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ABSTRACT

Engaging private investors and entrepreneurs through public-private partnership (PPP) in constructing and operating transportation facilities has emerged as one of the viable options to meet the challenges of funding the development and maintenance of transportation systems. PPP developments lead to additional capacities without (directly) using public funding, faster delivery of projects, risk sharing with the private sector and more efficient operations and management of facilities. However, the profit-maximizing private sector may compromise public interests by, e.g., imposing higher toll rates or failing to offer high quality of service. A rigorous up-front analysis is needed to better protect public interests prior to entering into a PPP arrangement.

This report considers the problem of selecting highway projects for the PPP development with the objective of improving the social benefit while ensuring the marketability of those selected. The problem has a structure of a tri-level leader-follower game and is formulated as a mixed integer program with equilibrium constraints. Without solving the associated problem, we show that optimal tolls and travel times on selected PPP highway projects can be determined from their attributes under mild assumptions. This leads to an efficient heuristic algorithm for solving the project selection problem.

EXECUTIVE SUMMARY

The current approach to funding the development and maintenance of surface transportation systems is no longer able to address the serious challenges we face today. Engaging private investors and entrepreneurs through public-private partnership (PPP) in constructing and operating transportation facilities has emerged as one of the viable options to meet those challenges. PPP developments lead to additional capacities without (directly) using public funding, faster delivery of projects, risk sharing with the private sector and more efficient operations and management of facilities. However, the profit-maximizing private sector may compromise the public interests by, e.g., imposing higher toll rates or failing to offer high quality of service. A more rigorous up-front analysis is needed to better protect public interests prior to entering into a PPP arrangement.

From a government's perspective, a PPP development process typically involves three stages. The first one is the project selection stage where the responsible governmental agency (or, more simply, the government) selects PPP projects that benefit society and are attractive to private firms at the same time. In the second stage, the government grants PPP concessions to private firms, typically those submitting the best bids at an auction. Finally, the government must negotiate the terms of contracts during the third stage to protect public interests during the concession period.

The objective of this project is to provide a methodology to solve the first-stage problem, i.e., the PPP project selection problem. To our best knowledge, this is the first attempt to solve such a problem. The project selection problem can be viewed as a tri-level leader-follower game with three groups of players. The first group consists of a single player, the government, who is the leader in the game and makes decisions to which others must react. As the leader, the government selects, from a pool of candidates, projects for PPP developments to maximize the social benefit while ensuring the marketability of those selected. Specifically, the toll revenue from a marketable project should be enough to cover its construction, operation and maintenance costs during the concession period. The government will offer those selected projects for bidding at an auction. Of the remaining two groups, one consists of private firms who bid for (or react to) the PPP projects offered by the government. Finally, road users are the last group of followers who must choose the route and pay the necessary tolls to travel to their destinations. In other words, road users must react to private firms' decisions, ones that previously react to the project selection by the government.

With some mild assumptions, this problem can be formulated as a mixed integer mathematical programming problem with equilibrium constraints, which is very difficult to solve mathematically. In order to efficiently solve this problem, we first analyze the private firms' behavior under the PPP setting and find that tolls and link travel times for the candidate links for PPP developments can be pre-determined under assumptions common in the literature. Based on this finding, we develop a heuristic algorithm to solve the PPP project selection problem very effectively. Results from three different networks are provided in this report to validate the model and demonstrate the efficiency of the proposed algorithm.

1. BACKGROUND

Financing the nation's surface transportation infrastructure has never been more challenging than it is today (Rosenbloom, 2009). A weakening economy, the ever-improving fuel efficiency of vehicles and other factors have acted together to erode the funding sources for transportation systems. The Highway Trust Fund, the principal mechanism for providing federal funds for highway and transit programs, is soon to be bankrupt (GAO, 2009). Similar to the last year, its highway account balance was approaching zero in June 2009. Congress subsequently approved legislation in July 2009 to appropriate $7 billion from the General Fund of the Treasury to replenish the account. Given that it is politically unpopular to raise fuel tax rates, state and local governments have been struggling to find new alternatives for transportation financing to meet the growing demand for new transportation capacity.

One alternative is to increase private sector participation in constructing and operating transportation facilities. Under this alternative, a private sector would enter into a public-private partnership (PPP) arrangement with a public agency and assume "a greater role in the planning, financing, design, construction, operation, and maintenance of a transportation facility compared to traditional procurement methods" (GAO, 2008). For example, the Florida Department of Transportation (FDOT) has recently executed a PPP contract with a private consortium headed by Spanish-owned ACS Infrastructure Development to design, build, finance, operate and maintain the $1.8 billion I-595 Corridor Improvements project (http://www.i-595.com). The concession period is 35 years and the financing includes a $603 million TIFIA loan, $750 million in private bank debt and over $200 million in private equity (Orski, 2009). It has been reported that around one-third of the Western European highway network is currently under concession, with most of them located in France, Spain, Italy and Portugal (Verhoef, 2007). Private toll roads have become a popular choice in Asia and South America as a way to add transportation capacities when governments have limited ability to finance road constructions (e.g., Tam, 1999).

Generally, PPP developments offer a variety of benefits, such as construction of new infrastructure without (directly) using public funding, faster delivery of projects, risk sharing with the private sector and more efficient operations and management of facilities. However, these benefits are not "free". The profit-maximizing private sector may compromise the public interests by, e.g., imposing higher toll rates or failing to offer high quality of service. Moreover, the PPP agreements can be more costly to the public than traditional procurement methods. Therefore, it is necessary to conduct more rigorous up-front analysis to better protect public interests prior to entering into a PPP arrangement. Unfortunately, no methodology for such analysis is readily available (GAO, 2008).

There are three critical stages for a PPP development: (1) selection of a marketable PPP project, (2) determination of a private concessioner and (3) negotiation of a concession contract. Previous studies have investigated various auction mechanisms to select concessionaires for PPP projects to protect the public interest. Given a pre-determined concession period, Verhoef (2007) and Ubbels and Verhoef (2008) discussed the choice of capacity and toll by private investors in a competitive auction organized by the government. They compared various winner determination criteria, e.g., minimum toll charged for a pre-defined capacity, maximum capacity supplied, maximum patronage and minimum subsidy, and concluded that the criterion of maximum patronage replicates the second-best social optimum under a zero-profit constraint. On the other hand, Engel et al. (1997, 2001) pointed out that fixing the length of concession period in the actions does not generally yield optimal outcomes and it leads to the frequent contract renegotiations observed in practice. Instead, they suggested an alternative winner determination criterion, i.e., the least present value of revenue, which allows the concession length to be adjustable based on effective demand. Along the same line, Nombela and De Rus (2004) proposed another auction mechanism based on a flexible-term contract and bi-dimensional bids for total revenue and maintenance cost. Tan et al. (2010) analyzed the performance of a PPP contract, which specifies the concession period, capacity and toll charge, on private firm's profit and the social welfare and concluded that any Pareto-efficient PPP contract requires that the concession period should be the whole life of the road. They also suggested that price-cap and rate-of-return regulations result in inefficient outcomes and both demand and markup charge regulations lead to Pareto-optimal outcomes. In summary, previous studies has primarily focused on determination of a private concessioner and negotiation of the PPP contract, i.e. the second and third stages of a PPP development, and . None or little has been done for the first stage. Moreover, the strategic interactions among multiple private firms have been largely ignored in the investigation of government regulatory regimes.

This study fills this void by analyzing the first-stage problem, which we refer to herein as the project selection problem. More specifically, it is to select PPP projects from a pool of candidates accounting for their spatial interactions. The projects, once completed, should substantially improve the social benefit. However, they have to be marketable such that private firms will have an interest to bid. The project selection problem has a tri-level structure and mathematically very hard to solve. In order to efficiently solve the problem, we first analyze the private firms' behaviors under the PPP developments. Based on the finding that the tolls and link travel times for private toll roads can be pre-determined under some common assumptions, an efficient heuristic algorithm is proposed to solve the project selection problem. For the remainder, Chapter 2 discusses the property associated with the private toll roads under PPP developments. Chapter 3 introduces the project selection problem and formulates the problem and develops an efficient heuristic algorithm upon this property, which is demonstrated with numerical examples. Chapter 4 concludes the report.

2. CHOICE OF CAPACITY AND TOLL ON PRIVATE TOLL ROAD

2.1 INTRODUCTION

This chapter analyzes private provision of roads (e.g. under PPP development) in an oligopolistic market. It is assumed that in a general traffic network, some or all of the roads are built and then operated by individual private firms. Each private firm only controls one road. For a private road, the concession period is pre-determined to be the road life and the firm simultaneously decides the capacity to construct and the toll rate to charge given its belief on other firms' choices in order to maximize profit. These private firms compete for the same travel demand of multiple origin–destination (OD) pairs and the underlying flow distribution is assumed to be in user equilibrium. Based on some other widely used assumptions, we prove that the level of service provided by a private firm on a particular road, represented by the volume–capacity (v/c) ratio, is independent of another competitor's choice of capacity and toll rate for another road, and is the same as the v/c ratio in the socially optimal provision of roads. We further prove the property still holds in a regulated market where the traffic authority regulates the generalized travel cost on a private road.

