routing policy has a random travel time, there exist multiple optimization criteria. The expected travel time is used in the remainder of the report, as generally it is the primary criterion in routing choices. Other criteria regarding travel reliability, such as expected schedule delay and travel time variance will be explored in future research, yet some criteria are harder to deal with then others.

In this section, it is shown that Bellman's principle of optimality does not hold for the three partial or no online information problems. A heuristic algorithm is then designed and computationally evaluated.

In all the studied problems, information coverage Q is determined by the current time, instead of the whole trajectory, therefore $\boldsymbol{EV}(t)$ is used instead of $\boldsymbol{EV}(Q)$. Time lag Δ in delayed information, departure time t_0 in pre-trip information and radio coverage B in radio information are treated as exogenous system parameters. In pre-trip information with departure time t_0, $\boldsymbol{EV}(t) = \boldsymbol{EV}(t_0)$, $\forall\ t \geq t_0$.

Except for delayed information, in all other four cases no more useful information is available during static period, i.e., Q does not grow beyond K–1, because either no information is provided (pre-trip and no online information), or additional information will not enlarge Q (radio and perfect online information). In the case of delayed information, a traveler continues receiving information in the static period until K-1+Δ, at which time Q=A×T. Let T^* denote the time beyond which a traveler receives no more useful information and Q remains unchanged. We then have T^*=K-1+Δ for delayed information, and T^*=K-1 for all other four cases.

5.1. Bellman's Principle of Optimality

Proposition 1. Bellman's principle of optimality does not hold for the delayed, pre-trip, radio or no online information case. In other words, if μ^* is optimal for a given initial event collection EV_0 at (j_0,t_0), and (j,t,EV) is an intermediate state during the execution of μ^*, then the remainder of μ^* is not necessarily optimal when EV is an initial event collection at (j,t).

Proof. This can be shown through an example in Figure 3 and Table 3. Note that only relevant link travel times are shown. The travel time on link (d, c) is always 0 and not listed. No online information is assumed, such that the routing decision only depends on the arrival time at each decision node, i.e, EV = {C^1, C^2} at any node and time. The problem is to find an optimal routing policy from node a to c for departure time 0.

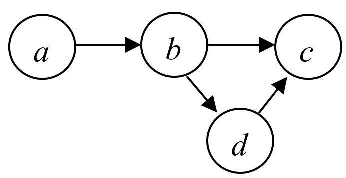

Fig. 3. An illustrative small network

Table 3. Support points for the small network (p_1 = p_2 = 1/2)

Time	Link	C^1	C^2
0	(a, b)	1	2
1	(b, c)	1	10
	(b, d)	3	3
2	(b, c)	10	1
	(b, d)	3	3

Link (a, b) has two possible travel times at time 0: 1 and 2, therefore the arrival time at node b can be either 1 or 2. As there are two alternatives to go from node b to c at each of the two possible arrival times, altogether there are four routing policies, listed in Table 4 along with the corresponding expected travel times.

Table 4. Routing policies from node *a* at time 0

	At node *a*	At node *b*		Expected travel time
		Arrival time 1	Arrival time 2	
Routing policy 1	Node *b*	Node *c*	Node *c*	2.5
Routing policy 2	Node *b*	Node *c*	Node *d*	3.5
Routing policy 3	Node *b*	Node *d*	Node *c*	3.5
Routing policy 4	Node *b*	Node *d*	Node *d*	4.5

The optimal routing policy from node *a* to *c* at departure time 0 is therefore *a-b-c* (actually a path). However, the optimal routing policy from node *b* to *c* at either departure time 1 or 2 is not the policy *b-c* with mean travel time 0.5(1+10), but *b-d-c* with mean travel time 3.

The key here is the treatment of the possibly large travel time on link (*b*, *c*). The travel time of 10 on link (*b*, *c*) can never be realized if the traveler leaves node *a* at time 0, due to the stochastic dependency between link (*a*, *b*) and (*b*, *c*). However if *b* is the origin, then the travel time of 10 is possible and should be taken into account. If link travel times are time-wise and link-wise independent, Bellman's optimality principle will hold and the no online information problem reduces to the ones studied by Miller-Hooks and Mahmassani (2000), Chabini (2000) and Miller-Hooks (2001).

Examples for the three partial online information cases can be constructed similarly. If *j* is an origin with *EV*, the calculation of expected travel time from *j* is not conditional on the past and thus includes all support points in *EV*. However, if *j* is an intermediate node, the calculation must be conditional on the traversed link travel times from the origin to the current node, which are not necessarily covered by the online information. Since link travel times are stochastically dependent, the conditional expected travel time might be different from the unconditional one. Examples can be constructed so that this discrepancy will lead to different optimal policies based on whether the node is an origin. Details of these examples are not presented due to space limit.
Q.E.D.

