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...LUATION OF FLOOD RISK FACTORS IN THE DESIGN OF HIGHWAY STREAM CROSSINGS

Vol. III. Finite Element Model for Bridge Backwater Computation

M. T. Tseng

April 1975

Final Report

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16. Abstract

A mathematical model describing the steady, two-dimensional subcritical flow in wide, heavily vegetated flood plains of bridge waterways has been developed using the finite element method of numerical analysis. The basic fluid equations comprising the model consist of the phenomenologic motion equations and the continuity equation, which are solved simultaneously by numerical methods to yield the spatial distribution of water surface elevations and velocities within the flow region for prescribing boundary conditions.

The model simulates flow characteristics of arbitrary geometry. Hydraulic computations for various highway stream crossing orientations can be performed by the model. The model also simulates flow overtopping roadway embankments and performs hydraulic computations for a series of bridges across a stream valley without requiring prior assumption of the flow distribution for each bridge opening. The model has been tested for two example problems: a field site near Laurel, Mississippi, and for hydraulic flume data. In both examples good agreement between the model and the observed data was demonstrated.

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I. INTRODUCTION

This report is the third in a series of five volumes comprising the final report for the study entitled *Evaluation of Flood Risk Factors in the Design of Highway Stream Crossings*, authorized by the Federal Highway Administration (FHWA) under Contract No. DOT-FH-11-7669. The overall objective of the study is to develop an engineering systems analysis technique to reduce flood damage to highway stream crossings on a sound, probabilistic basis by including economic risk analysis in the hydraulic and hydrologic design of bridge waterways.

Volume III is a description of the application of the Finite Element Method (FEM) to predict the backwaters resulting from constrictions of highway crossings on flood plains. The FEM is adapted for a two-dimensional computer model in order to more accurately simulate the complex hydrodynamics of turbulent flow. The model simulation has been demonstrated with data collected in a series of experiments using a large hydraulic flume with artificial large scale roughness and with field data gathered by the U.S. Geological Survey (USGS) in Mississippi. The results of the two-dimensional Finite Element Model using the field data for the flood risk evaluation are described in Volume IV, *Economic Risk Analysis for Design of Bridge Waterways*.

BACKGROUND

Recent field verification of current methods for backwater prediction has demonstrated that existing methods tend to underpredict the magnitude of backwater in many cases, particularly when bridges extend over wide valleys with heavily vegetated flood plains. In recognition of this problem, the FHWA directed that one of the principal objectives of this study would be to develop an improved method to compute backwater.

An effective method for determining backwater is the use of results obtained from field measurements of water surface elevation at various bridge sites during flood stage. There are three highway planning and research (HP&R) studies in Alabama, Mississippi and Louisiana in which the USGS is the principal investigator collecting field data at various bridge sites in these states. The USGS will develop design recommendations from these data. However, the extensive time and effort necessary for such field programs is a practical restriction on the number of cases to be observed. In addition, the data base thus gathered is probably too narrow in scope to provide a wide range of typical prototype conditions. It is thus important to have available a more usable and accurate method for determining the optimum bridge opening using techniques of risk analysis.

STUDY APPROACH

The approach adopted in this study is to:

1. Use to the maximum extent possible all available information and data related to the bridge backwater problem,

2. Develop otherwise unavailable data from hydraulic experiments in the laboratory to determine flow characteristics in the vicinity of the bridge opening, and

3. Use all data acquired in (1) and (2) to develop a two-dimensional mathematical model to simulate flood plain flow and bridge backwater. This computer model is then verified as far as possible by field data obtained from state highway agencies.

In other words, the WRE approach is to maximize the use of existing technology to model, both physically and mathematically, the prototype behavior in such a way as to minimize the uncertainty associated with bridge waterway hydraulics. It is not only uneconomical but practically impossible to physically model the entire river reach under the influence of bridge backwater, mainly due to the wide variation in the width-depth ratio existing in natural streams. A physical model, however, may be used with confidence to study the flow patterns adjacent to the bridge opening. Away from the opening, the flow conditions may vary markedly from site to site, depending upon the variations of roughness distribution and the topographic features of the stream. It is this area which the physical model is not able to reproduce accurately and which must be simulated by other means, in this case by a Finite Element Model. The use of this computer model may be further expanded once the model is verified or calibrated.

OBJECTIVES AND SCOPE OF THE MODELING EFFORT

The primary objective of the work reported here is to develop a mathematical model that will predict the water surface elevation within the stream reach affected by the flow constriction of highway approach embankments on flood plains. The model incorporates the two-dimensional flow characteristics under the bridge and along (or over) the embankments and channel banks during low flow and high flow conditions. The model will give accurate results to be used as a tool in an economic optimal design technique for highway stream crossings.

The scope of this modeling effort is to apply the Finite Element Method of numerical analysis to the two-dimensional flow equations satisfying the prescribed boundary conditions. The model is applicable to steady state, two-dimensional and subcritical flow conditions in open channel flow. The spatial variation in bottom configuration, roughness and boundary geometry within a flow region are considered by the model. The model computes the velocity components on the horizontal plane and the stage at various points throughout the flow system. The model is coded in FORTRAN for solution by digital computer.

II. BRIDGE BACKWATER MODELS

THE BRIDGE BACKWATER PHENOMENON

In building highways across wide river valleys, it is common practice to extend the bridge approach embankments onto the flood plains. This minimizes construction costs by minimizing bridge length, since unit length of earth fill costs less than unit length of bridge structure. In flood conditions, the embankments constrict the stream flow and thus cause an energy loss through the contraction and expansion phenomena in addition to that of the normal channel resistance. Thus, in order for the flow to maintain itself, there must be a source of potential energy upstream from the bridge. This potential energy takes the form of bridge backwater.

Laursen (1) has divided the flow region affected by a bridge constriction into four zones, as shown in Figure 1. According to conventional definitions in open-channel hydraulics, Zones 1 and IV are areas of gradually-varied flow in which the flow pattern is essentially governed by the channel resistance and Zones II and III are areas of rapidly-varying flow patterns. The geometry of the bridge constriction causes the flow to contract upstream and expand downstream and thus determines the flow patterns in these zones. Boundaries between Zones I and II and Zones III and IV may be considered transition zones wherein the channel resistance and the constriction geometry play equally important roles.

Zone I is the reach of river upstream from, but not immediately adjacent to, the bridge. It covers the flow region encompassed by the length of the classic M_1 backwater curve and the width of flood plain

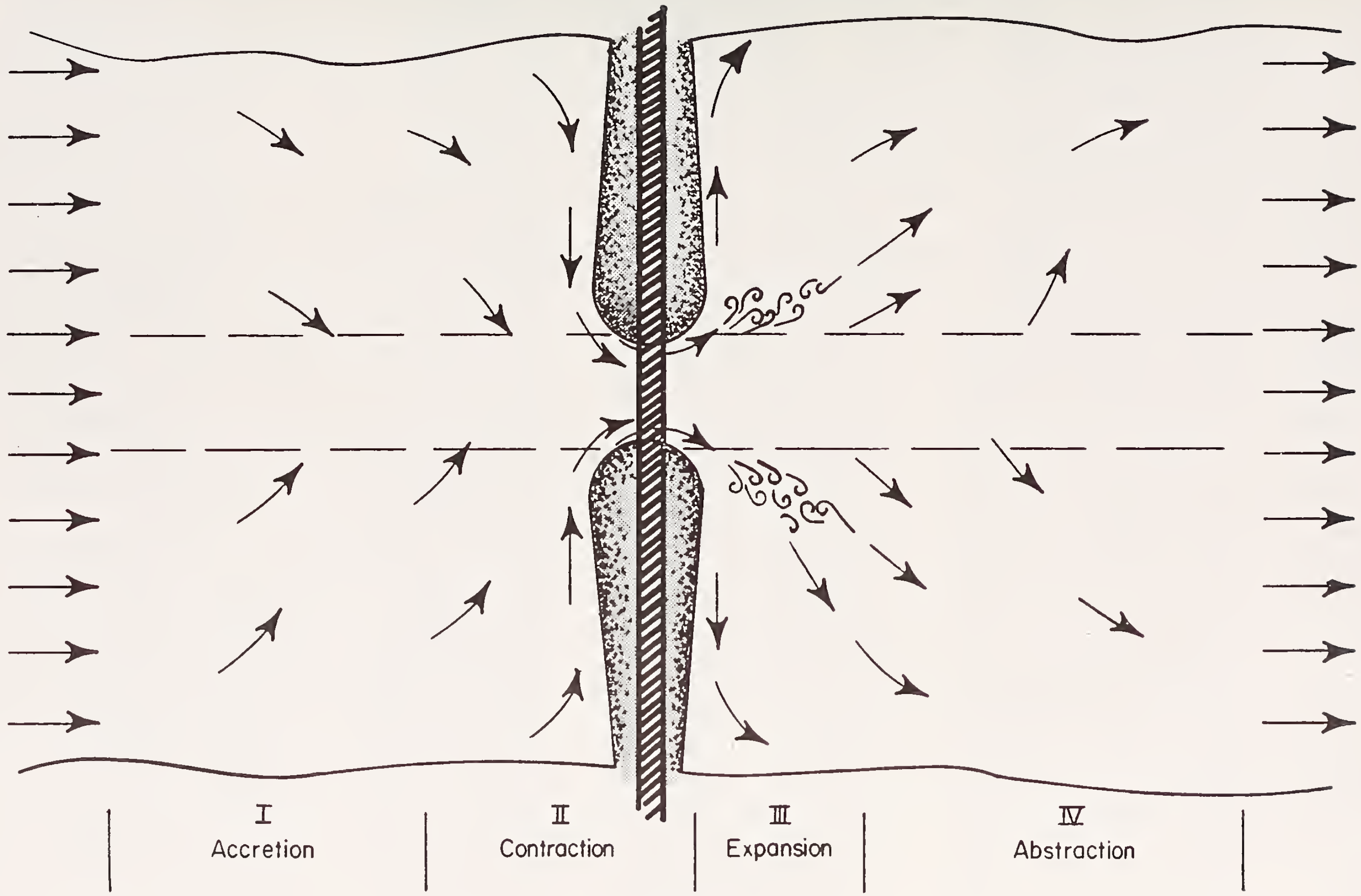


Figure 1. Schematic of Flow Region
Affected by Bridge Constriction

in which flows move laterally into the main channel. The upper boundary is at the point where the M_1 curve meets the normal flow profile.

The upper boundary of Zone II is just upstream from the bridge where the curvature of the water surface starts to increase rapidly in the vertical plane. This zone extends through the bridge to the section where the flow contraction ends. The flow pattern is approximated by the irrotational flow theory of a slot orifice with a free streamline separating from the abutment at the end of the embankment. Thus Zone II characterizes flow contraction and jet formation.

Zone III is the region in which the jet formed by the contracting flow of Zone II is expended through turbulent diffusion or mixing. A large amount of energy is lost in the course of flow expansion. If the channel resistance in Zone III is substantial, then the resultant energy loss associated with jet diffusion and the channel resistance may produce a total loss sufficient to cause backwater at the downstream side of the bridge opening. Such instances have been reported in a recent USGS report (2).

Zone IV may be considered the downstream counterpart of Zone I. In this reach the flow leaves the main channel and moves laterally across the flood plain. At some distance downstream the flow will be reestablished and resume its natural flow condition in the stream.