The property of constant v/c ratio of private toll roads is discovered by Xiao et al. (2007) who studied both toll and capacity competition in a network with one OD pair connected by parallel links. Yang et al. (2009) observed the same property in their numerical example on a simple but more general network. Verhoef et al. (2010) also derived the same results in their analysis of second best network problems. This chapter provides a theoretical proof of the existence of the constant v/c ratio property over general traffic networks and on different conditions. Furthermore, a method is proposed to predict the toll and capacity choice of private toll roads under PPP developments.

2.2 SIMULTANEOUS CHOICE OF CAPACITY AND TOLL UNDER NASH EQUILIBRIUM

Consider a general traffic network $G(N, A)$ where N is the set of nodes and A the set of links in the network. Let W be the set of OD pairs and K the set of paths connecting all OD pairs. Let d_w denote the travel demand for OD pair $w \in W$, f_k the flow on route $k \in K$ and v_a the flow on link $a \in A$.

It is assumed that there are multiple private toll roads in the network with each being controlled by one individual private firm. The firm simultaneously decides the road capacity to build and the toll charge to collect from road users. We denote the capacity and toll as c_a and τ_a respectively for link $a \in L$ where L is the set of private toll roads and $L \subseteq A$. We further assume that travelers will choose routes with the minimum generalized travel cost, which include travel time and toll charge. Consequently, the network flow distribution is in user equilibrium (e.g., Sheffi, 1985).

It is assumed that each firm attempts to choose both capacity and toll to maximize its profit and it can accurately predict the choices of other firms. Nash equilibrium will be achieved in such an oligopolistic market where no private firm is able to further increase its profit by unilaterally changing its choice on capacity or toll. In the following, we focus on the decision made by a private firm for a particular toll road, say link a, in the network. The profit-maximizing choice of toll and capacity can be obtained by solving a bi-level optimization problem below:

$$\max_{(\tau_a,c_a)} \quad \tau_a v_a - I(c_a)$$

$$s.t. \quad \tau_a, c_a \geq 0$$

where v_a is obtained by solving the following problem:

$$\min_{(x,v)} \quad \sum_{j\in A}\int_0^{v_j} t_j(\varpi, c_j)d\varpi + \sum_{j\in L} v_j.\tau_j$$

$$s.t. \quad \Lambda x^w = E^w d^w, \quad \forall w \in W$$

$$\sum_{w\in W} x_j^w = v_j, \quad \forall j \in A$$

$$x_j^w \geq 0, \quad \forall w \in W, j \in A$$

In the above, $I(c_a)$ is the amortized cost for constructing and maintaining capacity c_a; x^w is the vector of link flows for OD pair w; v is the vector aggregate link flows; $t_j(v_j, c_j)$ is the travel cost function of link j. It is assumed that the function is strictly increasing and convex with respect to v_j, and is strictly decreasing and convex with respect to c_j. Λ is the link-node incidence matrix. E^w is the “input-output” vector, i.e., a vector that has exactly two non-zero components, one with a value of 1 in the component corresponding to the origin node and the other with -1 in the component for the destination.

The upper-level problem represents the private firm’s behavior, determining capacity and toll to maximize its profit, i.e., toll revenue minus the amortized construction and maintenance cost. The lower-level problem is a tolled user equilibrium problem where c_j and τ_j for $j \in L, j \neq a$ are the choices of capacity and toll by other private firms. The lower-level problem essentially defines an implicit reaction function $v_a(\tau, c)$ for the upper-level problem where τ and c are the vector of tolls and capacities respectively. We have the following assumption regarding the function:

Assumption 2-1. The reaction function $v_a(\tau, c)$ defined by the lower-level tolled user equilibrium problem is continuously differentiable with respect to generalized link travel costs.

Lemma 2-1. If Assumption 2-1 holds, then the reaction function $v_a(\tau, c)$ satisfies the following:

$$\frac{\partial v_a / \partial c_a}{\partial v_a / \partial \tau_a} = \frac{\partial t_a}{\partial c_a}$$

Proof.

Denote the generalized cost of link $j \in A$ as $\bar{t}_j = t_j(v_j, c_j) + \tau_j$. Note that the reaction function $v_a(\tau, c)$ can be more explicitly written as $v_a(\bar{t}(\tau, c))$ where $\bar{t}(\tau, c)$ is the vector of generalized travel cost of all links in the network, because the link flow v_a is indirectly affected by the choices of toll and capacity via the generalized costs. Differentiating the reaction function with respect to the capacity and toll of link a yields:

$$\frac{\partial v_a}{\partial c_a} = \sum_{j \in A} \frac{\partial v_a}{\partial \bar{t}_j} \cdot \frac{\partial \bar{t}_j}{\partial c_a} = \frac{\partial v_a}{\partial \bar{t}_a} \cdot \frac{\partial t_a}{\partial c_a} \tag{2-1}$$

$$\frac{\partial v_a}{\partial \tau_a} = \sum_{j \in A} \frac{\partial v_a}{\partial \bar{t}_j} \cdot \frac{\partial \bar{t}_j}{\partial \tau_a} = \frac{\partial v_a}{\partial \bar{t}_a} \cdot 1 \tag{2-2}$$

The last equalities in the above two equations are due to the fact that for link $j \in A, j \neq a$, $\frac{\partial \bar{t}_j}{\partial c_a} = \frac{\partial \bar{t}_j}{\partial \tau_a} = 0$. From (2-1) and (2-2), it is straightforward to obtain:

$$\frac{\partial v_a / \partial c_a}{\partial v_a / \partial \tau_a} = \frac{\partial t_a}{\partial c_a} \tag{2-3}$$

This completes the proof of the property. □

We now discuss the condition for Assumption 2-1 to hold based on the results from sensitivity analysis of traffic equilibrium (e.g., Tobin and Friesz, 1988; Yang and Bell, 2007). Readers who have no interest may skip this paragraph. Yang and Bell (2007) defined a path connecting an OD pair as an equilibrated path if the path is one of the minimum-cost paths connecting the OD pair. Given a path flow solution to the (tolled) user equilibrium problem, if there exists at least one equilibrated path on which the path flow cannot be strictly positive, the solution is said to be degenerate. Otherwise it is a non-degenerate or regular equilibrium point. Yang and Bell (2007) further proved that if the (tolled) user equilibrium problem admits at least one non-degenerate or regular path flow solution, then the equilibrium link flows are once differentiable with respect to generalized travel costs. More specifically, the existence of a non-degenerate path flow solution is a sufficient (but not a necessary) condition for the differentiability of equilibrium link flows. Consequently, one can use the formula given by Tobin and Friesz (1988) and Yang and Bell (2007) to compute $\partial v_a / \partial c_a$ and $\partial v_a / \partial \tau_a$, and subsequently prove Lemma 2-1.

With Lemma 2-1, we now consider the choice of capacity and toll by one particular private firm under Nash equilibrium. Two additional assumptions are introduced as follows.

Assumption 2-2. The travel cost function $t_a(v_a, c_a)$ for the private toll link of interest is homogeneous of degree zero with respect to link flow v_a and capacity c_a. More specifically, $t_a(v_a, c_a) = t_a(\zeta v_a, \zeta c_a)$, where ζ is any positive scalar.

The above assumption is equivalent to assuming that the link travel cost only depends on the v/c ratio of the link. This is not a restrictive assumption, because, for example, the widely used BPR (Bureau of Public Roads) type functions satisfy it. With the assumption, the link cost function can be written as $t_a(v_a/c_a)$ (with a slight abuse of notation), leading to the following equations:

$$\frac{dt_a(v_a/c_a)}{d(v_a/c_a)} = c_a \cdot \frac{\partial t_a}{\partial v_a} = -\frac{c_a{}^2}{v_a}\frac{\partial t_a}{\partial c_a} \tag{2-4}$$

Assumption 2-3. There are neutral scale economies in capacity provision, namely, $\frac{dI_a}{dc_a} \cdot \frac{c_a}{I_a} = 1$. More specifically, $I_a = \kappa_a c_a$, where κ_a is an amortized cost for constructing and maintaining one unit of capacity.

Theorem 2-1. In a general network under Assumptions 2-1, 2-2 and 2-3, the v/c ratio provided by a profit-maximizing firm on a particular private link will be independent of another competitor's choice of capacity and toll for another link in the network.

