Gulf Coast Research Center for Evacuation and Transportation Resiliency

LSU / UNO University Transportation Center

A DYNAMIC FEEDBACK-CONTROL TOLL PRICING METHODOLOGY: A CASE STUDY ON INTERSTATE 95 MANAGED LANES

Final Report

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GULF COAST RESEARCH CENTER FOR EVACUATION AND TRANSPORTATION RESILIENCY

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16. Abstract

Recently, congestion pricing emerged as a cost-effective and efficient strategy to mitigate the congestion problem on freeways. This study develops a feedback-control based dynamic toll approach to formulate and solve for optimal tolls. The study compares the performance of the proposed methodology to that of the current strategy deployed on Interstate 95 express lanes. Two objectives are studied: one is to maximize the toll revenue while maintaining a minimum level of service on the managed lanes and the other is to maximize both revenue and throughput on the managed lanes while keeping a minimum level of service. The impact of drivers' value of time based on their income level is also examined. Three values ranging from 60% to 120% of the mean hourly income are used. The results show that for high demand, an increase in the probability of choosing managed lanes is obvious, with the highest increase observed for the case of 120%. Besides, the effects of distributions of drivers' value of time among drivers are addressed. Two numerical examples are provided to explain how the proposed strategy works under three driver groups and forty-four driver groups, and an external module is developed to execute the strategy in real time during VISSIM runtime. When compared to the currently adopted toll pricing strategy on I-95, the proposed strategy with both objectives produce steadier toll rate profiles, while keeping the speeds at 45 mph or more. The objective of revenue maximization produces larger toll revenue and objective of both revenue and throughput maximization produces higher throughput on the managed lanes.

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EXCUTIVE SUMMARY& REPORT CONTENT

Recently, congestion pricing emerged as a cost-effective and efficient strategy to mitigate the congestion problem on freeways. This study develops a feedback-control based dynamic toll approach to formulate and solve for optimal tolls. The study compares the performance of the proposed methodology to that of the current strategy deployed on Interstate 95 express lanes. Two objectives are studied: one is to maximize the toll revenue while maintaining a minimum level of service on the managed lanes and the other is to maximize both revenue and throughput on the managed lanes while keeping a minimum level of service. The impact of drivers' value of time based on their income level is also examined. Three values ranging from 60% to 120% of the mean hourly income are used. The results show that for high demand, an increase in the probability of choosing managed lanes is obvious, with the highest increase observed for the case of 120%. Besides, the effects of distributions of drivers' value of time among drivers are addressed. Two numerical examples are provided to explain how the proposed strategy works under three driver groups and forty-four driver groups, and an external module is developed to execute the strategy in real time during VISSIM runtime. When compared to the currently adopted toll pricing strategy on I-95, the proposed strategy with both objectives produce steadier toll rate profiles, while keeping the speeds at 45 mph or more. The objective of revenue maximization produces larger toll revenue and objective of both revenue and throughput maximization produces higher throughput on the managed lanes.

1. BACKGROUND AND OBJECTIVES

PROBLEM STATEMENT

Dynamic congestion pricing is a very complex topic which needs to be considered from different perspectives. An important concern is the objectives of the congestion pricing strategy. Previous researchers or road authorities have focused on many objectives of congestion pricing. For instance, the interstate 95 express toll lanes aims at providing the best traffic conditions possible on the managed lanes. Its toll rate only depends on the traffic conditions of express toll lanes. Other objectives include: maximizing the throughput of the freeway; minimizing the total travel time delay; maximizing the toll revenues; maximizing the travelers' utilities; maintaining a desirable traffic demand on freeway and others. Diverse objectives will lead to various toll pricing strategies. Another considerable issue is the heterogeneity of drivers' value of time. Due to the different socio-economic status, drivers have various values of time which result in distinct trip utilities and disparate route decisions. As a result, it is very important to incorporate the heterogeneity of drivers' value of time into the route decision model to reproduce realistic driver behaviors. Besides, questions such as "what's the appropriate frequency of change of toll rate?"; "how drivers will react to the change of toll rate and how road authorities can influence drivers' route decision through toll controlling?" should also be investigated when designing a dynamic toll pricing strategy. This study explored the dynamic congestion pricing in terms of three perspectives: the control mechanism of dynamic congestion pricing, the objectives of dynamic toll pricing and the impacts of heterogeneity of drivers' value of time on route decisions.

RESEARCH OBJECTIVES

The primary goal of this study is to develop a feedback-control based dynamic toll pricing strategy to formulate and solve optimal tolls with a focus on two distinct objectives of the road authority. The first objective of congestion pricing is to maximize toll revenues while maintaining a minimum desired level of service on managed lanes. The second is to maximize both toll revenue and throughput on managed lanes while keeping a minimum desired level of service on managed lanes. According to the Washington State Route SR-167 HOT Lane project, the minimum level of service requires the average speed on managed lanes to be larger than 45 miles per hour at peak period. The specific objectives of this phase of research are to:

- Review the state of the practice of dynamic toll pricing strategies and their application in other metropolitan areas in order to learn from similar experiences and identify the various strategies used thus far, as well as their points of strengths and weaknesses.
- Develop a feedback-control based dynamic toll strategy with a focus on two different objectives.
- Select a microscopic simulation platform (e.g. VISSIM) and build the simulation network for the study area.
- Test the proposed dynamic toll strategies with a focus on two different objectives in microscopic simulation and compare their performances with that of the current toll strategy on Interstate 95.

SCOPE

The scope of the study is limited to the state of Florida. The study area boundary includes a seven-mile segment of southbound I-95 between NW151 Street and I-395.

BACKGROUND

This section presents a comprehensive review of some of the research studies on dynamic toll pricing strategies. The concept of congestion pricing was first introduced in "The Economics of Welfare" (Pigou, 1920) and was later greatly promoted both theoretically and practically (e.g. William Vickrey, 1968). Congestion pricing consists in imposing a fixed or variable toll on motorists for using a particular lane or roadway segment in an attempt to influence travel demand by encouraging motorists to either switch to alternative routes or changing their trip time (Kachroo, 2011). In the U.S., a common form of congestion pricing is used in the form of managed lanes or express toll lanes. The toll price on managed lane can be fixed or dynamic. In recent years, along with the advancement of technologies such as detectors, and electronic toll collection devices, a few states have deployed dynamic toll pricing systems such as the "San Diego I-15 Fas Trak" with a toll changing every 6 minutes, the Orange County, CA,SR-91 with a toll changing every hour, the Minnesota I-39 with a toll changing as frequently as every 3 minutes (Kachroo, 2011), and the I-95 express toll lanes in Florida updating toll rates every 15 minutes.

Static and Dynamic Congestion Pricing Strategies

Li and Govind (2003) developed an optimization model for assessing the pricing strategies of managed lanes. Although the study considered demand elasticity on managed lanes, it assumed total traffic demand for the whole study corridor to be static and only allowed fixed toll rate. The paper provided examples to illustrate the application of the model for searching optimal toll pricing strategies with a focus on five different objectives including maintaining a minimum speed on managed lanes or general purpose lanes, maximizing traffic flow on managed lanes or general purpose lanes and maximizing toll revenues or minimizing emissions. Results showed that a toll rate of $3.50 can satisfy several objectives at the same time. However some objectives may conflict, for example, maximizing toll revenues can only be achieved at sacrificing environmental benefits.

Verhoef (2002) studied the second-best problem of congestion pricing for general static traffic networks where not all links can be tolled. It was a bi-level optimization problem where the road authority wanted to maximize the total social welfare, given that individual travelers tried to maximize their own benefits. The social welfare was defined as total benefits minus total costs. The problem was defined as a standard non-linear programming problem for interior second-best optimal toll, which was as an optimal toll for which the set of relevant paths does not change due to marginal changes in any of the tolls available. An example was illustrated to demonstrate that interior second-best optimal toll did not always exist yet it was not a major concern for practical applications since the non-existence occurred in extreme conditions.

May, et al. (2002) conducted research on designing a static optimal toll price over a real traffic network with the goal of benefit maximization. The study presented judgmental approaches of

pricing optimization adopted by transportation planners to outline the theoretical basis of pricing design. The proposed methodology was tested in a simple hypothetic network with 5 links and was then applied to a traffic network of medium sized European city for morning peak hour.

Fan (2008) studied day-to-day dynamic pricing schemes with elastic traffic demand. The study investigated the continuous time dynamics in a discrete time fashion. Based on a family of day-to-day dynamics in literature, the study formulated a new dynamic marginal toll that can force the day-to-day dynamic traffic flows to achieve the status of system optimum while keeping the dynamic social net benefit increasing along the day-to-day traffic flow trajectory. Elasticity of traffic demand as well as drivers' day-to-day route choice adjustment process was considered so that the toll scheme could levy an optimal toll over dynamic traffic flows.

Dusica, et al. (2006) formulated the dynamic optimal toll problem as a bi-level optimization problem with the road authority setting tolls and drivers responding to the tolls by adjusting decisions of routes and departure time. They applied the mathematical program with equilibrium constraints (MPEC) to formulate the toll design problem over a dynamic traffic network where only parts of the links were tolled. The study illustrated examples where a simply network was applied to demonstrate the application of the proposed dynamic toll scheme. Three case studies were considered: fixed toll rate and route choice only; fixed toll rate and route and departure time choice; variable toll rate and route and departure time choice. The study also addressed the complexity of modeling and solving varying toll over dynamic traffic network and concluded that due to the non-linearity and non-convexity of objective functions, it was difficult to find optimal solution.

Dimitra, et al. (2011) developed a robust and proactive dynamic toll approach for high occupancy/toll lanes considering real-time traffic conditions. The proposed methodology aimed at keeping a free-flow condition on the managed lanes while maximizing freeway's throughput. There were two important parts of the proposed toll pricing scheme: "system inference" and "toll optimization". In the first step, the real-time traffic data such as speed and flow were used to infer the "travelers' willingness-to-pay" and forecast the traffic demand. In the second step, an optimal toll that could meet the objectives would be determined based on information acquired through the first step. The methodology was studied and compared to Interstate 95's current dynamic toll strategy in a simulation study. Simulation results revealed that their robust toll approach produced a smoother toll rate pattern and smoother throughputs, less queues as well as higher average speeds on toll lanes.

Travelers' Value of Time

Besides the status of toll rate reviewed in the previous section, another important concern of congestion pricing problem is the heterogeneity of drivers' value of time. This heterogeneity influences the cost function in drivers' route decision model and thus influences subsequent traffic flows as well as toll rates on toll lanes. In the literature, some researchers have addressed this issue. For instance, Cheng, et al. (2006) addressed the problem of heterogeneous users' response to toll charges in their bi-criterion dynamic user equilibrium traffic assignment model (BDUE) for dynamic toll pricing evaluation. The study assumed drivers' value of time (VOT) as a continuously distributed random variable among the population. Based on the distribution of

drivers' value of time, the BDUE problem was modeled as a group of infinite dimensional variation inequalities (VI) and was solved using a generalized Frank-Wolfe method. Numerical experiments were conducted to investigate how VOT distributions affect the path flow pattern and toll road usage with different dynamic toll strategies. Results showed that compared to a normal distribution of VOT, the constant VOT model overestimated the use of a toll road under a low toll rate and underestimated the use of toll lanes under a high toll rate.

Guevara (2012) pointed out that due to the heterogeneity in drivers' value of time, full internalization of congestion externalities by Pigouvian's Equilibrium actually reduced the social welfare. The study first described a simple example of traffic assignment with heterogeneous values of time. (Explaining briefly what Pigouvian's Equilibrium is at this point would be appropriate and helpful) Two equilibrium methods were applied respectively to solve the optimal congestion pricing strategy for the traffic assignment: Users' Equilibrium (UE) and Pigouvian' Equilibrium (PE). Although the PE produced an increase in social welfare compared to UE, the PE had its limitations such as the social benefits from pricing strategy would disappear when the total toll collection exceeded a certain amount, specifically $195 in the illustrated example. (You need to describe the example if you quote it. Also, what is $195? Is the VOT $195/hr?) The study then proved that such drawbacks of PE were caused by the non-convexity of the traffic assignment problem, while the non-convexity of the traffic assignment problem was due to heterogeneity of drivers' value of time. As a result, the study called for other policies to relieve the traffic congestion rather than blindly believing in congestion pricing.

Devarasetty, et al (2012) studied route choice behavior of drivers on managed lanes through a stated preference of Houston's Katy Freeway travelers. The study adopted three different survey design techniques: Db-efficient, random level generation and an adaptive random approach and compared the accuracy of the value of time (VOT) and value of reliability (VOR) produced by each survey technique. The survey questionnaire consisted of five parts: the first part asked the respondents detailed information about their most recent trips on the Katy Freeway; the second asked the respondents about their usage of the managed toll lanes; the third provided a risk-aversion question; the fourth included stated preference questions, and the last part asked about respondents' socio-economic status. The study concluded that drivers' VOT could be estimated as 63 percent, 132 percent, and 108 percent of the mean hourly wage using the survey results of the three design techniques: Db-efficient, random level generation, and adaptive random designs yet only the Db-efficient design could estimate the VOR.

Brownstone, et al. (2002) estimated drivers' willingness to pay for reducing travel time using revealed stated preference data of Interstate-15(I-15) congestion pricing project in San Diego. The data was collected from a panel survey of drivers who used I-15 in the morning when the express lanes were open. Individual drivers were assumed to have a fixed departure time in response to the change of congestion and toll fees. Drivers were asked to choose between three alternative route choices: 1). Solo driving on the general purpose lanes; 2). Solo driving on the express lanes; 3). Carpooling; based on the estimated traveling time and the given toll price. This research found that I-15 users had a median willingness to pay of $30 to reduce travel time by one hour. The paper stated that the value of $30 was upward biased because the users on the express lane also had the benefits of safety since the express lane was separated from other lanes.

Zumd, et al. (2007) used information from an evaluation study of a high occupancy toll lane (HOT) project in Minnesota to exam drivers' willingness to pay based on stated preference (SP) analysis. The evaluation study included an attitudinal panel survey which covered issues of changes in travel behavior, mode choice, and route choice and willingness to pay for the priced lane before-and-after the project implementation. Based on the results of the panel survey, the paper designed a stated preference survey to examine drivers' willingness to pay for using HOT lanes and conducted SP analysis on the survey results. In the end, the study concluded that respondents tended to a "homogenized" willingness to pay if an SP survey was done before respondents had experienced actual HOT lanes yet when the SP survey was done after the actual HOT system was opened to respondents, respondents would have a clearer idea of whether they would use HOT lane or not, and thus the willingness to pay for using HOT lane tended to vary.

Some researchers developed methods to infer drivers' value of time based on the data from loop detectors or information from dynamic toll system. For instance, Liu, et al. (2007) investigated the time-dependent effects of the variation of drivers' value of travel time (VOT) and the value of reliability (VOR) on drivers' route decision. The study formulated drivers' route decision process as a function of travel time, reliability and toll rate based on a mixed logit model. Their research used a genetic algorithm to determine parameters that represent drivers' different preferences in the route decision model so as to reproduce traffic volumes closest to the traffic volumes collected on loop detectors. Quasi-Monte Carlo simulation was applied to ease the calculation of the mixed logit model and to approximate drivers' probabilities of choosing toll lanes. The proposed algorithm was applied to data drawn from California State Route 91 (SR-91) to estimate VOT and VOR of users on SR-91.

Objectives of Congestion Pricing

In addition, objectives of toll strategies play an important role in the problem of congestion pricing. Objectives vary among researches of congestion problem and lead to various toll schemes. Throughout the literature, researchers have developed toll schemes with a focus on distinct toll objectives. Wie (2007) developed a Stackelberg game based dynamic toll scheme with an aim of shaping the travelers' flow pattern to one that can maximize net consumer surplus. The study formulated the process of road authority setting toll rate and travelers choosing routes in response to a toll rate as a nonzero-sum Stackelberg game model and developed a heuristic iterative algorithm to solve the optimal tolls.

Hranac, (2006) suggested the goals of high occupancy toll lanes (HOT) could be grouped into three categories: corridor efficiency, revenue and congestion management. The study analyzed the three major objectives and their implications on the determination of the toll price.

Sumalee, et al. (2005) analyzed the design of optimal toll rate and the optimal toll location on the cordons with a focus on different objectives using a genetic based algorithm. The study stated that the critical characteristics of maximizing the social welfare for a cordon was to concentrate on the area with highest marginal cost where reducing the trips would make most contributions to the improvement of the social welfare. Maximizing the revenue requires the cordon to have a large number of crossings so that the drivers' possibility of re-routing to avoid the toll can be minimized.

Leonhardt, et al. (2012) developed a feedback based dynamic toll controlling method for high occupancy toll (HOT) lanes with the aim of keeping specified level of traffic volume on HOT lanes. Some researchers considered two or more objectives while designing the toll strategies. For instance, Yin and Lou (2009) proposed two dynamic toll strategies with the focus on maintaining a free-flow condition on toll lanes while maximizing a freeway's throughput.

In summary, the literature review focused on three critical issues of congestion pricing: the status of toll pricing, the heterogeneity of drivers' value of time, and the objectives of congestion pricing. The status of toll pricing could be static, where fixed toll rates are applied for all time periods, and dynamic, where toll rates are changed based on the time-of-day or real-time traffic conditions. Early research studied fixed toll rates in a hypothetical static network where the traffic flows, speeds and densities are uniform along the road and independent of time. This was not practical. Later on, researchers focused on dynamic toll pricing strategies. Many theories from different disciplines have been applied to investigate the problem of dynamic toll pricing including game theory, robust optimization, second-best optimization, feedback control and others. Dynamic toll pricing is capable of dealing with the dynamic traffic network where traffic demand is elastic.

The heterogeneity of drivers' value of time (VOT) is an important concern in the problem of congestion pricing. Throughout the literature, different researchers have treated heterogeneity of VOT differently. For instance, some researches ignored the heterogeneity of drivers' VOT and used a single value of VOT for all drivers, some grouped drivers' VOT into several categories and assigned a distinct value to each category, while others used continuous distributions of VOT among all drivers. Various algorithms have been developed to estimate drivers' VOT such as estimating from the stated preference survey and inferring from measured traffic condition and toll rate information by loop detectors.

Different objectives result in distinct design of toll pricing strategies. The literature review illustrated several studies on congestion pricing that focus on diverse goals. The most common goals are maximizing toll revenue, maximizing the travelers' utility and maximizing the total social surplus. Examples of the formulation of these common objectives by previous researchers were provided in the literature review.

2. DATA COLLECTION

This research focuses on a seven-mile segment of southbound I-95 between NW151 Street and I-395. It has three general purpose and two express lanes, as shown in FIGURE 1. The general purpose lanes have eight intermediate entrances (on-ramps) and seven exits (off-ramps), while the managed lanes have only one entrance and one exit, and therefore, vehicles cannot switch back to the general purpose lanes before they reach the end of managed lanes. In this study, simulation was used to model the operation of the I-95 section with managed lanes. All geometric characteristics of the freeway segment including locations of on-ramps, off-ramps, weaving sections, and the number, length and width of lanes were coded in the simulation model.

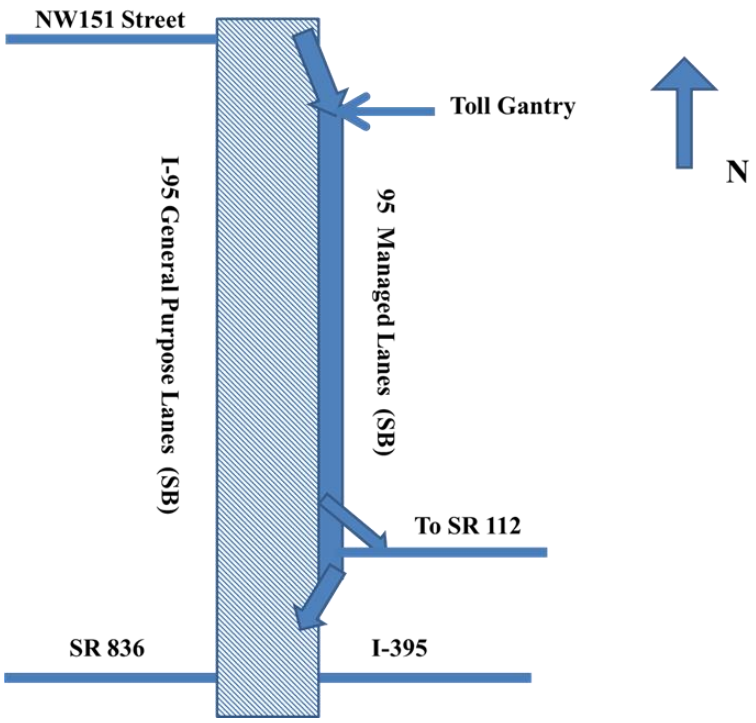

FIGURE 1 Map of study area

Data of the traffic demand in this study is retrieved from the database "Statewide Transportation Engineering Warehouse for Archived Regional Data" (STWARD). More specifically, two months' worth of traffic demand data for Wednesdays and Thursdays were extracted from the database in 15-minute intervals to generate an average traffic demand profile for the two weekdays. Traffic demand of morning pre-peak and peak hours from 5:00 Am to 7:30 AM was used in the study shown in FIGURE 2. The selected period ensures variation in demand from low to high in order to examine the performance of the proposed strategy under varying traffic conditions. Traffic demand for the on-ramps along the general purpose lanes was assumed to be the 20% of the overall traffic demand entering the study segment.