Bellman's principle of optimality is valid for the perfect online information case (stated formally later by combining Propositions 2 and 3). Note that in this case the online information covers everything that happened in the past, including the traversed link travel times to any intermediate node. Therefore the expected travel time with perfect online information does not depend on whether the node is an origin.

5.2. Necessary Conditions for Optimality

Proposition 1 indicates that we cannot generate an optimal routing policy by compositing the optimal next node and the optimal policy from the next node. We then present the necessary conditions for the optimal solutions in Proposition 2. Any feasible solution to the optimal routing policy problem provides an upper bound on the minimal expected travel time, yet one that satisfies the necessary conditions for optimality conceivably provides a tighter upper bound than an arbitrary solution. Therefore a heuristic algorithm is proposed to solve for the necessary conditions, and its effectiveness in terms of closeness to optimal solutions evaluated computationally. The heuristic is a generalization of the algorithm for the perfect online information problem in Gao and Chabini (2006), with a distinction in the major recursive equation.

Let $e_\mu(j,t,EV)$ be the expected travel time to the destination node d by following routing policy μ, if the departure from origin node j happens at time t with the event collection EV. $S_\mu(j,t,r)$ is the travel time to the destination node d if support point r is realized with a departure from node j (origin or intermediate) at time t by following routing policy μ. The relationship between $e_\mu(j,t,EV)$ and $S_\mu(j,t,r)$ is as follows:

$$e_\mu(j,t,EV) = \sum_{r \in EV} S_\mu(j,t,r)\Pr(r \mid EV) \tag{1}$$

where Pr(A) is the probability of event A. Note that the algorithm in Gao and Chabini (2006) for perfect online information deals with $e_\mu(j,t,EV)$ only, while $S_\mu(j,t,r)$ is needed for partial and no online information cases to correctly calculate expected travel times.

A routing policy is defined based on event collections, not support points, where an event collection includes a number of support points compatible with revealed information at the decision node and time. Conceivably an event collection is equivalent to a support point if the traveler is omnipotent and knows exactly what will happen in each day at the beginning of the day. Generally this is impossible and one has to deal with a set of possible support points, although the set size will likely decrease over time during the trip. For each support point (at the end of a day), a routing policy is manifested as a path with a deterministic travel time. For a given time t and support point r, there is one and only one corresponding event collection $EV(t,r)$, since $\boldsymbol{EV}(t)$ is a partition of the universal set of support points. This ensures that the next node of routing policy μ at (j,t,r) can be uniquely retrieved as $\mu(j,t,EV(t,r))$, and $S_\mu(j,t,r)$ can be obtained by executing μ in support point r. In the example of Figure 1 and Table 2, for a traveler with radio information on (a,b), the routing decision at node a and time 0 can only be made based on the event collection $\{C^1,C^2,C^3\}$. Let $\mu\{a,0,\{C^1,C^2,C^3\}\}=c$. The travel time by following routing policy μ starting from node a at time 0 is a random variable with possible different outcomes in different support points: $S_\mu(a,0,C^1)=3$, $S_\mu(a,0,C^2)=3$, and $S_\mu(a,0,C^3)=2$.

The recursive relationship between S_μ at node j and the succeeding node k by following μ is critical to solving the optimal routing policy problem. $S_\mu(j,t,r)$ is defined for a trip leaving node j at time t. For all the information schemes except for pre-trip, the information coverage is not a function of departure time, and thus event collections at time t and node j are the same no matter whether j is an origin or intermediate node. In this case,

$$S_\mu(j,t,r)=C^r_{jk,t}+S_\mu(k,t+C^r_{jk,t},r),\text{ where } k=\mu(j,t,EV(t,r)). \tag{2}$$

With perfect online information, the travel time on the next link (j,k) at time t, $C^r_{jk,t}$ is the same for all support points in a given EV (denoted as $\pi^{EV}_{jk,t}$), and thus taking an expectation of both sides of (2) over EV gives the following:

$$\begin{aligned}
e_\mu(j,t,EV) &= \sum_{r\in EV} S_\mu(j,t,r)\Pr(r\mid EV)\\
&= \sum_{r\in EV}\left(\pi^{EV}_{jk,t}+S_\mu(k,t+\pi^{EV}_{jk,t},r)\right)\Pr(r\mid EV)\\
&= \pi^{EV}_{jk,t}+\sum_{EV'\in \boldsymbol{EV}(t+\pi^{EV}_{jk,t})}\sum_{r\in EV'} S_\mu(k,t+\pi^{EV}_{jk,t},r)\Pr(r\mid EV')\Pr(EV'\mid EV)\\
&= \pi^{EV}_{jk,t}+\sum_{EV'\in \boldsymbol{EV}(t+\pi^{EV}_{jk,t})} e_\mu(k,t+\pi^{EV}_{jk,t},EV')\Pr(EV'\mid EV)
\end{aligned} \tag{3}$$

where $k=\mu(j,t,EV)$. In the third equality, support points at a later time $t+\pi^{EV}_{jk,t}$ is re-partitioned into finer event collections EV'. In the fourth equality, support point travel times in each EV' are summarized as the expected travel time.