Proof.
Consider the above bi-level program for the choice of capacity and toll by a profit-maximizing firm under Nash equilibrium. Using the implicit reaction function $v_a(\tau, c)$, the profit-maximizing problem or PM can be simplified as follows:

PM:

$$\max_{(\tau_a, c_a)} \quad \tau_a v_a(\tau, c) - \kappa_a c_a$$

$$s.t. \quad \tau_a, c_a \geq 0$$

Since we are only interested in the situation where the capacity provided by the private firm is positive[1], i.e., $\tau_a, c_a > 0$, the corresponding first-order optimality condition is:

$$v_a(\tau, c) + \tau_a \frac{\partial v_a(\tau, c)}{\partial \tau_a} = 0 \tag{2-5}$$

$$\tau_a \frac{\partial v_a(\tau, c)}{\partial c_a} = \kappa_a \tag{2-6}$$

Eliminating τ_a from the above two equations yields:

$$\kappa_a = -v_a(\tau, c) \frac{\frac{\partial v_a(\tau, c)}{\partial c_a}}{\frac{\partial v_a(\tau, c)}{\partial \tau_a}}$$

From Lemma 2-1, we have:

$$\kappa_a = -v_a(\tau, c) \frac{\partial t_a(v_a, c_a)}{\partial c_a} \tag{2-7}$$

Combining (2-4) and (2-7) yields:

$$\left(\frac{v_a}{c_a}\right)^2 \frac{dt_a(v_a/c_a)}{d(v_a/c_a)} = \kappa_a \tag{2-8}$$

The left-hand side of (2-8) is a function of v_a/c_a and the right-hand side is a constant. Because $t_a(v_a/c_a)$ is strictly convex with respect to v_a/c_a, the v/c ratio is uniquely determined by (2-8). The v/c ratio provided for the link depends on its own travel cost function t_a and unit capacity cost κ_a. Therefore, for this specific road, the v/c ratio will be constant and independent of another competitor's choice of capacity and toll for another link in the network. □

2.3 SOCIALLY OPTIMAL CHOICE OF CAPACITY AND TOLL

The social optimum condition of the network is the one where the determination of capacities and tolls are centrally planned to maximize total social welfare. The objective is equivalent to minimizing total system travel cost plus the total cost for capacity provision. With previous assumptions, the condition can be obtained by solving the following convex system optimum or SO problem:

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[1] There may be two distinct cases where the capacity provided by the private firm is zero. One is that the road is not profitable and the firm decides not to build it. The other is that the firm is a monopolist and the travel demand of the OD pair is fixed. In this situation, theoretically the firm can provide a road with (near) zero capacity and charge users an infinite amount of toll. If the demand is elastic or there exist other alternative roads connecting the same OD pair, this case will not happen.

SO:

$$\min_{(v,c)} \sum_{j\in A} v_j t_j(v_j, c_j) + \sum_{j\in L} I(c_j)$$

$$s.t. \quad \Lambda x^w = E^w d^w, \quad \forall w \in W$$

$$\sum_{w\in W} x_j^w = v_j, \quad \forall j \in A$$

$$x_j^w \geq 0, \quad \forall w \in W, j \in A$$

$$c_j \geq 0, \quad \forall_j \in L$$

By examining the first-order optimality condition of the above problem, it is easy to establish that the social optimal choice of toll charge is the so-called marginal-cost pricing toll, i.e., $\tau_j^{so} = v_j \left.\frac{\partial t_j(v_j, c_j)}{\partial v_j}\right|_{v_j = v_j^{so}}$ where the superscript SO indicates the solution to the SO problem. For a particular link a with strictly positive capacity, we have the following:

$$v_a \frac{\partial t_a(v_a, c_a)}{\partial c_a} + \kappa_a = 0 \qquad (2\text{-}9)$$

Theorem 2-2. In a general network under Assumptions 2-1, 2-2 and 2-3, the socially optimal provision of v/c ratio on one particular link is equal to the v/c ratio provided by a profit-maximizing private firm.

Proof.
Substituting (2-4) into (2-9) yields:

$$\left(\frac{v_a}{c_a}\right)^2 \frac{dt_a(v_a/c_a)}{d(v_a/c_a)} = \kappa_a \qquad (2\text{-}10)$$

Equation (2-10) is exactly the same as Equation (2-8) in the proof of Theorem 2-1. Therefore, we conclude that the v/c ratio under the social optimum condition is the same as the v/c ratio provided by a profit-maximizing private firm. □

Theorem 2-2 requires that all links in the network are optimally tolled. It is worth noting that the theorem still holds for one particular link even if the capacities of other links in the network are not chosen optimally. To see this, assuming that the capacities of other links are fixed to certain (not necessarily optimal) values, the objective function of the SO model changes to:

$$\min_{(v,c_a)} \sum_{j\in A} v_j t_j(v_j, c_j) + I(c_a)$$

where link a is the link of interest. Examining the optimality condition of the restricted SO problem, we still obtain (2-9) and thus Theorem 2-2.

As the v/c ratio of a road represents the level of service offered on the road, Theorem 2-2 implies that a profit-maximizing private firm will be able to provide the same level of service, i.e., the same quality of product to the traveling public as a welfare-maximizing centralized traffic authority does. However, the prices charged for the same level of service will be different. Below is the price charged by the profit-maximizing firm, obtained by substituting (2-7) into (2-6), and then applying (2-4):

$$\tau_a^{PM} = v_a \frac{\partial t_a(v_a, c_a)}{\partial v_a} \frac{\frac{v_a}{c_a}}{\frac{\partial v_a}{\partial c_a}}\Bigg|_{\substack{v_a = v_a^{PM} \\ c_a = c_a^{PM}}} = v_a \frac{\partial t_a(v_a, c_a)}{\partial v_a} \frac{1}{E_{v_a}^{c_a}}\Bigg|_{\substack{v_a = v_a^{PM} \\ c_a = c_a^{PM}}}$$

where $E_{v_a}^{c_a}$ is the elasticity of the demand (flow) for link a with respect to the capacity provided at the link and the superscript PM indicates the solution to the profit maximization problem or PM in Section 2.2.

2.4 SIMULTANEOUS CHOICE OF CAPACITY AND TOLL IN A REGULATED MARKET

The above discussion has been focused on unregulated markets. To protect the public interests, government agencies may regulate the market of private road provision by setting up additional requirements, e.g., the ceiling for toll level, minimum traffic flow and maximum travel cost (e.g., Tsai and Chu, 2003; Ubbels and Verhoef, 2008 and Tan et al., 2010). In this section, we examine the choice of capacity and toll by a profit-maximizing firm in an oligopolistic market regulated by an upper bound on the generalized travel cost of the private toll road. The regulation is proposed from the perspective of protecting individual travelers. The regulation will ensure travelers' travel costs on the private road less than or equal to an upper bound. We prove that such a regulation will not affect the level of service, i.e., the *v*/*c* ratio, provided by the private firm.

The regulation can be represented as:

$$t_a + \tau_a \le \hat{t}_a \tag{2-11}$$

Theorem 2-3. In a general network under Assumptions 2-1, 2-2 and 2-3, if the upper bound of generalized travel cost on one particular private link is large enough such that the choice of capacity is still strictly positive, the v/c ratio provided by the profit-maximizing firm will be the same as the one in the unregulated market.

Proof.

With a strictly positive choice of capacity (and thus toll), the first-order condition of the profit maximization problem, i.e., PM in Section 2.2, with one additional constraint (2-11) is:

$$v_a(\tau,c)+\tau_a\frac{\partial v_a(\tau,c)}{\partial \tau_a}-\mu=0 \tag{2-12}$$

$$\tau_a\frac{\partial v_a(\tau,c)}{\partial c_a}-\kappa_a-\mu\frac{\partial t_a}{\partial c_a}=0 \tag{2-13}$$

where μ is the multiplier associated with constraint (2-11).

From (2-13), we have:

$$\tau_a=\frac{\kappa_a+\mu\frac{\partial t_a}{\partial c_a}}{\frac{\partial v_a(\tau,c)}{\partial c_a}} \tag{2-14}$$

Substituting (2-14) into (2-12) yields:

$$v_a(\tau,c)+\frac{\kappa_a+\mu\frac{\partial t_a}{\partial c_a}}{\frac{\partial v_a(\tau,c)}{\partial c_a}}\cdot\frac{\partial v_a(\tau,c)}{\partial \tau_a}-\mu=0 \tag{2-15}$$

Applying the result from Lemma 2-1, we arrive at:

$$-v_a\frac{\partial t_a}{\partial c_a}=\kappa_a \tag{2-16}$$

Substituting (2-4) into (2-16), we will once again obtain:

$$\left(\frac{v_a}{c_a}\right)^2\frac{dt_a(v_a/c_a)}{d(v_a/c_a)}=\kappa_a$$

which is the same as (2-8) and (2-10) in the proof of Theorems 2-1 and 2-2 respectively. We thus complete the proof. □

An intuitive explanation for Theorem 2-3 is that when the constraint of the generalized cost is active, profit maximization still entails minimization of the sum of the capacity cost and travel time experienced by users since this allows the firm to charge the highest possible toll for the given demand. Note that minimization of the sum of travel time and capacity cost coincides with welfare maximization. Thus, the v/c ratio provided by the private firm will be the one in the social optimum, i.e., the one in the unregulated market.