ONE-DIMENSIONAL BACKWATER MODELS

Eichert (3) has recently published the results of a survey of programs for computing water surface profiles in natural channels. The survey covers 20 leading water resource agencies, including federal and state agencies and several academic institutions. Eleven computer models were selected for comprehensive review. The conclusions of the review for six of these models are presented in Eichert's paper. In the review the eleven models were compared in 19 major areas of computer program capability. All of the models are one-dimensional and use the standard step method for backwater computation. Most of the models were developed for subcritical flow only. A detailed comparison of the methods of computing energy losses through bridge flow constriction was made for all eleven models. Only five calculate those energy losses. Of these, one program, the U.S. Geological Survey model, uses the Bureau of Public Roads (4) criteria to compute the bridge backwater. The four other programs use either the Bernoulli equation, the Yarnell energy principles (5), the Koch and Carstanjen momentum theories (6), or the Kindsvater method (7) to determine the changes in water surface elevation through the bridge for subcritical flow.

Eichert and Peters (8) describe in detail a procedure for determining water surface profiles through bridge structures. This method is incorporated by the U.S. Army Engineers as a bridge flow routine in the Hydrologic Engineering Center (HEC) Program on Water Surface Profiles. The procedure uses the momentum theory under low flow conditions to compute the pier losses; and the contraction and expansion losses at the bridge are evaluated by multiplying a loss coefficient times the absolute difference in velocity heads inside and outside of the bridge constriction. Methods for determining the existence of pressure flow, weir flow and combinations of flow conditions passing the bridge section are also described in the Eichert and Peters paper (8). While the method may be adequate as a routine for determining the water surface profiles through bridges in the HEC model for natural channels, it may not be accurate enough for bridge waterway design. This is because the one-dimensional method assumes that the water surface profile is horizontal across the stream cross section perpendicular to the flow. Furthermore, with this method it is not possible to compute the water surface profiles through skewed or eccentric highway crossings.

The Bureau of Public Roads criteria for computing bridge backwater were revised and reissued in 1970 (9). Included in the new issue are revised base curves for the backwater coefficient, USGS field data for streams with large width to depth ratios, and material on partially inundated superstructures, proportioning of spur dikes at bridge abutments, and supercritical flow under a bridge. However, the single most important reason for revising the criteria is the marked discrepancy of backwater coefficient base curves between the model data and the USGS field data. This discrepancy has been attributed to the limited range of width to depth ratios under which the model data were obtained. For longer bridges and wider flood plains the model data were entirely inadequate. A comparison of model data and field data reveals that for contraction ratios less than 0.55 the model curve flattens while the field data show a rising trend. The field data, however, verified the model base curves for contraction ratios between 0.55 to 1.0.

Although the Bureau of Public Roads criteria have been improved by these changes, the basic approach is still one-dimensional, which ignores the variation of water surface elevation in the transverse direction. In the computation of bridge backwater for width to depth ratios as high as 700, the one-dimensional approximation may not be adequate for design purposes.

The complex variations in natural streams coupled with varied types of highway stream crossings found in the field make it difficult to gather sufficient empirical expressions for effective implementation of a one-dimensional model. While one-dimensional methods have been used to predict backwater with satisfactory results in many instances, there is still a need in highway design for a model which can be applied in such situations as multiple bridges across main channel and flood plain, islands or other obstructions in the waterway, meandering streams, skewed and eccentric crossings and the nonuniform distribution of vegetation in the stream bed. Such a need has recently been expressed by state highway departments (10). A feasible solution to the problem of wide variations in stream characteristics and types of highway crossings is the application of a two-dimensional mathematical model.

TWO-DIMENSIONAL MODELS

There are two major types of two-dimensional models which are applicable to the problem. The first type is the Finite Difference Models and the second is the Finite Element Models. In the Finite Difference Models the general approach is to discretize the flow region into a system of grid networks composed of either rectangular or square cells. The size of the cells is governed by the accuracy and stability of the specific numerical scheme applied to the solution of differential equations describing the flow. In the Finite Element Models the flow region is represented by a grid network of either triangular or polygonal elements, with triangular elements the most popular. Again the size of the elements is largely determined by the accuracy of the solution scheme but generally not by numerical stability.

In two-dimensional flow problems, the basic set of differential equations to be solved consists of the continuity equation and the equations of motion in the two space dimensions. The dependent variables are the depth and velocity (or flow) components in each of the two space dimensions. Conceptually the Finite Difference Models approximate the solution of differential equations in a discrete manner. Once the governing differential equations are discretized, the set of differentials becomes a set of difference equations. This set of difference equations is then solved using either an explicit or an implicit method for prescribed boundary conditions. A Finite Element Model, on the other hand, solves the differential equations without discretization. The Finite Element Models solve by either:

1. Minimizing a functional, an integral quantity which is a function of unknown functions, or

2. Approximating the finite elements directly from the differential equations governing the problem using weighted residual methods over the flow region and part of its boundary.

The advantage of the Finite Difference Models is that their numerical solution scheme is more conventional and thus better known to program users than the Finite Element Models. However, they have limited solution efficiency, lack of flexibility in representing irregular flow boundaries, difficulties in treating boundary conditions, and are likely to have numerical stability problems for many schemes. The Finite Element Models, on the other hand, are readily adapted to represent irregular flow boundaries, have a more efficient solution process, and are more reliable in numerical stability. The Finite Element Models are more flexible for laying out grid size and in representing flow boundaries. This is particularly important in bridge backwater computation: a coarse network is sufficient to represent the low variation flow in Zones I and IV and a fine network is necessary for the rapid variation flow in Zones II and III. The major disadvantage of the Finite Element Models is their sophisticated solution technique.

A potential advantage associated with the Finite Element Models is the computation of the contraction scour at the bridge site since this type of model gives detailed information on depths and velocities throughout the flow region. The Finite Element Models thus have the potential for computing contraction scour and bridge backwater simultaneously, a very important feature for design engineers.

DEVELOPMENT OF THE FINITE DIFFERENCE AND FINITE ELEMENT MODELS

In the early stages of development, the finite-difference schemes were broadly applied to many one-dimensional and some two-dimensional flow problems. In dealing with two-dimensional flow fields the Marker-and-Cell (MAC) method (11) applied the finite difference representation to the complete Navier-Stokes equations, including all nonlinear terms, to obtain approximate solutions. Other finite difference methods to solve two-dimensional flow problems have been given by Shubinski *et al.* (12), Liggett and Hadjitheodarou (13), Masch (14) and Laendertse (15). With these methods various solutions have been developed using either the complete equations with simplified boundary conditions or more general boundary conditions with restricted forms of the equations.

The Finite Element Method (FEM) was originally developed by structural analysts seeking the solution to the complex problems of stress analysis of a continuum. The initial theory of FEM was based upon physical intuition and a balancing of forces, i.e., the flexibility introduced by cutting the system and reconnecting it only at discrete points would be balanced by the restricted displacement pattern for each element. Theoretical proofs of convergence to the true solution soon followed (16, 17, 18). The first applications of the method were in the aerospace industry where the resistance capability of continuous aircraft skins had to be maximized in keeping with the overall objective of weight minimization. The solution of such continuum systems using traditional structural analysis methods would have required an infinite number of joints, each connecting infinitesimally small elements.

In the FEM, the continuum area is represented by a series of discrete polygons interconnected at a finite number of locations called nodal points. These nodal points are directly equivalent to joints in structural problems, and the elements, although two dimensional, are equivalent to beams and columns. In the first applications the elements were allowed to respond in restricted and simple displacement patterns in order to maintain continuity along all interfaces between elements, but force transfer was allowed at the nodes. Thus, with the system approximated, the method of assemblage of elements and solutions of equations was exactly that of conventional structural analysis.

The first practical application of this technique outside of the aerospace industry was to the stress analysis of a concrete gravity dam (19). Application to problems in hydrodynamics was slow due primarily to the complexity of equations describing the processes. Some applications of the FEM to flow problems include:

- Doctors (20), and Chen and Larock (21) solved problems of potential flows,
- Loziuk *et al.* (22) attempted to solve problems involving slow viscous flow,
- Norton *et al.* (23) applied the FEM to solve density stratified flow problems in reservoirs,
- Gallagher *et al.* (24) used the method to analyze the problems of lake circulation, and
- Zienkiewicz *et al.* (25) applied the method to seepage flow in porous media.

Of special interest is the report of Franques (26) which describes the solution of the two-dimensional flow through bridge constrictions, the same problem addressed in this report. Franques' approach is to solve for flow distribution in a two-dimensional flow region using the equation

$$-\frac{D\xi}{Dt} = \frac{\partial}{\partial x}\left(\frac{C_f\ |V|}{d}\ \frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{C_f\ |V|}{d}\ \frac{\partial\psi}{\partial y}\right) \tag{1}$$

and then solving for flow depths along a stream line by the equation

$$\frac{V^2}{2g} + z - \int_0^{\ell} \frac{F}{g}\, d\ell = B(\psi) \tag{2}$$

where ξ = scalar quantity or vorticity,

C_f = a depth dependent resistance coefficient,

V = velocity vector in the flow field,

ψ = stream function,

d = depth,

z = water surface elevation,

F = body force,

ℓ = distance along a stream line,

B = a function dependent on ψ,

t = time,

x = distance along x-coordinate, and

y = distance along y-coordinate.

Equation 1 was derived from the Navier-Stokes equation by neglecting the viscous terms, and Equation 2 is the Bernoulli equation along a stream line. A linearized form of Equation 1 was solved by the FEM to obtain the flow distribution. Through the use of Manning's equation and the computed flow distribution, the depth and water surface elevation along a stream line were computed numerically from Equation 2. In Franques' approach the linearized differential equation (Equation 1) was transformed to an integral form through variational calculus from which the finite element representation was constructed. Franques demonstrated this method for two flood events at Tallahala Creek at State Highway 528 near Bay Springs, Mississippi. The computed results for flow distribution showed fairly good agreement with field data. However, the comparison of computed water surface elevation with measured data was reported to be inconclusive.

III. GOVERNING EQUATIONS

The derivation of the equations describing the incompressible turbulent free-surface flow begins with a statement of the continuity equations and the momentum equations. Each of the basic equations is developed in three dimensions employing traditional methods of differential analysis. The Eulerian (fixed in space) description is used with the standard Cartesian reference system. In the derivations the horizontal plane coincides with the x-y plane.

The derivations are first shown for a general three-dimensional fluid volume at an arbitrary location in a fluid body of infinite extent. Next, several terms are introduced which reflect the fact that the fluid body is not really infinite and is located on a rotating sphere. Finally, a summary of the basic two-dimensional forms employed in the mathematical model is provided for future reference.

BASIC EQUATIONS

Consider an element of fluid volume with dimensions dx, dy and dz. The continuity equation and the equations of motion for an incompressible fluid assume the forms of

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (3)$$

and

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \mathbf{X} - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \mathbf{Y} - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{4}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial} + v\frac{\partial w}{\partial} + w\frac{\partial w}{\partial}\right) = \mathbf{Z} - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

where p = instantaneous value of pressure,

μ = dynamic viscosity of fluid,

ρ = fluid density,

u, v, w = instantaneous velocity components in x, y, and z directions, and

$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ = the body force acting on x, y and z directions.

The equations grouped in (4) are the so-called Navier-Stokes equations. They are second order differential equations because of the viscous terms and quadratic because of the inertial terms. These equations describe the fluid motion in laminar viscous flows and must be modified if they are to be used to study the turbulent flow fields. This modification is necessary to describe the rapid fluctuation of the velocities and pressures over both time and space characteristic of turbulent flow fields. In the present analysis the turbulent flow will be analyzed for mean motion only.