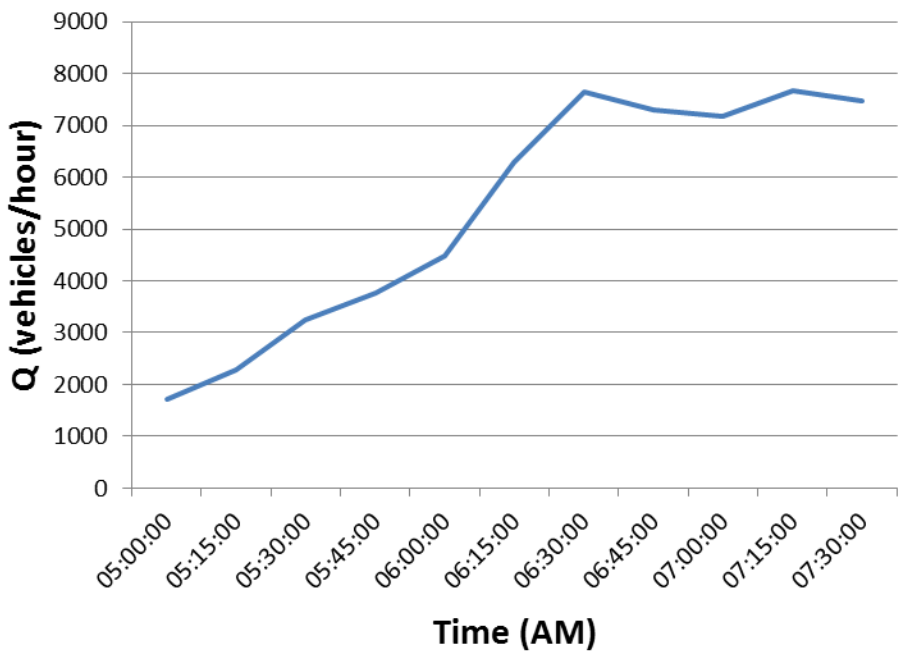

FIGURE 2 Traffic demand profile

The vehicle class distributions on Interstate 95 are obtained from Florida Department of Transportation and the information is shown in TABLE 1. However this study will only use the passenger cars as the traffic composition since passenger cars account for more than ninety percent of the vehicle class distribution and passenger cars are the major customers of toll fees.

TABLE 1 Vehicle Class Distributions on Interstate 95

		Commercial Vehicles			
Year	Passenger Cars	Single Unit	Multi-Unit	Total	Total Vehicles
2003	93.8%	3.0%	3.2%	6.2%	100.0%

Since the study considered the impacts of drivers' value of time on route decision behaviors, it is important to estimate drivers' value of time properly. The study assumed that drivers' value of time was determined by their annual income levels only. The annual income levels of drivers on Interstate 95 are assumed to follow the income distributions of U.S. population which is obtained from "U.S. Census Bureau, Current Population Survey, 2010 Annual Social and Economic Supplement" for male (30) and female (31). The source data has multiple income levels, starting with income under $2500, $2500-$4999, $5000-$7499, and ending with $250,000 or above. For each income level, the mean income and the number of people belonging to that level were provided. Population percentage of each level can be calculated by dividing the total number of people in that income category by the total population. This study incorporated income levels of male and female and generated a combined distribution of annual income shown in FIGURE 3.

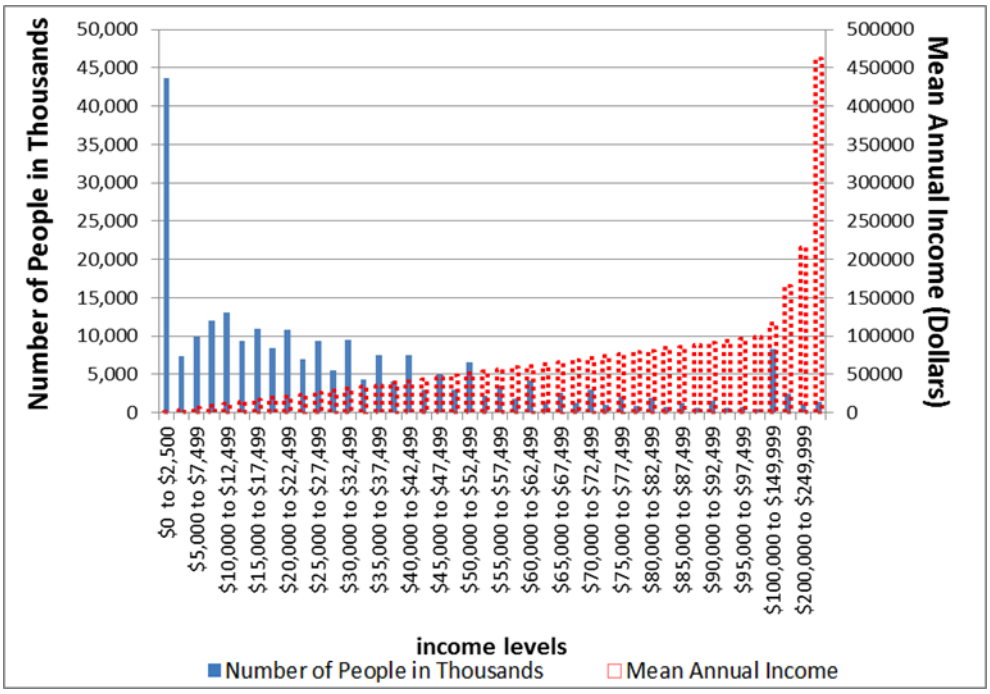

FIGURE 3 Distribution of population's annual income

3. METHODOLOGY

This chapter extensively explains the algorithm of the proposed dynamic toll pricing model developed in this study. The algorithm consists of four main components: VISSIM simulation, external managed lanes model, drivers' route choice model, and the feedback control mechanism. The first section introduces the application of VISSIM simulation to the proposed dynamic toll strategy. The second section describes an external model of managed lanes which was developed to overcome the limitations of VISSIM's built-in toll calculation model. The third section explains the logit model used in this study to model drivers' route choice behavior. The last section explains the feedback control mechanism of the proposed methodology which is the core of the strategy. The essential idea is to classify the traffic conditions on managed lanes into two cases based on the average speed measured on the managed lanes, and then construct a specific control rule for each case to achieve the tolling objectives.

VISSIM SIMULATION EXPERIMENTS

VISSIM, a microscopic traffic simulation model, was selected to model the operation of traffic on the study segment under the proposed toll pricing methodology. VISSIM has adequate capabilities of simulating traffic and reproducing acceptable driver behavior on common facilities such as freeways (e.g. Gomes, et al, 2004; Zhang, et al, 2009). VISSIM is a discrete, stochastic, time-scan based traffic simulation platform model. The system employs a psycho-physical car following model for longitudinal vehicle movement and a rule-based algorithm for lateral movements. The model is based on the continued work of Wiedemann (Karlsruhe, 2011). Since its commercial debut in 1993 (Chitturi, 2011), VISSIM has gained more attention and is now more widely accepted by practitioners and researchers.

In this study, VISSIM was used to simulate the freeway network for I-95 from Golden Glades Interchange to I-95 at I-395. A background image was generated by composite photographs downloaded from Google Maps and was scaled to match the real length of freeway. Links and link connectors were traced on this background image. In addition, every half a mile, a detector was placed on each of the general purpose lanes and the managed lanes, with a total of 70 detectors along the entire segment. These detectors were used to collect real-time traffic data during the simulation runs such as traffic volume, average travel speed and other performance measures. In VISSIM, vehicle population is classified into different types. A single vehicle type shares the same performance attributes including mode, length, desired speed distribution, maximum and minimum acceleration or deceleration, etc. Since the passenger cars account for more than ninety percent of the vehicle class distribution on Interstate 95 as shown inTABLE 1, this study only used passenger cars. The speed associated with this type was assumed to follow a uniform distribution with minimum speed of 60 mph and maximum speed of 80 mph.

EXTERNAL TOLL PRICING MODULE

VISSIM's built-in toll calculation function allows users to utilize travel time savings and average speed on the managed lanes to control the toll price. FIGURE 4 shows the toll pricing

calculation model. The right part lists all toll pricing calculation models. Multiple calculation models can be used in one simulation and each calculation model has a unique number although they can have the same names. The left part is detail of each toll pricing calculation model which can be set and modified. The first row is "No.:" and "Name:" where users can give a specific number and name to a toll calculation model. The second row is a checkbox with "COM Script". If this is checked, users can apply their own external toll calculation model programs with the format of COM script or *DLL. This allows users to set the toll rate as a function of travel time savings or the average speed on the managed lanes. The third row is a checkbox with "Traffic Responsive". If this is checked, the toll calculation method below the "Traffic Responsive" will be applied. Users can assign a fixed toll rate to a certain interval of travel time savings or to a certain interval of the average speed on the managed lanes or to a combination of travel time savings interval and average speed interval. The travel time savings is calculated as the travel time on the managed lanes minus the travel time on the general purpose lanes and if the result is less than zero, it will be treated as zero.

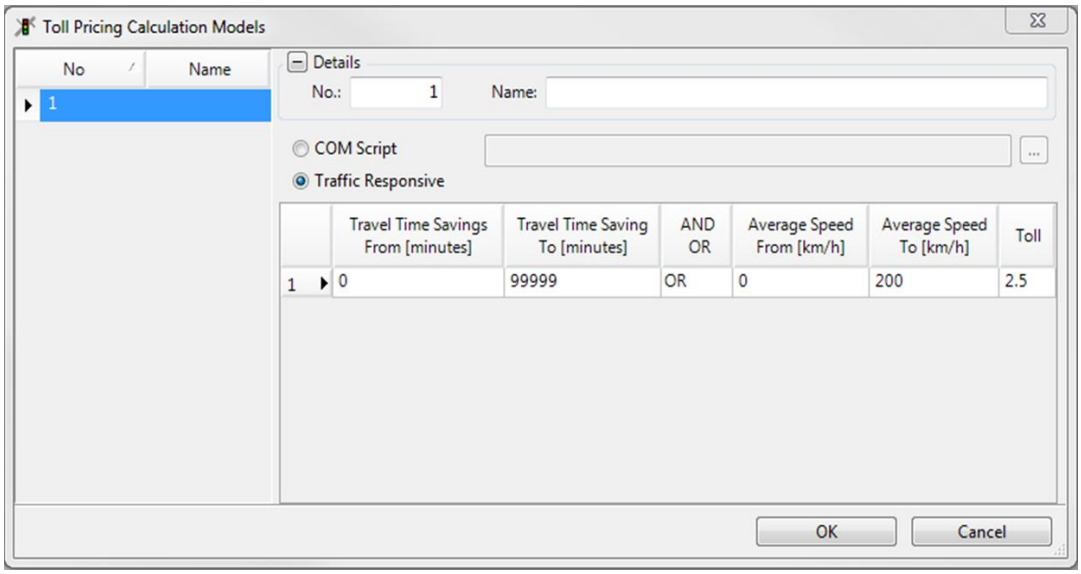

FIGURE4 VISSIM's built-in toll calculation table

However, VISSIM's built-in toll price calculation is limited to two parameters only from previous time intervals (travel time savings and average speed on managed lanes). Unfortunately, this does not meet all requirements of the toll pricing methodology proposed in this study. For instance, the built-in function does not consider the toll revenue or average speed on the general purpose lanes from previous time intervals. As a result, this research developed an external toll pricing module with the Visual Basic programming language that integrates well with VISSIM through the VISSIM_COM interface (Karlsruhe, 2011). The external module allows the toll price to be updated every three minutes and the calculation of toll rate is based on the objectives as well as the traffic conditions on both toll lanes and general purpose lanes detected from previous time intervals.

The process of toll calculation of the external module is depicted graphically in FIGURE 5. As the visual basic module (VBA) starts, it calls for VISSIM to run one single step for the first time interval. The parameters such as average speed on managed lanes are then exported from

VISSIM back to the external module at the end of the simulation step. The VBA toll calculation model then estimates the new toll rate for the next time interval based on the proposed toll strategy and passes the results back to the VBA route decision model, from which the flow ratio of vehicles on the general purpose lanes and managed lanes for the next time interval are generated and passed on to VISSIM simulation as input parameters for the next simulation step.

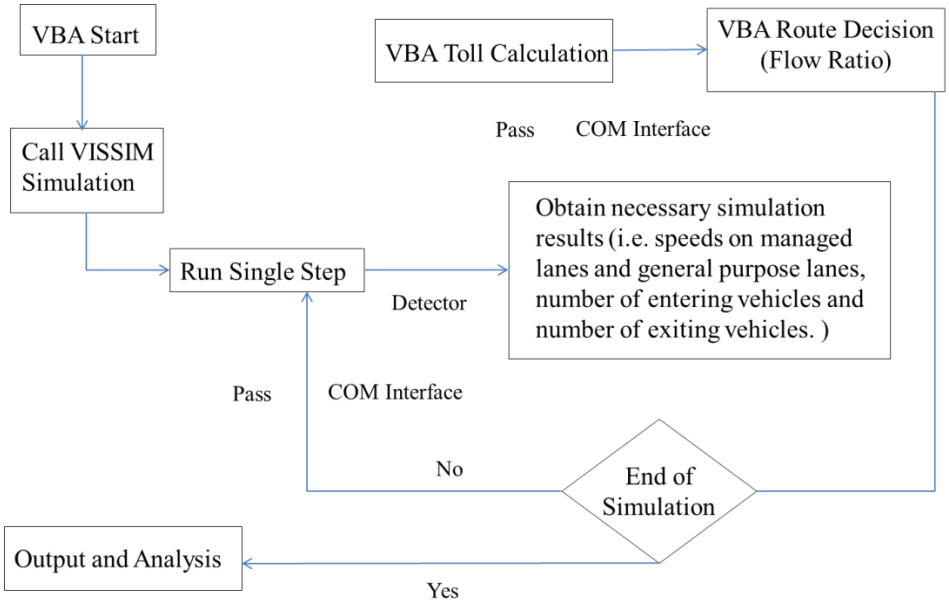

FIGURE 5 Flow chart of dynamic toll calculation

MODELING DRIVERS' ROUTE CHOICE BEHAVIOR

A Logit model was applied to model the drivers' decision making process as follows:

$$\boldsymbol{P(i,t) = \frac{e^{U_m(i,t)}}{e^{U_m(i,t)}+e^{U_g(i,t)}} = \frac{1}{1+e^{-\Delta U(i,t)}}} \quad \textbf{(1)}$$

Where,
$U_m(i,t)$= the utility of choosing the managed lanes for driver i at time interval t;
$U_g(i,t)$= the utility of choosing the general purpose lanes for driver i at time interval t;
$P(i,t)$= The probability of choosing the managed lanes by a particular driver i at time interval$t$;
$\Delta U(i,t)$= $U_m(i,t) - U_g(i,t)$ = the difference between the utility of choosing the managed lanes and the general purpose lanes for a particular driver i at time interval t.

Also, the utility of choosing managed lanes can be expressed as a function of the toll rate and the total travel time on the managed lane section as follows:

$$\boldsymbol{U_m(i,t) = -\alpha(i) * c(t) - \beta(i) * T_m(t)} \quad \textbf{(2)}$$

Where $T_m(t)$ is the travel time on the managed lanes at time interval t and $c(t)$ is the toll rate on the managed lanes at time interval t. $\alpha(i)$is the rate of change of utility for a particular driver i

per unit change of toll rate. The negative sign associated with $\alpha(i)$implies a disutility since the increase in the toll rate reduces the utility of the managed lanes. $\beta(i)$is the rate of change of utility for a particular driver i per unit change of travel time saving or drivers' value of time. The negative sign implies that as travel time on the managed lanes increases, the utility of managed lanes decreases. Since the general purpose lanes are free, their utility can be expressed as a function of the total travel time on the general purpose lanes as follows:

$$\boldsymbol{U_g(i,t) = -\beta(i) * T_g(t)} \quad \textbf{(3)}$$

Where $T_g(t)$ is the travel time on the general purpose lanes at time interval t.

Given, $\Delta T(t) = T_g\ (t) - T_m(t)$ = the travel time saving of choosing the managed lanes over the general purpose lanes, the utility difference and logit model can be rewritten as follows:

$$\boldsymbol{\Delta U(i,t) = -\alpha(i) * c(t) + \beta(i) * \Delta T(t)} \quad \textbf{(4)}$$

$$\boldsymbol{P(i,t) = \frac{1}{1+e^{\propto(i)C(t)-\beta(i)\Delta T(t)}}} \quad \textbf{(5)}$$

The positive sign of $\beta(i)$ in (11) implies that the utility difference increases when the travel time saving increases. It should be noted that the values of $\alpha(i)$ and $\beta(i)$ are user specific. Some studies assumed certain values for these parameters for simplification. For instance, Yin et al. (15) assumed that $\alpha(i)$=1 and $\beta(i)$=0.5. To better understand drivers' behavior, some studies developed methods to estimate the drivers' value of travel time savings $\beta(i)$ and value of toll costs $\alpha(i)$. Chand, et al. (2012) designed a stated preference survey of Houston's Katy Freeway travelers using three different survey techniques to estimate travelers' value of travel time savings (VTTS). Their study found that VTTS could be estimated as 63%, 132% and 108% of the mean hourly wage rate of the sample based on survey results of each survey technique.

In this research, the value of $\alpha(i)$ is assumed to be 1 while $\beta(i)$ will be estimated through drivers' annual income with the assumption that individuals with the similar annual incomes have the same value of time. The paper first divides individual drivers into three groups based on their annual income levels shown in FIGURE 3 such that each group has a fixed value of time. The value of time for each driver group will be calibrated using 90% of drivers' mean hourly income as a base case according to NCHRP report 463 (Weisbrod, et al, 2001). The study regrouped the income levels into three categories: high-income ($80,000 and above), medium-income ($40,000-$79,999), and low-income (below $40,000). The population percentage and mean income of each category are recalculated from the original population percentages and mean incomes of all sub levels within that category. For each of the three income categories, the driver characteristics are assumed to be similar, and therefore, the values of $\alpha(i)$ and $\beta(i)$ for all drivers within that income category are assumed constant. For all drivers in income category j, the values of α and β are denoted by $\alpha(j)$ and $\beta(j), \forall j = 1,2,3$. TABLE 2 shows the characteristics of each driver group.

Besides, the study extended the driver income groups to the number of drivers' original income groups from FIGURE 3. The information of the extended driver income groups is shown in **TABLE 2**.

TABLE 2 Information of Driver Groups

Driver Groups	Description	Income level	Percentage	mean hourly income
1	high value of time, (low value of toll rate cost)	>= $80000	10	$49.8
2	middle value of time, (middle value of toll rate cost)	$40000-$79999	24	$28.8
3	low value of time, (high value of toll rate cost)	<= $39999	66	$9.6

FEEDBACK CONTROL MECHANISM

Feedback control has been applied to various fields such as satellites, robots, industrial processes, to name a few. Enlightened by its various applications, this research develops a feedback control rule to calculate the optimal toll rate for each particular time interval t, such that a high level of service on the managed lanes is maintained (speed greater than 45 mph) and the total toll revenue is maximized.