The above theorem implies that the regulation does not impact the level of service provided, thus the travel time experienced by the travelers. The regulation essentially sets up a ceiling for the toll level. However, interestingly, if the toll level is directly regulated, i.e.,

$$\tau_a \leq \hat{\tau}_a \tag{2-17}$$

the v/c ratio will become larger if the above constraint is active, as demonstrated below. With a strictly positive choice of capacity and toll, the first-order optimality condition of PM with one additional constraint (2-17) includes:

$$v_a(\tau, c) + \tau_a \frac{\partial v_a(\tau, c)}{\partial \tau_a} - \lambda = 0 \tag{2-18}$$

$$\tau_a \frac{\partial v_a(\tau, c)}{\partial c_a} - \kappa_a = 0 \tag{2-19}$$

where λ is the multiplier associated with constraint (2-17).

Eliminating τ_a from (2-18) and (2-19) yields:

$$v_a(\tau, c) + \frac{\kappa_a}{\frac{\partial v_a(\tau, c)}{\partial c_a}} . \frac{\partial v_a(\tau, c)}{\partial \tau_a} - \lambda = 0 \tag{2-20}$$

Substituting (2-3) and (2-4) into (2-20) leads to:

$$\left(\frac{v_a}{c_a}\right)^2 \frac{dt_a(v_a/c_a)}{d(v_a/c_a)} = \kappa_a + \lambda \frac{v_a}{c_a^2} . \frac{dt_a(v_a/c_a)}{d(v_a/c_a)} \tag{2-21}$$

The second term in the right-hand side of (2-21) is strictly positive if constraint (2-17) is active. Since the left-hand side is monotonically increasing, the resulting v/c will be larger than the level in the situations we have discussed so far. This implies that the regulation of price cap may decrease the quality of service provided by the private firm.

2.5 SECOND-BEST CHOICE OF CAPACITY AND TOLL WITH REVENUE NEUTRAL CONSTRAINT

Section 2.3 describes the socially optimal choice of capacities and tolls of toll roads. The choice can improve the total social benefit in an optimal way. However, the choice does not guarantee that the toll revenue generated from a toll road is able to cover the construction and maintenance cost of the road, which eventually make the toll road unsustainable. It makes more practical sense to seek for a second-best choice of capacities and tolls of the toll roads to ensure that each toll road can generate enough toll revenue to pay off its cost. In this section, we discuss a model for the second-best capacity and toll choice where toll roads in the network are required to be revenue neutral.

The second-best system optimal problem can be formulated as the following bi-level problem:

$$\min_{(\tau,c)} \sum_{j\in A} v_j t_j(v_j, c_j) + \sum\nolimits_{j\in L} I(c_j)$$

$$s.t. \quad \tau_j v_j = I(c_j), \qquad \forall j \in L \tag{2-22}$$

$$\tau_j, c_j \geq 0 \tag{2-23}$$

where v is obtained by solving the following problem:

$$\min_{(x,v)} \sum_{j\in A} \int_0^{v_j} t_j(\varpi, c_j) d\varpi + \sum_{j\in L} v_j . \tau_j$$

$$s.t. \quad \Lambda x^w = E^w d^w, \quad \forall w \in W$$

$$\sum_{w\in W} x_j^w = v_j, \quad \forall j \in A$$

$$x_j^w \geq 0, \quad \forall w \in W, j \in A$$

Using the implicit reaction function $v_j(\tau, c)$ defined by the lower-level problem for each link j in the network, the above problem can be rewritten as a single-level problem:

SSO:

$$\min_{(\tau,c)} \sum_{j\in A} v_j(\tau,c) t_j(v_j(\tau,c), c_j) + \sum\nolimits_{j\in L} I(c_j)$$

$s.t.$ (2-22) and (2-23)

Now, we extend Assumption 2-1 and Lemma 2-1 to consider all the links in the network.

Assumption 2-4. The reaction function $v_j(\tau, c)$ for any link j defined by the lower-level tolled user equilibrium problem is continuously differentiable with respect to generalized link travel costs.

Lemma 2-2. If Assumption 2-4 holds, then the reaction function $v_j(\tau, c)$ satisfies the following:

$$\frac{\partial v_j/\partial c_a}{\partial v_j/\partial \tau_a}=\frac{\partial t_a}{\partial c_a},\qquad \forall j\in A \tag{2-24}$$

where link a is a private toll road.

Proof.
The proof is similar to the proof of Lemma 2-1.□

Theorem 2-4. In a general network under Assumptions 2-2, 2-3 and 2-4, for a toll road a, if the constraint $c_a \geq 0$ is non-binding at an optimal solution to SSO and the multipliers associated with (2-22), λ_a, does not equal -1[2], then the v/c ratio provided by the toll road under the second-best social optimal condition with revenue neutral constraint will be the same as first-best social optimal condition, and also the same as the one provided by the private profit-maximizing firm in the unregulated market.

Proof.
Consider the toll road a. Because $c_a \geq 0$ is non-binding, it is straightforward to obtain from constraint (2-22) that $\tau_a \neq 0$. Then the first-order optimality conditions for SSO reduce to the following:

$$\sum_{j\in A}\frac{\partial v_j}{\partial \tau_a}t_j(v_j,c_j)+\sum_{j\in A}v_j\frac{\partial t_j(v_j,c_j)}{\partial v_j}\frac{\partial v_j}{\partial \tau_a}-\sum_{j\in L}(\lambda_j\frac{\partial v_j}{\partial \tau_a}\tau_j)-\lambda_a v_a=0 \tag{2-25}$$

$$\begin{aligned}&\sum_{j\in A}\frac{\partial v_j}{\partial c_a}t_j(v_j,c_j)+\sum_{j\in A}v_j\frac{\partial t_j(v_j,c_j)}{\partial v_j}\frac{\partial v_j}{\partial c_a}+v_a\frac{\partial t_a(v_a,c_a)}{\partial c_a}+\kappa_a\\&-\sum_{j\in L}(\lambda_j\frac{\partial v_j}{\partial c_a}\tau_j)+\lambda_a\kappa_a=0\end{aligned} \tag{2-26}$$

Combining (2-24), (2-25) and (2-26) yields:

$$v_a\frac{\partial t_a(v_a,c_a)}{\partial c_a}+\kappa_a+\lambda_a\kappa_a+\lambda_a v_a\frac{\partial t_a}{\partial c_a}=0$$

After rearranging the terms on the left and using the assumption that $\lambda_a \neq -1$, we have the following:

$$\left(\frac{\partial t_a(v_a,c_a)}{\partial c_a}v_a+\kappa_a\right)(\lambda_a+1)=0$$

[2] See Appendix A for discussions on the situation when $\lambda_a=-1$.

$$\frac{\partial t_a(v_a, c_a)}{\partial c_a} v_a + \kappa_a = 0 \qquad (2\text{-}27)$$

From Assumption 2-2 and (2-4), the above is equivalent to:

$$\left(\frac{v_a}{c_a}\right)^2 \frac{dt_a(v_a/c_a)}{d(v_a/c_a)} = \kappa_a$$

which is the same as (2-8) and (2-10) in the proof of Theorems 2-1 and 2-2 respectively. We thus complete the proof. □

Theorem 2-4 is a very useful result, especially for the PPP project selection problem described in Chapter 3. It provides a way to directly determine the v/c ratio of a toll road without solving the capacity and toll optimization problem.

3. OPTIMAL SELECTION OF PPP PROJECTS ON TRANSPORTATION NETWORKS

3.1 PROBLEM DESCRIPTION

In this chapter, we focus on formulating the first-stage decision in the PPP development as described in Chapter 1, i.e., how to select highway links in a transportation network for PPP developments in order to maximize the social benefit.

We view the project selection problem as a tri-level leader-follower game with three groups of players. The first group consists of a single player, the government, who is the leader in the game and makes decisions to which others must react. As the leader, the government selects, from a pool of candidates, projects for PPP developments to maximize the social benefit while ensuring the marketability of those selected. Specifically, the toll revenue from a marketable project should be enough to cover its construction, operation and maintenance costs during the concession period. The government will offer those selected projects for bidding at an auction. Our formulation in the next section assumes that the concession period is pre-determined and bids are in terms of, e.g., the capacity of and the toll rate for the highway to be constructed. Of the remaining two groups, one consists of private firms who bid for (or react to) the PPP projects offered by the government. Finally, road users are the last group of followers who must choose the route and pay the necessary tolls to travel to their destinations. In other words, road users must react to private firms' decisions, ones that previously react to the project selection by the government.