MEAN VALUES IN A TURBULENT FLOW

In a turbulent flow the velocities and pressure are functions of the time and space coordinates. The instantaneous value at a point fixed in space may be considered to be the sum of a mean value with respect to some time interval and some fluctuating component. Mathematically this is expressed as

$$u = \bar{u} + u' \tag{5}$$

$$v = \bar{v} + v' \tag{6}$$

$$w = \bar{w} + w' \tag{7}$$

$$p = \bar{p} + p' \tag{8}$$

where $\bar{u}, \bar{v}, \bar{w}$ = the mean velocity in the x, y and z directions over some time interval, Δt,

$\bar{p}$ = the mean pressure over the time interval, Δt,

u', v', w' = the velocity fluctuations with respect to the mean values in the x, y, and z directions, and

p' = the fluctuation in pressure.

Further, the following definitions are introduced to clarify the statistical nature of this approach to turbulent flow.

$$\bar{u} = \frac{1}{\Delta t} \int_0^{\Delta t} u dt \tag{9}$$

$$\bar{v} = \frac{1}{\Delta t} \int_0^{\Delta t} v dt \tag{10}$$

$$\bar{w} = \frac{1}{\Delta t} \int_0^{\Delta t} w dt \tag{11}$$

$$\bar{p} = \frac{1}{\Delta t} \int_0^{\Delta t} p dt \tag{12}$$

$$\bar{u}' = \frac{1}{\Delta t} \int_0^{\Delta t} u' dt = 0 \tag{13}$$

$$\bar{v}' = \frac{1}{\Delta t} \int_0^{\Delta t} v' dt = 0 \tag{14}$$

$$\bar{w}' = \frac{1}{\Delta t} \int_0^{\Delta t} w' dt = 0 \tag{15}$$

$$\bar{p}' = \frac{1}{\Delta t} \int_0^{\Delta t} p'dt = 0 \tag{16}$$

From the above definitions it can be seen that turbulent motion is to be considered as the superposition of a mean motion and a fluctuating and disorderly motion, random in nature, which is not well known and can be described only in terms of statistical values.

CONTINUITY EQUATION

Substitution of Equations 5 through 8 in Equation 3 yields

$$\frac{\partial}{\partial x} (u' + \bar{u}) + \frac{\partial}{\partial y} (v' + \bar{v}) + \frac{\partial}{\partial z} (w' + \bar{w}) = 0 \tag{17}$$

or

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \tag{18}$$

Now, to evaluate the averaging process as it relates to the various terms of Equation 18, consider the two following examples (bar notation indicates the averaging process):

$$\overline{\frac{\partial \bar{u}}{\partial x}} = \frac{1}{\Delta t} \int_0^{\Delta t} \frac{\partial u}{\partial x} dt = \frac{\partial}{\partial x} \frac{1}{\Delta t} \int_0^{\Delta t} \bar{u} \; dt = \frac{\partial \bar{u}}{\partial x} \tag{19}$$

and

$$\overline{\frac{\partial \bar{u}'}{\partial x}} = \frac{1}{\Delta t} \int_0^{\Delta t} \frac{\partial u'}{\partial x} dt = \frac{\partial}{\partial x} \frac{1}{\Delta t} \int_0^{\Delta t} u'dt = 0 \tag{20}$$

Thus, the continuity equation for mean motion in a turbulent flow becomes

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \tag{21}$$

MOMENTUM EQUATIONS

By the substitution of $\bar{u} + u'$ for u, $\bar{v} + v'$ for v, etc., in Equation 4, there results a set of general momentum equations for turbulent flow:

$$\rho \left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z}\right) = \mathbf{x} - \frac{\partial \bar{p}}{\partial x} + \mu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{u'u'}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z}\right) \tag{22}$$

$$\rho \left(\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial z}\right) = \mathbf{y} - \frac{\partial \bar{p}}{\partial y} + \mu\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'v'}}{\partial y} + \frac{\partial \overline{v'w'}}{\partial z}\right) \tag{23}$$

$$\rho \left(\frac{\partial \bar{w}}{\partial t} + \bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial z}\right) = \mathbf{z} - \frac{\partial \bar{p}}{\partial z} + \mu\left(\frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{\partial^2 \bar{w}}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{u'w'}}{\partial x} + \frac{\partial \overline{v'w'}}{\partial y} + \frac{\partial \overline{w'w'}}{\partial z}\right) \tag{24}$$

These equations are the so-called Reynold's equations of motion for turbulent flow. They are similar to the Navier-Stokes equations with the exceptions that (1) an additional term has been added to the inertial forces due to the turbulent fluctuation and (2) the other forces are now expressed in terms of mean values. The turbulent fluctation forces, which are the final terms on the right-hand side of the Reynold's equations, are often called the "Reynold's stresses" and are the subject of a great deal of work in advanced fluid mechanics. A simplified approach to the evaluation of the Reynold's stresses, which is essential to the solution of the momentum equations, is now presented.

To make the Reynold's equations mathematically tractable, Boussinesq has introduced the concept of a turbulence exchange coefficient, ε, which is dimensionally equal to the coefficient of viscosity, μ. In the case of uniform flow in the x direction, the exchange coefficient is defined by:

$$\rho \, \overline{u'v'} = - \varepsilon \frac{\partial \bar{u}}{\partial y} \tag{25}$$

Employing this definition, the fluid shear stress becomes

$$\tau = (\mu + \varepsilon) \frac{\partial \bar{u}'}{\partial y} \tag{26}$$

From this definition it may be seen that the Reynold's stress terms would act similarly to the viscous friction term, with their effect added linearly. Experimental data have shown that μ and ε are of different orders of magnitude, and that, in general, $\varepsilon >>> \mu$. And, for engineering purposes, it is possible to make the approximation:

$$\tau = \varepsilon \frac{\partial \bar{u}}{\partial y} \tag{27}$$

Traditionally the main drawback to applying this approximation of Equation 26 to the Reynold's stresses was that ε is known to vary in space and time. This condition precludes its inclusion in any traditional method of analysis. As the limitation is relaxed by the numerical methods described in this report, it is possible to use the Boussinesq approximation. When the approximation is substituted into the Reynold's equations the result is given as:

$$\rho \left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} \right) = \mathbf{X} - \frac{\partial \bar{p}}{\partial x} + \varepsilon_x \left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2} \right) \tag{28}$$

$$\rho \left(\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial z} \right) = \mathbf{Y} - \frac{\partial \bar{p}}{\partial y} + \varepsilon_y \left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2} \right)$$

$$\rho\left(\frac{\partial \bar{w}}{\partial t} + \bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial z}\right) = \mathbf{z} - \frac{\partial \bar{p}}{\partial z} + \varepsilon_z\left(\frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{\partial^2 \bar{w}}{\partial z^2}\right) \qquad (30)$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ = the turbulence exchange coefficients in the x, y, and z directions, respectively.

The above equations, which are sometimes referred to as the phenomenologic equations, are the basic form of the general momentum equations which have been incorporated into the computer model described in this report.

TWO-DIMENSIONAL HORIZONTAL FLOW MODEL

GOVERNING EQUATIONS AS FUNCTIONS OF VELOCITY, PRESSURE AND DEPTH

In the absence of an electromagnetic field, gravity is the only body force encountered in fluid flow. For two-dimensional horizontal flow, $\mathbf{x} = \mathbf{y} = 0$, and the basic phenomenological and continuity equations (bar notation dropped) may be stated as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\varepsilon_x}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0 \qquad (31)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\varepsilon_y}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = 0 \qquad (32)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (33)$$

The above equations have been derived for an elemental fluid volume contained in a large fluid mass. As we wish to derive more complete forms for flow in the horizontal plane, we must modify the basic equations to allow for a free water surface, variable depth, and the effect of bottom friction. This may be done by integrating Equations 31 and 32 through the full depth.

By introducing the definitions for the local water depth, h, and elevation of the bottom surface, z_o, as shown in Figure 2, it is possible to express the effect of friction along the boundary between the flowing water and its confining boundary as

$$F_{b_x} = \rho \frac{gu}{C^2 h} (u^2 + v^2)^{1/2} \tag{34}$$

$$F_{b_y} = \rho \frac{gv}{C^2 h} (u^2 + v^2)^{1/2} \tag{35}$$

where F_{b_x} = the unit force due to bottom friction in the x direction,

F_{b_y} = the unit force due to bottom friction in the y direction,

g = the gravitational constant,

C = the Chezy coefficient, and

h = the local water depth.

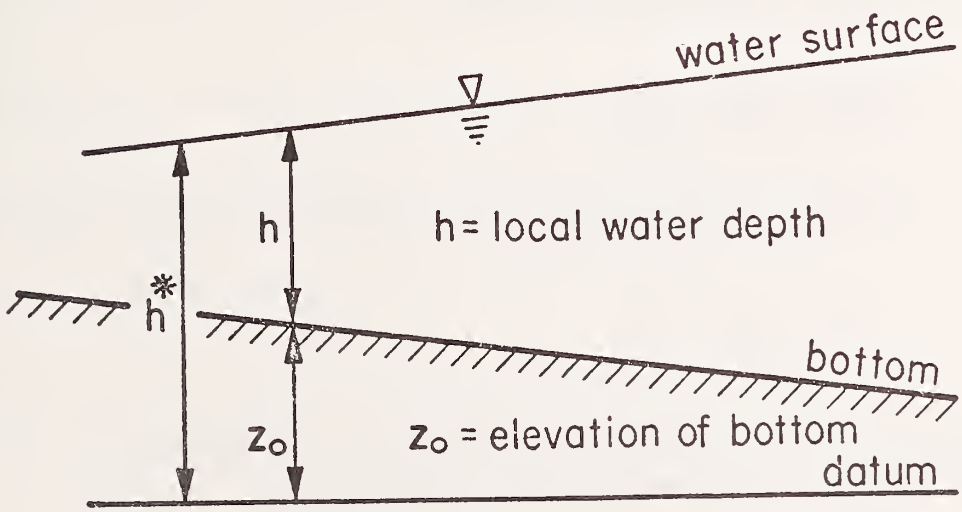


Figure 2. Definition Sketch for Horizontal Flow Model

The inclusion of the bottom friction completes the basic statement of the horizontal flow model employed in the finite element formulation. Prior to writing the complete forms, however, we shall introduce a different form for the pressure term in Equations 31 and 32 and modify Equation 33 to reflect the effects of a moving water surface.

Again referring to Figure 2, the pressure term $\frac{1}{\rho}\frac{\partial p}{\partial x}$ of Equation 31 may be written for a flat bottom as

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{1}{\rho h}\frac{\partial}{\partial x}\left(\rho g \frac{h^2}{2}\right) \tag{36}$$

If g = constant, and

$h^* = h + z_o$, then for a sloping bottom

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{1}{\rho h}\frac{\partial}{\partial x}\left(\rho g \frac{h^2}{2}\right) + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho g z_o) \tag{37}$$

Equation 37 can be expanded to the form

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = g\frac{\partial h}{\partial x} + g\frac{\partial z_o}{\partial x} \tag{38}$$

In a similar fashion, the pressure term of Equation 32 takes the form

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = g\frac{\partial h}{\partial y} + g\frac{\partial z_o}{\partial y} \tag{39}$$

The above expressions, Equations 38 and 39, are the forms used in the final statement of the phenomenologic equations.