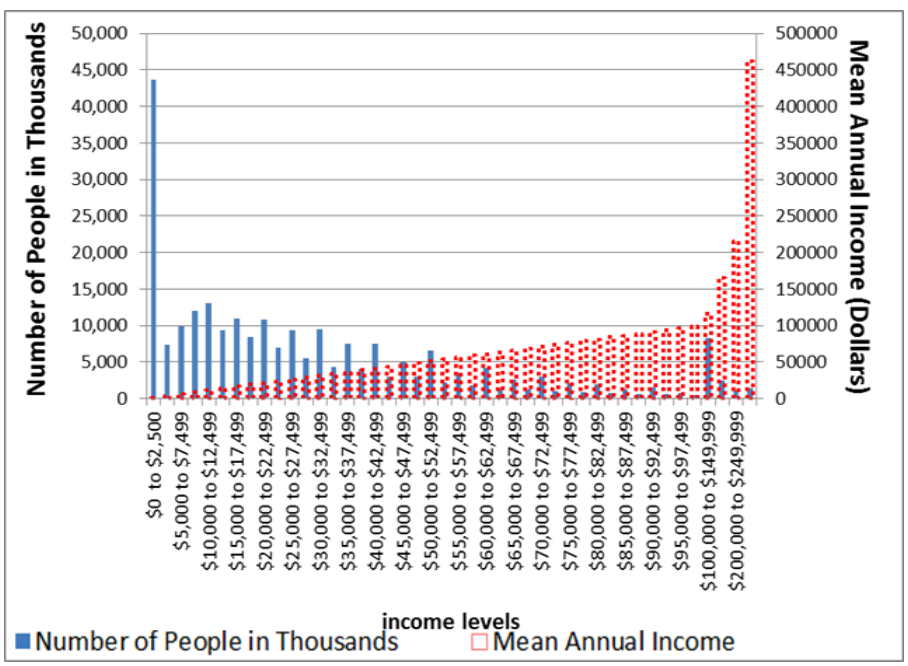

FIGURE6 Information of driver groups

The speed of managed lanes has been used as a key performance measure for managed lanes by many studies, and therefore, it is adopted in this research as a variable in the feedback control mechanism. Besides, to address the interaction between managed lanes and general purpose lanes, the travel time savings (TTS) of managed lanes over general purpose lanes are also used in the control mechanism. The control principle of the strategy is to classify the traffic condition of managed lanes into two cases based on the average speed measured on the managed lanes, $S_m(t)$, and then construct a specific control rule for each case to achieve the toll objectives. For case A, $S_m(t) > 45mph$, the managed lanes are somewhat underutilized, and therefore, the lanes can accommodate additional traffic before speed drops to the lower limit of 45 mph. In this case, the toll rate can be either increased or decreased to achieve the maximum toll revenue. If $S_m(t) \gg 45mph$, the toll rate should be decreased to attract more users to the managed lanes and thus increase the overall toll revenue. If $S_m(t)$ is slightly higher than 45 mph, the toll rate may need to be increased to discourage more vehicles from choosing the managed lanes such that the minimum level of service can be maintained.

For case B, $S_m(t) \leq 45mph$, the level of service on the managed lanes has already dropped below the minimum desirable limit, and therefore, the toll should be increased to discourage vehicles from using the managed lanes. For condition B, the main goal is to restore the minimum level of service on the managed lanes as quickly as possible. Therefore, the toll rate should be increased by a relatively larger amount than that for speeds slightly higher than 45 mph. Different feedback control rules are developed for these two conditions and are formulated as follows:

$$\boldsymbol{c(t+1) = c(t) + \Delta c^*(t) = c(t) + \begin{cases} \gamma_1^*(t) * \Delta T(t) S_m(t) > 45mph \\ \gamma_2^*(t) * \Delta T(t) S_m(t) \leq 45mph \end{cases}} \quad \textbf{(6)}$$

Where $c(t+1)$ is the calculated optimal toll rate for the managed lanes during time interval $t+1$. This new rate is obtained by adjusting the current toll rate, $c(t)$, by $\Delta c^*(t)$, where $\Delta c^*(t)$ is the optimal change that can satisfy the maximization goals of the road authority at time $t+1$while keeping the average speed on the managed lanes greater than 45 mph. As mentioned before, the paper will focus on two different maximization goals of the road authority: 1). Maximizing the toll revenue which is expressed as$r(t+1)$; 2). Maximizing both the toll revenue and the throughputs on managed lanes which is expressed as $r(t+1)$+$\theta \times Q(t+1)$, where the$Q(t+1)$ indicates the throughput on managed lanes of the time interval $t+1$ and θ is a parameter that indicates the monetary value of throughput. The unit of θ is in dollars per vehicle, as a result, the unit of $\theta \times Q(t+1)$ will also be in dollars. This study arbitrarily sets the value of θ equal to 0.5.(Couldn't you use 90% of income?)$\gamma_1^*(t)$and$\gamma_2^*(t)$ are parameters that indicate the change in toll rate per unit of travel time saving, $\Delta T(t)$ for time interval t. In order to obtain the optimal change in toll rate at time t, $\Delta c^*(t)$ is estimated as follows
For toll revenue maximization

$$\boldsymbol{\Delta c^*(t) = Argmax\,\{\hat{r}(t+1), \Delta c(t)\}} \quad \textbf{(7)}$$

$$\boldsymbol{\hat{r}(t+1) = \hat{c}(t+1) * \hat{N}(t+1) * \sum_{j=1}^{3}\left[\hat{P}(j, t+1) * q(j)\right]} \quad \textbf{(8)}$$

For maximization of both toll revenue and the throughputs on managed lanes

$$\Delta \boldsymbol{c}^*(\boldsymbol{t}) = \boldsymbol{Argmax}\,\{\hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1}) + \widehat{\boldsymbol{Q}}(\boldsymbol{t}+\mathbf{1}), \Delta \boldsymbol{c}(\boldsymbol{t})\} \tag{9}$$

$$\hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1}) + \boldsymbol{\theta} \times \widehat{\boldsymbol{Q}}(\boldsymbol{t}+\mathbf{1}) = \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1}) * \widehat{\boldsymbol{N}}(\boldsymbol{t}+\mathbf{1}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}}\left[\widehat{\boldsymbol{P}}(\boldsymbol{j},\boldsymbol{t}+\mathbf{1}) * \boldsymbol{q}(\boldsymbol{j})\right] + \boldsymbol{\theta} \times \{\widehat{\boldsymbol{N}}(\boldsymbol{t}+\mathbf{1}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}}\left[\widehat{\boldsymbol{P}}(\boldsymbol{j},\boldsymbol{t}+\mathbf{1}) * \boldsymbol{q}(\boldsymbol{j})\right] + \boldsymbol{N_m}(\boldsymbol{t}) - \Delta \boldsymbol{N_{out}}(\boldsymbol{t})\} \tag{10}$$

Where $\hat{r}(t+1)$ is the estimated total toll revenue for time interval $t+1$, $\hat{c}(t+1)$ is the estimated feasible toll rate for time interval $t+1$,$\hat{Q}(t+1)$ is the estimated throughput on managed lanes, $N_m(t)$ represents the number of vehicles that remained on the managed lanes at time interval t, $\Delta N_{out}(t)$ is the number of vehicles that exited the managed lanes at time interval $t$and $\widehat{N}(t+1)$ is the expected number of vehicles to choose between the general purpose lanes and the managed lanes during time interval $t+1$. For simplification, $N(t)$ is used as an approximation for $\widehat{N}(t+1)$ when estimating the total toll revenue. This is because the time interval used is relatively short (e.g. 3 minutes), and therefore, changes in traffic demand from one time interval to the next may not be significant. $q(j)$is the percentage of drivers in driver group j and $\hat{P}(j,t+1)$ is the estimated probability of choosing the managed lanes by drivers of driver group j during time interval $t+1$. This is estimated from the expected utility of the general purpose lanes and the managed lanes as follows:

$$\widehat{\boldsymbol{P}}(\boldsymbol{j},\boldsymbol{t}+\mathbf{1}) = \frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\widehat{\boldsymbol{T}}(\boldsymbol{t}+\mathbf{1})}} \tag{11}$$

Where $\Delta\widehat{\mathrm{T}}(t+1)$ is the estimated travel time savings for time interval t+1. Again, for simplification, $\Delta\widehat{\mathrm{T}}(t+1)$ is approximated with $\Delta\mathrm{T}(t)$, which can be directly estimated from the detectors for time interval t. $\widehat{\mathrm{P}}(j,t+1)$can be rewritten as:

$$\widehat{\boldsymbol{P}}(\boldsymbol{j},\boldsymbol{t}+\mathbf{1}) = \frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\boldsymbol{T}(\boldsymbol{t})}} \tag{12}$$

$\hat{r}(t+1)$can be rewritten as:

$$\hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1}) = \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1}) * \boldsymbol{N}(\boldsymbol{t}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}}\left[\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\boldsymbol{T}(\boldsymbol{t})}} * \boldsymbol{q}(\boldsymbol{j})\right] \tag{13}$$

$\hat{r}(t+1) + \theta \times \hat{Q}(t+1)$can be rewritten as:

$$\hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1}) + \boldsymbol{\theta} \times \widehat{\boldsymbol{Q}}(\boldsymbol{t}+\mathbf{1}) = \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1}) * \boldsymbol{N}(\boldsymbol{t}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}}\left[\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\boldsymbol{T}(\boldsymbol{t})}} * \boldsymbol{q}(\boldsymbol{j})\right] + \boldsymbol{\theta} \times \{\boldsymbol{N}(\boldsymbol{t}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}}\left[\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\boldsymbol{T}(\boldsymbol{t})}} * \boldsymbol{q}(\boldsymbol{j})\right] + \boldsymbol{N_m}(\boldsymbol{t}) - \Delta\boldsymbol{N_{out}}(\boldsymbol{t})\} \tag{14}$$

In addition to maximizing the toll revenue, another objective of this dynamic toll pricing strategy is to maintain a minimum level of service on the managed lanes by ensuring that the average speed on the managed lanes is greater than 45 mph. Consequently, the estimated feasible toll rate $\hat{\mathrm{c}}(t+1)$ should result in attracting the right proportion of vehicles on the managed lanes such that the estimated average speed for time interval t+1, $\hat{\mathrm{s}}_{\mathrm{m}}(t+1)$, is above 45 mph. The estimated average speed $\hat{\mathrm{S}}_{\mathrm{m}}(t+1)$ can be calculated from:

$$\hat{s}_m(t+1) = S_f\left(1 - \frac{\hat{k}_m(t+1)}{k_m^*}\right) > 45 \quad \textbf{(15)}$$

Where S_f is the free-flow speed on the managed lanes, $\hat{k}_m(t+1)$ is the estimated density per mile per lane for time interval $t+1$, and k_m^* is the jam density for the managed lanes. $S_f$andk_m^* are characteristics of the freeway section. $\hat{k}_m(t+1)$can be obtained from:

$$\hat{k}_m(t+1) = \frac{\Delta\hat{n}_{in}(t+1) + N_m(t) - \Delta N_{out}(t)}{L * \mathrm{n}} \quad \textbf{(16)}$$

Where L denotes the length of the managed lanes and n represents the number of managed lanes, which is equal to two for this study segment. $\Delta\hat{N}_{in}(t+1)$denotes the expected number of vehicles that will enter the managed lanes at time interval t+1, $N_m(t)$ represents the number of vehicles that remained on the managed lanes at time interval t, $\Delta N_{out}(t)$ is the number of vehicles that exited the managed lanes at time interval t. This is also used as an approximation for the number of vehicles expected to exit the managed lanes at time interval t+1. $\Delta\hat{N}_{in}(t+1)$can be calculated as follows:

$$\Delta\hat{n}_{in}(t+1) = N(t) * \sum_{j=1}^{3}\left[\hat{P}(j, t+1) * q(j)\right] \quad \textbf{(17)}$$

Where $N(t)$ is the number of vehicles making a decision to choose between the managed lanes and the general purpose lanes at time interval t. This is also used to approximate the expected number of vehicles to make a choice between the managed lanes and the general purpose lanes at time interval t+1. Therefore,

$$\Delta\hat{n}_{in}(t+1) = N(t) * \sum_{j=1}^{3}\left[\frac{1}{1+e^{\alpha(j)*\hat{c}(t+1)-\beta(j)*\Delta T(t)}} * q(j)\right] \quad \textbf{(18)}$$

SIMPLIFIED NUMERICAL EXAMPLE

In this section, two numerical examples are illustrated to demonstrate the applications of the proposed strategy. For each of the example, two objectives of the toll strategy are considered: one is to maximize the toll revenue while keeping a minimum level of service on the managed lanes and the other is to maximize both the toll revenue and the throughput on the managed lanes while maintaining a desired level of service on the managed lanes. In the first example the distribution of driver's value of time is simplified. Only three driver income groups are considered which is regrouped from drivers' original income categories. In the second example the distribution is extended to a forty-four driver income group which is the same as drivers' original income categories.

The general information of traffic condition and the toll rate that are used by the numerical examples is as following: For time interval t, (1) the current toll rate $c(t) = \$2$, (2) the number of vehicles choosing between managed lanes and general purpose lanes $N(t) = 1200$, (3) the travel time saving $\Delta T(t) = 5$ min, (4) the number of vehicles occupying the managed lanes

$N_m(t) = 500$, and (5) the number of vehicles leaving the managed lanes $\Delta N_{out}(t) = 50$. The length of the managed lanes and the jam density are given as L=6.5 miles, $k_m^* = 200$ vpmpl.

The first example uses three driver groups with three different values of time based on drivers' mean income. Assuming drivers' value of toll rate, $\alpha = \{1, 1, 1\}$ and a value of time equivalent to 90% of the mean hourly income according to NCHRP report 463 (Weisbrod, 2001), the value of time for the three driver groups shown in TABLE 2are $\beta(j) = \{0.75, 0.43, 0.14\}$. The percentage of drivers in each driver group is assumed to follow the data in TABLE 2, that is $r = \{0.1, 0.24, 0.66\}$. The algorithm to calculate the toll rate for the next time interval $t + 1$ is executed for conditions A and B as follows:

1. Let $P_{max} = 0.99$and $P_{min} = 0.01$, which denote the maximum and minimum probability of choosing the managed lanes by any particular driver.

2. From Eq. (21), $\hat{c}(j, t + 1)$, the toll rate corresponding to $\hat{P}(j, t + 1)$, can be determined from:

$$\boldsymbol{\hat{c}(j, t + 1) = \frac{ln\left(\frac{1-\hat{P}(j,t+1)}{\hat{P}(j,t+1)}\right)+\beta(j)*\Delta T(t)}{\alpha(j)} \qquad \forall j = 1, 2, 3} \tag{19}$$

Substituting P_{max} and P_{min} into Eq. (29) yields

$$\boldsymbol{\hat{c}_{min}(j, t + 1) = \frac{ln\left(\frac{1-P_{max}}{P_{max}}\right)+\beta(j)*\Delta T(t)}{\alpha(j)} = \frac{-4.6+\beta(j)*\Delta T(t)}{\alpha(j)} \qquad \forall j = 1, 2, 3} \tag{20}$$

$$\boldsymbol{\hat{c}_{max}(j, t + 1) = \frac{ln\left(\frac{1-P_{min}}{P_{min}}\right)+\beta(j)*\Delta T(t)}{\alpha(j)} = \frac{4.6+\beta(j)*\Delta T(t)}{\alpha(j)} \qquad \forall j = 1, 2, 3} \tag{21}$$

3. From Eq. (6), $\gamma_1(j, t)$ can be estimated from

$$\boldsymbol{\begin{cases} \gamma_1(j, t) = \frac{\hat{c}(j,t+1)-c(t)}{\Delta T(t)} S_m(t) > 45mph \\ \gamma_2(j, t) = \frac{\hat{c}(j,t+1)-c(t)}{\Delta T(t)} S_m(t) \leq 45mph \end{cases}} \tag{22}$$

And therefore,

$$\boldsymbol{\begin{cases} \gamma_{1,min}(j, t) = \frac{1}{\Delta T(t)}\left[\frac{-4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right] \\ \gamma_{1,max}(j, t) = \frac{1}{\Delta T(t)}\left[\frac{4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right] \end{cases} S_m(t) > 45mph, \forall j = 1, 2, 3} \tag{23}$$

$$\boldsymbol{\begin{cases} \gamma_{2,min}(j, t) = \frac{1}{\Delta T(t)}\left[\frac{-4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right] \\ \gamma_{2,max}(j, t) = \frac{1}{\Delta T(t)}\left[\frac{4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right] \end{cases} S_m(t) \leq 45mph, \quad \forall j = 1, 2, 3} \tag{24}$$

4. Substituting for $\alpha = \{1,\ 1, 1\}$ and $\beta = \{0.75, 0.43, 0.14\}$ into (33) and (34),

$$\begin{cases} \gamma_{1,min}(t) = \{-0.57, -0.89, -1.18\} \\ \gamma_{1,max}(t) = \{1.27, 0.95, 0.66\} \end{cases} S_m(t) > 45mph$$

$$\begin{cases} \gamma_{2,min}(t) = \{-0.57, -0.89, -1.18\} \\ \gamma_{2,max}(t) = \{1.27, 0.95, 0.66\} \end{cases} S_m(t) \leq 45mph$$

Merging $\gamma_{1,min}(t)$ and $\gamma_{1,max}(t)$ into one set yields

$$\gamma_{1,min}(t) \cup \gamma_{1,max} = \{1.27, 0.95, 0.66, -0.57, -0.89, -1.18\}$$

Merging $\gamma_{2,min}(t)$ and $\gamma_{2,max}(t)$ into one set yields

$$\gamma_{2,min}(t) \cup \gamma_{2,min}(t) = \{1.27, 0.95, 0.66, -0.57, -0.89, -1.18\}$$

5. Find the upper and lower boundaries of the two union sets. For union set of γ_1, the upper boundary is set to be $max\{\gamma_{1,min}(t) \cup \gamma_{1,max}(t)\} = 1.27$ and the lower boundary is set to be $min\{\gamma_{1,min}(t) \cup U\gamma_{1,max}(t)\} = -1.18$. However the lower boundary of union set of γ_2 should be zero since for condition B the average speed on the managed lanes is less than or equal to 45 mph. The toll rate should be increased to discourage more vehicles from selecting the managed lanes and to restore the level of service. The upper boundary of union set of γ_2 is determined by $\max\{\gamma_{2,min}(t) \cup \gamma_{2,max}(t)\} = 1.27$.
6. In this example, the upper boundary (1.27) and lower boundary (-0.6 or zero) define the search region for the optimum value $\gamma_1^*(t)\ or\ \gamma_2^*(t)$ that maximizes the toll revenue or maximizes the throughput on the managed lanes during the next time interval. The search region is thus divided into small intervals with an increment (say 0.01) such that:

$$\begin{cases} \boldsymbol{\gamma_1(t) = \{-1.18, -1.19, \ldots, 1.25, 1.26, 1.27\} S_m(t) > 45mph} \\ \boldsymbol{\gamma_2(t) = \{0, 0.01, 0.02, \ldots, 1.25, 1.26, 1.27\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(25)}$$

7. From Eq. (6), the set of estimated toll rates $\hat{C}'(t+1)$for time interval $t+1$ can be calculated from

$$\begin{cases} \boldsymbol{\widehat{C}'(t+1) = c(t) + \gamma_1(t) * \Delta T(t) S_m(t) > 45mph} \\ \boldsymbol{\widehat{C}'(t+1) = c(t) + \gamma_2(t) * \Delta T(t) S_m(t) \leq 45mph} \end{cases} \quad \textbf{(26)}$$

Substituting for $\gamma_1(t) = \{-1.18, -1.17, \ldots, 1.25, 1.26, 1.27\}$,
$\gamma_2(t) = \{0, 0.01, 0.02, \ldots, 1.25, 1.26, 1.27\}, C(t) = \$2,\ \Delta T(t) = 5\ min$into equation (36), the set of estimated toll rates is obtained:

$$\begin{cases} \boldsymbol{\widehat{C}'(t+1) = \{-3.9, -3.85, \ldots, 8.25, 8.3, 8.35\} S_m(t) > 45mph} \\ \boldsymbol{\widehat{C}'(t+1) = \{2, 2.05, 2.1, \ldots, 8.3, 8.35\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(27)}$$