Such a relationship between expected travel times at adjacent nodes generally does not exist for partial or no online information, since the derivation in (3) depends on the fact that the travel time on the next link given the current EV is fixed.

For the pre-trip information, the information coverage depends on the departure time, and thus there is an ambiguity as to which event collection r belongs to at a given time t. A different variable $S_\mu(j,t,r;t_0)$ can be defined as the travel time from node j and time t to the destination node if support point r is realized by following routing policy μ, with a departure time t_0. Similarly $e_\mu(j,t,EV;t_0)$ and $\mu(j,t,EV;t_0)$ can be defined. In this case,

$$S_\mu(j,t,r;t_0) = C^r_{jk,t} + S_\mu(k,t+C^r_{jk,t},r;t_0) \text{ where } k=\mu(j,t,EV(t,r);t_0);$$

$$e_\mu(j,t,EV;t_0) = \sum_{r\in EV} S_\mu(j,t,r;t_0)\Pr(r\mid EV)$$

We propose the following system of recursive equations to solve for the perfect online, delayed, radio and no online information problems based on the recursive equation in (2).

$$e_{\mu^*}(j,t,EV) = \min_{k\in A(j)} \left\{ \sum_{r\in EV} (C^r_{jk,t} + S_{\mu^*}(k,t+C^r_{jk,t},r))\Pr(r\mid EV) \right\} \tag{4}$$

$$\mu^*(j,t,EV) = \arg\min_{k\in A(j)} \left\{ \sum_{r\in EV} (C^r_{jk,t} + S_{\mu^*}(k,t+C^r_{jk,t},r))\Pr(r\mid EV) \right\} \tag{5}$$

$$\forall j\in N\backslash\{d\},\ \forall t,\ \forall EV\in \boldsymbol{EV}(t)$$

where $A(j)$ the set of downstream nodes out of node j. The boundary conditions are:

a) At the destination: $S_{\mu^*}(d,t,r)=0$, $\mu^*(d,t,EV)=d$, $\forall t$, $\forall EV\in \boldsymbol{EV}(t)$, $\forall r\in EV$.

b) Beyond T^*: $\mu^*(j,t\geq T^*,EV)=\mu^*(j,T^*,EV)$, $\forall j$, $\forall EV\in \boldsymbol{EV}(T^*)$, $T^*=K-1+\Delta$ for delayed information, and $T^*= K-1$ for other three cases (radio, perfect and no online information).

Note that, $S_{\mu^*}(j,t,r) = C^r_{jk^*,t} + S_{\mu^*}(k^*,t+C^r_{jk^*,t},r)$, where $k^*=\mu^*(j,t,EV(j,t))$. $S_{\mu^*}(d,t,r)$ is the travel time of the solution routing policy μ^* in support point r, not the minimum travel time calculated using a deterministic shortest path algorithm in support point r. $S_{\mu^*}(d,t,r)$ is obtained by executing μ^* after μ^* is generated.

For the pre-trip problem, a similar system of equations can be solved to obtain a solution from all nodes and all possible event collections, but with departure time t_0 only.

Proposition 2. Conditions (4) and (5) are necessary for μ^* to be an optimal routing policy for all possible initial states for the perfect online, delayed, radio and no online information problems.

Proof. Trivially, if the boundary conditions at the destination node are not satisfied, μ^* is not optimal.

At time period T^* and beyond, information coverage includes all links at all time periods. Therefore there are R event collections, each with one support point. The optimal routing policy beyond T^* is not a function of time t, as travel times and event collections do not change over time. $\mu^*(j,t\geq T^*,EV)=\mu^*(j,T^*,EV)$, $\forall j$, $\forall EV\in \boldsymbol{EV}(T^*)$. Conditions (4) and (5) become

$$e_{\mu^*}(j,T^*,\{r\}) = \min_{k\in A(j)} \{C^r_{jk,T^*} + e_{\mu^*}(k,T^*,\{r\})\} \tag{6}$$

$$\mu^*(j,T^*,\{r\}) = \arg\min_{k\in A(j)} \{C^r_{jk,T^*} + e_{\mu^*}(k,T^*,\{r\})\} \quad (7)$$

$$\forall j\in N\backslash\{d\},\ \forall r$$

plus boundary conditions. These are the optimality conditions of a static shortest path problem in a deterministic network where link travel times are C^r_{jk,T^*}, $\forall(j,k)$. If μ^* is optimal, it must manifest as a shortest path in each deterministic network defined by a support point beyond T^*, and thus (6) and (7) must be satisfied.