The continuity equation for two-dimensional flow with a free surface is

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(wh) = 0$$

With all the necessary additions and modifications now set forth, the basic equations for the horizontal flow model can be restated as

Motion Equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} + g\frac{\partial z_o}{\partial x} - \frac{\varepsilon_x}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{gu}{C^2 h}\left(u^2 + v^2\right)^{1/2} = 0 \tag{40}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y} + g\frac{\partial z_o}{\partial y} - \frac{\varepsilon_y}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{gv}{C^2 h}\left(u^2 + v^2\right)^{1/2} = 0 \tag{41}$$

Continuity Equation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0 \tag{42}$$

GOVERNING EQUATIONS AS FUNCTIONS OF FLOW AND DEPTH

Because of the order of approximations set forth for the finite element formulation, it is desirable to make a change of variables and make the primary variables flow and depth rather than velocity and pressure. This change of variables is accomplished by introducing the following substitutions:

$$r = uh \tag{43}$$

$$s = vh \tag{44}$$

where r, s = unit flow in the x and y directions, respectively.

Bypassing undue detail, it can be seen that from Equations 43 and 44 we can derive all necessary substitutions for the governing equations. For example, from Equation 43, we can write:

$$u = r\ h^{-1} \tag{45}$$

$$\frac{\partial u}{\partial x} = h^{-1}\ \frac{\partial r}{\partial x} - rh^{-2}\ \frac{\partial h}{\partial x} \tag{46}$$

$$\frac{\partial^2 u}{\partial x^2} = h^{-1}\ \frac{\partial^2 r}{\partial x^2} - rh^{-2}\ \frac{\partial^2 h}{\partial x^2} - 2h^{-2}\ \frac{\partial h}{\partial x}\frac{\partial r}{\partial x} + 2rh^{-3}\left(\frac{\partial h}{\partial x}\right)^2 \tag{47}$$

With the appropriate change in variables now defined, the final form of the governing differential equations can be written as follows:

Motion Equations

Substituting Equations 43 and 44 into Equations 40 and 41 and integrating with depth the result is

$$\begin{aligned} h^{-1}\frac{\partial r}{\partial t} &- h^{-2}r\frac{\partial h}{\partial t} + rh^{-2}\frac{\partial r}{\partial x} - r^2h^{-3}\frac{\partial h}{\partial x} + sh^{-2}\frac{\partial r}{\partial y} - srh^{-3}\frac{\partial h}{\partial y} \\ &+ g\frac{\partial h}{\partial x} + g\frac{\partial z_o}{\partial x} - \frac{\varepsilon_x}{\rho}\left[h^{-1}\frac{\partial^2 r}{\partial x} - rh^{-2}\frac{\partial^2 h}{\partial x^2} - h^{-2}\frac{\partial h}{\partial x}\frac{\partial r}{\partial x}\right. \\ &\left. + rh^{-3}\left(\frac{\partial h}{\partial x}\right)^2\right] - \frac{\varepsilon_x}{\rho}\left[h^{-1}\ \frac{\partial^2 r}{\partial y^2} - rh^{-2}\frac{\partial^2 h}{\partial y^2} - h^{-2}\frac{\partial h}{\partial y}\frac{\partial r}{\partial y}\right. \\ &\left. + rh^{-3}\left(\frac{\partial h}{\partial y}\right)^2\right] + \frac{gh^{-3}}{C^2}\ r(r^2 + s^2)^{1/2} = 0 \end{aligned} \tag{48}$$

and

$$h^{-1}\frac{\partial s}{\partial t} - h^{-2}s\frac{\partial h}{\partial t} + rh^{-2}\frac{\partial s}{\partial x} - rsh^{-3}\frac{\partial h}{\partial x} + sh^{-2}\frac{\partial s}{\partial y} - s^2h^{-3}\frac{\partial h}{\partial y}$$

$$+ g\frac{\partial h}{\partial y} + g\frac{\partial z_o}{\partial y} - \frac{\varepsilon_y}{\rho}\left[h^{-1}\frac{\partial^2 s}{\partial x^2} - sh^{-2}\frac{\partial^2 h}{\partial x^2} - h^{-2}\frac{\partial h}{\partial x}\frac{\partial s}{\partial x}\right.$$

$$\left. + sh^{-3}\left(\frac{\partial h}{\partial x}\right)^2\right] - \frac{\varepsilon_y}{\rho}\left[h^{-1}\frac{\partial^2 s}{\partial y} - sh^{-2}\frac{\partial^2 h}{\partial x} - h^{-2}\frac{\partial h}{\partial y}\frac{\partial s}{\partial y}\right.$$

$$\left. + sh^{-3}\left(\frac{\partial h}{\partial y}\right)^2\right] + \frac{gh^{-3}}{C^2}s\left(r^2 + s^2\right)^{1/2} = 0 \qquad (49)$$

Next, in order to eliminate the time derivatives we shall introduce the linear acceleration assumption. Using the x direction flow, r, as an example, we can define the relationship

$$r^{t+\Delta t} = r^t + \frac{\Delta t}{2}\left[\left(\frac{\partial r}{\partial t}\right)^{t+\Delta t} + \left(\frac{\partial r}{\partial t}\right)^t\right] \qquad (50)$$

where r^t, $r^{t+\Delta t}$ = the x direction flow at time t and time $t+\Delta t$, respectively,

$\left(\frac{\partial r}{\partial t}\right)^t$, $\left(\frac{\partial r}{\partial t}\right)^{t+\Delta t}$ = the time rate of change of x direction flow at some time t and time $t+\Delta t$, respectively, and

Δt = time interval.

Then

$$\left(\frac{\partial r}{\partial t}\right)^{t+\Delta t} = \frac{2}{\Delta t}r^{t+\Delta t} - \frac{2}{\Delta t}r^t - \left(\frac{\partial r}{\partial t}\right)^t \qquad (51)$$

Further, if in general the vectors r^t and $\left(\frac{\partial r}{\partial t}\right)^t$ are known at time t, we can define the following relationship (dropping the superscript)

$$\frac{\partial r}{\partial t} = \alpha r - \beta \qquad (52)$$

where $\frac{\partial r}{\partial t}$ = the time rate of change of the x direction flow at the end of some time interval,

r = the x direction flow at the end of some time interval, and

α = a coefficient, and

$$\beta = \left(\frac{2}{\Delta t} r + \frac{\partial r}{\partial t}\right)^t$$

If we now interpret the governing equations as being defined at the end of some known time interval, with appropriately stated variables known at the beginning of the interval, we can write

$$\frac{\partial r}{\partial t} = \alpha r - \beta_1 \tag{53}$$

$$\frac{\partial s}{\partial t} = \alpha s - \beta_2 \tag{54}$$

$$\frac{\partial h}{\partial t} = \alpha h - \beta_3 \tag{55}$$

$$\frac{\partial \rho}{\partial t} = \alpha \rho - \beta_4 \tag{56}$$

The governing equations can be put into final form by multiplying Equations 48 and 49 by h^3 and substituting Equations 53, 54, 55 and 56 for the time terms, with the result:

$$\begin{aligned} h^2(\alpha r - \beta_1) - hr\left(\alpha h - \beta^3\right) + rh\frac{\partial r}{\partial x} - r^2\frac{\partial h}{\partial x} + sh\frac{\partial r}{\partial y} - sr\frac{\partial h}{\partial y} \\ + gh^3\frac{\partial h}{\partial x} + gh^3\frac{\partial z_o}{\partial x} - \frac{\varepsilon_x}{\rho}\left[h^2\frac{\partial^2 r}{\partial x^2} - rh\frac{\partial^2 h}{\partial x^2}\right. \\ \left. - h\frac{\partial h}{\partial x}\frac{\partial r}{\partial x} + r\left(\frac{\partial h}{\partial x}\right)^2\right] - \frac{\varepsilon_x}{\rho}\left[h^2\frac{\partial^2 r}{\partial y^2} - rh\frac{\partial^2 h}{\partial y^2}\right. \\ \left. - h\frac{\partial h}{\partial y}\frac{\partial r}{\partial y} + r\left(\frac{\partial h}{\partial y}\right)^2\right] + \frac{gr}{C^2}\left(r^2 + s^2\right)^{\frac{1}{2}} = 0 \end{aligned} \tag{57}$$

and

$$\begin{aligned} h^2(\alpha s - \beta_2) \quad & hs\left(\partial h - \beta_3\right) + rh\frac{\partial s}{\partial x} - rs\frac{\partial h}{\partial x} + sh\frac{\partial s}{\partial y} - s^2\frac{\partial h}{\partial y} \\ & + gh^3\frac{\partial h}{\partial y} + gh^3\frac{\partial z_o}{\partial y} - \frac{\varepsilon_y}{\rho}\left[h^2\frac{\partial^2 s}{\partial x^2} - sh\frac{\partial^2 h}{\partial x^2}\right. \\ & \left. - h\frac{\partial h}{\partial x}\frac{\partial s}{\partial x} + s\left(\frac{\partial h}{\partial x}\right)^2\right] - \frac{\varepsilon_y}{\rho}\left[h^2\frac{\partial^2 s}{\partial y^2} - sh\frac{\partial^2 h}{\partial y^2}\right. \\ & \left. - h\frac{\partial h}{\partial y}\frac{\partial s}{\partial y} + s\left(\frac{\partial h}{\partial y}\right)^2\right] + \frac{gs}{C^2}\left(r^2 + s^2\right)^{1/2} = 0 \end{aligned} \tag{58}$$

Continuity Equation

$$(\alpha h - \beta_3) + \frac{\partial r}{\partial x} + \frac{\partial s}{\partial y} = 0 \tag{59}$$

These last three equations, Equations 57, 58 and 59, are the basic differential equations used to describe fluid movement in the horizontal plane. When boundary conditions are specified, the values of h, r and s can be obtained by solving Equations 57, 58 and 59. In these equations the unsteady terms have been retained to maintain generality.

IV. THE FINITE ELEMENT METHOD

CONDITIONS FOR THE FINITE ELEMENT METHOD

The first step in the application of the Finite Element Method is the construction of acceptable single-element approximations to the physical system. A suitable element will have an approximating function with the same number of independent variables as the number of interconnection points to other elements. The variation of the approximating function along the boundary of an element must exactly match that of an adjacent element. For example, a triangular element with nodal point degrees of freedom at the vertices must have a linear approximating function (three coefficients $\phi = a + bx + cy$). The second condition is met because the function is linear along the sides of the element and the adjacent elements match exactly. Similarly, if mid-side nodes are used a quadratic approximating function may be used.

A necessary condition for convergence of the finite element approximation with decreasing element size is that the fundamental equations do not contain derivatives two orders higher than those derivatives that match across the element boundary. Also, the function must be capable of representing a constant value of the function or its derivatives present in the integral equation describing the element.

FINITE ELEMENT REPRESENTATION USING METHOD OF WEIGHTED RESIDUALS (GALERKIN PROCESS)

The conventional basis for formulating a finite element representation is an integral form which, in the sense of a variational principle, corresponds to the governing differential equation. Such an integral form is known as a "functional." The correct solution of the problem is that which minimizes the functional over a certain domain.