Our formulation in the next section is an optimization problem whose solution identifies projects selected for PPP developments (i.e., to be offered at an auction). To make the problem tractable, the formulation relies on several assumptions that are mild and common in the literature (see, e.g. as Verhoef, 2007; Ubbels and Verhoef, 2008). It is assumed that there are a sufficient number of private firms interested in bidding for the PPP projects and thus the auction is under a perfect competition. Consequently, concessioners with the winning bids will receive, theoretically, a zero profit. We further assume that the auction mechanism is well designed so that the winning bids replicate the second-best social optimum under the zero-profit constraint. For road users, we assume that they always switch to a route with a lower generalized cost when one exists. Thus, the social benefit in our formulation is calculated based on traffic flows that are in Wardropian user equilibrium.

To describe the action of road users, let $G = (N, A)$ be a directed network where N and A are the set of nodes and directed links, respectively. For the latter, let L represent the set of candidate links for PPP developments and $L \subseteq A$. Each link $j \in A$ has an associated travel time function, $t_j(v_j, c_j)$, that depends on link flow v_j and capacity c_j. For link $j \in A - L$, the capacity is given while for those in L, their capacities will be decision variables. We assume that

$t_j(v_j, c_j)$ is differentiable and monotonically increasing with respect to v_j and decreasing with respect to c_j. The set of OD pairs is denoted as W and d^w represents the demand for OD pair $w \in W$, which is assumed to be fixed. To satisfy demands, the variable x_j^w denotes the flow on link j for OD pair w and v_j is an aggregation of x_j^w. Then, the set of all feasible flow distributions for the network, denoted as Φ, can be described as follows:

$$Ax^w = E^w d^w, \quad \forall w \in W, \tag{3-1}$$

$$\sum_{w \in W} x_j^w = v_j, \quad \forall j \in A, \tag{3-2}$$

$$v_j \leq M b_j, \quad \forall j \in L, \tag{3-3}$$

$$x_j^w \geq 0, \quad \forall w \in W, j \in A, \tag{3-4}$$

where A is the link-node incidence matrix and E^w is the "input-output" vector, i.e., a vector that has exactly two non-zero components, one with a value of 1 in the component corresponding to the origin node and the other with -1 in the component for the destination. Constraint (3-1) ensures that flows emanating from and terminating at every node are balanced. Constraint (3-2) computes the aggregate flow v_j from individual flow x_j^w. As stated above, Φ depends on b_j, a binary variable representing the decision whether to select a candidate link $j \in L$ for PPP development. Specifically, $b_j = 1$ indicates that j is selected and $b_j = 0$ says otherwise. In addition, M is a sufficiently large number to allow a flow on link $j \in L$ when $b_j = 1$.

As assumed earlier, the government selects projects to maximize the social benefit. Because the travel demand is constant, maximizing the social benefit is equivalent to minimizing the system cost or the sum of the total travel time (into monetary units) and the cost of building and maintaining the PPP projects. Then, the project selection problem (PSP) can be formulated as follows:

PSP:

$$\min_{\tau,c,b,v} \sum_{j \in A} \theta v_j t_j(v_j, c_j) + \sum_{j \in L} I_j(c_j)$$

$$s.t. \quad \sum_{j \in A} \theta t_j(v_j, c_j)(u_j - v_j) + \sum_{j \in L} \tau_j (u_j - v_j) \geq 0, \qquad \forall u \in \Phi \tag{3-5}$$

$$\tau_j v_j = I_j(c_j), \qquad \forall j \in L \tag{3-6}$$

$$\sum_{j \in L} b_j \leq s \tag{3-7}$$

$$b_j = \{0,1\}, \qquad \forall j \in L \tag{3-8}$$

$$\tau_j, c_j \geq 0, \qquad \forall j \in L \tag{3-9}$$

$$v \in \Phi$$

In the above, $I_j(c_j)$ is the amortized cost for providing and maintaining capacity c_j on link j, τ_j is the toll rate charged on a PPP link j and the parameter s is the maximum number of links that can be selected for PPP developments.

The objective function of PSP is to minimize the system cost as defined previously. Constraint (3-5) is a variational inequality and ensures that v_j satisfies the equilibrium conditions with tolls τ_j and capacities c_j (Patriksson, 1994). Constraint (3-6) is the zero-profit condition under perfect competition. Constraint (3-7) limits the total number of selected PPP projects to s. Constraints (3-8) and (3-9) force the selection variables to be binary and the associated tolls and capacities to be nonnegative.

As formulated, PSP is a mathematical program with equilibrium constraints (MPEC) that contains integer decision variables. In optimization, MPEC is a class of problems difficult to solve because its feasible region is non-convex and none of its feasible solution satisfies the Magasarian-Fromovitz constraint qualification (see, e.g., Chen and Florian, 1995; Scheel and Scholtes, 2000). Moreover, the presence of binary variables makes PSP more complex than MPEC with only continuous variables.

The next section discusses the properties of a problem that is related and useful in developing algorithms for solving PSP. Specifically, we separate the decision variables of PSP and examine a reduced problem where the candidate links have been selected. Given a selection of PPP projects, i.e. given the value of b_j, PSP reduces to a problem that determines the capacity and toll rate for each PPP link and evaluates the system cost associated with the selection. More specifically, it is a sub problem of PSP that only considers the behavior of the private toll road and road users. Below, we refer to this sub-problem as the system cost evaluation problem or SCEP.

3.2 PROPERTIES OF THE SYSTEM COST EVALUATION PROBLEM

Given the government's selection of candidate PPP projects to put up for bids at an auction, the cost associated with such action can be determined by solving the System Cost Evaluation Problem as formulated below:

SCEP:

$$\min_{\tau,c,v} \sum_{j\in A} v_j t_j(v_j, c_j) + \sum_{j\in \hat{L}} I_j(c_j)$$

$$s.t. \sum_{j\in A} t_j(v_j, c_j)(u_j - v_j) + \sum_{j\in \hat{L}} \tau_j(u_j - v_j) \geq 0, \quad \forall u \in \Phi \tag{3-10}$$

$$\tau_j v_j = I_j(c_j), \quad \forall j \in \hat{L} \tag{3-11}$$

$$\tau_j, c_j \geq 0, \quad \forall j \in \hat{L} \tag{3-12}$$

$$v \in \Phi$$

In the above, $\hat{L}$ represents the set of PPP projects selected for the auction and $\hat{L} \subseteq L$. In terms of PSP, $b_j = 1, \ \forall j \in \hat{L}$.

As formulated, SCEP is another MPEC with a structure similar to a continuous network design problem, an extensively studied problem in the literature. Algorithms proposed for the design problem include the sensitivity- analysis-based algorithm (Friesz et al., 1990), the pattern search method (Abdulaal and LeBlanc, 1979), the gap-function-based approach (Meng et al., 2001), meta-heuristics (Yin, 2000) and relaxation method (Ban et al., 2006). See Yang and Bell (1998) for a more extensive review.

Note that SCEP is essentially the same problem as discussed in Section 2.5 where the toll road capacities and tolls are chosen to maximize the social benefit subject to the revenue neutral constraint. Thus, Theorem 2-4 is applicable to SCEP, i.e., the v/c ratio of each toll link can be predetermined. We further show below that the travel time and the toll charge can also be pre-determined. Base on this result, the optimal solution to SCEP can be obtained by solving a convex optimization problem to be formulated.

Corollary 3-1. Under the same assumptions as Theorem 2-4, the travel time on link $a \in \hat{L}$ can be pre-determined.

Proof. Because the link travel time only depends on the v/c ratio, the corollary follows immediately from Theorem 2-4.□

Corollary 3-2. Under the same assumptions as Theorem 2-4, the toll on link $a \in \hat{L}$ must equal its marginal external cost and can be pre-determined.

Proof. From (2-27), for toll link a

$$\frac{\partial t_a(v_a, c_a)}{\partial c_a} v_a + \kappa_a = 0 \tag{3-13}$$

Multiplying c_a to both sides of (3-13) and applying Assumption 2-2 yield:

$$v_a\left(v_a\frac{\partial t_a(v_a,c_a)}{\partial v_a}\right)=\kappa_a c_a \tag{3-14}$$

Comparing (3-14) to (3-11), it follows that

$$\tau_a=v_a\frac{\partial t_a(v_a,c_a)}{\partial v_a}$$

From the zero-profit constraint (3-11), we have:

$$\tau_a=\frac{\kappa_a}{v_a/c_a}$$

Because the v/c ratio is constant, the above τ_a is also constant and depends only on the attributes of the link itself. □

Although the tolls in Corollary 3-2 are in the form of marginal-cost tolls, they are not necessarily first-best (e.g. Button, 1993) because not allowing tolls on links not in $\hat{L}$ effectively makes the solution to SCEP second-best.

Given the above corollaries, we show below that the optimal solutions to SCEP can be constructed, under some conditions, from the solutions to a convex nonlinear program. A similar idea is presented in Verhoef et al. (2010) without proof.