Assume by contradiction that (4) and (5) are not satisfied for some state with a departure time earlier than T^*. Let (j,t,EV) be such a state. Therefore there must exist an outgoing node $k\in A(j)$, such that $\sum_{r\in EV}(C^r_{jk,t} + S_{\mu^*}(k,t+C^r_{jk,t},r))\Pr(r\mid EV) < \sum_{r\in EV}(C^r_{jk^*,t} + S_{\mu^*}(k^*,t+C^r_{jk^*,t},r))\Pr(r\mid EV)$ A different routing policy μ can be constructed such that $\mu(j,t,EV)=k$, and $\mu=\mu^*$ for all other states. Then the following is obtained:

$$\begin{aligned}
&e_\mu(j,t,EV) = \sum_{r\in EV} S_\mu(j,t,r)\Pr(r\mid EV) = \sum_{r\in EV}(C^r_{jk,t} + S_\mu(k,t+C^r_{jk,t},r))\Pr(r\mid EV)\\
&= \sum_{r\in EV}(C^r_{jk,t} + S_{\mu^*}(k,t+C^r_{jk,t},r))\Pr(r\mid EV)\\
&< \sum_{r\in EV}(C^r_{jk^*,t} + S_{\mu^*}(k^*,t+C^r_{jk^*,t},r))\Pr(r\mid EV) = e_{\mu^*}(j,t,EV)
\end{aligned}$$

The third equality is due to the fact that μ and μ^* are the same at all times later than t. The equation contradicts with the fact that μ^* is optimal, therefore (4) and (5) must be satisfied for $t < T^*$. **Q.E.D.**

Proposition 3. Conditions (4) and (5) are sufficient for μ^* to be an optimal routing policy for all possible initial states in the perfect online information problem, and equivalent to the optimality conditions in Gao and Chabini (2006).

Proof. With perfect online information, $C^r_{jk,t}$ is the same for all support points in a given EV, and thus taking expectations of both sides of (4) over EV and changing (5) accordingly gives the optimality conditions in Gao and Chabini (2006), similar to the derivation in (3). The sufficiency of (4)(5) then follows from the optimality of the conditions in Gao and Chabini (2006).

5.3. Algorithm DOT-PART

In this section we design a heuristic algorithm to solve the system of equations (4)(5). The evaluation of $e_{\mu^*}(j,t,EV)$ only depends on $S_{\mu^*}(j,t',r)$ from a later time $t'>t$, due to the positive and integral link travel time assumption. Therefore the labels can be set in a decreasing order of time, making use of the acyclic property of the network along the time dimension (Chabini, 1998). At time T^* and beyond, any deterministic static shortest path algorithm can be used to compute $e_{\mu^*}(j,t,EV)$, $\forall j \in N$, $\forall t \geq T^*$, $\forall EV \in EV(T^*)$. The procedure to generate event collections carry out partitions of the universal set of support points in an increasing order of time. At time t, a partition is made on $\boldsymbol{EV}(t\text{-}1)$ based on each (link, time) pair in the incremental information coverage, $Q(t)\backslash Q(t\text{-}1)$. Note that Q is written as a function of t, because in all the five cases, Q only depends on t, not the trajectory.

Generate_Event_Collection

$D = \{C^1, \ldots, C^R\}$
If information scheme = no online, $\boldsymbol{EV}(t) \leftarrow D$, $t = 0$ to K-1, **STOP.**
For $t = 0$ to T^*
    If information scheme = perfect online, $Q(t) = A \times \{0,1,\ldots,t\}$
    If information scheme = delayed, $Q(t) = A \times \{0,1,\ldots,t - \Delta\}$
    If information scheme = pre-trip, $Q(t) = A \times \{0\}$
    If information scheme = radio, $Q(t) = B \times \{0,1,\ldots,t\}$
$Q(-1) = \varnothing$ //a proxy for convenience of representation
For $t = 0$ to T^*
    For each (link, time) pair $((j,k),t') \in Q(t) \setminus Q(t\text{-}1)$
        For each disjoint subset $S \in D$
            $D' \leftarrow$ A partition of S based on $\tilde{C}_{jk,t'}$
        $D \leftarrow$ Union of all D'
    $\boldsymbol{EV}(t) \leftarrow D$;

Algorithm DOT-PART

(Generic for perfect online, delayed, pre-trip, radio and no online information)

Initialization

Step 1:

If information scheme = delayed, $T^* = K - 1 + \Delta$; else $T^* = K - 1$.