In many physical problems, however, a functional may not exist or has not been discovered with the differential equations governing the problem. For example, there are no convenient functionals for the Navier-Stokes equations and the Reynolds equations of motion. In such cases the transformation of the governing differential equations to integral form is made through the use of weighted residuals rather than through variational calculus. In this report the method of weighted residuals will be applied to the differential equations described in Chapter III to form the finite element representation. Before this procedure is discussed, however, it is advantageous to demonstrate the solution of the quasi-harmonic problem, which has as a special case the well-known Laplace and Poisson equations. This exercise will serve as an outline for the solution in Chapter V of the equations governing fluid flow (see Chapter III). Methods of numerical area integration techniques will also be reviewed.

The general form of a quasi-harmonic equation in a two-dimensional field is

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) + Q = 0 \qquad (60)$$

in which ϕ is the unknown function, assumed to be a single value, and k_x, k_y and Q are known or specified functions of x and y (Figure 3a).

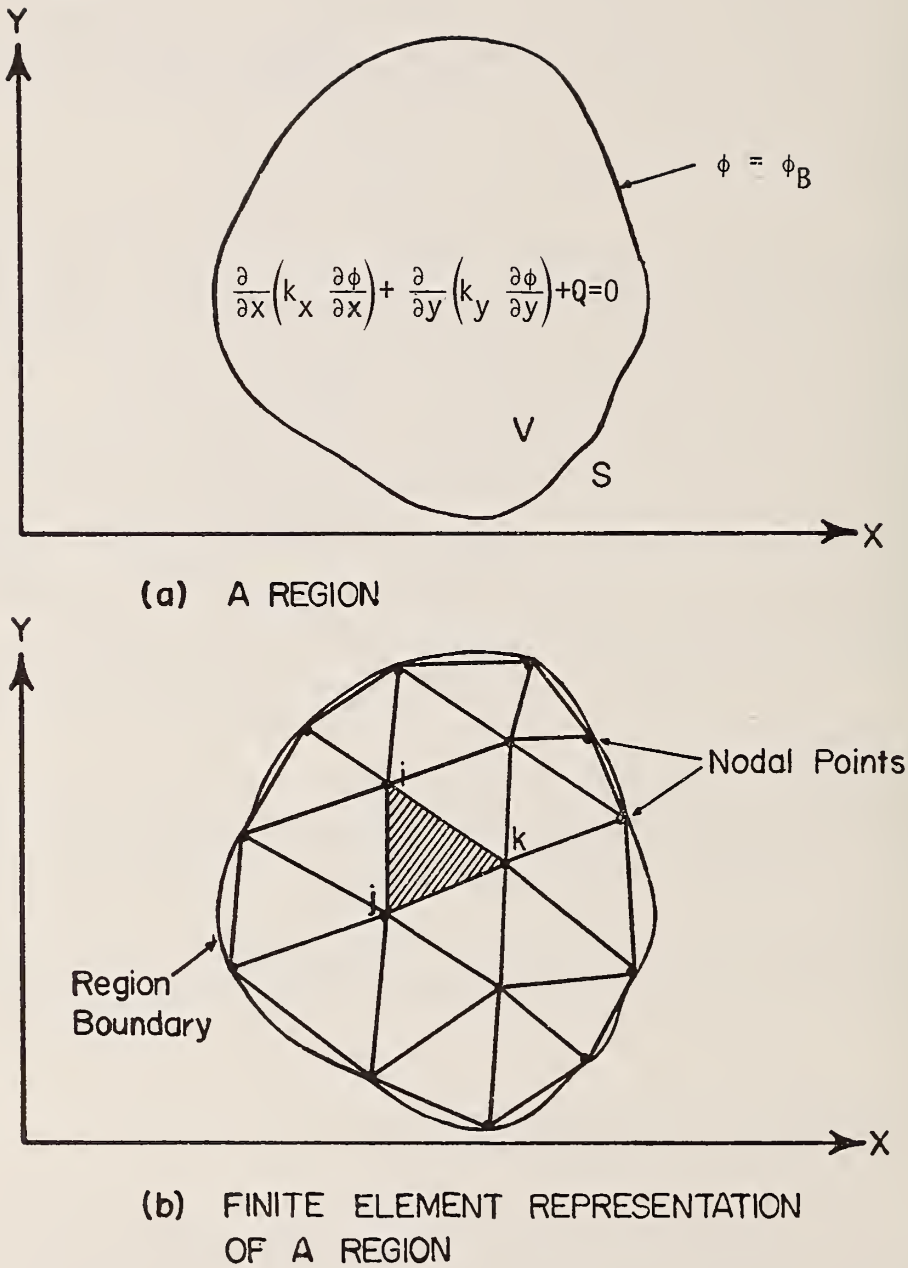


Figure 3. Finite Element Representation

To obtain the approximate solution of Equation 60 by the Finite Element Method, the region is divided into triangular elements, as shown in Figure 3b. Over each element the function is allowed to vary in a limited, prespecified form and elements are interconnected only at nodal points. In the conceptual representation of the element and a region, let the nodal points be the vertices of the triangle. Then for a typical triangle i,j,k, an approximating trial function may be written

$$\phi = [N_i, N_j, N_k] \begin{Bmatrix} \phi_i \\ \phi_j \\ \phi_k \end{Bmatrix} \tag{61}$$

or

$$\phi = [N] \{\phi_n\}$$

in which N_i is a linear function[1] of x and y where N_i = unity at node i and zero at nodes j and k. A similar relationship exists for N_j and N_k. N_i is the so-called "shape function." In all finite element applications the shape functions have the same definition, i.e.,

N_i is continuous over the element and

$$N_i = \begin{matrix} 1 \text{ at node } i \\ 0 \text{ at all other nodes.} \end{matrix}$$

---

[1]For a triangle in which a quadratic approximation is desired, nodes on the mid-side of each element are required and

$$\phi = [N_i, N_l, N_j, N_m, N_k, N_n] \begin{Bmatrix} \phi_i \\ \phi_l \\ \phi_j \\ \phi_m \\ \phi_k \\ \phi_n \end{Bmatrix}$$

where N is a quadratic function of x and y, but also a known function of element geometry.

The coefficients of the linear expansions for N_i and other N functions are all known functions of the element geometry. For example, in an element with vertices at locations (0,0) (2,0) (0,2) as shown in Figure 4, the shape functions are

$$N_1 = \frac{1}{2}(2 - x - y) \tag{62}$$

$$N_2 = \frac{1}{2}x \tag{63}$$

$$N_3 = \frac{1}{2}y \tag{64}$$

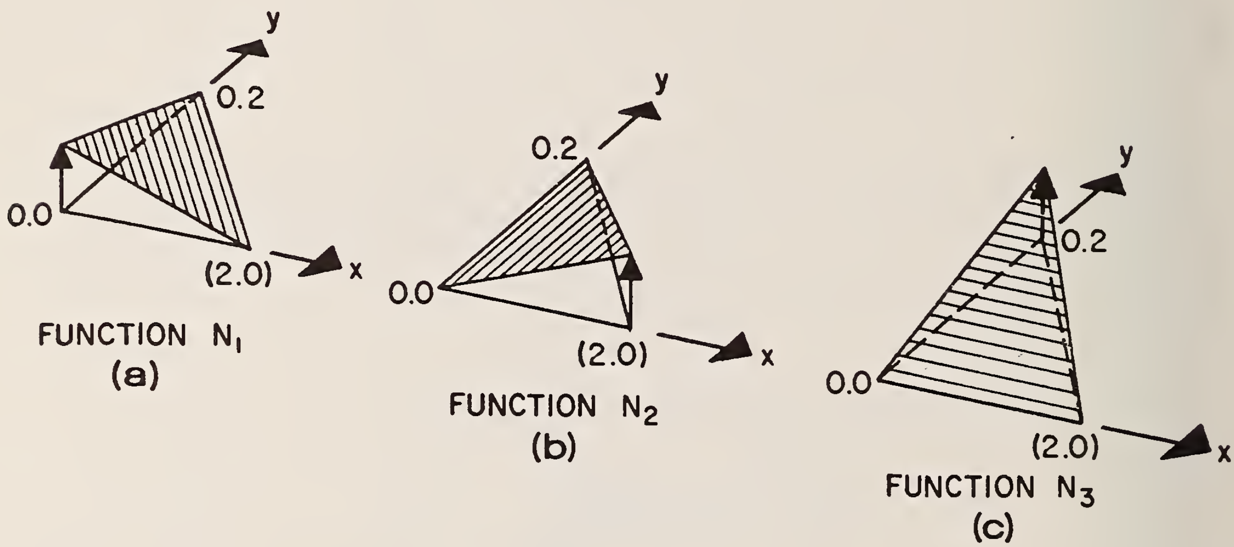


Figure 4. Shape Functions for a 45° Triangle With Base Length Equal 2

Similar functions may be written for the general linear triangle i,j,k, i.e.,

$$N_i = a_i + b_i x + c_i y \tag{65}$$

where a_i, b_i, c_i are purely geometric constants.

Let Equation 60 be written as a function $h(\phi, x, y) = 0$:

$$h = \frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) + Q = 0 \tag{66}$$

Then the general method of weighted residuals states that

$$\iint w\ h\ dxdy = 0 \tag{67}$$

where h is evaluated for a location function ϕ_a and w is any geometric weighting function. In terms of discrete elements, Equation 67 may be written as

$$\sum_e \iint w\ h_e\ dxdy = 0 \tag{68}$$

Then for any element the approximating function $\phi = N\ \phi_n$ may be selected where ϕ_n are the nodal values and N is a geometric slope function, allowing h to be expressed as a function of nodal values ϕ_n. The Galerkin variation of the Method of Least Residuals states that the best approximation to the true solution will be obtained when the shape functions are used as weighting factors, i.e., $w = N_i$.

The final result of the Galerkin process will be one equation for each node. That equation consists of the sum of the contributions for all adjacent elements. Again, it is convenient to form the equations on an element-by-element basis. To form the equation for one such element, we use the Poisson equation and compute the contributions to all nodes of the element as follows:

$$\iint [N]^T \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) + Q\right] dxdy \tag{69}$$

where $\quad [N]^T = \{N\}$.

Since ϕ is assumed to be continuous only along element sides, values of $\frac{\partial \phi}{\partial x}$ may be discontinuous, but integrals of such discontinuities will be zero. The value of $\frac{\partial^2 \phi}{\partial x^2}$, however, tends toward infinity at element interfaces and the integral of such values across the boundary will be nonzero.

It is necessary, therefore, to transform the contribution of Equation 69 to eliminate second order differentials. Using Green's Transformation, Equation 69 becomes

$$\int [N]^T \left(k_x \frac{\partial \phi}{\partial x} c_x + c_y k_y \frac{\partial \phi}{\partial y} \right) dS - \iint \left[[N,_x]^T k_x \frac{\partial \phi}{\partial x} + [N,_y]^T k_y \frac{\partial \phi}{\partial y} - [N]^T Q \right] dxdy \tag{70}$$

where c_x and c_y = the direction cosines of the outward normal and the x and y directions, respectively,

$$[N,_x] = \frac{\partial [N]}{\partial x}, \text{ and}$$

$$[N,_y] = \frac{\partial [N]}{\partial y} .$$

The surface integral will disappear for all internal element interfaces and only appear on the boundary of the system where the specified boundary conditions will provide sufficient information to evaluate the integral. A strict statement of the Poisson functional would have included a boundary surface integral and a similar definition of boundary conditions. Thus, it can be shown that the functional approach and the Galerkin approach can lead to identical finite element formulations.