Let $\overline{t_j}$ and $\overline{\tau_j}$ be the pre-determined optimal link travel time and toll for $j\in\hat{L}$ according to Corollaries 3-1 and 3-2. Then, the following problem (the tolled user equilibrium problem or TUEP) determines the equilibrium flow when tolls are present:

TUEP:

$$\min_{v}\quad\sum_{j\in A-L}\int_0^{v_j}t_j(\omega)d\omega+\sum_{j\in\hat{L}}\overline{t_j}\,v_j+\sum_{j\in\hat{L}}\overline{\tau_j}\,v_j$$

$$s.t.\quad v\in\Phi$$

Given v^*, an optimal solution to TUEP, we construct a triplet $(v^*,c^*,\overline{\tau})$ by setting $c_j^*=v_j^*/\overline{\gamma_j}$, $\forall j\in\hat{L}$, where $\overline{\gamma_j}$ is the optimal v/c ratio according to Theorem 2-4, and $c_j^*=0$, $\forall j\in L-\hat{L}$. Because the solution to TUEP may not be unique, Lemma 3-1 below states that all the triplets constructed from solving TUEP yield the same system cost. Lemma 3-2 further states that $(v^*,c^*,\overline{\tau})$ is feasible to SCEP. Theorem 3-1 then shows that the triplet is optimal to SCEP under some conditions.

Lemma 3-1. When TUEP admits multiple optimal solutions, all the triplets constructed as above must yield the same system cost.

Proof. Assume that v^* and v' are two optimal solutions to TUEP, and two triplets, i.e., $(v^*, c^*, \bar{\tau})$ and $(v', c', \bar{\tau})$, can be constructed accordingly. It follows from the Theorem 2.5 of Patriksson (1994) that the optimal link travel times from TUEP are unique. Because $t_j(v_j, c_j)$ is assumed to be strictly increasing with respect to v_j for all $j \in A - L$, the optimal link flows for these links are also unique, i.e., $v_j^* = v_j'$, $\forall j \in A - L$. Consequently,

$$\sum_{j \in A-L} \int_0^{v_j^*} t_j(\omega) d\omega = \sum_{j \in A-L} \int_0^{v_j'} t_j(\omega) d\omega$$

Because v^* and v' yield the same objective value of TUEP, we then have

$$\sum_{j \in \hat{L}} \bar{t}_j \, v_j^* + \sum_{j \in \hat{L}} \bar{\tau}_j \, v_j^* = \sum_{j \in \hat{L}} \bar{t}_j \, v_j' + \sum_{j \in \hat{L}} \bar{\tau}_j \, v_j'$$

Consequently,

$$\sum_{j \in A-L} t_j(v_j^*) \, v_j^* + \sum_{j \in \hat{L}} \bar{t}_j \, v_j^* + \sum_{j \in \hat{L}} \bar{\tau}_j \, v_j^* = \sum_{j \in A-L} t_j(v_j') v_j' + \sum_{j \in \hat{L}} \bar{t}_j \, v_j' + \sum_{j \in \hat{L}} \bar{\tau}_j \, v_j'$$

Thus the two triplets yield the same system cost.□

Lemma 3-2. $(v^*, c^*, \bar{\tau})$ is feasible to SCEP.

Proof. Because TUEP is essentially a tolled user equilibrium problem associated with c^* and $\bar{\tau}$, v^* must satisfy (3-10) in SCEP.

By construction, $(c^*, \bar{\tau})$ also satisfies the zero-profit constraint (3-11). Thus, $(v^*, c^*, \bar{\tau})$ is feasible to SCEP.□

Theorem 3-1. If there exists an optimal solution to SCEP in which $c_j > 0$, $\forall j \in \hat{L}$, then $(v^*, c^*, \bar{\tau})$ solves SCEP.

Proof. To obtain a contradiction, assume that $(v^*, c^*, \bar{\tau})$ is not an optimal solution to SCEP. Let $(\hat{v}, \hat{c}, \hat{\tau})$ be the optimal solution to SCEP and $\hat{c} > 0$. From Theorem 4-2, it follows that $\bar{t}_j = t_j(\hat{v}_j, \hat{c}_j)$ and $\hat{\tau}_j = \bar{\tau}_j$, $\forall j \in \hat{L}$. $\hat{v}$ is the tolled user equilibrium flow distribution associated with $\bar{t}$ and $\bar{\tau}$, thus $\hat{v}$ solves TUEP. Also, Lemma 3-1 implies that $(v^*, c^*, \bar{\tau})$ and $(\hat{v}, \hat{c}, \hat{\tau})$ must yield the same system cost. From Lemma 3-2, $(v^*, c^*, \bar{\tau})$ must be feasible to SCEP. Thus, $(v^*, c^*, \bar{\tau})$ is optimal to SCEP, which contradicts the earlier assumption. In other words, $(v^*, c^*, \bar{\tau})$ solves SCEP.□

Theorem 3-1 suggests that if there exists a solution to SCEP with $c_j^* > 0$ for all $j \in \hat{L}$, then the same solution can be obtained by solving the convex optimization problem TUEP. However, the converse may not hold. Specifically, the solution constructed from TUEP, i.e., $(v^*, c^*, \bar{\tau})$, may not be an optimal solution to SCEP even when $c^* > 0$. The following example demonstrates this situation.

Consider a network in Figure 3-1 with four nodes and one OD pair, (1,4), with a demand of 4 units. The functions next to the links determine their travel times and the value of travel time, θ, is 1. The link (3, 2) is the only candidate for PPP development with an amortized cost 0.3 per unit capacity.

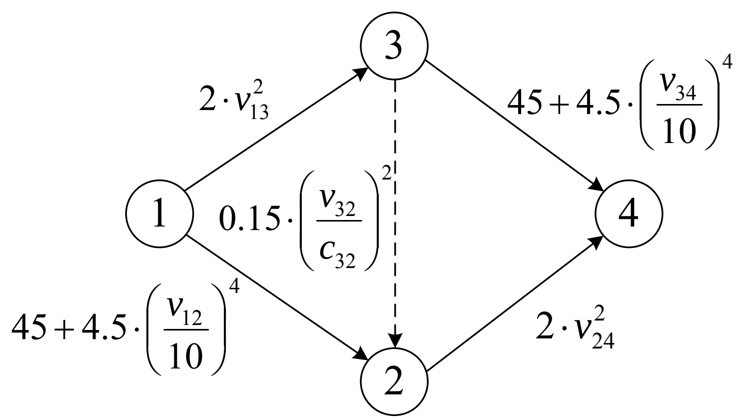

Figure 3-1: Four-Node Network

It is easy to verify that the network in Figure 3-1 satisfies Assumptions 2-2, 2-3 and 2-4. If the PPP link (3, 2) is chosen to be built, i.e., $c_{32} > 0$ in the optimal solution to SCEP, its v/c ratio and toll rate should be 1 and 0.3, respectively according to Theorem 2-4 and Corollary 3-2. With this information, we formulate TUEP, whose optimal solution leads to an optimal capacity of 4 units for link (3, 2) and the total system cost of 257.8. However, solving SCEP directly produces a total system cost of 212.0 and a zero capacity for the PPP link instead.

The network in Figure 3-1 is an instance of Braess' Paradox (Braess et al., 2005). It demonstrates that if the optimal solution to SCEP requires some links not to be built (i.e., private firms have no interest in them), the solution from TUEP only provides an upper bound to SCEP. One way to obtain an exact solution is to enumerate the subset of the current selection (i.e., setting b_j from 1 to 0 for some $j \in \hat{L}$) and solve a series of TUEP. The solution with the minimum system cost will be the exact solution to SCEP. This process can be time consuming. The next section proposes an algorithm that avoids such an enumeration for computational efficiency.

3.3 ALGORITHM

This section presents a heuristic procedure for solving PSP. It is similar in spirit to the active set algorithm proposed by Zhang et al. (2009) for the discrete network design problem. We attempted to apply the active set algorithm in a more direct way, but found that it is not as efficient as the one presented below.

Given an initial selection of PPP projects, we solve SCEP to evaluate the selection. The procedure then estimates the marginal benefit of changing the status of each PPP candidate link from the current selection via a finite differencing scheme. Based on these marginal benefits, a knapsack problem is solved to find a plan for updating the current project selection that yields the most decrease in the system cost. The procedure continues until the system cost cannot be further reduced.

Denote an initial selection of PPP candidate links as b^0. Instead of solving SCEP directly, we determine the optimal travel times and tolls of those PPP links according to Corollaries 3-1 and 3-2, and then formulate TUEP accordingly. Solving TUEP yields an optimal system cost, denoted as $z(b^0)$. Let e_j denote a vector of size $|L|$whose element j equals 1 and the others are 0. The marginal benefit of changing the status of PPP project j is calculated as $q_j = z(|e_j - b^0|) - z(b^0)$ via solving another TUEP with $|e_j - b^0|$. The process repeats $|L|$ times until all the marginal benefits are obtained.