Discarding the negative values, the series $\hat{C}'(t+1)$ is reduced to $\hat{C}'_{+}(t+1)$, $\hat{C}'_{+}(t+1) = \{\hat{C}'(n,t+1) | \hat{C}'(n,t+1) > 0, n = 1,2,3,\dots,m\}$, where m is the length of $\hat{C}'(t+1)$. Thus,

$$\begin{cases} \boldsymbol{\widehat{C}'_{+}(t+1) = \{0.05, 0.1, 0.15, \dots, 8.25, 8.3, 8.35\} S_m(t) > 45mph} \\ \boldsymbol{\widehat{C}'_{+}(t+1) = \{2, 2.05, 2.1, \dots, 8.3, 8.35\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(28)}$$

8. From Eq. (28), the set representing the number of vehicles that will likely choose the managed lanes at time interval $t+1$, $\Delta\widehat{N}_{in}(t+1)$, can be estimated from:

$$\boldsymbol{\Delta\widehat{N}_{in}(t+1) = N(t) * \sum_{j=1}^{3} \left[\frac{1}{1+e^{\alpha(j)*\widehat{C}'_{+}(t+1) - \beta(j)*\Delta T(t)}} * q(j) \right]} \quad \textbf{(29)}$$

Substituting for $\hat{C}'_{+}(t+1)$ from Eq. (38), $\Delta T(t) = 5$ min, $q = \{0.1, 0.24, 0.66\}$, $N(t) = 1200$, $\alpha = \{1,1,1\}$ and $\beta = \{0.75, 0.43, 0.14\}$into Eq.(39),

$$\begin{cases} \boldsymbol{\Delta\widehat{N}_{in}(t+1) = \{894.04, 883.47, 872.71, \dots, 2.38, 2.26, 2.15\} S_m(t) > 45mph} \\ \boldsymbol{\Delta\widehat{N}_{in}(t+1) = \{426.63, 415.71, 404.94, \dots, 2.26, 2.15\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(30)}$$

9. From Eq. (26), the set representing the estimated vehicle densities on the managed lanes at time interval $t+1$, $\widehat{K}_m(t+1)$ can be calculated from

$$\boldsymbol{\widehat{K}_m(t+1) = \frac{\Delta\widehat{N}_{in}(t+1) + N_m(t) - \Delta N_{out}(t)}{L*2}} \quad \textbf{(31)}$$

Substituting for $\Delta\widehat{N}_{in}(t+1)$, $N_m(t) = 500$, $\Delta N_{out}(t) = 50$, L = 6.5 miles into Eq. (41) yields

$$\begin{cases} \boldsymbol{\widehat{K}_m(t+1) = \{103.39, 102.57, 101.75, \dots, 34.80, 34.79, 34.78\} S_m(t) > 45mph} \\ \boldsymbol{\widehat{K}_m(t+1) = \{67.43, 66.59, 65.76, \dots, 34.79, 34.78\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(32)}$$

10. From Eq. (25), the set representing the estimated average speeds on the managed lanes at time interval $t+1$, $\hat{S}_m(t+1)$, can be calculated from

$$\boldsymbol{\widehat{S}_m(t+1) = S_f \left(1 - \frac{\widehat{K}_m(t+1)}{k_m^*}\right) > 45} \quad \textbf{(33)}$$

Substituting for $S_f = 70\ mph$ (assumed free-flow speed),$\widehat{K}_m(t+1)$,$k_m^* = 200\ vpmpl$ (assumed jam density) into Eq. (43) yields

$$\begin{cases} \boldsymbol{\widehat{S}_m(t+1) = \{33.81, 34.10, 34.39, \dots, 57.82, 57.82, 57.83\} S_m(t) > 45mph} \\ \boldsymbol{\widehat{S}_m(t+1) = \{46.40, 46.69, 46.98, \dots, 57.82, 57.83\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(34)}$$

11. Find the set of feasible toll rates for time interval $t+1$, $\hat{C}(t+1)$, such that the estimated average speed on the managed lanes is greater than 45 mph.

$$\boldsymbol{\widehat{C}(t+1) = Arg\{\widehat{S}_m(n,t+1) > 45 | \widehat{S}_m(n,t+1) \in \widehat{S}_m(t+1), \widehat{C}'_{+}(t+1)\}} \quad \textbf{(35)}$$

This results in

$$\begin{cases} \hat{\mathbf{C}}(\mathbf{t}+\mathbf{1}) = \{\mathbf{1.80}, \mathbf{1.85}, \mathbf{1.90}, \dots, \mathbf{8.25}, \mathbf{8.3}, \mathbf{8.35}\} & \mathbf{S_m(t)} > \mathbf{45}\boldsymbol{mph} \\ \hat{\mathbf{C}}(\mathbf{t}+\mathbf{1}) = \{\mathbf{2}, \mathbf{2.05}, \mathbf{2.10}, \dots, \mathbf{8.3}, \mathbf{8.35}\} & \mathbf{S_m(t)} \leq \mathbf{45mph} \end{cases} \quad \textbf{(36)}$$

As all the feasible toll rates have been identified in Eq.(43), we can now determine the optimal toll rate that will satisfy the objective of the toll strategy. Two maximization objectives are considered: toll revenue maximization and both toll revenue and throughput on the managed lanes maximization. The road authority can chose one of the maximization goals and follow the optimal toll searching procedure of that objective.

If the objective of road authority is to maximize the toll revenue while keeping a minimum level of service on the managed lanes, the following steps are searching procedure of the optimal toll given that all feasible toll rates have been identified.

12. (a) From Eq. (20) the set representing the estimated toll revenues for time interval $t+1$, $\hat{R}(t+1)$ can be calculated from

$$\boldsymbol{\widehat{R}(t+1) = \widehat{C}(t+1) * N(t) * \sum_{j=1}^{3}\left[\frac{1}{1+e^{\alpha(j)*\widehat{C}(t+1)-\beta(j)*\Delta T(t)}} * q(j)\right]} \quad \textbf{(37)}$$

Substituting for the toll rates set $\hat{C}(t+1)$from Eq. (46), $q = \{0.1, 0.24, 0.66\}$, $N(t) = 1200, \alpha = \{1,1,1\}$, $\beta = \{0.75, 0.43, 0.14\}$ and $\Delta T(t) = 5$ min into Eq. (47) yields

$$\begin{cases} \boldsymbol{\widehat{R}(t+1)} = \{\mathbf{849.23}, \mathbf{851.54}, \dots, \mathbf{19.63}, \mathbf{18.79}, \mathbf{17.99}\}\boldsymbol{S_m(t)} > \mathbf{45}\boldsymbol{mph} \\ \boldsymbol{\widehat{R}(t+1)} = \{\mathbf{853.27}, \mathbf{852.20}, \mathbf{850.37}, \dots, \mathbf{18.79}, \mathbf{17.99}\}\boldsymbol{S_m(t)} \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(38)}$$

13. (a) Calculate the maximum toll revenue $\hat{r}(t+1)$ such that $\hat{r}(t+1) = \max\{\hat{R}(t+1)\}$. In this example, the estimated maximum toll revenue and corresponding toll rates for the next three-minute time interval are

$$\begin{cases} \boldsymbol{\hat{r}(t+1)} = \$\mathbf{853.53},\ \boldsymbol{\hat{c}(t+1)} = \$\mathbf{1.85}\ \boldsymbol{S_m(t)} > \mathbf{45}\boldsymbol{mph} \\ \boldsymbol{\hat{r}(t+1)} = \$\mathbf{853.27},\ \boldsymbol{\hat{c}(t+1)} = \$\mathbf{2}\quad \boldsymbol{S_m(t)} \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(39)}$$

If the objective of road authority is to maximize both the toll revenue and the throughput on the managed lanes while keeping a minimum level of service, the following steps are searching procedure of the optimal toll given that all feasible toll rates have been identified.

14. (b) From Eq. (21) the set representing the estimated revenue and throughput on managed lanes for time interval $\mathrm{t}+1$, $\widehat{\mathrm{R}}(\mathrm{t}+1) + \theta \times \widehat{\mathrm{Q}}(\mathrm{t}+1)$ can be calculated from

$$\widehat{\mathbf{R}}(\mathbf{t}+\mathbf{1}) + \boldsymbol{\theta} \times \{\mathbf{N(t)} * \sum_{\mathbf{j=1}}^{\mathbf{3}}\left[\frac{\mathbf{1}}{\mathbf{1+e^{\alpha(j)*\hat{c}(t+1)-\beta(j)*\Delta T(t)}}} * \mathbf{q(j)}\right] + \mathbf{N_m(t)} - \Delta\mathbf{N_{out}(t)}\} \quad \textbf{(40)}$$

15. (b) Substituting for the $\theta = 0.5$, $q = \{0.1, 0.24, 0.66\}$, $N(t) = 1200$, $\alpha = \{1, 1, 1\}$, $\beta = \{0.74, 0.43, 0.14\}, \Delta T(t) = 5, N_m(t) = 500, \Delta N_{out}(t) = 50$ and $\widehat{R}(t+1)$ from Eq. (45) into Eq. (47) we get

$$\widehat{\mathbf{R}}(\mathbf{t}+\mathbf{1}) + \boldsymbol{\theta} \times \widehat{\mathbf{Q}}(\mathbf{t}+\mathbf{1}) = \begin{cases} \{\mathbf{1310.13}, \mathbf{1306.69}, \dots, \mathbf{244.92}, \mathbf{244.07}\} & \mathbf{S_m(t)} > \mathbf{45}\boldsymbol{mph} \\ \{\mathbf{1291.59}, \mathbf{1285.06}, \dots, \mathbf{244.92}, \mathbf{244.07}\} & \mathbf{S_m(t)} \leq \mathbf{45mph} \end{cases} \quad \textbf{(41)}$$

16. Calculate the maximum sum of toll revenues and throughputs on managed lanes $\widehat{R}(t+1) + \theta \times \hat{q}(t+1)$ such that $\hat{r}(t+1) + \theta \times \hat{q}(t+1) = \max\{\widehat{R}(t+1) + \theta \times \widehat{Q}(t+1)\}$. The estimated maximum sum of toll revenues and throughput as well as the corresponding toll rates for the next three-minute time interval are

$$\begin{cases} \hat{\mathbf{r}}(\mathbf{t}+\mathbf{1}) + \hat{\mathbf{q}}(\mathbf{t}+\mathbf{1}) = \mathbf{1310.13},\ \hat{\mathbf{c}}(\mathbf{t}+\mathbf{1}) = \$\mathbf{1.8} & \mathbf{S_m(t)} > \mathbf{45}\boldsymbol{mph} \\ \hat{\mathbf{r}}(\mathbf{t}+\mathbf{1}) + \hat{\mathbf{q}}(\mathbf{t}+\mathbf{1}) = \mathbf{1291.59},\ \hat{\mathbf{c}}(\mathbf{t}+\mathbf{1}) = \$\mathbf{2} & \mathbf{S_m(t)} \leq \mathbf{45mph} \end{cases} \quad \textbf{(42)}$$

This example illustrates the procedure of calculating the toll price for the next time interval by the proposed strategy with two objectives. The first objective is revenue maximization and the second objective is both revenue and throughput on the managed lanes maximization. Two traffic conditions are considered separately. For traffic condition A, the average speed on the managed lanes of the current time interval is greater than 45 mph and for traffic condition B, the average speed on the managed lanes for the current time interval is less than or equal to 45 mph. Only three driver income groups are considered. Results are summarized in TABLE 3. The toll rate of the traffic condition A is lower than that of traffic condition B. This is because under the traffic condition B, it is more important to discourage vehicles from using the managed lanes so as to recover the level of service than to encourage vehicles entering the managed lanes to collect more toll revenues. Under traffic condition A, the toll revenue produced by the first objective is higher than that of the second objective although the number of vehicles choosing the managed lanes of the first objective is lower than that of the second objective. This can be explained by the fact that the first objective only focuses on the revenue maximization while the second objective also considers the throughput on the managed lanes. Under traffic condition B, the toll rate, toll revenue and number of vehicles choosing the managed lanes are the same for both objectives. This can be explained that under traffic condition B, since the average speed is lower than 45 mph, the constraint of keeping a minimum level of service on the managed lanes has the priority in toll rate determination.

TABLE 3 Results of Simplified Example

	Toll Rate ($)	Toll Revenue ($/3 min)	Number of vehicles choosing the managed lanes (vehicles/3 min)
Revenue maximization ($S_m(t) > 45mph$)	1.85	853.53	461.4
Revenue maximization ($S_m(t) \leq 45mph$)	2	853.27	426.6
Revenue & Throughput maximization ($S_m(t) > 45mph$)	1.8	849.23	471.8
Revenue & Throughput maximization ($S_m(t) \leq 45mph$)	2	853.27	426.6

EXTENDED NUMERICAL EXAMPLE

In the previous numerical example, only three driver groups are considered. However the methodology can be applied to as many driver groups as users want. This part extends the number of driver groups to the actual number of drivers' annual income categories.The mean hourly income and the percentage of drivers for driver group are shown in FIGURE 6.

Assuming drivers' value of toll rate $\alpha = \{1, 1, \dots, 1\}$ and a value of time equivalent to 90% of the mean hourly income according to NCHRP report 463 (Weisbrod, 2001), the value of time for the 44 driver groups shown in FIGURE 6 are $\beta(j) = \{0.001, 0.028, 0.049, \dots, 1.288, 1.68, 3.603\}$. The percentage of drivers in each driver group is assumed to follow the data in FIGURE 6, that is $r = \{0.18, 0.031, 0.041, \dots, 0.01, 0.004, 0.006\}$. The algorithm to calculate the toll rate for the next time interval $t + 1$ is executed for conditions A and B as follows:

1. Let $P_{max} = 0.99$and $P_{min} = 0.01$, which denote the maximum and minimum probability of choosing the managed lanes by any particular driver.

2. From the first three steps of the example one we have:

$$\begin{cases} \boldsymbol{\gamma_{1,min}(j,t) = \frac{1}{\Delta T(t)}\left[\frac{-4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right]} \\ \boldsymbol{\gamma_{1,max}(j,t) = \frac{1}{\Delta T(t)}\left[\frac{4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right]} \end{cases} \boldsymbol{S_m(t) > 45mph,\ \forall j = 1,2,3} \quad \textbf{(43)}$$

$$\begin{cases} \boldsymbol{\gamma_{2,min}(j,t) = \frac{1}{\Delta T(t)}\left[\frac{-4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right]} \\ \boldsymbol{\gamma_{2,max}(j,t) = \frac{1}{\Delta T(t)}\left[\frac{4.6+\beta(j)*\Delta T(t)}{\alpha(j)} - c(t)\right]} \end{cases} \boldsymbol{S_m(t) \leq 45mph,\ \forall j = 1,2,3} \quad \textbf{(44)}$$

3. Substituting for $\alpha = \{1, 1, 1, \dots, 1$ and $\beta(j) = \{0.002, 0.032, 0.054, \dots, 1.431, 1.866, 4.004\}$, $c(t) = 2$, $\Delta T(t) = 5$ into (50) and (51),

$$\begin{cases} \gamma_{1,min}(t) = \{-1.32, -1.28, \dots, -0.03, 0.36, 2.28\} \\ \quad \gamma_{1,max}(t) = \{0.52, 0.55, \dots, 1.81, 2.2, 4.12\} \end{cases} S_m(t) > 45mph$$

$$\begin{cases} \gamma_{2,min}(t) = \{-1.32, -1.28, \dots, -0.03, 0.36, 2.28\} \\ \quad \gamma_{2,max}(t) = \{0.52, 0.55, \dots, 1.81, 2.2, 4.12\} \end{cases} S_m(t) \leq 45mph$$

Merging $\gamma_{1,min}(t)$ and $\gamma_{1,max}(t)$ into one set yields

$$\gamma_{1,min}(t) \cup \gamma_{1,max} = \{4.12, 2.28, 2.2, 1.81, \dots, 0.55, 0.52, -0.03, -1.28, -1.32\}$$

Merging $\gamma_{2,min}(t)$ and $\gamma_{2,max}(t)$ into one set yields

$$\gamma_{2,min}(t) \cup \gamma_{2,min}(t) = \{4.12, 2.28, 2.2, 1.81, \dots, 0.55, 0.52, -0.03, -1.28, -1.32\}$$

4. Find the upper and lower boundaries of the two union sets. For union set of γ_1, the upper boundary is set to be $max\{\gamma_{1,min}(t) \cup \gamma_{1,max}(t)\} = 4.12$ and the lower boundary is set to be $min\{\gamma_{1,min}(t) \cup U\gamma_{1,max}(t)\} = -1.32$. However the lower boundary of union set of γ_2 should be zero since for condition B the average speed on the managed lanes is less than or equal to 45 mph. The toll rate should be increased to discourage more vehicles from selecting the managed lanes and to restore the level of service. The upper boundary of union set of γ_2 is determined by $\max\{\gamma_{2,min}(t) \cup \gamma_{2,max}(t)\} = 4.12$.

5. In this example, the upper boundary (4.12) and lower boundary (-1.32 or zero) define the search region for the optimum value $\gamma_1^*(t)$ *or* $\gamma_2^*(t)$ that maximizes the toll revenue or maximizes the throughput on the managed lanes during the next time interval. The search region is thus divided into small intervals with an increment (say 0.01) such that:

$$\begin{cases} \boldsymbol{\gamma_1(t) = \{-1.32, -1.31, \dots, 4.10, 4.11, 4.12\} S_m(t) > 45mph} \\ \boldsymbol{\gamma_2(t) = \{0, 0.01, 0.02, \dots, 4.10, 4.11, 4.12\} S_m(t) \leq 45mph} \end{cases} \quad \textbf{(45)}$$

6. From Eq. (6), the set of estimated toll rates $\hat{C}'(t+1)$for time interval $t+1$ can be calculated from

$$\begin{cases} \boldsymbol{\widehat{C}'(t+1) = c(t) + \gamma_1(t) * \Delta T(t) S_m(t) > 45mph} \\ \boldsymbol{\widehat{C}'(t+1) = c(t) + \gamma_2(t) * \Delta T(t) S_m(t) \leq 45mph} \end{cases} \quad \textbf{(46)}$$

Substituting for

$\gamma_1(t) = \{-1.32, -1.31, \dots, 4.10, 4.11, 4.12\}$, $\gamma_2(t) = \{0, 0.01, 0.02, \dots, 4.10, 4.11, 4.12\}$, $C(t) = \$2$, $\Delta T(t) = 5\ min$into equation (60), the set of estimated toll rates is obtained:

$$\begin{cases} \widehat{\boldsymbol{C}}'(\boldsymbol{t}+\mathbf{1}) = \{-\mathbf{4.6}, -\mathbf{4.55}, \dots, \mathbf{22.5}, \mathbf{22.55}, \mathbf{22.6}\}\boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \widehat{\boldsymbol{C}}'(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{2}, \mathbf{2.05}, \mathbf{2.1}, \dots, \mathbf{22.5}, \mathbf{22.55}, \mathbf{22.6}\}\boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(47)}$$

Discarding the negative values, the series $\hat{C}'(t+1)$ is reduced to $\hat{C}'_+(t+1)$, $\hat{C}'_+(t+1) = \{\hat{C}'(n,t+1) | \hat{C}'(n,t+1) > 0, n = 1,2,3, \dots, m\}$, where m is the length of $\hat{C}'(t+1)$. Thus,

$$\begin{cases} \widehat{\boldsymbol{C}}'_+(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{0.05}, \mathbf{0.1}, \mathbf{0.15}, \dots, \mathbf{22.5}, \mathbf{22.55}, \mathbf{22.6}\}\boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \widehat{\boldsymbol{C}}'_+(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{2}, \mathbf{2.05}, \mathbf{2.1}, \dots, \mathbf{22.5}, \mathbf{22.55}, \mathbf{22.6}\}\boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(48)}$$

7. From Eq. (62), the set representing the number of vehicles that will likely choose the managed lanes at time interval $t+1$, $\Delta\widehat{N}_{in}(t+1)$, can be estimated from:

$$\Delta\widehat{\boldsymbol{N}}_{\boldsymbol{in}}(\boldsymbol{t}+\mathbf{1}) = \boldsymbol{N}(\boldsymbol{t}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}} \left[\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\widehat{\boldsymbol{C}}'_+(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\boldsymbol{T}(\boldsymbol{t})}} * \boldsymbol{q}(\boldsymbol{j})\right] \quad \textbf{(49)}$$