As stated previously, no convenient functional approach exists for the solution of the Navier-Stokes equations. Hence the Galerkin method will be used in all finite element derivations in this report. One significant feature of the Galerkin method as opposed to the functional approach is that the simultaneous equations derived from a functional are always symmetrical, whereas no such guarantee exists in the Galerkin method.

NUMERICAL INTEGRATION

One of the most important features of a finite element model is the ability to perform integration over the area of the element as functions of x and y. For the particular case of the linear approximation to the Poisson equation the integration is trivial since the order of integration is either constant or linear. However, when quadratic shape functions are used, the order of integration is quadratic or higher and algebraic area integration is often too cumbersome for practical application. There is, however, a convenient numerical approach, called numerical integration, which yields exact answers for any desired degree of integration.

The process of numerical integration is (1) evaluating the function at specified locations within the triangle and then (2) multiplying the value of the function by appropriate weighting factors, i.e.,

$$\iint_A h(x,y)\ dxdy = A \sum_{i=1}^{q} w_i h(x_i, y_i) \tag{71}$$

where
A = area of the triangle,
w_i = specified weighting factor,
x_i, y_i = specified coordinate locations for evaluation of h,
q = number of integration points, and

x_i and y_i are invariant with respect to the actual shape of the triangle.

Appropriate numbers of points and their locations for given orders of error are available in the literature (27). A quintic order integration method is used for the application of the finite element method to the Navier-Stokes equations. There are seven locations for integration and the error is of the sixth polynomial order. The reader is referred to O. C. Zienkiewicz (Ref. 27), page 151, for the exact locations and weighting functions.

V. METHOD OF SOLUTION

In this chapter the Galerkin process of the Method of Weighted Residuals will be applied to develop a finite element formulation for the coupled continuity and momentum equations describing the two-dimensional flow in the horizontal plane. Since the present work is focussed on the steady state condition, the time dependent terms will be omitted in this derivation. The Newton-Raphson scheme will be used to solve the nonlinear inertial terms in the flow equations.

BASIC ELEMENT AND SHAPE FUNCTIONS

The basic element shape selected for the fluid mechanics model is the triangle since irregular boundaries may be better approximated with such a shape and the complete model can be developed in terms of one basic element without the special cases that are assoicated with a quadrilateral.

Six nodes are associated with each triangle: three at the vertices and three on the mid-sides. The system is described in terms of unit flow components r and s at all six nodes and in terms of depth at the vertices only (see Figure 5). This is consistent with the assumption that r and s vary as quadratic functions over the element but that depth varies linearly. When the coefficients are constructed for an element there are 15 equations comprised of six unknown values of r and of s and three unknown values of h. The assumption of linear depth variation is necessary for consistency of the order of solution.

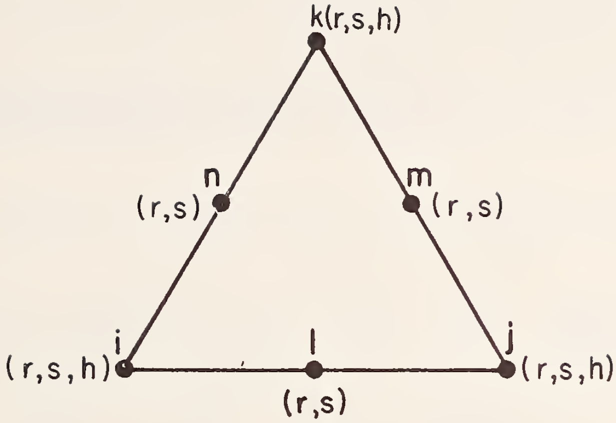


Figure 5. Basic Element for Fluid Flow Equations

APPLICATION OF THE FINITE ELEMENT METHOD TO THE BASIC EQUATIONS

The first step in applying the Finite Element Method to governing equations is to employ Galerkin's criterion to the method of weighted residuals. This is done by making the weighting function equal to the shape function defined for the Finite Element Method. In the following equations, the basic definitions are implied by the method and the integral equations are all written in matrix form. For example,

$$s = [N] \; \{s\}$$

where $$[N] = [N_i \; N_j \; N_k \; N_l \; N_m \; N_n]$$

and

$$\{s\} = \begin{Bmatrix} s_i \\ s_j \\ s_k \\ s_l \\ s_m \\ s_n \end{Bmatrix}$$

and $[N]^T = \{N\}$

With these definitions, the Galerkin statement of the governing equations becomes for the

Motion Equations

$$\int_V [N]^T \Bigg\{ rh\frac{\partial r}{\partial x} - r^2\frac{\partial h}{\partial x} + sh\frac{\partial r}{\partial y} - sr\frac{\partial h}{\partial y} + gh^3\frac{\partial h}{\partial x} + gh^3\frac{\partial z_o}{\partial x}$$
$$- \frac{\varepsilon_x}{\rho}\left[h^2\frac{\partial^2 r}{\partial x^2} - rh\frac{\partial^2 h}{\partial x^2} - h\frac{\partial h}{\partial x}\frac{\partial r}{\partial x} + r\left(\frac{\partial h}{\partial x}\right)^2\right]$$
$$- \frac{\varepsilon_x}{\rho}\left[h^2\frac{\partial^2 r}{\partial y^2} - rh\frac{\partial^2 h}{\partial y^2} - h\frac{\partial h}{\partial y}\frac{\partial r}{\partial y} + r\left(\frac{\partial h}{\partial y}\right)^2\right]$$
$$+ \frac{gr}{C^2}(r^2 + s^2)^{1/2}\Bigg\} dV = 0 \qquad (72)$$

and

$$\int_V [N]^T \Bigg\{ rh\frac{\partial s}{\partial x} - rs\frac{\partial h}{\partial x} + sh\frac{\partial s}{\partial y} - s^2\frac{\partial h}{\partial y} + gh^3\frac{\partial h}{\partial y} + gh^3\frac{\partial z_o}{\partial y}$$
$$- \frac{\varepsilon_y}{\rho}\left[h^2\frac{\partial^2 s}{\partial x^2} - sh\frac{\partial^2 h}{\partial x^2} - h\frac{\partial h}{\partial x}\frac{\partial s}{\partial x} + s\left(\frac{\partial h}{\partial x}\right)^2\right]$$
$$- \frac{\varepsilon_y}{\rho}\left[h^2\frac{\partial^2 s}{\partial y^2} - sh\frac{\partial^2 h}{\partial y^2} - h\frac{\partial h}{\partial y}\frac{\partial s}{\partial y} + s\left(\frac{\partial h}{\partial y}\right)^2\right]$$
$$+ \frac{gs}{C^2}(r^2 + s^2)^{1/2}\Bigg\} dV = 0 \qquad (73)$$

and for the *Continuity Equation*

$$\int_V [M]^T\left(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\right) dV = 0 \qquad (74)$$

where
$[N]$ = a quadratic weighting function defined at six nodes,
$[M]$ = a linear weighting function defined at three corner nodes, and
V = element area.

The Motion Equations (72 and 73) contain second order derivatives. It is necessary to reduce the higher order derivatives in each of the governing equations by an application of Green's Transformation. Typical transformations take the form

(1) $$\int_V [N]^T h^2 \frac{\partial^2 r}{\partial x^2}\, dV = \int_S [N]^T h^2 \frac{\partial r}{\partial x} c_x dS - \int_V [N]^T 2h \frac{\partial r}{\partial x} \frac{\partial h}{\partial x}\, dV - \int_V [N,_x]^T h^2 \frac{\partial r}{\partial x}\, dV \qquad (75)$$

(2) $$\int_V [N]^T rh \frac{\partial^2 h}{\partial x^2}\, dV = \int_S [N]^T rh \frac{\partial h}{\partial x} c_x dS - \int_V [N]^T r\left(\frac{\partial h}{\partial x}\right)^2 dV - \int_V [N]^T h \frac{\partial r}{\partial x} \frac{\partial h}{\partial x}\, dV - \int_V [N,_x]^T rh \frac{\partial h}{\partial x}\, dV \qquad (76)$$

(3) $$\int_V [N]^T gh^3 \frac{\partial h}{\partial x}\, dV = \frac{g}{4} \int_S [N,_x]^T h^4 c_x dS - \frac{g}{4} \int_V [N,_x]^T h^4 dV \qquad (77)$$

where c_x = the direction cosine normal to the surface s, and

S = contour.

When the relationships defined by Equations 75 through 77 are substituted into the Motion Equations, we arrive at the final forms for the finite element method statement of the horizontal flow model.

Motion Equations

$$\{f_1\} = \int_V \Bigg\{ [N]^T \Bigg(rh\frac{\partial r}{\partial x} - r^2\frac{\partial h}{\partial x} + sh\frac{\partial r}{\partial y} - rs\frac{\partial h}{\partial y} + gh^3\frac{\partial z_o}{\partial x}$$

$$- \frac{\varepsilon_x}{\rho}\left[2r\left(\frac{\partial h}{\partial x}\right)^2 + 2r\left(\frac{\partial h}{\partial y}\right)^2 - 2h\left(\frac{\partial h}{\partial x}\frac{\partial r}{\partial x} + \frac{\partial h}{\partial y}\frac{\partial r}{\partial y}\right)\right]$$

$$+ \frac{gr}{C^2}(r^2 + s^2)^{\frac{1}{2}}\Bigg) + [N,_x]^T\left(hr\frac{\partial h}{\partial x} - h^2\frac{\partial r}{\partial x} - \frac{g}{4}h^4\right)$$

$$+ [N,_y]^T\left(hr\frac{\partial h}{\partial y} - h^2\frac{\partial r}{\partial y}\right)\Bigg\}dV - \int_S [N]^T\left[h^2\left(\frac{\partial r}{\partial x} + \frac{\partial r}{\partial y}\right)\right.$$

$$\left. - rh\left(\frac{\partial h}{\partial x} + \frac{\partial h}{\partial y}\right) - \frac{g}{4}h^4\right] dS = 0 \tag{78}$$

$$\{f_2\} = \int_V \Bigg\{ [N]^T \Bigg(rh\frac{\partial s}{\partial x} - rs\frac{\partial h}{\partial x} + sh\frac{\partial s}{\partial y} - s^2\frac{\partial h}{\partial y} + gh^3\frac{\partial z_o}{\partial y}$$

$$- \frac{\varepsilon_y}{\rho}\left[2s\left(\frac{\partial h}{\partial x}\right)^2 + 2s\left(\frac{\partial h}{\partial y}\right)^2 - 2h\left(\frac{\partial h}{\partial x}\frac{\partial s}{\partial x} + \frac{\partial h}{\partial y}\frac{\partial s}{\partial y}\right)\right]$$

$$+ \frac{gs}{C^2}(r^2 + s^2)^{\frac{1}{2}}\Bigg) + [N,_x]^T\left(hs\frac{\partial h}{\partial x} - h^2\frac{\partial s}{\partial x}\right)$$

$$+ [N,_y]^T\left(hs\frac{\partial h}{\partial y} - h^2\frac{\partial s}{\partial y} - \frac{g}{4}h^4\right)\Bigg\}dV$$

$$- \int_S [N]^T\left[h^2\left(\frac{\partial s}{\partial x} + \frac{\partial s}{\partial y}\right) - sh\left(\frac{\partial h}{\partial x} + \frac{\partial h}{\partial y}\right) - \frac{g}{4}h^4\right] dS = 0 \tag{79}$$

Continuity Equation

$$\{f_3\} = \int_V [M]^T\left(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\right) dV = 0 \tag{80}$$

NONLINEAR SOLUTION METHODS

Equations 78 and 79 contain nonlinear terms in the integrals. Typical nonlinear terms are in the form of

$$\int_V [N]^T \, rh\frac{\partial r}{\partial x} \, dV$$

$$\int_V [N]^T \, sh\frac{\partial r}{\partial y} \, dV$$

$$\int_V [N]^T \, rh\frac{\partial s}{\partial x} \, dV$$

and

$$\int_V [N]^T \, sh\frac{\partial s}{\partial y} \, dV$$

A general expression for the finite element formulation of Equations 78 through 80 may be written

$$[K] \{u_n\} + [D(u)] \{u_n\} = \{R\} \tag{81}$$

where $[K]$ = the contribution of linear terms,
$[D(u)]$ = the contribution of nonlinear terms,
$\{u_n\}$ = a generalized variable vector, and
$\{R\}$ = a right-hand side vector.