Construct $\boldsymbol{EV}(t)$, $t=0,\ldots,T^*$ by calling Generate_Event_Collection.

Step 2:

Compute $e_{\mu^*}(j,T^*,EV)$ and $\mu^*(j,T^*,EV)$, $\forall j \in N$, $\forall EV \in \boldsymbol{EV}(T^*)$ with a static deterministic shortest path algorithm in a converted static deterministic network where link travel times are replaced by their means at time T^*.

Compute $S_{\mu^*}(j,T^*,r)$ by executing μ^* in the original static stochastic network, $\forall j \in N$, $\forall r \in EV$; set $S_{\mu^*}(j,t>T^*,r)=S_{\mu^*}(j,T^*,r)$

Step 3:
$e_{\mu^*}(j, t, EV) \leftarrow +\infty, \forall j \in N \backslash \{d\}, \forall t < T^*, \forall EV \in \mathbf{EV}(t)$
$e_{\mu^*}(d, t, EV) \leftarrow 0, S_{\mu^*}(d, t, r) \leftarrow 0, \forall t < T^*, \forall EV \in \mathbf{EV}(t), \forall r \in EV$

Main Loop
For $t = T^*$-1 down to 0 and for each $EV \in \mathbf{EV}(t)$
    For each link $(j, k) \in A$
        $temp = \sum_{r \in EV} (C^r_{jk,t} + S_{\mu^*}(k, t + C^r_{jk,t}, r)) \Pr(r \mid EV)$
        If $temp < e_{\mu^*}(j, t, EV)$ then
            $e_{\mu^*}(j, t, EV) = temp$
            $\mu^*(j, t, EV) = k$
    For each $r \in EV$ and each $j \in N$
        $k^* = \mu^*(j, t, EV)$
        $S_{\mu^*}(j, t, r) = C^r_{jk^*,t} + S_{\mu^*}(k^*, t + C^r_{jk^*,t}, r)$

According to Propositions 2 and 3, Algorithm DOT-PART is exact for the perfect online information case. It generates approximate solutions with all initial states for delayed, radio and no online information, and with departure time 0 for pre-trip information. In order to solve pre-trip case with all departure times, a loop over all departure times t_0 has to be added outside the main loop, and the main loop will be executed from T^*-1 to t_0 (not shown in the algorithm statement).

Following a similar analysis as in Gao and Chabini (2006), Algorithm DOT-PART (including Generate_Event_Collection) has a time complexity of $O(mKR\ln R + R \times \text{SSP})$ except for pre-trip information and $O(mK^2R\ln R + R \times \text{SSP})$ for pre-trip information, where SSP is the time complexity of the static deterministic shortest path algorithm. The algorithm is strongly polynomial in R, the number of support points. For real life applications, time-dependent travel time observations on all (random) links from each day can be viewed as one support point. Such data are available with the advent of advanced sensor and surveillance technologies, such as GPS and probe vehicles. The number of support points might seem exponential in the number of links, however, if we consider the high stochastic dependencies among link travel times and use observations from each day as a support point, we can safely have several years' data with the number of support points in the thousands, similar to the number of links in a medium-sized network and much less than its exponential. Running time tests are conducted with randomly generated networks that confirm the complexity analysis. The reader is referred to Gao and Huang (2009) for a detailed account of the running time test results.

5.4. Computational Tests

The objectives of the computational tests are to 1) systematically investigate the effectiveness of the heuristic, Algorithm DOT-PART in generating optimal solutions to the partial and no online information problems; and 2) study the (approximate) value of information empirically as a complement to the theoretical study in Section 4.

Algorithm DOT-PART provides upper bounds of the minimal expected travel times in partial and no online information cases since it generates (conceivably good) feasible solutions. The upper bound however can be arbitrarily loose by constructing an example similar to that in Proposition 1. We are more interested in its effectiveness on average through a systematic test over a large number of instances. We do not have an exact solution algorithm to the partial or no online information cases. However, Theorem 1 states that the optimal solution under perfect online information scheme is at least as good as the optimal solution under any partial or no online information scheme, since the former coverage is larger with any given trajectory. Therefore the optimal solution with perfect online information, which can be computed exactly by Algorithm DOT-PART, provides a lower bound of the optimal solution with any partial or no online information. The error of the heuristic, which is difference between the unknown exact solution to a partial or no online information case and the heuristic solution, is then bounded above by the difference between the perfect online information solution and the heuristic solution. Furthermore, we can also view the same difference as an upper bound on the value of perfect information compared to partial or no online information. A schematic view of these relationships for any given partial or no online information case is shown in Figure 4.