Substituting for $\hat{C}'_+(t+1)$ from Eq. (55), $\Delta T(t) = 5$ min, $q = \{0.18, 0.031, 0.041, \dots, 0.01, 0.004, 0.006\}$, $N(t) = 1200, \alpha = \{1, 1, 1, \dots, 1\}$ and $\beta = \{0.001, 0.028, 0.049, \dots, 1.288, 1.68, 3.603\}$into Eq.(63),

$$\begin{cases} \Delta\widehat{\boldsymbol{N}}_{\boldsymbol{in}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{843}, \mathbf{831.9}, \mathbf{820.81}, \dots, \mathbf{0.077}, \mathbf{0.073}\}\boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \Delta\widehat{\boldsymbol{N}}_{\boldsymbol{in}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{393.6}, \mathbf{383.82}, \mathbf{374.13}, \dots, \mathbf{0.077}, \mathbf{0.073}\}\boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(50)}$$

8. From Eq. (26), the set representing the estimated vehicle densities on the managed lanes at time interval $t+1$, $\widehat{K}_m(t+1)$ can be calculated from

$$\widehat{\boldsymbol{K}}_{\boldsymbol{m}}(\boldsymbol{t}+\mathbf{1}) = \frac{\Delta\widehat{\boldsymbol{N}}_{\boldsymbol{in}}(\boldsymbol{t}+\mathbf{1}) + \boldsymbol{N_m}(\boldsymbol{t}) - \Delta\boldsymbol{N_{out}}(\boldsymbol{t})}{\boldsymbol{L}*\mathbf{2}} \quad \textbf{(51)}$$

Substituting for $\Delta\widehat{N}_{in}(t+1)$from Eq. (65), $N_m(t) = 500$, $\Delta N_{out}(t) = 50$, L = 6.5 miles into Eq. (66) yields

$$\begin{cases} \widehat{\boldsymbol{K}}_{\boldsymbol{m}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{99.46}, \mathbf{98.61}, \mathbf{97.75}, \dots, \mathbf{34.62}, \mathbf{34.62}\}\boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \widehat{\boldsymbol{K}}_{\boldsymbol{m}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{64.89}, \mathbf{66.14}, \mathbf{63.39}, \dots, \mathbf{34.62}, \mathbf{34.62}\}\boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(52)}$$

9. From Eq. (25), the set representing the estimated average speeds on the managed lanes at time interval $t+1$, $\hat{S}_m(t+1)$, can be calculated from

$$\widehat{\mathbf{S}}_{\mathbf{m}}(\mathbf{t}+\mathbf{1}) = \mathbf{S_f}\left(\mathbf{1} - \frac{\widehat{\mathbf{K}}_{\mathbf{m}}(\mathbf{t}+\mathbf{1})}{\mathbf{k}^*_{\mathbf{m}}}\right) > \mathbf{45} \quad \textbf{(53)}$$

Substituting for $S_f = 70\ mph$ (assumed free-flow speed),$\widehat{K}_m(t+1)$,$k^*_m = 200\ vpmpl$ (assumed jam density) into Eq. (60) yields

$$\begin{cases} \widehat{\boldsymbol{S}}_{\boldsymbol{m}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{35.19}, \mathbf{35.49}, \mathbf{35.79}, \dots, \mathbf{57.88}, \mathbf{57.88}\}\boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \widehat{\boldsymbol{S}}_{\boldsymbol{m}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{47.29}, \mathbf{47.55}, \mathbf{47.81}, \dots, \mathbf{57.88}, \mathbf{57.88}\}\boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \quad \textbf{(54)}$$

10. Find the set of feasible toll rates for time interval $t+1$, $\hat{C}(t+1)$, such that the estimated average speed on the managed lanes is greater than 45 mph.

$$\widehat{\boldsymbol{C}}(\boldsymbol{t}+\mathbf{1}) = \boldsymbol{Arg}\{\widehat{\boldsymbol{S}}_{\boldsymbol{m}}(\boldsymbol{n},\boldsymbol{t}+\mathbf{1}) > \mathbf{45}|\widehat{\boldsymbol{S}}_{\boldsymbol{m}}(\boldsymbol{n},\boldsymbol{t}+\mathbf{1}) \in \widehat{\boldsymbol{S}}_{\boldsymbol{m}}(\boldsymbol{t}+\mathbf{1}), \widehat{\boldsymbol{C}}'_{+}(\boldsymbol{t}+\mathbf{1})\} \tag{55}$$

This results in

$$\begin{cases} \widehat{\boldsymbol{C}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{1.60}, \mathbf{1.65}, \mathbf{1.70}, \dots, \mathbf{22.5}, \mathbf{22.55}, \mathbf{22.6}\} & \boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \widehat{\boldsymbol{C}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{2}, \mathbf{2.05}, \mathbf{2.10}, \dots, \mathbf{22.55}, \mathbf{22.6}\} & \boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \tag{56}$$

As all the feasible toll rates have been identified in Eq.(63), we can now determine the optimal toll rate that will satisfy the objective of the toll strategy. Two maximization objectives are considered: toll revenue maximization as well as both toll revenue and throughput on the managed lanes maximization. The road authority can chose one of the maximization goals and follow the optimal toll searching procedure of that objective.

If the objective of road authority is to maximize the toll revenue while keeping a minimum level of service on the managed lanes, the following steps are searching procedure of the optimal toll given that all feasible toll rates have been identified.

11 (a) From Eq. (63), the set representing the estimated toll revenues for time interval $t+1$, $\hat{R}(t+1)$ can be calculated from

$$\widehat{\boldsymbol{R}}(\boldsymbol{t}+\mathbf{1}) = \widehat{\boldsymbol{C}}(\boldsymbol{t}+\mathbf{1}) * \boldsymbol{N}(\boldsymbol{t}) * \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}} \left[\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{e}^{\boldsymbol{\alpha}(\boldsymbol{j})*\widehat{\boldsymbol{C}}(\boldsymbol{t}+\mathbf{1})-\boldsymbol{\beta}(\boldsymbol{j})*\Delta\boldsymbol{T}(\boldsymbol{t})}} * \boldsymbol{q}(\boldsymbol{j})\right] \tag{57}$$

Substituting for the toll rates set $\hat{C}(t+1)$, $q = \{0.18, 0.031, 0.041, \dots, 0.01, 0.004, 0.006\}$, $N(t) = 1200, \alpha = \{1, 1, 1, \dots, 1\}$ and $\beta = \{0.001, 0.028, 0.049, \dots, 1.288, 1.68, 3.603\}$ and $\Delta T(t) = 5$ min into Eq. (64) yields

$$\begin{cases} \widehat{\boldsymbol{R}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{764.77}, \mathbf{770.46}, \mathbf{775.27}, \dots, \mathbf{1.74}, \mathbf{1.65}\}\boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \widehat{\boldsymbol{R}}(\boldsymbol{t}+\mathbf{1}) = \{\mathbf{787.2}, \mathbf{786.83}, \mathbf{785.67}, \dots, \mathbf{1.74}, \mathbf{1.65}\}\boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \tag{58}$$

12 (a) Calculate the maximum toll revenue $\hat{r}(t+1)$ such that $\hat{r}(t+1) = \max\{\hat{R}(t+1)\}$. In this example, the estimated maximum toll revenue and corresponding toll rates for the next three-minute time interval are

$$\begin{cases} \hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1}) = \$\mathbf{843.7},\ \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1}) = \$\mathbf{1.85} & \boldsymbol{S_m}(\boldsymbol{t}) > \mathbf{45}\boldsymbol{mph} \\ \hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1}) = \$\mathbf{787.2},\ \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1}) = \$\mathbf{2} & \boldsymbol{S_m}(\boldsymbol{t}) \leq \mathbf{45}\boldsymbol{mph} \end{cases} \tag{59}$$

If the objective of road authority is to maximize both the toll revenue and the throughput on the managed lanes while keeping a minimum level of service, the following steps are searching procedure of the optimal toll given that all feasible toll rates have been identified.

(b) From Eq. (21) the set representing the estimated revenue and throughput on managed lanes for time interval $\mathbf{t+1}$, $\widehat{\mathbf{R}}(\mathbf{t+1}) + \boldsymbol{\theta} \times \widehat{\mathbf{TP}}(\mathbf{t+1})$ can be calculated from

$$\widehat{\mathbf{R}}(\mathbf{t}+\mathbf{1})+\boldsymbol{\theta}\times\{\mathbf{N}(\mathbf{t})*\sum_{\mathbf{j}=\mathbf{1}}^{\mathbf{3}}\left[\frac{\mathbf{1}}{\mathbf{1}+\mathbf{e}^{\boldsymbol{\alpha}(\mathbf{j})*\hat{\mathbf{c}}(\mathbf{t}+\mathbf{1})-\boldsymbol{\beta}(\mathbf{j})*\Delta\mathbf{T}(\mathbf{t})}}*\mathbf{q}(\mathbf{j})\right]+\mathbf{N}_{\mathbf{m}}(\mathbf{t})-\Delta\mathbf{N}_{\mathbf{out}}(\mathbf{t})\} \quad \textbf{(60)}$$

(b) Substituting for the$\boldsymbol{\theta}=\mathbf{0.5}$, $\mathbf{q}=\{\mathbf{0.1},\mathbf{0.24},\mathbf{0.66}\}$, $\mathbf{N}(\mathbf{t})=\mathbf{1200}$, $\boldsymbol{\alpha}=\{\mathbf{1},\mathbf{1},\mathbf{1}\}$, $\boldsymbol{\beta}=\{\mathbf{0.74},\mathbf{0.43},\mathbf{0.14}\}$, $\Delta\mathbf{T}(\mathbf{t})=\mathbf{5}$, $\mathbf{N}_{\mathbf{m}}(\mathbf{t})=\mathbf{500}$, $\Delta\mathbf{N}_{\mathbf{out}}(\mathbf{t})=\mathbf{50}$ and $\widehat{\mathbf{R}}(\mathbf{t}+\mathbf{1})$

from Eq. (66)into Eq. (67) we get

$$\widehat{\boldsymbol{R}}(\boldsymbol{t}+\mathbf{1})+\boldsymbol{\theta}\times\widehat{\boldsymbol{TP}}(\boldsymbol{t}+\mathbf{1})=\begin{cases}\{\mathbf{1228.76},\mathbf{1228.93},\mathbf{1228.29},\ldots,\mathbf{226.78},\mathbf{226.69}\} & \boldsymbol{S_m}(\boldsymbol{t})>\mathbf{45}\boldsymbol{mph}\\ \{\mathbf{1209},\mathbf{1203.74},\ldots,\mathbf{226.78},\mathbf{226.69}\} & \boldsymbol{S_m}(\boldsymbol{t})\leq\mathbf{45}\boldsymbol{mph}\end{cases} \quad \textbf{(61)}$$

11. Calculate the maximum sum of toll revenues and throughputs on managed lanes $\hat{R}(t+1)+\theta\times\hat{Q}(t+1)$ such that $\hat{r}(t+1)+\theta\times\hat{q}(t+1)=\max\{\hat{R}(t+1)+\theta\times\hat{Q}(t+1)\}$. In thisexample, the estimated maximum sum of toll revenues and throughputs as well as the corresponding toll rates for the next three-minute time interval are

$$\begin{cases}\hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1})+\hat{\boldsymbol{q}}(\boldsymbol{t}+\mathbf{1})=\mathbf{1300.5},\ \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})=\$\mathbf{1.75} & \boldsymbol{S_m}(\boldsymbol{t})>\mathbf{45}\boldsymbol{mph}\\ \hat{\boldsymbol{r}}(\boldsymbol{t}+\mathbf{1})+\hat{\boldsymbol{q}}(\boldsymbol{t}+\mathbf{1})=\mathbf{1209},\ \hat{\boldsymbol{c}}(\boldsymbol{t}+\mathbf{1})=\$\mathbf{2} & \boldsymbol{S_m}(\boldsymbol{t})\leq\mathbf{45}\boldsymbol{mph}\end{cases} \quad \textbf{(62)}$$

This extended example has the same conditions of traffic, toll rate, toll objectives as the previous example, except that it has forty-four driver income groups instead of the simplified three driver income groups. Results are summarized in TABLE 4. Again, the toll rate of the traffic condition A is lower than that of traffic condition B. Also, under both traffic condition A and B, the toll revenues and the number of vehicles choosing the managed lanes have the same observations as the simplified example.

TABLE 4 Results of Extended Example

	Toll Rate ($)	Toll Revenue ($/3 min)	Number of vehicles choosing the managed lanes (vehicles/3 min)
Revenue maximization ($S_m(t)>45mph$)	1.85	843.7	456
Revenue maximization ($S_m(t)\leq 45mph$)	2	787.2	393.6
Revenue & Throughput maximization ($S_m(t)>45mph$)	1.75	836.5	478
Revenue & Throughput maximization ($S_m(t)\leq 45mph$)	2	787.2	393.6

Results of the two numerical examples are compared in TABLE 5. As we can see, there are no big differences of the toll rate, toll revenue or number of vehicles choosing the managed lanes between the two numerical examples. We might conclude that the number of driver income

groups does not have a significant impact on the performance of the proposed toll pricing strategy. However, the traffic conditions given in these examples are greatly simplified that cannot properly represent the real-world traffic situation. Besides, the numerical examples only consider one time interval thus the results do not reflect the trend when the time period is much longer than a single three-minute interval. As a result, it is still necessary to consider two distributions of drivers' value of time in the simulation study.

TABLE 5 Results Comparison of Two Numerical Examples

	Toll Rate ($)		Toll Revenue ($/3 min)		Number of vehicles choosing the managed lanes (vehicles/3 min)	
	Simplified	Extended	Simplified	Extended	Simplified	Extended
Revenue maximization ($S_m(t) > 45mph$)	1.85	1.85	853.53	843.7	461.4	456
Revenue maximization ($S_m(t) \leq 45mph$)	2	2	853.27	787.2	426.6	393.6
Revenue & Throughput maximization ($S_m(t) > 45mph$)	1.8	1.75	849.23	836.5	471.8	478
Revenue & Throughput maximization ($S_m(t) \leq 45mph$)	2	2	853.27	787.2	426.6	393.6

4. SIMULATION AND ANALYSIS OF RESULTS

This chapter tests the performance of the proposed strategy in simulation. The first section introduces the input parameters of the simulation study. The second section addresses the impacts of the value of $\beta(i)$on drivers' probability of choosing the managed lanes as well as on the subsequent traffic and toll conditions. The third section discusses the impacts of the distribution of drivers' value of time over their route choice decision and also compares the performances of the proposed strategy under two distributions of drivers' value of time. The last section compares the performances of the proposed strategy with that of the current toll strategy adopted by the I-95.

Simulation Input Parameters

Before running the simulation scenarios, basic input parameters need to be set: the free-flow speed and the jam density. The free-flow speed is based on the user-defined speed distribution of vehicles. In this study, the free-flow speed was assumed to follow a uniform distribution with a minimum speed of 60 mph and a maximum speed of 80 mph. Therefore, the free-flow speed is 70 mph, which is the mean speed of the distribution. The jam density was simply measured from VISSIM by making the traffic on a link congested to stop and calculating the density using the total number of vehicles on that link divided by the length of the link. The jam density was found to be 240 passenger cars per mile per lane.

In addition, the data of traffic demand was retrieved from real traffic volume data on I-95 at NW151, which is provided by the database "Statewide Transportation Engineering Warehouse for Archived Regional Data" (STWARD). The traffic demand profile was shown in FIGURE 2. The simulation period begins at 5:00 AM and ends at 7:30 AM. This period ensures variation in demand from low to high in order to examine the performance of the proposed strategy under varying traffic conditions. FIGURE 2shows that from 5:00 AM to 6:30 AM, the traffic demand increases gradually from low: 1800 vph to high: 7500 vph, and remains at a high level from 6:30 AM to 7:30 AM. (Traffic demand for the on-ramps along the general purpose lanes was assumed to be the 20% of the overall traffic demand entering the study segment.

Effect of $\boldsymbol{\beta}(\mathbf{i})$

After all of the parameters for VISSIM simulation are determined, we still need to choose the value of $\beta(i)$ associated to the drivers' route choice model. The parameter $\beta(i)$ represents the monetary value of travel time savings and has an important effect on drivers' route decisions. In order to set a proper value of $\beta(i)$, this section investigates the sensitivity of drivers' route decisions and subsequent traffic and toll conditions to $\beta(i)$ when the toll rate was determined by the proposed toll pricing strategy with different objectives. Two objectives were examined: the first was to maximize the toll revenue while keeping a minimum level of service on the managed lanes and the second was to maximize both the revenue and the throughput on the managed lanes while maintaining a minimum level of service. Initial value of $\beta(i)$ is set to be 90% of the mean

hourly income of drivers according to the data from NCHRP report 463 (Weisbrod ,2001). This value is then changed to 60% and 120% to quantify its effect on the proposed strategy focused on three different objectives. The drivers' income groups were assumed with the same information presented in TABLE 2.

FIGURE 7 shows that no travel time savings could be gained from using the managed lanes at low traffic demand. FIGURE shows the effect of $\beta(i)$ on the collective probability of drivers choosing the managed lanes under the proposed strategy with the revenue maximization objective. At low traffic demand, the probability of choosing managed lanes is not impacted by the value of $\beta(i)$. However, as traffic demand increases, the collective probability increases since congestion on the general purpose lanes leads to higher travel time savings, which increases the utility of managed lanes. FIGURE proves that higher value of $\beta(i)$ leads to a higher collective probability of drivers choosing the managed lanes.

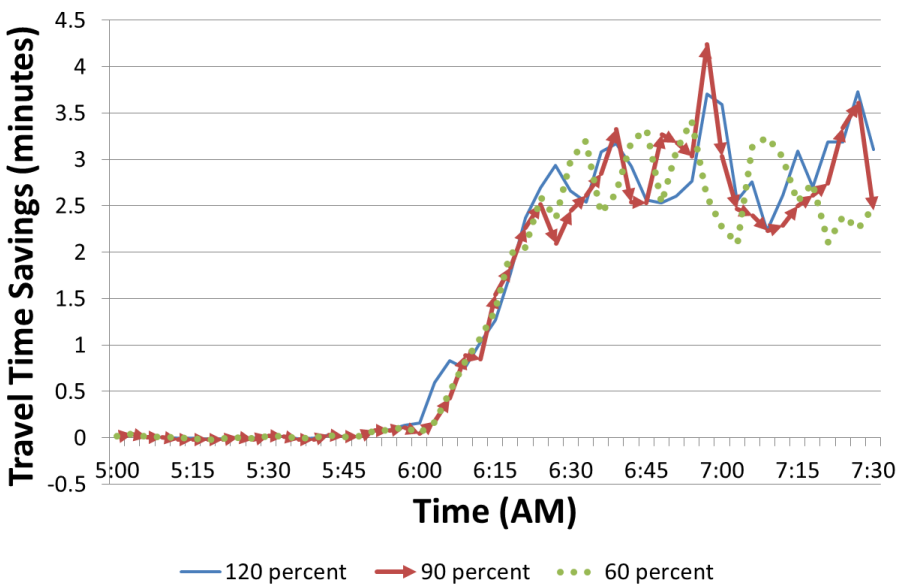

FIGURE 7 Travel time savings under the objective of revenue maximization

FIGURE shows the effect of $\beta(i)$ on the toll revenue. The observations are consistent with the effect of $\beta(i)$ on the probability of choosing managed lanes. FIGURE shows that higher value of $\beta(i)$ leads to higher toll revenue since the collective probability of drivers choosing the managed lanes is higher. When the $\beta(i)$ is at the value of 120% of mean income, the toll revenue is highest. FIGURE 8 shows the flow on the managed lanes did not impact their level of service. Because the average speed on the managed lanes is still above 45 mph at the highest oll revenue. Therefore, this value will be adopted in the remaining analysis for the proposed strategy with the objective of toll revenue maximization.