The following three methods represent the most attractive possibilities for solving Equation 81:

1. Successive approximations using a modified coefficient matrix,
2. Successive approximations using a constant coefficient matrix, and
3. The Newton-Raphson scheme.

All three methods are iterative in that they require an estimate of the value of the unknown before solution can proceed, and the process is repeated using progressively better estimates of the solution. Among the three methods it has been found that methods 1 and 2 were not only unstable but also unreliable in solving the flow equations. These methods are generally useful only when the effects of nonlinearity are small, a situation which is not usually true in fluid problems. The Newton-Raphson scheme, on the other hand, has proved to be convergent for all problems that have shown acceptable answers in the initial step. Therefore, the Newton-Raphson scheme has been selected for the model described in this report. A detailed description of this method follows.

THE NEWTON-RAPHSON SCHEME

The Newton-Raphson scheme replaced Equation 81 with a derivative approach based upon the error of the last solution. If an error function f is defined as

$$\{f^m\} = [K]\,\{u_n^{\,m}\} + [D(u^m)]\,\{u_n^{\,m}\} - \{R\} \tag{82}$$

and solution is based upon solving for the correction, Δu, to the previous solution, then

$$[J^m]\,\{\Delta u_n^{\,m+1}\} = \{f^m\} \tag{83}$$

where $\{u_n^{\,m+1}\} = \{u_n^{\,m} + \Delta u_n^{\,m+1}\}$ and

$$\left[J_{ij}^{\,m}\right] = \left[\frac{\partial f_i^{\,m}}{\partial u_j}\right] \tag{84}$$

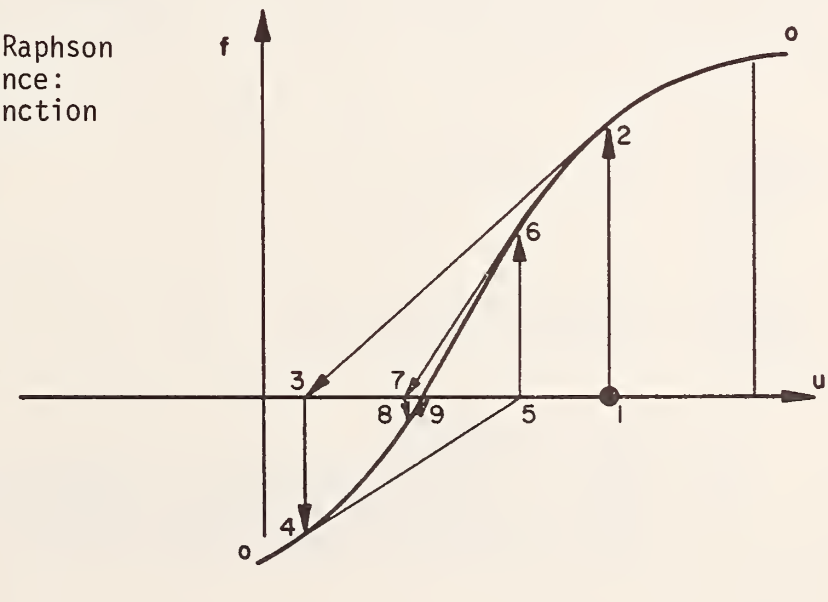
$[J^m]$ is the so-called Jacobian of the system at step m, and terms of this matrix indicate the instantaneous slope of the function f with respect to changes in each of the variables. Actual solution thus consists of evaluating all the error functions {f}, and computing the corrections to {u} based upon the instantaneous slopes to eliminate all the errors.

Graphically the Newton-Raphson procedure may be represented in a one-degree-of-freedom system with values of f following the curve 0-0 (see Figure 6) by the following steps:

1. Point 1 represents the initial guess u_i,
2. Point 2 represents the value of the function f for u_i,
3. Point 3 represents the adjusted value of u based upon the slope at 2, and
4. Point 4 represents the adjusted value of f, etc., until convergence is achieved at point 9.

Note that in a multi-degree-of-freedom system there are n values for f and each f has n slopes. The solution process must therefore adjust all n values of f simultaneously by variation of the n variables.

Figure 6. Newton-Raphson Iterative Convergence: One-Dimensional Function



APPLICATION OF THE NEWTON-RAPHSON SCHEME TO THE FLUID FLOW FINITE ELEMENT

Consider a nonlinear term $u\frac{\partial v}{\partial x}$ with a finite element contribution of the form

$$\{f\} = \int_A [N]^T [N] \{u_n\} [N,_x] \{v_n\} dA \tag{85}$$

The derivatives of this integral must be taken with respect to each of the variables u_n and v_n, i.e., typically:

$$\left\{\frac{\partial f}{\partial u_j}\right\} = \int_A [N]^T [N,_x] \{v_n\} [N_j] dA \tag{86}$$

$$\left\{\frac{\partial f}{\partial v_j}\right\} = \int_A [N]^T [N] \{u_n\} [N_{j,x}] dA \tag{87}$$

The interior matrix products $[N,_x]\{v_n\}$ and $[N]\{u_n\}$ are the values $\frac{\partial v}{\partial x}$ and u, respectively. When numerical integration is used to evaluate the integrals of Equations 86 and 87, they may be rewritten as a summation

$$\left\{\frac{\partial f}{\partial u_j}\right\} = A\sum w_g [N]^T \left(\frac{\partial v}{\partial x}\right)_g [N_j] \tag{88}$$

$$\left\{\frac{\partial f}{\partial v_j}\right\} = A\sum w_g [N]^T u_g [N_{j,x}] \tag{89}$$

where A = area of triangle,

w_g = numerical integration weighting factor at location g, and g indicates evaluation at the specified integration point (often called a Gauss Point).

Equations 88 and 89 are the submatrices that are generated by the $u\frac{\partial v}{\partial x}$ term, and they appear in columns appropriate to u and v of the final coefficient matrix.

JACOBIAN FORMS OF BASIC EQUATIONS

A complete list of Jacobian forms of the Galerkin process of the basic equations (78 through 80) can be derived using a method similar to that applied to Equation 85. These forms are:

$$\begin{aligned}\left[\frac{\partial f_1}{\partial v}\right] = \int_V \Bigg\{ & [N]^T \left[[N] \left(h\frac{\partial r}{\partial x} - 2r\frac{\partial h}{\partial x} - s\frac{\partial h}{\partial y}\right) + [N,_x]\ rh + [N,_y]\ sh\right] \\ & - \frac{\varepsilon_x}{\rho} \left([N]^T \left[[N]\ 2\left(\frac{\partial h}{\partial x}\right)^2 + [N]\ 2\left(\frac{\partial h}{\partial y}\right)^2 - [N,_x]\ 2h\frac{\partial h}{\partial x}\right.\right. \\ & \left.\left. - [N,_y]\ 2h\frac{\partial h}{\partial y}\right] + [N,_x]^T \left([N]\ h\frac{\partial h}{\partial x} - [N,_x]\ h^2\right)\right) \\ & + [N,_y]^T \left([N]\ h\frac{\partial h}{\partial y} - [N,_y]\ h^2\right) \\ & + [N]^T \left([N]\ \frac{g}{C^2}\ (2r^2 + s^2)(r^2 + s^2)^{-\frac{1}{2}}\right)\Bigg\}\ dV \\ & + \int_S [N]^T \left[[N] \left(h\frac{\partial h}{\partial x} + h\frac{\partial h}{\partial y}\right) - [N,_x]\ h^2 - [N,_y]\ h^2\right] dS \end{aligned} \qquad (90)$$

$$\left[\frac{\partial f_1}{\partial s}\right] = \int_V [N]^T \left[[N] \left(h\frac{\partial r}{\partial y} - r\frac{\partial h}{\partial y}\right) + [N]\ \frac{g}{C^2}\ rs\ (r^2 + s^2)^{-\frac{1}{2}}\right] dV \qquad (91)$$

$$\left[\frac{\partial f_1}{\partial h}\right] = \int_V \Bigg\{[N]^T \left[[M] \left(r\frac{\partial r}{\partial x} + s\frac{\partial r}{\partial y}\right) - [M,_x]\, r^2 - [M,_y]\, rs\right]$$

$$+ [N]^T \left[[M] \left(3g\frac{\partial z_o}{\partial x}\, h^2\right)\right] - [N,_x]^T \left[[M]\, gh^3\right]$$

$$- \frac{\varepsilon_x}{\rho} \Bigg([N]^T \Bigg[[M] \left(-2\frac{\partial h}{\partial x}\,\frac{\partial r}{\partial x} - 2\frac{\partial h}{\partial y}\,\frac{\partial r}{\partial y}\right)$$

$$+ [M,_x] \left(4r\frac{\partial h}{\partial x} - 2h\frac{\partial r}{\partial x}\right) + [M,_y] \left(4r\frac{\partial h}{\partial y} - 2h\frac{\partial r}{\partial y}\right)\Bigg]\Bigg)$$

$$+ [N,_x]^T \left[[M] \left(r\frac{\partial h}{\partial x} - 2h\frac{\partial r}{\partial x}\right) + [M,_x]\, rh\right]$$

$$+ [N,_y]^T \left[[M] \left(r\frac{\partial h}{\partial y} - 2h\frac{\partial r}{\partial y}\right) + [M,_y]\, rh\right]\Bigg\}\, dV$$

$$- \int_S [N]^T \Bigg\{[M] \left[2h\left(\frac{\partial r}{\partial x} + \frac{\partial r}{\partial y}\right) - r\left(\frac{\partial h}{\partial x} - \frac{\partial h}{\partial y}\right)\right]$$

$$- [M,_x]\, rh - [M,_y]\, rh\Bigg\}\, dS \tag{92}$$

$$\left[\frac{\partial f_2}{\partial r}\right] = \int_V [N]^T \left[[N] \left(h\frac{\partial s}{\partial x} - s\frac{\partial h}{\partial x}\right) + [N]\, \frac{g}{C^2}\, rs\, (r^2 + s^2)^{-1/2}\right] dV \tag{93}$$

$$\left[\frac{\partial f_2}{\partial s}\right] = \int_V \Bigg\{[N]^T \left[[N] \left(h\frac{\partial s}{\partial y} - 2s\frac{\partial h}{\partial y} - r\frac{\partial h}{\partial x}\right) + [N,_x]\, hr + [N,_y]\, sh\right]$$

$$- \frac{\varepsilon_y}{\rho} \Bigg([N]^T \Bigg[[N]\, 2\left(\frac{\partial h}{\partial x}\right)^2 + [N]\, 2\left(\frac{\partial h}{\partial y}\right)^2 - [N,_x]\, 2h\frac{\partial h}{\partial x}$$