The individual marginal benefits may not accurately predict the change in the system cost for two reasons: (a) the solution constructed from TUEP may not be the exact solution to SCEP, but provides an upper bound; (b) multiple variables in b can be changed simultaneously. However, they can be used as indicators of descent directions for the problem. A negative q_j indicates that changing the value of b_j will reduce the system cost. However, doing so may make the new selection violate constraint (3-7). Therefore, we formulate the following knapsack problem to determine an improved update of the selection of PPP projects.

KP:

$$\min_{g} \sum_{j \in L} q_j g_j$$

$$s.t. \sum_{j \in L/P(b)} (b_j + g_j) + \sum_{j \in P(b)} (b_j - g_j) \leq s \tag{3-15}$$

$$\sum_{j \in S_1^h} g_j + \sum_{j \in S_0^h} (1 - g_j) \leq m - 1, \qquad \forall j = 1, \dots, \bar{h} \tag{3-16}$$

$$g_j \in \{0,1\}, \qquad \forall j \in L$$

where m is the total number of candidate links and g_j is a binary variable indicating whether to "flip" the selection decision of link j. If $g_j = 1$, b_j is changed from 0 to 1 or from 1 to 0. If $g_j = 0$, b_j is unchanged. The objective of KP is to maximize the reduction in the system cost by adjusting the current selection of PPP projects. A negative objective value suggests that the new selection decision may be able to reduce the system cost. Constraint (3-15) guarantees the adjusted selection remains feasible. The set $P(b)$ is the set of links with b_j equal to 1.

Without (3-16), KP is the standard binary knapsack problem, a problem well-studied in the literature (e.g. Martello and Toth, 1990). Constraints (3-16) are the canonical cuts discussed by Balas and Jeroslow (1972). Each constraint cuts one solution away from the feasible region. Initially, $\bar{h}$ is set to be zero and the constraint set is empty. Solving KP generates an adjustment plan g^* with the most negative change based on the individual marginal benefits q_j. When an adjustment plan leads to a reduction in the system cost, it is used to update the current selection of projects.

On the other hand, if the adjustment plan does not lead to a reduction in the system cost, the knapsack problem needs to be re-solved to find the next "best" plan. For this purpose, constraints (3-16) are introduced to prevent plans already examined from being feasible. Specifically, when an adjustment plan does not reduce the total system cost, the algorithm below sets $\bar{h} = \bar{h} + 1$, and $S_1^{\bar{h}} = \{j \in L: g_j = 1\}$, $S_0^{\bar{h}} = \{j \in L: g_j = 0\}$. In the next iteration of Step 3b, constraint set (3-16) includes one more constraint that excludes the previous adjustment from the feasible region. In this manner, solving KP in successive iterations is guaranteed to generate a new adjustment plan. If no adjustment plan from KP can improve the system cost, the algorithm terminates.

Below is a sketch of the proposed algorithm that contains the essential ideas, not the implementation details:

1. Initialization
   Set $b_j^0 = 0$, $\forall j \in L$, formulate and solve TUEP and calculate the system cost z^0. Set $k = 0$.
2. Calculate marginal benefits q^k.
3. Update the selection of PPP projects.
   a. Set $\bar{h} = 0$. Set $P(b^k) = \left\{j \in L: b_j^k = 1\right\}$.
   b. Let (g^k) solve KP. If the optimal objective value is greater than or equal to zero, stop and return the current solution b^k. Otherwise, go to Step 3c.
   c. Set

$$\bar{h} = \bar{h} + 1,$$
$$S_{g,0}^{\bar{h}} = \{j \in L: g_j^k = 0\},$$
$$S_{g,1}^{\bar{h}} = \{j \in L: g_j^k = 1\},$$
$$b_j^{k,\bar{h}} = b_j^k - g_j^k, \quad \forall j \in P(b^k),$$
$$b_j^{k,\bar{h}} = b_j^k + g_j^k, \quad \forall j \in L - P(b^k).$$

d. Formulate and solve TUEP using $b^{k,\bar{h}}$ and calculate the system cost $z^{k,\bar{h}}$. If $z^{k,\bar{h}} < z^k$, set $b^{k+1} = b^{k,\bar{h}}$, $z^{k+1} = z^{k,\bar{h}}$, $k = k + 1$ and go to Step 2. Otherwise, update KP and go to Step 3b.

Theorem 3-2. The proposed heuristic algorithm terminates after a finite number of iterations.

Proof. If the algorithm does not stop at Step 3b, the new selection $\mathrm{b}^{\mathrm{k}+1}$ obtained from Step 3d will lead to a better solution, i.e. $\mathrm{z}^{\mathrm{k}+1} < \mathrm{z}^{\mathrm{k}}$, implying that the new selection is distinct. Because the number of possible selections is finite, the number of the outer iteration must be finite.

For each inner iteration at Step 3b, the solution g is distinct because of (3-16). Since the total number of possible combinations of g is finite, the number of the inner iterations within an outer iteration must be finite. Thus the algorithm will terminate after a finite number of iterations.□

3.4 NUMERICAL EXAMPLES

To illustrate its effectiveness, the algorithm in the previous section was implemented in GAMS (Brooke et al., 2005) with CONOPT (Drud, 1995) and CPLEX (IBM, 2009) as the solver for TUEP and KP, respectively. All the computations were on a Dell personal computer with a 3.4GHz Intel CPU and 2GB RAM.

Three networks (nine-node, Sioux Falls and Hull) from the literature are used in our experiment. The value of travel time is assumed to be one dollar per minute for all the three cases. The description of each network is given below.

Nine-Node Network: This network is shown in Figure 3-2. It contains 9 nodes, 18 existing links and 4 OD pairs. The pair of numbers next to each link are its free-low travel time and capacity. There are 18 candidate links for PPP developments and all are parallel to the existing ones. The free-flow travel time for each candidate link is the same as the existing link parallel to it. The amortized cost for one unit capacity is 0.5 and at most three links can be selected for PPP development, i.e., $s = 3$.

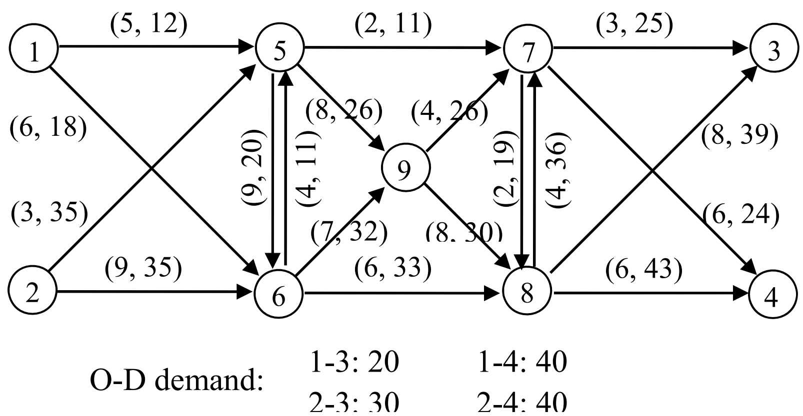

Figure 3-2: Nine-Node Network

Sioux Falls Network: This network is shown in Figure 3-3. It consists of 24 nodes, 76 existing links and 528 OD pairs. The OD demands are the ones reported in LeBlanc et al. (1975) multiplied by 0.09. There are 11 candidate links (shown in Figure 3-3 as dashed links) and their free-flow travel times are listed in Table 3-1. The amortized cost of one unit capacity of each link is 0.01 and $s = 3$.

Table 3-1: Free-flow travel time of candidate links

Link number	77	78	79	80	81	82
Free flow travel time	1.2	1.8	2.4	2.4	1.2	2.4
Link number	83	84	85	86	87	
Free flow travel time	2.4	1.2	2.4	1.8	1.8	