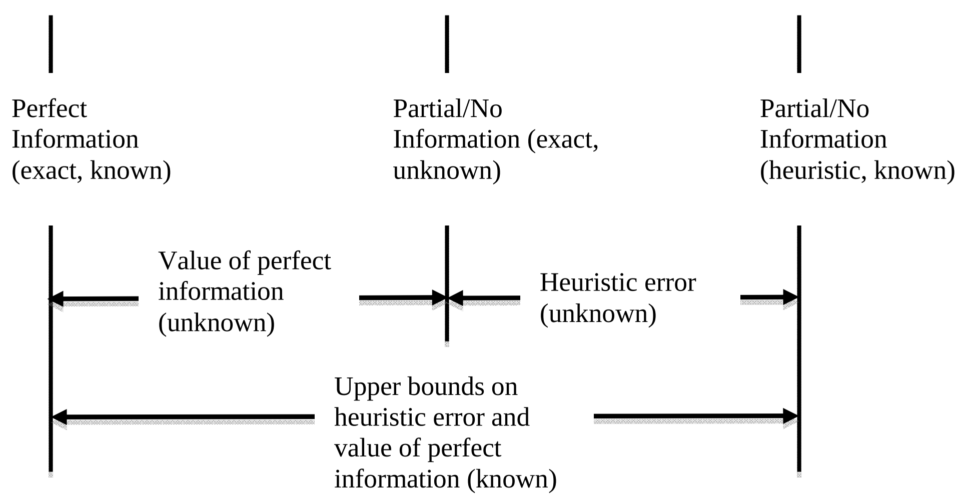

Fig. 4. Relationships between heuristic and exact solutions

The first test network is shown in Fig. 5 with 6 nodes and 8 directed links. There are diversion possibilities at nodes O, 1 and 2. The study period is from 6:30am to 8:00am. The time resolution is 1 minute for departures and arrivals at intermediate nodes, and there are 90 time periods in total. The travel time is in seconds.

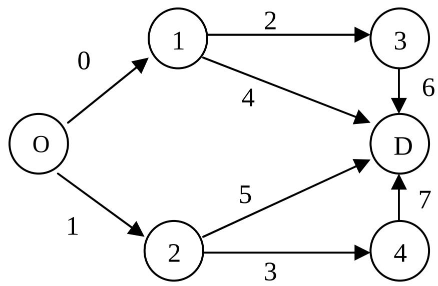

Fig. 5. The test network

The link travel time distribution is generated through an exogenous simulation with the mesoscopic supply simulator of DynaMIT (Ben-Akiva et al., 2001). The demand between the origin and destination is low from 6:30am to 7:00am and higher later on. There are random incidents in the network that result in 37 support points. Details of the network can be found in Gao (2005).

Algorithm DOT-PART is run for the three partial online, no online and perfect online information cases to derive the (upper bounds of) minimum expected travel times for each of them from node O to D for all departure times and all event collections. The results are aggregated by departure time, by taking expectations over all event collections at a given time.

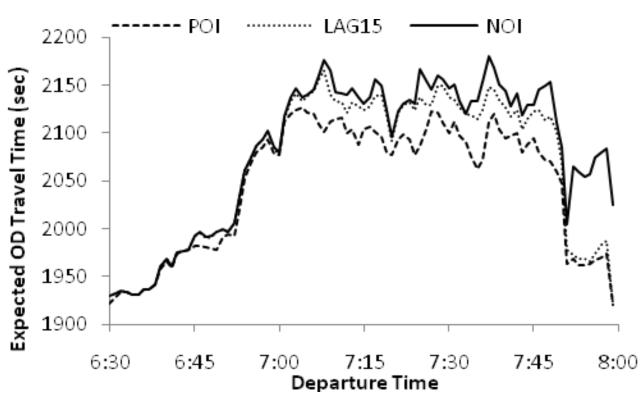

Fig. 6. Results for the 15-min delayed (LAG15) vs. perfect (POI) and no online information (NOI)

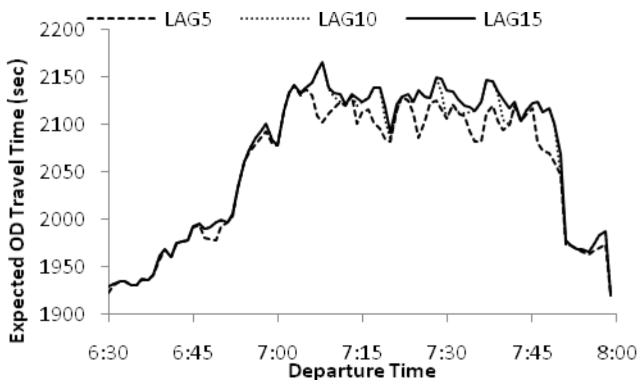

Fig. 7. Results for delayed information with 5 (LAG5), 10 (LAG10) and 15-min time lags