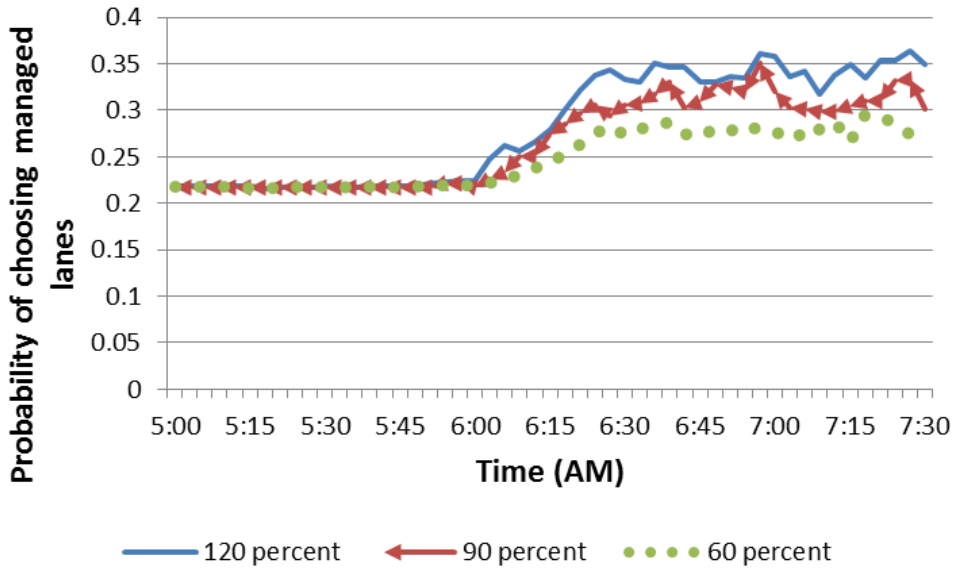

FIGURE 8 Probability of choosing managed lanes under the objective of revenue maximization

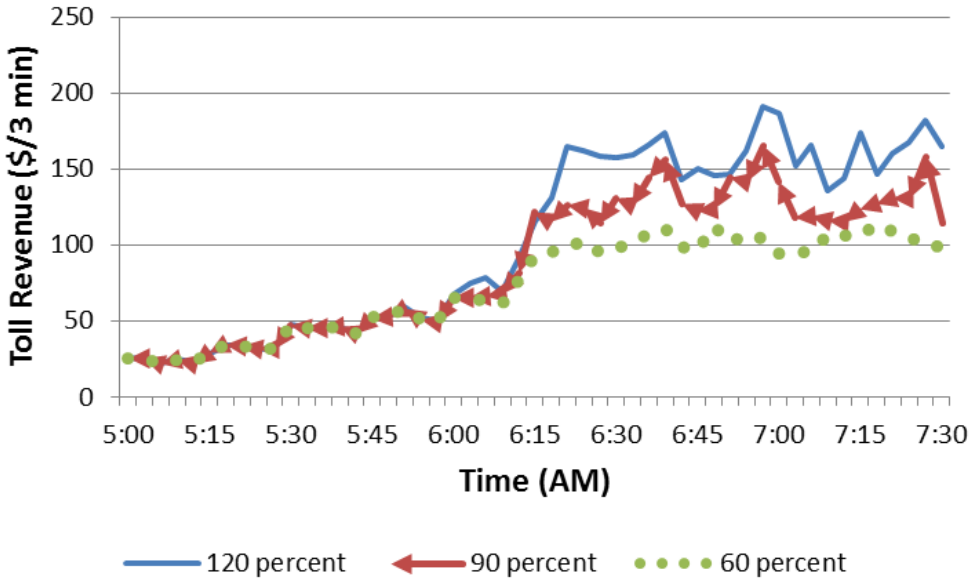

FIGURE 7 Toll revenue under the objective of revenue maximization

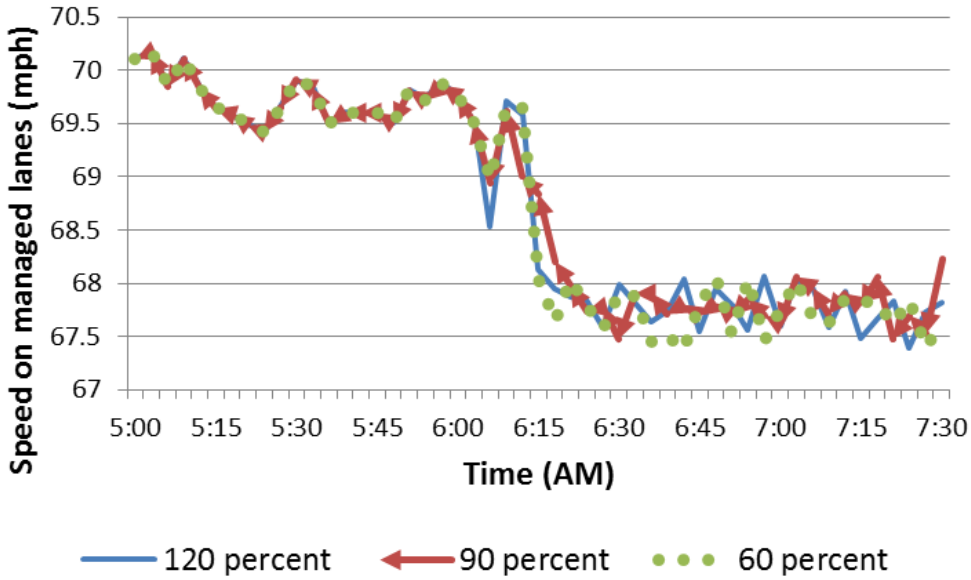

FIGURE 8 Average speeds on the managed lanes under the objective of revenue maximization

FIGURE 9 to FIGURE 12 demonstrates the effect of $\beta(i)$ on the collective probability of drivers choosing the managed lanes, the sum of revenue and monetary value of throughput on the managed lanes, travel time savings and the average speed on the managed lanes when the toll price is determined by the proposed methodology with the objective of maximizing both revenue and monetary value of throughput on the managed lanes.The probability of choosing the managed lanes in FIGURE 9 was not impacted by when the traffic demand is low since no travel time savings at low demand. A sudden drop of probability was observed at the onset of high traffic demand. The reason can be that before the onset of high traffic demand, there were sufficient vehicles on the managed lanes since the probability of choosing the managed lanes was high at low demand, so when the demand increased greatly, the toll strategy actually imposed a relatively high toll rate to discourage vehicles from entering the managed lanes in order to keep the minimum level of service on the managed lanes. After the sudden drop, the probability increased gradually as the traffic demand increased since the number of vehicles on the managed lanes was sufficiently low to keep the minimum level of service and the travel time savings in FIGURE 10 increased as the demand increased.

FIGURE 11 indicates that the sum of the revenue and the monetary value of throughput on managed lanes increase as the traffic demand increases. The higher value of $\beta(i)$ produces the higher summation. FIGURE 12 indicates the average speed on the managed lanes is not impacted by the value of $\beta(i)$. The reason can be vehicles on the managed lanes are not sufficient enough to cause congestion. Because when the value of $\beta(i)$ is 120% of mean incomethe sum of monetary value of throughput and the revenue is highest and the average speed on the managed lanes is above 45 mph, this value will be adopted in the remaining analysis of the proposed methodology with the objective of maximizing both revenue and the throughput.

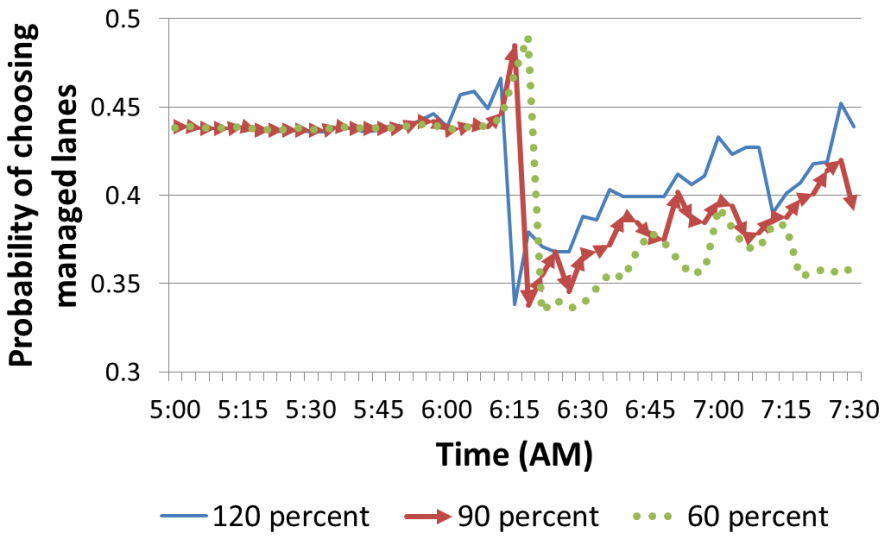

FIGURE 9 Probability of choosing the managed lanes under revenue and throughput maximization

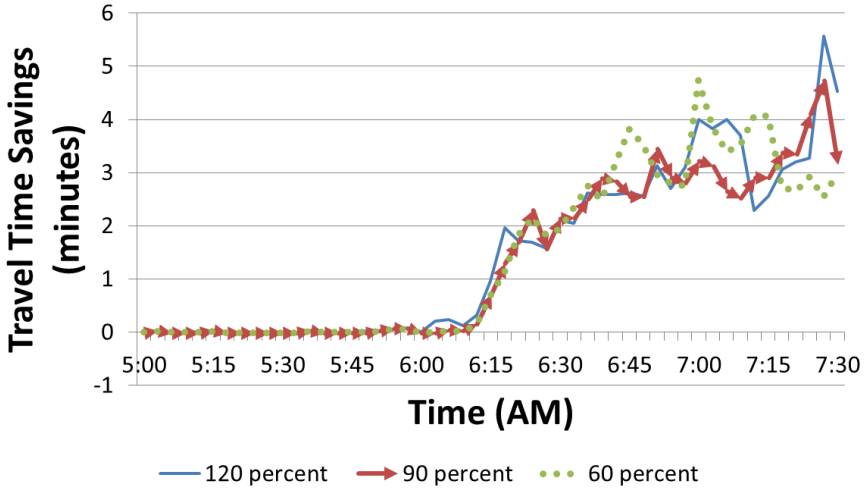

FIGURE 10 Travel time savings under revenue and throughput maximization

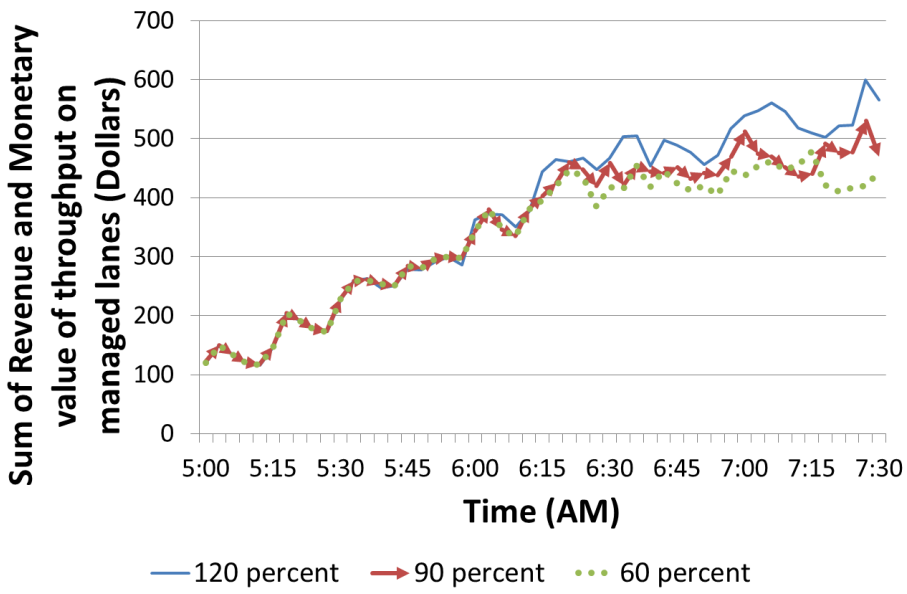

FIGURE 11 Sum of revenue and monetary value of throughput on the managed lanes under revenue and throughput maximization

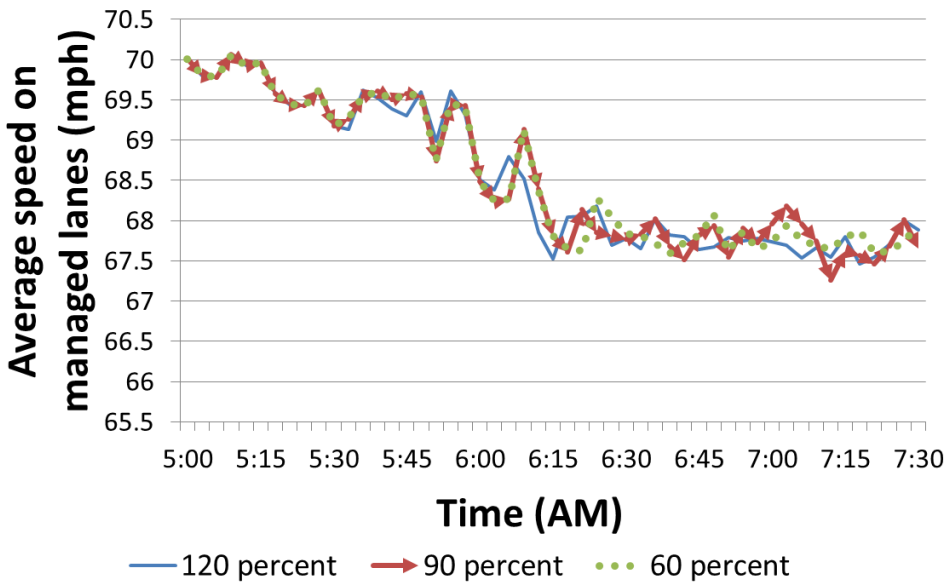

FIGURE 12 Average speeds on the managed lanes under revenue and throughput maximization

Drivers' Income Groups

In the literature review, many studies assumed the homogeneity of drivers in terms of value of travel time savings. This was not practical since due to different socio-economic statues, drivers would have distinct values of travel time savings, $\beta(i)$. As mentioned before, this study used drivers' annual income levels to estimate $\beta(i)$ among drivers.

In previous section, the value was determined to be 120 percent of drivers' mean hourly income. This section addresses the impacts of drivers' income groups on the route decisions followed by a performance comparison of the proposed strategy under two distributions of drivers' income groups.

Effects of drivers' income groups on the route decision

FIGURE 13 compares the probabilities of choosing the managed lanes by five distributions of drivers' income groups under the proposed toll strategy with the objective of revenue maximization. Information of groups is from TABLE 2 and FIGURE 6. Insignificant effect of income group on the probability of choosing the managed lanes is observed at low traffic demand. The reason can be insignificant travel time saving and therefore, the difference between the utility of managed lanes and general purpose lanes perceived by drivers is negligible. As demand increases and the toll rate changes, the effect of income on probability becomes apparent among the different groups. Higher probability is observed for group 1, which belongs to the highest mean income, and lower probability for group 3 with the lowest mean income. This is expected since the same travel time saving would attract more high income users than low income users for the same toll rate. The probability of the combined three driver income groups is very close to that of the combined forty-four driver income groups. FIGURE 14 compares the probabilities of choosing the managed lanes by five distributions of drivers' income groups under the proposed toll strategy with the objective of both revenue and throughput on the managed lanes maximization. It has similar observations as FIGURE 13 except that there is a big difference between the probability of the combined three driver groups and that of the forty-four driver groups.

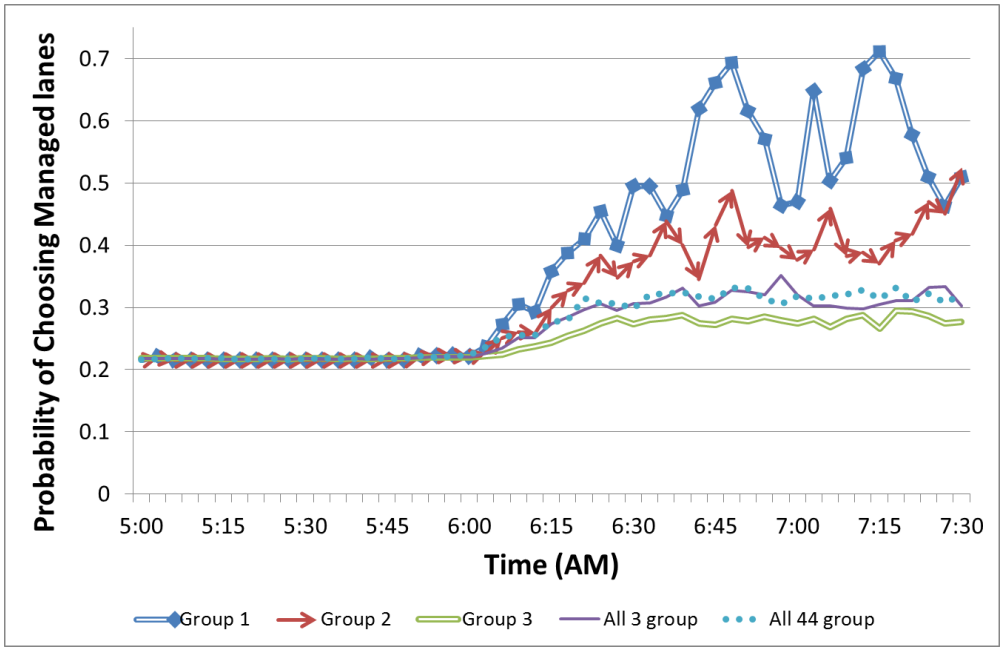

FIGURE 13 Collective route decisions by different driver groups under proposed toll method with the objective of revenue maximization

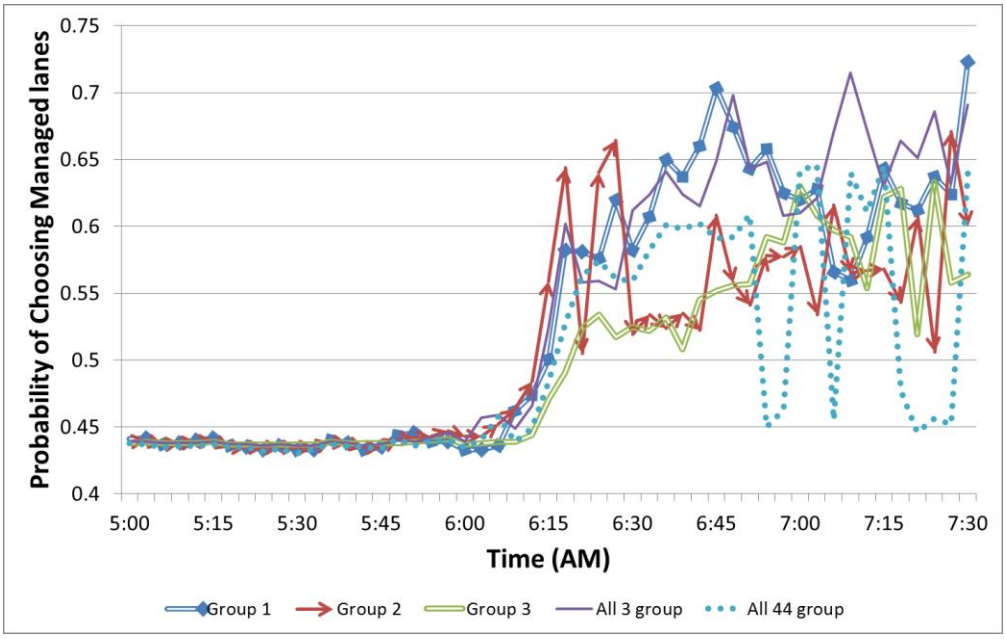

FIGURE 14 Collective route decisions by different driver groups under proposed toll method with the objective of revenue andthroughputs maximization

Effects of drivers' income groups on the toll strategy

FIGURE 15 shows that the toll rate produced by the two distributions are the same at low traffic demand and the toll rate of the forty-four driver groups becomes slightly higher at high traffic demand. The travel time savings of the both distributions have the same trend: at the low traffic demand, no travel time savings are observed. Onset of high traffic demand, the travel time savings increase greatly. As the traffic demand becomes stable only small fluctuations occur in travel time savings. FIGURE 16 shows the two distributions generate the same throughput and toll revenues on the managed lanes at low traffic demand since no travel time savings are gained.