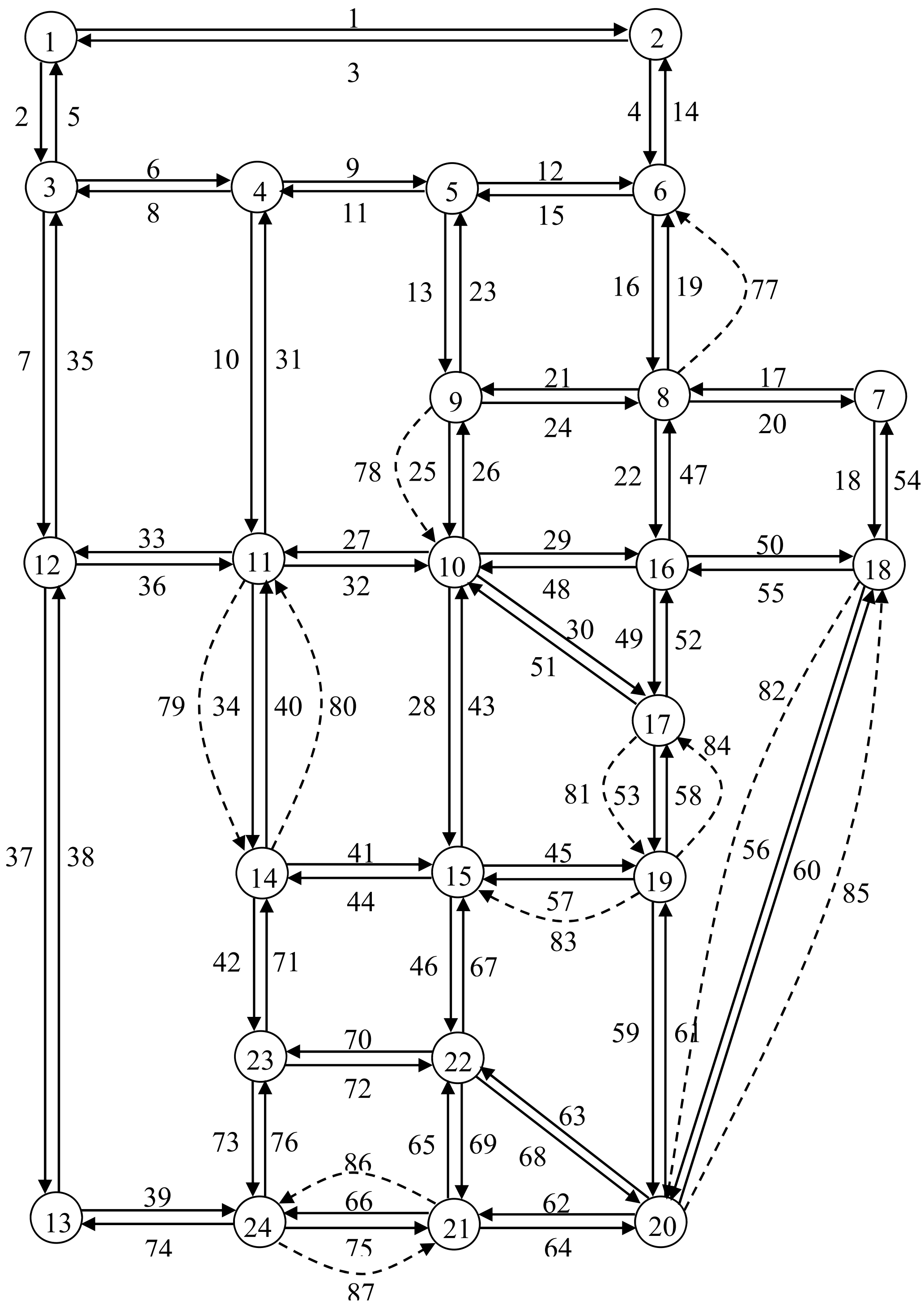

Figure 3-3: Sioux Falls Network

Hull Network: Because of its size, this network is not displayed and its topology is the same as the one in Florian et al. (1987). On the other hand, we modified link data to make it more convenient for implementations. Specifically, fractional exponents of some travel time functions in Florian et al. (1987) are rounded to the nearest integers. The network has 798 links, 501 nodes and 158 OD pairs. There are 50 candidate links and they are parallel to 50 most congested existing links in the network under the user equilibrium distribution. The travel time functions for the candidate links are the same as the existing parallel links. The unit capacity cost is assumed to be 0.05 multiplied by the free-flow travel time of each candidate link. Finally, we set $s = 5$ because Hull is a larger network.

Below are results for our experiments. Table 3-2 compares the solution quality and computation efficiency from TUEP against those from solving SCEP directly using the relaxation scheme in Ban et al. (2006). Although, both provide similar results, the execution time TUEP is less than 1% of the time required to solve SCEP by the relaxation scheme. This confirms that solving TUEP yields high quality solutions to SCEP while requiring much less computational time.

Table 3-2: Comparison of different algorithms for solving SCEP

		Relaxation	TUEP
Nine Node	Total system cost	1959.88	1959.88
	Execution time (sec)	2.90	0.02
Sioux Falls	Total system cost	1885.78	1885.78
	Execution time (sec)	111.64	0.59
Hull	Total system cost	33877.05	33874.92
	Execution time (sec)	47199.77	104.97

The proposed heuristic algorithm was then used to solve PSP for all three test networks. Table 3-3 compares the results with the ones obtained from enumerating all possible combinations. For nine-node and Sioux Falls, the heuristic algorithm provides solutions as good as or close to the optimal solutions obtained by enumerating all PPP development possibilities. At the same time, the algorithm requires solving a much smaller number of TUEP. For Hull, there are over two million possible combinations and it is too time-consuming to enumerate all PPP development possibilities. However, the heuristic algorithm still provides a solution that reduces the system cost by more than 23%, while requiring only 306 TUEP evaluations.

Table 3-3: Results of test networks

	Nine Node		Sioux Falls		Hull	
Original cost	4860.64		2067.12		51350.74	
	Heuristic	Enum	Heuristic	Enum	Heuristic	Enum
Optimal cost	2456.74	2456.74	1985.12	1984.76	39159.17	N/A
Cost reduction (%)	49.5	49.5	4.0	4.0	23.7	N/A
# TUEP solved	76	987	48	231	306	2369935

Table 3-4 reports the optimal PPP project selections and the corresponding capacities and tolls from the heuristic algorithm for the nine-node and Sioux Falls network.

Table 3-4: Optimal PPP project selections

Nine Node			Sioux Falls		
Selected link	Capacity	Toll	Selected link	Capacity	Toll
(1,5)	34.2	1.25	79	29.6	1.62
(2,5)	26.4	1.12	82	27.8	1.62
(5,7)	99.6	1.04	85	27.5	1.62

4. CONCLUDING REMARKS

This report focuses on the first-stage decision of the PPP project developments, i.e., the optimal selection of highway projects for PPP development from a pool of candidates to achieve highest social benefit while guaranteeing that the selected projects is self-sustainable and is able to attract private companies to participate in the roadway development.

This report first analyzes the private companies' behavior in deciding the toll road capacities and tolls in general transportation networks. It is found that under some mild assumptions commonly used in the literature, the v/c ratio provided by the toll road controlled by a profit-maximizing private company would be the same as the one provided by a central government that maximizing total social benefit. It is also shown the same property will still hold even if the private toll road is regulated by a cap of generalized travel cost. More importantly, the v/c ratio will be the same if the capacities and tolls are selected to achieve a second-best social optimal with revenue-neutral constraints.

The report then formulates the PPP project selection problem as an MPEC with binary decision variables. The problem maximizes social benefit and can be viewed as a tri-level leader-follower game. As it is shown that tolls and link travel times for candidate links for PPP developments can be pre-determined under some mild assumptions, a heuristic algorithm is proposed to efficiently solve the selection problem. Results from three different networks validate the model and confirm the effectiveness and efficiency of the proposed algorithm

Appendix A. Situations When $\lambda_a = -1$ in Theorem 2-4

We now briefly discuss the condition when $\lambda_a = -1$ in Theorem 2-4. If $\lambda_a = -1$, the first-order conditions of SSO become:

$$\sum_{j\in A} \frac{\partial v_j}{\partial \tau_a} t_j(v_j, c_j) + \sum_{j\in A} v_j \frac{\partial t_j(v_j, c_j)}{\partial v_j} \frac{\partial v_j}{\partial \tau_a} + \sum_{j\in L} (\frac{\partial v_j}{\partial \tau_a} \tau_j) + v_a = 0 \tag{A.1}$$

$$\sum_{j\in A} \frac{\partial v_j}{\partial c_a} t_j(v_j, c_j) + \sum_{j\in A} v_j \frac{\partial t_j(v_j, c_j)}{\partial v_j} \frac{\partial v_j}{\partial c_a} + v_a \frac{\partial t_a(v_a, c_a)}{\partial c_a} + \sum_{j\in L} (\frac{\partial v_j}{\partial c_a} \tau_j) = 0 \tag{A.2}$$

It is easy to verify that conditions (A.1) and (A.2) are the first-order conditions of the following mathematical program:

$$\min_{(\tau,c)} \sum_{j\in A} v_j(\tau_a, c_a) t_j\big(v_j(\tau_a, c_a), c_j\big) + \sum\nolimits_{j\in L} I(c_j)$$

$s.t.$ (2-23)

The above program minimizes the total user cost (travel cost and toll) without considering the cost for constructing new PPP links. If $\lambda_a = -1$, the optimal solution to SCEP2 is the optimal solution to the above problem. Because of the zero-profit constraint (2-22), the optimal objective values of the two problems are the same, i.e., the optimal total user cost is exactly the optimal system cost. In other words, when $\lambda_a = -1$, the solution to the above program, a second-best choice of toll and capacity, is self-financing.

However, such a situation is rare, because a second-best toll and capacity choice normally leads to a deficit under neural economies of scale (Verhoef, et al., 2010). If the construction cost is not a concern, the solution to the above program will likely lead to free-flow travel times and a zero toll on the private links. Because the construction cost cannot be recovered by the toll revenue, the total user cost is always lower than the system cost, i.e., the two objectives do not coincide.

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