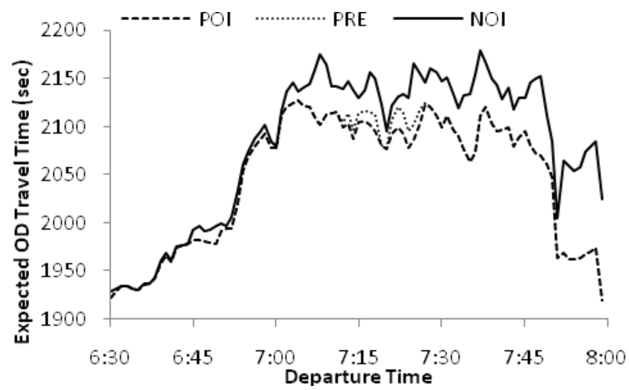

Fig. 8. Results for pre-trip (PRE) vs. perfect and no online information

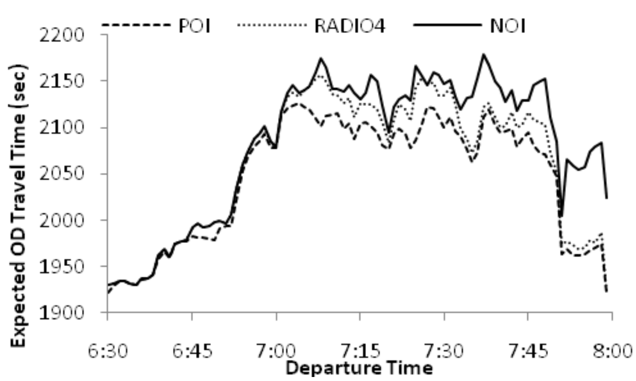

Fig. 9. Results for radio on link 4 vs. perfect and no online information

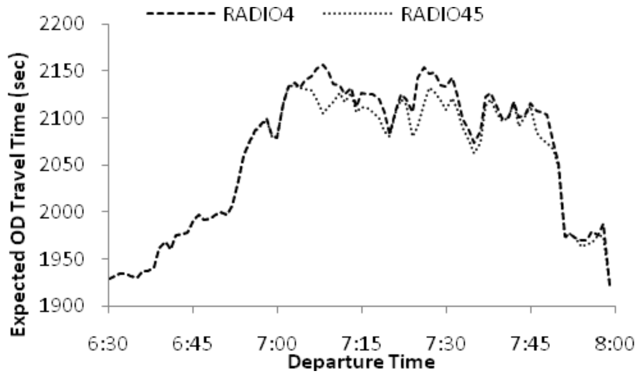

Fig. 10. Results for radio information with different radio coverage

Figures 6 through 10 show the expected OD travel times for the no online, 5-min delay, 10-min delay, 15-min delay, pre-trip, radio on link 4 and radio on links 4&5 cases. RADIO4 indicates that only traffic condition information on link 4 is available and RADIO45 on links 4 and 5. It is shown that the upper bounds generated by Algorithm DOT-PART are relatively tight: within 3% of the (unknown) exact solution. Also shown is that in the specific settings, global pre-trip information is nearly as good as perfect online information. Another interesting observation is that although the solutions to partial and no online information are not exact, they do exhibit the trend that "more error-free information is better in a flow-independent network". For example, the expected travel times with delayed information decreases when the delay decreases from 15 to 10 and from 10 to 5 minutes; and those with radio covering both links 4 and 5 are better than with radio covering only link 4. However this should not be viewed as a verification of Theorem 1.

Additional tests are conducted on larger randomly generated networks to investigate the effectiveness of the heuristics. The random network generator takes the following as input: 1) the number of nodes; 2) the number of links; and 3) the number of time periods. Four levels of the number of nodes are considered: 50, 100, 250, and 500. The number of links is always three times of the number of nodes, i.e., 150, 300, 750, and 1500. Three levels of the duration of the peak period are considered: 25, 50, and 100 time intervals. Other parameters include the number of support points fixed as 300, the range of link travel time fixed as [0, 10], and the maximum in-degree and out-degree fixed as 5. The topology of the network is randomly generated. The travel time on each link at each time interval for each support point is generated from a uniform distribution within the fixed range. More details on the random network generation can be found in Gao (2005).