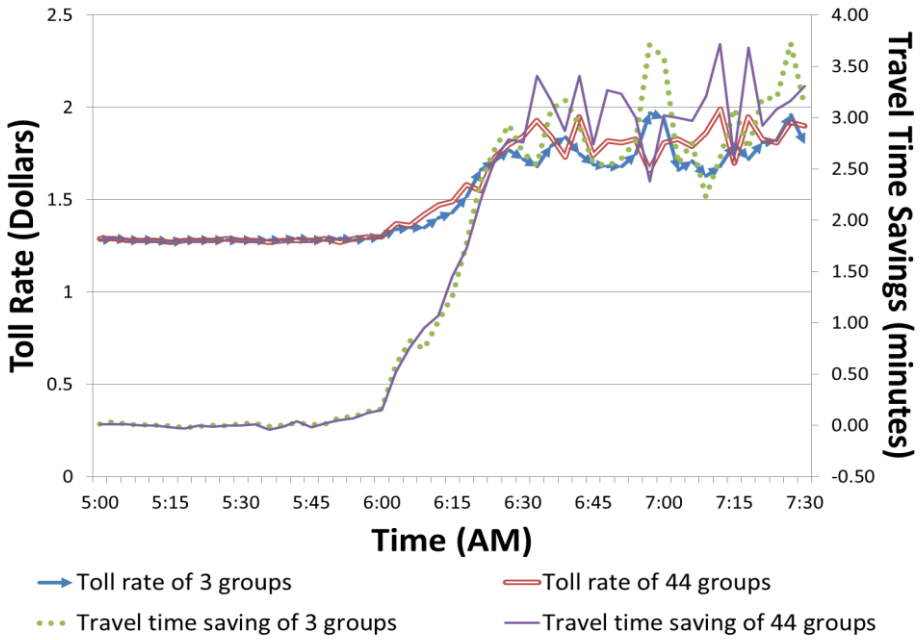

FIGURE 15 Toll rates and travel time savings under revenue maximization

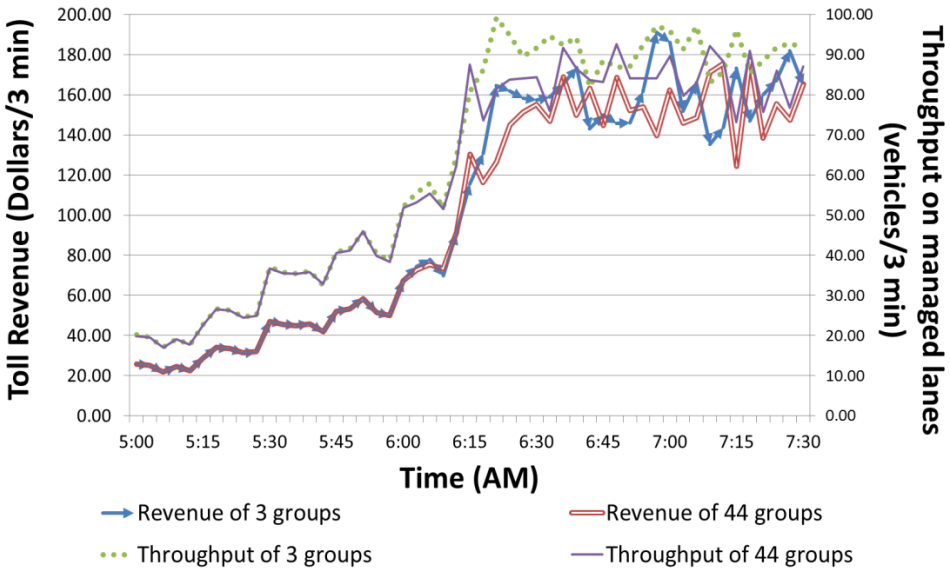

FIGURE 16 Toll revenues and throughput on the managed lanes under revenue maximization

At high traffic demand, the throughputs are still close though the three driver groups has slightly higher throughput. FIGURE 17 displays the very close rates and travel time savings for three driver groups and the forty-four driver groups. However in FIGURE 18, the throughput on the managed lanes produced by forty-four driver groups is much higher than that of the three driver groups and thus results in higher toll revenue. This might imply that under the same toll rate the three driver groups have smaller collective probability of choosing the managed lanes than the

forty-four driver groups do. In the other words, the three driver groups actually lower the collective value of time of drivers.

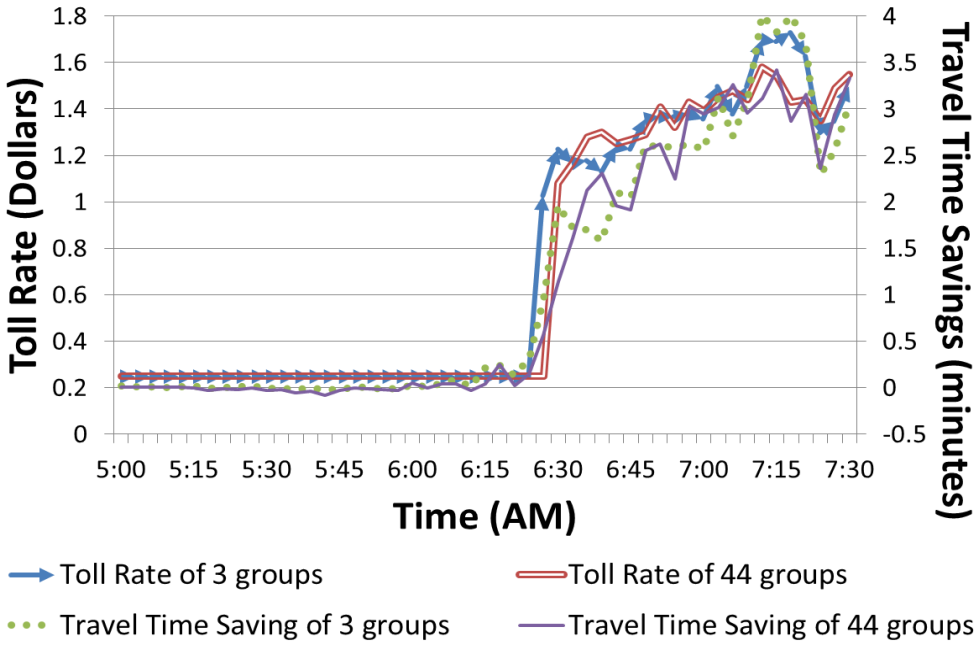

FIGURE 17 Toll rates and travel time savings under both revenue and throughput maximization

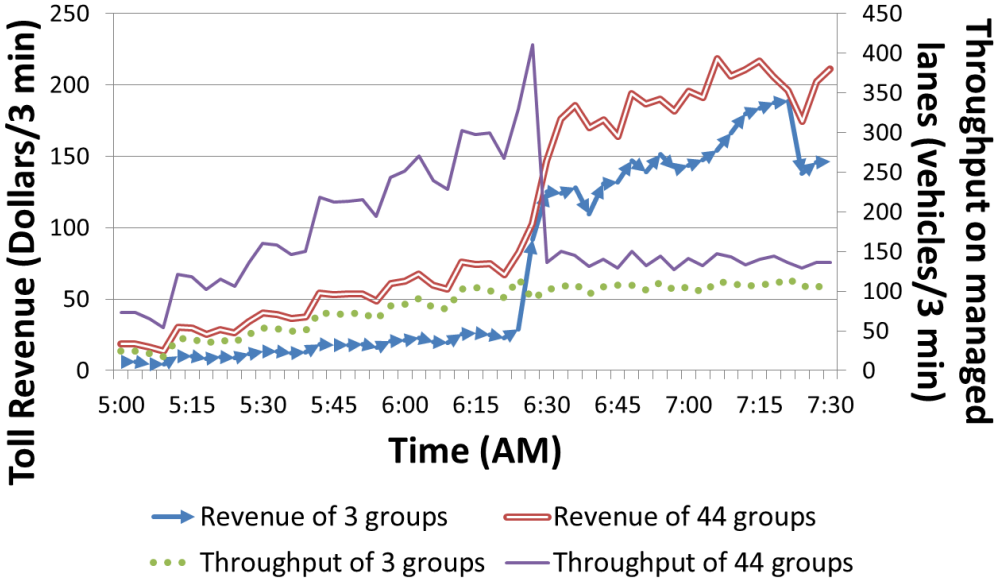

FIGURE 18 Toll revenues and throughput on managed lanesunder both revenue and throughput maximization

Comparative Evaluation

This section first compares the performances of the proposed dynamic toll pricing strategy to that of the currently adopted dynamic toll strategy on I-95under two different objectives. As mentioned in previous chapters, one objective is to maximize the toll revenue and the other is to maximize both revenue and throughput on the managed lanes. Both two objectives have the constraint of maintaining a minimum level of service on the managed lanes. Besides, two distributions of drivers' income groups are used: three driver income groups from TABLE 2 and forty-four driver income groups from FIGURE 6. Performance measures include the toll rates, toll revenue, average speeds on both managed and general purpose lanes, travel time savings, and throughput on the managed lanes.

FIGURE 19 to FIGURE 24 compares the performance of the proposed methodology focusing on two different objectives and that of the current dynamic toll strategy when only three driver groups are considered. FIGURE 19 shows that the proposed strategy focusing on both objectives produce steadier toll rate profiles than the currently I-95's strategy. At low traffic demand, the proposed strategy focusing on revenue maximization shows highest initial toll rate while the proposed strategy focusing on both revenue and throughput maximization and the current strategy produce similar relatively low toll rate. As demand increases, the current strategy produces highest toll rate, the proposed strategy with revenue maximization produces second highest toll rate and the proposed strategy with both revenue and throughput maximization produces lowest toll rate.

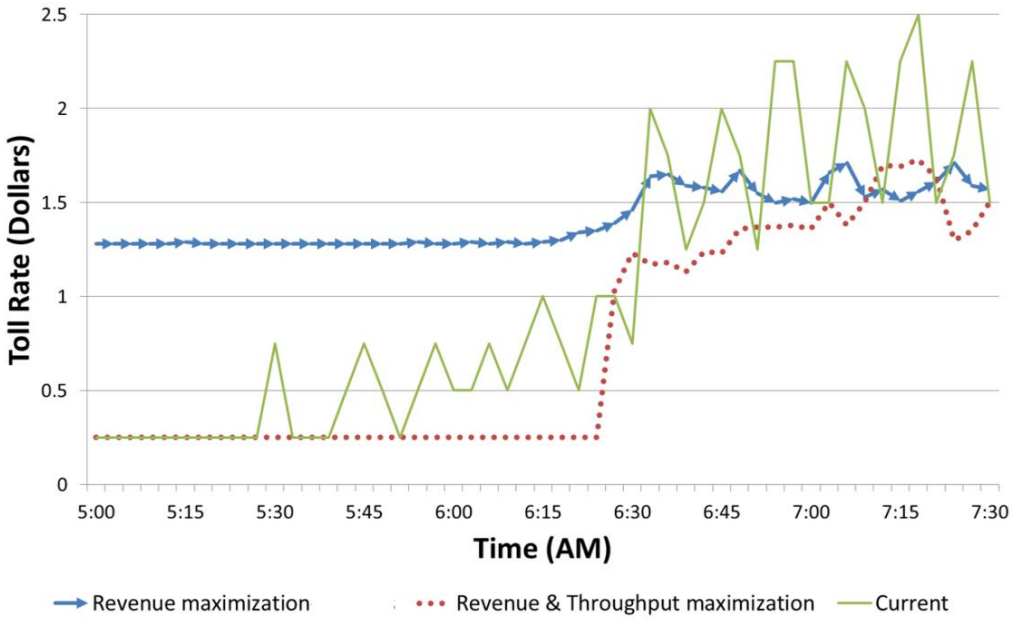

FIGURE 19 Toll rate under three driver income groups

Meanwhile, as shown in FIGURE 20, the throughput on the managed lanes produced by the proposed strategy with the objective of both revenue and throughput maximization is the highest at low and high traffic demand. The throughput on the managed lanes produced by the proposed strategy with a focus on revenue maximization is initially lower than that of the current strategy

though the trend is reversed as demand goes up. FIGURE 21 indicates that the toll revenue produced by the proposed strategy with a focus on revenue maximization is the highest at low and high traffic demand. The revenue produced by the proposed strategy with a focus on both revenue and throughput on the managed lanes is lower than that of the current strategy at low traffic demand and the trend reverses when the traffic demand is high.

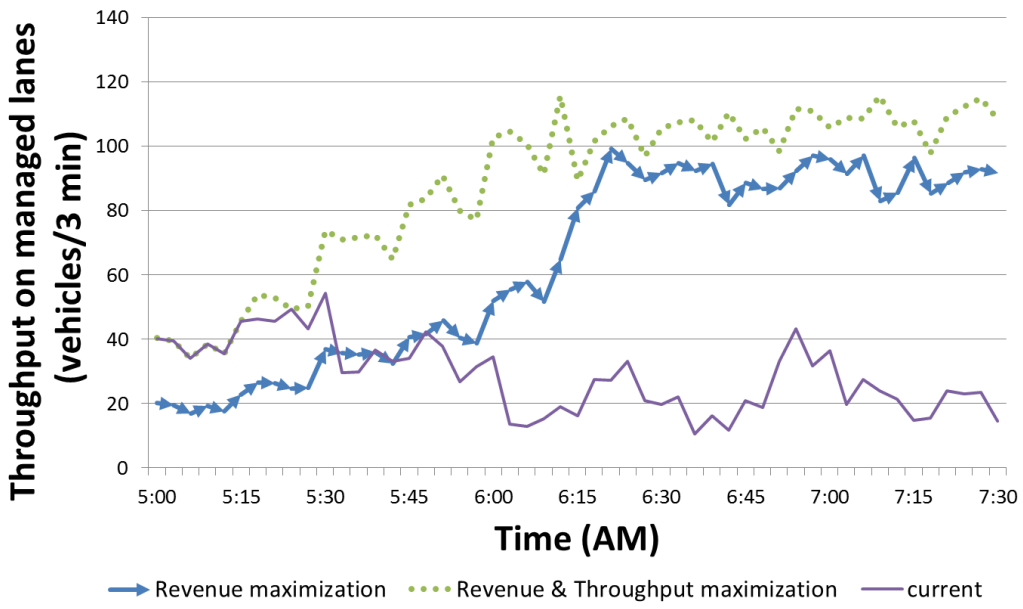

FIGURE 20 Throughput on the managed lanes under three driver income groups

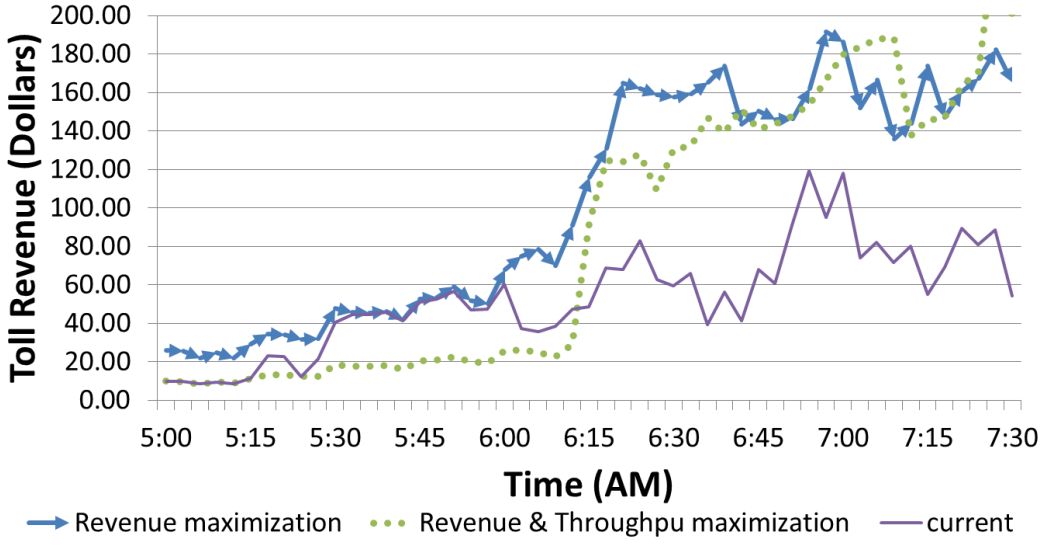

FIGURE 21 Toll revenue under three driver income groups

The higher revenue and throughput on managed lanes produced by the proposed strategies with two objectives are achieved without compromising the level of service on the managed lanes, as

shown in FIGURE 22. FIGURE 22 shows that the average speed on the managed lanes for all strategies followed the same trend and remained above 67 mph. For the general purpose lanes, the average speed drops below 40 mph as congestion develops when demand increases. FIGURE 23 shows slight differences in speed on the general purpose lanes between the two strategies. While the proposed strategy attracted more vehicles from the general purpose lanes into the managed lanes, this difference in flow rate appears to be offset by the effect of merging and diverging maneuvers near on- and off-ramps along the general purpose lanes, which could explain the insignificant change in traffic conditions. In terms of travel time savings in minutes, FIGURE 24 shows very close values for both strategies, except at the onset of congestion when slightly higher savings are observed for the proposed strategies.

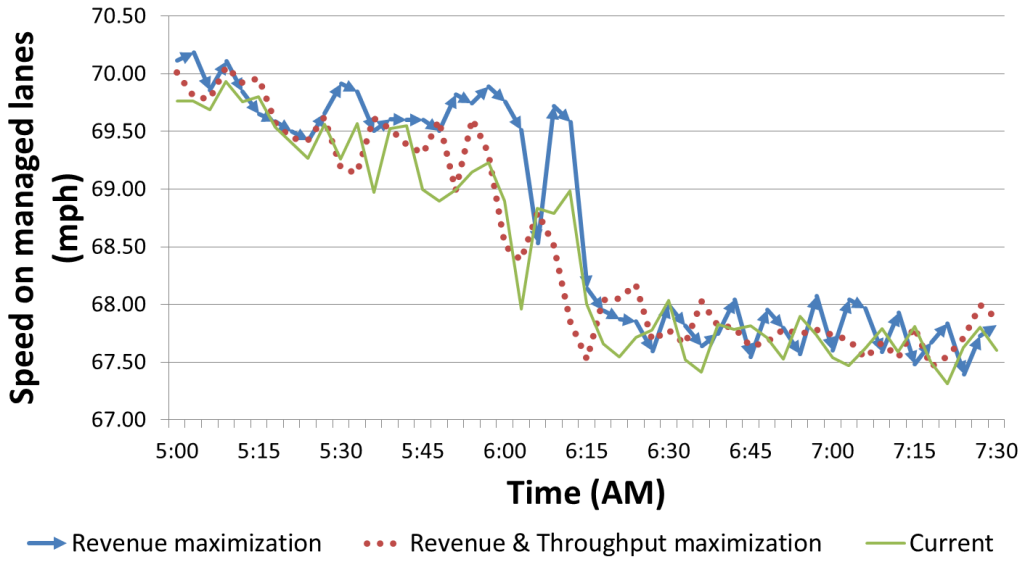

FIGURE 22 Average speeds on the managed lanes under three driver income groups

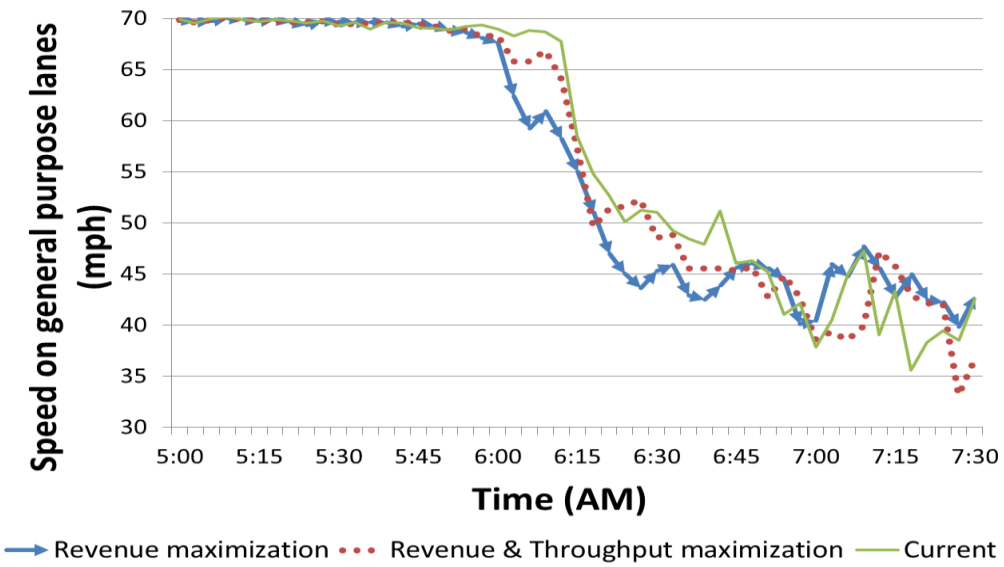

FIGURE 23 Average speeds on general purpose lanes under three driver income groups