Table 5. Upper bounds of heuristic errors (% difference from perfect online information)

Nodes	***Links***	***Time Periods (K)***	**No Online**	**Pre-trip**	**Delayed by 0.5*K***	**Delayed by 0.25*K***	**Radio on link 1**
50	150	25	40.3	0	14.9	6.1	2.2
50	150	50	26.6	0	11.2	4.2	0.5
50	150	100	22.3	0	10.5	4.9	0.3
100	300	25	13.8	0	5.3	2.3	0.9
100	300	50	24.4	0	10.5	4.1	0.6
100	300	100	26.0	0	12.8	6.1	0.4
250	750	25	31.4	0	12.0	5.1	1.8
250	750	50	33.9	0	14.3	5.6	0.8
250	750	100	27.0	0	12.4	5.6	0.3
500	1500	25	21.6	0	6.5	2.3	0.8
500	1500	50	26.5	0	11.4	4.5	0.7
500	1500	100	28.8	0	13.3	6.0	0.3
		Average	26.9	0	11.2	4.7	0.8

There are 12 different combinations of inputs, and 10 random networks are generated for each combination. Table 5 shows the upper bounds of heuristic errors, defined as the percentage difference of partial or no online information result from that of perfect online information. The errors are averaged over all departure times (except for pre-trip where only departure time 0 results are reported) and all origins to a single destination for each network, and then averaged over the 10 networks. The radio information covers only one link, randomly sampled 10 times for each of the 10 random networks. Thus in the radio column, the errors are averages over 100 runs.

Algorithm DOT-PART as a heuristic performs better than predicted by the theoretical worst case (arbitrarily large errors), with errors within 15% for partial online cases and 30% for most no online information. Note that these are upper bounds of errors, and the heuristic might perform better than these bounds. Future research is needed to design an exact algorithm and a more comprehensive evaluation of the effectiveness of the heuristic can then be carried out. It will also be interesting to investigate the effectiveness of the heuristic with real-world data, which is an important step towards its practical application.

We also see the same trend that "more error-free information is better in a flow-independent network". For example, information delayed for 0.25*K* unit time produces smaller expected travel time than information delayed for 0.5K unit time, which in turn is smaller than no online information. Pre-trip information is as good as perfect online information in all test scenarios, and radio information is almost as good. On the other hand, delayed information seems to perform not as well. This might suggest that up-to-date information is more valuable than information that covers a large area. However, again, since the solutions are not exact, these observations should be viewed with caution.

6. Conclusions and Future Directions

In this research a generic representation of online information in a general stochastic network is developed, based on which three types of information schemes are specialized: delayed global information, global pre-trip information, and radio information on a subset of links without time lag. The scope limitations of an information system on both the temporal and spatial dimensions are taken into account. A theoretical proof of the non-negative value of error-free traveler information for adaptive routing in a flow-independent stochastic network is presented. It is shown that Bellman's principle of optimality does not apply to the optimal routing policy problem with partial or no online information. A heuristic algorithm is then designed based on a set of necessary conditions for optimality and its effectiveness is tested empirically and shown to be satisfactory.

Other interesting information schemes will be studied in the future, e.g., VMS, which is one of the most common types of ATIS. The problem with VMS is more involved than those discussed in this report, as the information is trajectory-based rather time-based only. This could significantly complicate the algorithm design. The noise level of the information will also be considered, such that the information is no longer error-free. Theoretical studies will be conducted to establish the conditions (if existing) under which noisy information systems are comparable.

Predictive information (Bovy and van der Zijpp, 1999; Bottom, 2000; and Dong et al., 2006) that provides estimates of future travel times is not explicitly studied under the online information framework in this research. Mathematically one can easily build an information scheme where the coverage $Q(t)$ contains realized travel times beyond t, and all the analyses and algorithm in this research apply. The more fundamental question is whether an analysis framework built upon error-free information assumption is good for predictive information.

Although the error in measuring realized travel times can be reasonably assumed approaching zero with the ever-increasing accuracy of traffic surveillance, the same cannot be said for predictive information. Therefore the effort to model predictive information should be joined with that on noisy information as mentioned in the previous paragraph.

As mentioned in the introduction, the interaction between demand and supply needs to be considered to assess the value of real-time information with a large market penetration of information. In a congested un-priced network, information could be detrimental, as shown in Gao (2005) and many other studies (e.g., Arnott et al., 1991, 1999, Levinson, 2003). The next step of the research would be studies of the value of various types of information systems in a congested network. An equilibrium dynamic traffic assignment model or a day-to-day dynamic process model is to be applied.

Another interesting direction would be a theoretical quantification of the value of traveler information as a function of an array of information system and network characteristics. This would enable the cross comparison of different types of information systems. For example, is up-to-date spatially-limited information better than delayed global information? Answers to this type of questions can be obtained computationally as shown in Section 6, however a theoretical solution would provide valuable insights and guidelines for, e.g., optimal investment in ATIS.

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