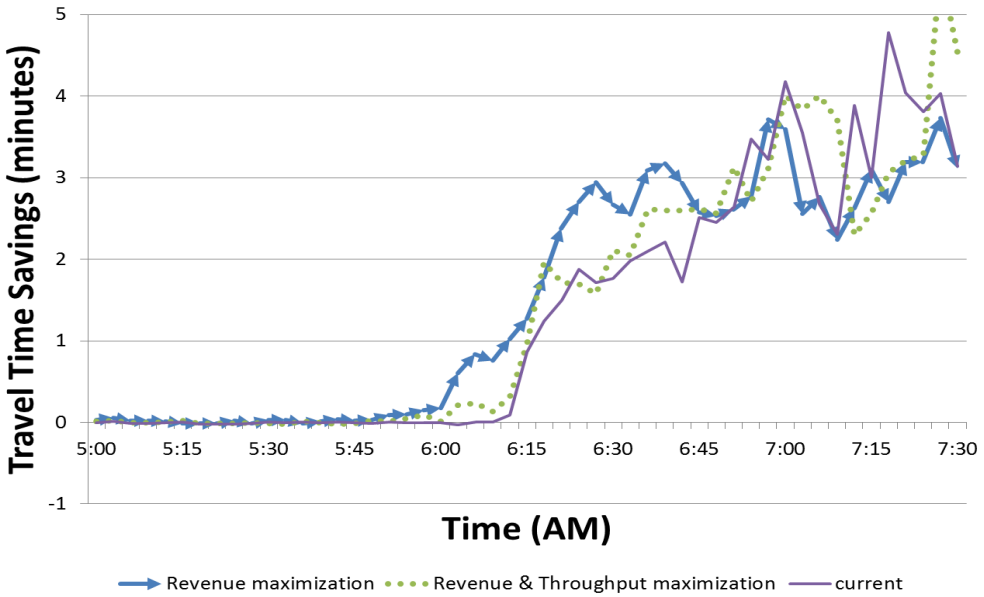

FIGURE 24 Travel time savings under three driver income groups

FIGURE 25 to FIGURE 30compare the performances of the proposed methodology focusing on two different objectives and that of the current dynamic toll strategy under forty-four driver income groups. FIGURE 25 shows that the proposed strategies focusing on two objectives both produce steadier toll rate profiles than the currently I-95's strategy. At low traffic demand, the proposed strategy focusing on revenue maximization shows highest initial toll rate while the proposed strategy focusing on both revenue and throughput maximization and the current strategy produce relatively lower toll rate. As demand increases, the current strategy produces highest toll rate, the proposed strategy with revenue maximization produces second highest toll

rate and the proposed strategy with both revenue and throughput maximization produces lowest toll rate.

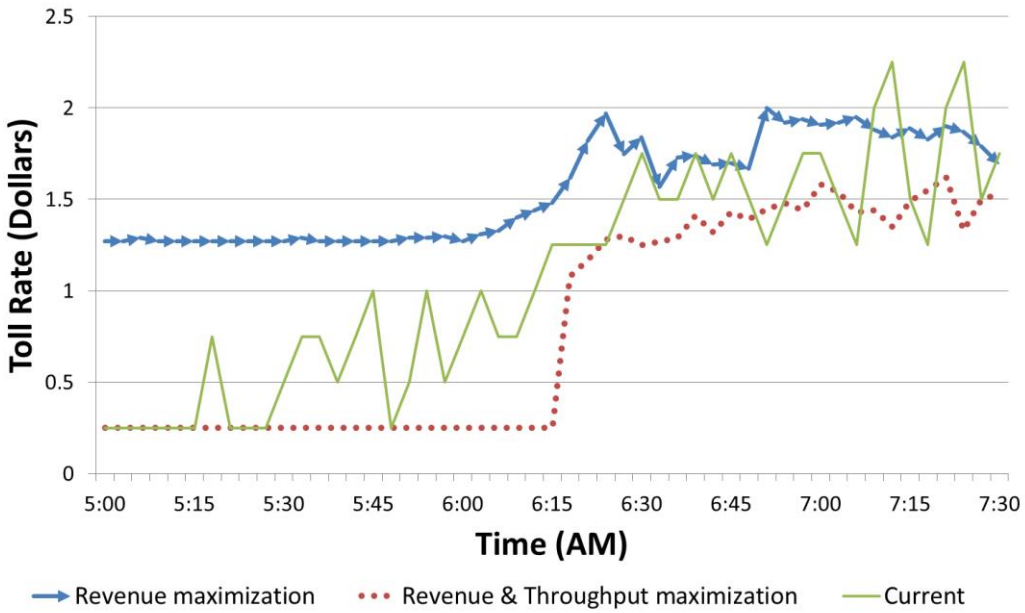

FIGURE 25 Toll rates under forty-four driver groups

As shown in FIGURE 26, the throughput on the managed lanes produced by the proposed strategy with the objective of both revenue and throughput maximization is the highest at low and high traffic demand. The throughput produced by the proposed strategy with a focus on revenue maximization is initially higher than that of the current strategy though the difference becomes insignificant as demand goes up. FIGURE 27 indicates that the toll revenue produced by the proposed strategy with a focus on revenue maximization is the highest while the toll revenue of the current strategy is the second highest one and the revenue of the proposed strategy with a focus on both revenue and throughput maximization is the lowest one. The revenue differences between these three methodologies are big at the low traffic demand yet the differences become insignificant as demand increases.

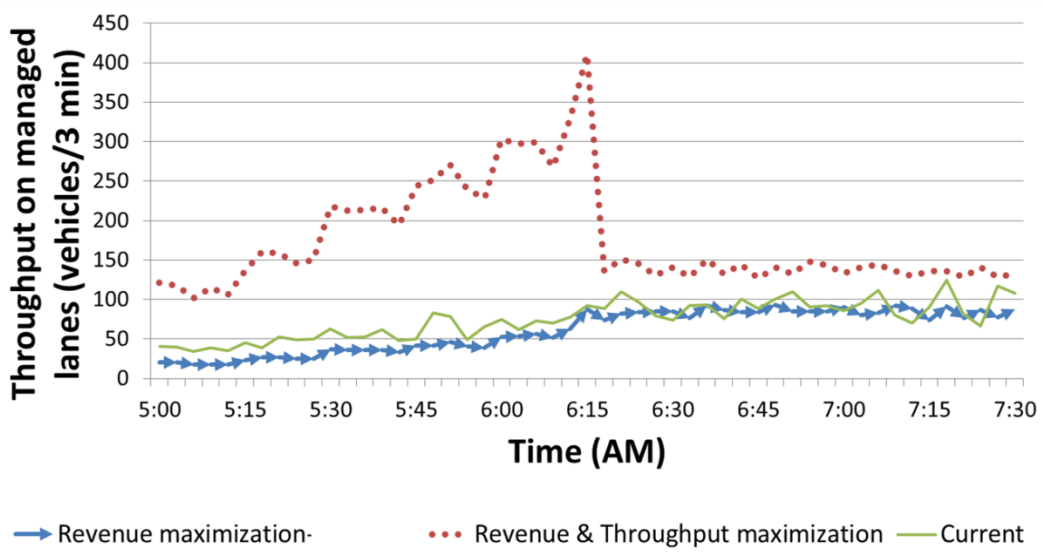

FIGURE 26 Throughput on the managed lanes under forty-four driver groups

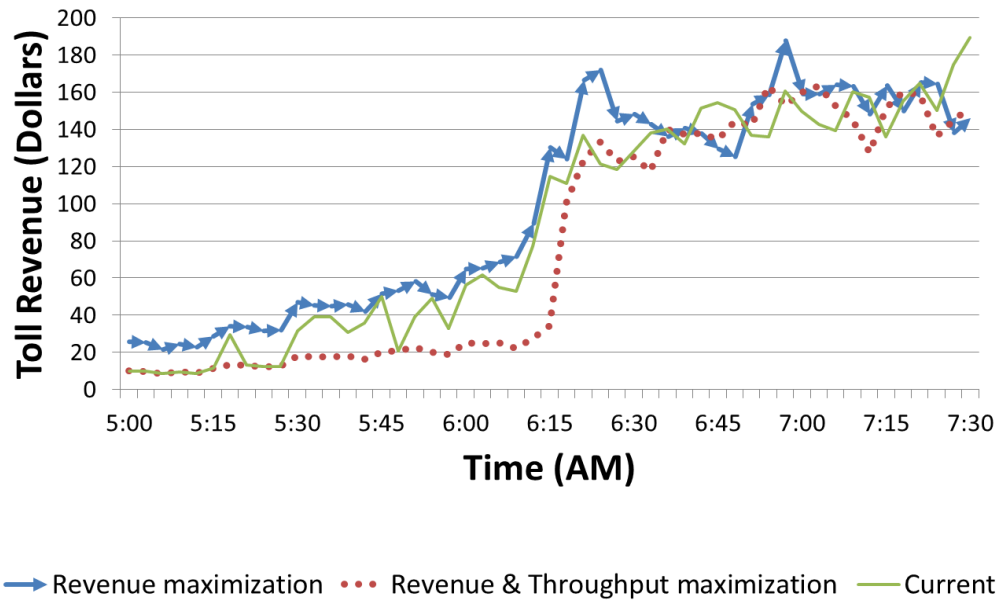

FIGURE 27 Toll revenue under forty-four driver groups

Compared to the performance of the current strategy, the higher revenue and throughput on managed lanes produced by the proposed strategies are achieved without compromising the level of service on the managed lanes, as shown in FIGURE 28. FIGURE 28 shows that the average speed on the managed lanes for all strategies followed the same trend and remained above 67 mph. For the general purpose lanes, the average speed drops below 40 mph as congestion develops when demand increases. FIGURE 29 shows slight differences in speed on the general purpose lanes among all strategies. While the proposed strategies attracted more vehicles from

the general purpose lanes into the managed lanes, this difference in flow rate appears to be offset by the effect of merging and diverging maneuvers near on- and off-ramps along the general purpose lanes, which could explain the insignificant change in traffic conditions. In terms of travel time savings in minutes, FIGURE 30 shows very close values for all strategies, except at the onset of congestion when slightly higher savings are observed for the new strategy with a focus on revenue maximization.

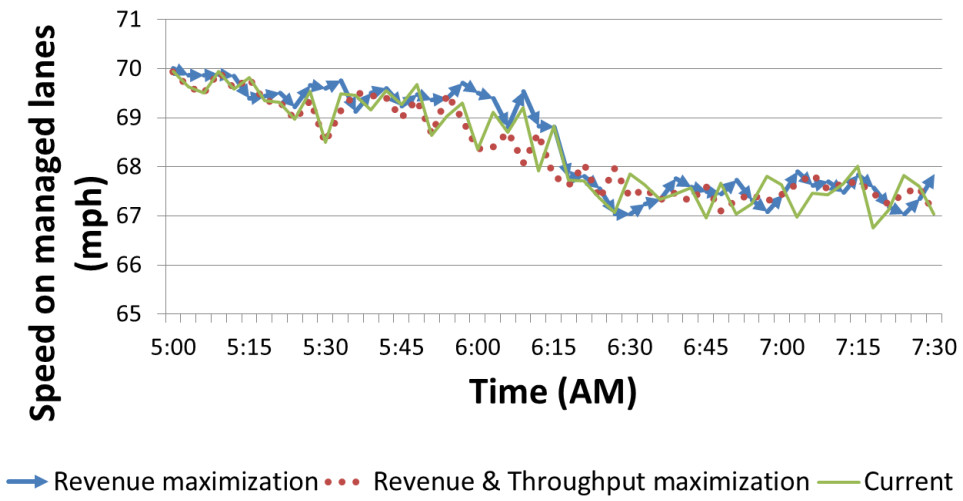

FIGURE 28 Average speeds on managed lanes under forty-four driver groups

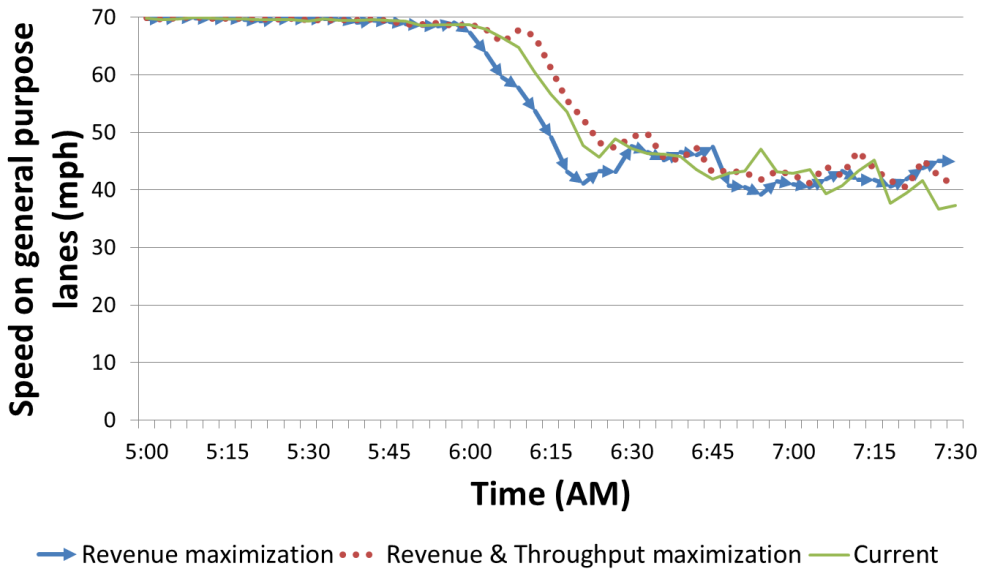

FIGURE 29 Average speed on general purpose lanes under forty-four driver groups

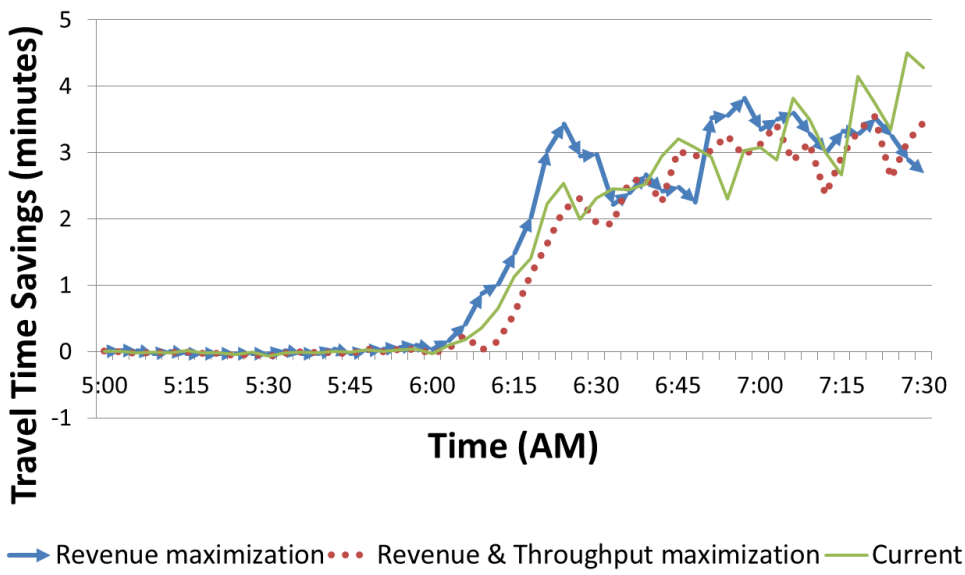

FIGURE 30 Travel time savings under forty-four driver groups

This chapter addresses the impacts of drivers' value of travel time savings $\beta(i)$as well as the impacts of the distributions of drivers' income groups on the probability of choosing the managed lanes and therefore on the performance of the proposed toll pricing strategy under two objectives in the simulation study. Results show that when the value of $\beta(i)$ is set to be 120% of the mean hourly income of drivers, the proposed toll strategy has the best performance for both objectives. As to the impacts of drivers' income groups, under the objective of revenue maximization, the proposed strategy has very close performances for the three driver groups and the forty-four driver groups. However, under the objective of both revenue and throughput on the managed lanes maximization, the difference in the probability of choosing the managed lanes by the two driver income distributions is significant at high traffic demand.

The performance of the proposed strategy is also compared to that of the current strategy on I-95. Again two toll objectives and two distributions of the drivers' income groups are examined in the performance comparison. Simulation results indicate that under both three and forty-four driver income groups, the proposed strategy with both objectives produce steadier toll rate profile than that of the current strategy. The proposed strategy with revenue maximization gains higher revenue than that of the current one without sacrificing the level of service on the managed lanes. Also the proposed strategy with both revenue and throughput on the managed lanes maximization has higher throughput on the managed lanes than the current strategy and is still able to keep the desired level of service on the managed lanes.

5. CONCLUSIONS

SUMMARY

This study develops a feedback-control based dynamic toll approach to formulate and solve for optimal tolls with a focus on two objectives. One is to maximize the toll revenue and the other is to maximize both revenue and throughput on the managed lanes. Both objectives have the constraints of maintaining a minimum level of service on the managed lanes. The proposed strategy consists of four main components: VISSIM simulation, external managed lanes module, drivers' route choice model and the feedback control mechanism. The essential control principle of the proposed strategy is to classify the traffic conditions of the managed lanes into two cases based on the measured average speed and then construct a specific control rule for each case to achieve the toll objectives. The strategy addresses the interactions between the managed lanes and the general purpose lanes by incorporating the travel time savings into the control rules.

FINDINGS

Performances of the proposed strategy under two objectives were examined and compared to that of the current toll method deployed on Interstate 95 express lanes in VISSIM simulation. Results showed that the proposed strategies with both objectives produced steadier toll rate profiles over low to high demand than that of the current method. The objective of revenue maximization produced higher toll revenue than the current method and objective of both revenue and throughput on the managed lanes maximization had higher throughput on the managed lanes from low to high traffic demand than that of the current method. Both objectives were able to keep the speed on the managed lanes at 45 mph or more. Other performance measures also indicated that the proposed strategy generally improved the operation of managed lanes and general purpose lanes.

Besides, a sensitivity analysis was conducted to demonstrate the effect of variation in the value of time on the probability of choosing the managed lanes and the corresponding toll rate and toll revenue. Three values ranging from 60% to 120% of the mean income were used. The results show that for low demand, such variation did not impact the probability of the managed lanes since no travel time savings could be gained. However, for high demand, an increase in the probability was obvious, showing the highest increase for a value of time at 120% of the mean hourly income. The distribution of drivers' income groups on the probability of choosing the managed lanes and the corresponding performance of the proposed strategy under two objectives were also examined. Results showed that under the objective of revenue maximization, the proposed strategy has very close performances for the three driver groups and the forty-four driver groups. However, under the objective of revenue and throughput on the managed lanes maximization, the differences in the probability of choosing the managed lanes, in the corresponding toll revenue and throughput on the managed lanes by the two driver income distributions is significant at high traffic demand. As a result, the distribution of drivers' income groups has more significant influence on the proposed strategy with objective of maximizing the

revenue and throughput on the managed lanes than on the objective of maximizing revenue alone.

FUTURE RESEARCH

Congestion pricing is a very complex topic. Due to the scope of the study, some issues related to congestion pricing are not addressed but are worth for research in the future. For instance, the study only uses drivers' income level to estimate drivers' value of time. However drivers' value of time can be influenced by many other factors such as trip purposes, traveling period and others. It might be more accurate to incorporate these factors into the determination of drivers' value of time. Besides, this study only considers passenger cars on the freeway network. In reality, some commercial vehicles also use the managed lanes and they are usually charged a different toll fee from the passenger cars. Although compared to the passenger cars, the commercial vehicles account for much smaller percentage of all vehicle classes on the freeway it is still worth to take into account of the impacts of commercial vehicles.

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