$$- [N,_y]\, 2h\frac{\partial h}{\partial y}\Bigg]\Bigg) + [N,_x]^T \left[[N]\, h\frac{\partial h}{\partial x} - [N,_x]\, h^2\right]$$

$$+ [N,_y]^T \left[[N]\, h\frac{\partial h}{\partial y} - [N,_y]\, h^2\right]$$

$$+ [N]^T \left[[N] \frac{g}{C^2} (2s^2 + r^2)(r^2 + s^2)^{-\frac{1}{2}} \right] \Bigg\} \, dV$$

$$+ \int_S [N]^T \left[[N] \left(h\frac{\partial h}{\partial x} + h\frac{\partial h}{\partial y} \right) - [N,_x] \, h^2 - [N,_y] \, h^2 \right] dS \tag{94}$$

$$\left[\frac{\partial f_2}{\partial h} \right] = \int_V \Bigg\{ [N]^T \left[[M] \left(s\frac{\partial s}{\partial z} + r\frac{\partial s}{\partial x} \right) - [M,_x] \, rs - [M,_y] \, s^2 \right]$$

$$+ [N]^T \left[[M] \left(3g\frac{\partial z_o}{\partial y} \, h^2 \right) \right]$$

$$- \frac{\varepsilon_y}{\rho} \left([N]^T \left[[M] \left(-2\frac{\partial h}{\partial x} \frac{\partial s}{\partial x} - 2\frac{\partial h}{\partial y} \frac{\partial s}{\partial y} \right) + [M,_x] \left(4s\frac{\partial h}{\partial x} - 2h\frac{\partial s}{\partial x} \right) \right.\right.$$

$$\left.\left. + [M,_y] \left(4s\frac{\partial h}{\partial y} - 2h\frac{\partial s}{\partial y} \right) \right] \right) + [N,_x]^T \left[[M] \left(s\frac{\partial h}{\partial x} - 2h\frac{\partial s}{\partial x} \right) \right.$$

$$\left. + [M,_x] \, sh \right] + [N,_y]^T \left[[M] \left(s\frac{\partial h}{\partial y} - 2h\frac{\partial s}{\partial y} \right) + [M,_y] \, sh \right] \Bigg\} \, dV$$

$$- \int_S [N]^T \Bigg\{ [M] \left[2h\left(\frac{\partial s}{\partial x} + \frac{\partial s}{\partial y} \right) - s\left(\frac{\partial h}{\partial x} + \frac{\partial h}{\partial y} \right) \right]$$

$$- [M,_x] \, sh - [M,_y] \, sh \Bigg\} \, dS \tag{95}$$

Continuity Equation

$$\left[\frac{\partial f_3}{\partial r} \right] = \int_V [M]^T \, [N,_x] \, dV \tag{96}$$

$$\left[\frac{\partial f_3}{\partial s} \right] = \int_V [M]^T \, [N,_y] \, dV \tag{97}$$

$$\left[\frac{\partial f_3}{\partial h}\right] = 0 \qquad (98)$$

The resulting element coefficient matrix can be expressed in the form

$$\begin{array}{cc} & \begin{array}{ccc} \text{6 cols.} & \text{6 cols.} & \text{3 cols.} \end{array} \\ \begin{array}{c} \text{6 rows} \\ \\ \text{6 rows} \\ \\ \text{3 rows} \end{array} & \begin{bmatrix} \dfrac{\partial f_1}{\partial r} & \dfrac{\partial f_1}{\partial s} & \dfrac{\partial f_1}{\partial h} \\ \dfrac{\partial f_2}{\partial r} & \dfrac{\partial f_2}{\partial s} & \dfrac{\partial f_2}{\partial h} \\ \dfrac{\partial f_3}{\partial r} & \dfrac{\partial f_3}{\partial s} & 0 \end{bmatrix} \begin{bmatrix} \Delta r \\ \\ \Delta s \\ \\ \Delta h \end{bmatrix} = \begin{bmatrix} f_1 \\ \\ f_2 \\ \\ f_3 \end{bmatrix} \end{array} \qquad (99)$$

Note that each term is, in fact, a submatrix of the sizes indicated. When numerical integration is carried out at each integration point for the Jacobian form of Equations 90 through 98, the final solution of Equation 99 can be obtained for Δr, Δs and Δh.

MODEL OPERATION

Each element coefficient matrix is assembled into the main coefficient matrix by direct addition. It should be noted at this time that in contrast to most structural applications, the main matrix is not symmetrical and does not have nonzero diagonal terms in all locations. Thus, special care is required in ordering the equations.

When the main coefficient matrix is fully assembled, the boundary conditions are inserted and the equations are solved simultaneously for all the changes of r, s, and h, using the error functions as a right-hand side.

The modified r, s, and h are then used to generate a new set of error functions and the process is repeated until satisfactory convergence is reached.

The model may be summarized by the following steps:

1. Describe the system by a series of triangular elements.
2. Make an initial guess for the distribution of r, s, and h.
3. Develop the error functions of all four equations at each node.
4. Develop the Jacobian matrix of instantaneous slopes for each element.
5. Assemble main coefficient matrix or Jacobian for the entire system.
6. Solve simultaneously for correction to r, s, and h and update r, s, and h.
7. Repeat steps 3 through 6 until convergence is achieved.

BOUNDARY CONDITIONS

The boundary conditions to be specified in the model are either the unit flows in the x and y directions or the water surface elevations at boundary points, depending upon which information is available or prescribed. In the case of bridge backwater hydraulic computations, the water surface elevations along a cross-section downstream from the bridge site and the flow of the stream are given. These prescribed flow conditions lead to the specification of the water surface elevation along the downstream free flow boundary, and the flow components along the upstream free boundary.

The boundary conditions along both banks are specified so that the flow will follow the outer edge of the boundary elements. This implies that there is no flow normal to the boundary edge. At the corner of the edges it is assumed that flow is stagnant, therefore, $Q_x = Q_y = 0$.

VI. APPLICATION OF FINITE ELEMENT MODEL TO BRIDGE BACKWATER COMPUTATION

In this chapter two example problems are described in order to demonstrate the application of the Finite Element Model to bridge backwater computations. The first example is the calibration of the model to simulate the water surface profile obtained from the data of a laboratory model test. The prismatic nature of the test flume and the bridge alignments plus the accuracy of controlled data measurement make this example an ideal case for testing the model validity.

The second example is the application of the Finite Element Model to field conditions. In this example a flood of medium size was first selected for model calibration. Two flood flows, one larger and the other smaller than the medium flood, were then simulated using the calibrated model parameters. Reasonably good results were observed from the model simulations.

In both examples the model did not include the effect of scour and change of river bed form on backwater. Additional model development to incorporate algorithms for bed scour is needed as many field conditions may warrant such considerations.

FLOW CONDITIONS

TYPE OF FLOW

There are four major types of flow conditions encountered in bridge waterways, as shown in Figure 7. Briefly these are:

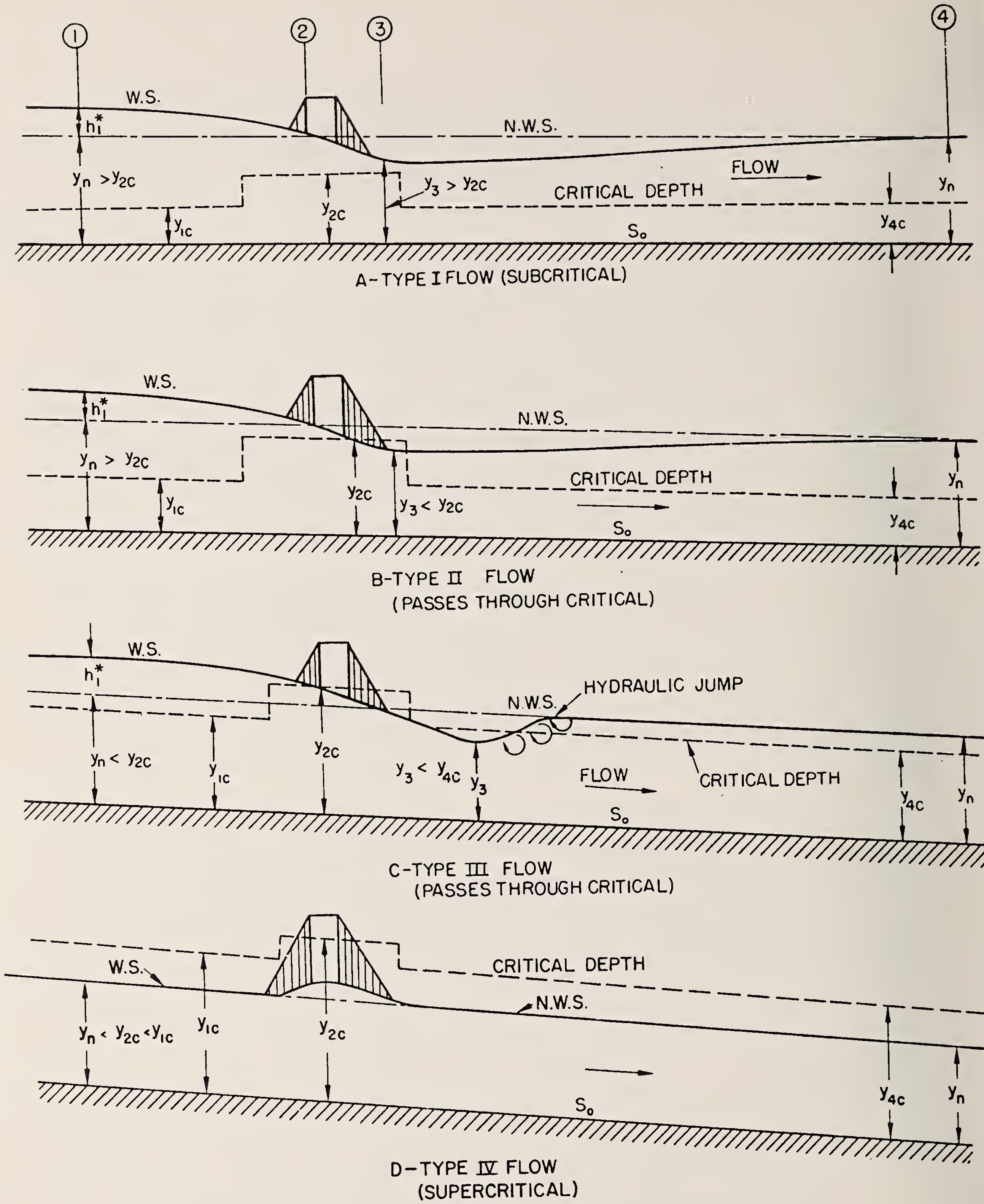


Figure 7. Flow Profile Near a Bridge Site
(Source: Ref. 9)

Type I - Subcritical flow throughout the flow reaches,

Type II - Subcritical flow upstream and downstream from the bridge, and water surface passes through critical depth in the constriction,

Type III - Subcritical flow upstream from the bridge, critical at constriction, supercritical at immediate downstream from the bridge, recovering to subcritical flow again in the downstream reaches, and

Type IV - Supercritical flow throughout the flow reaches.

The occurrence of each flow type is largely governed by the slope of the stream bed, roughness and the degree of contraction at the bridge site. Types I through III usually exist in streams with mild slope located in flat coastal plain regions. Type IV exists in streams with steep slope which are commonly found in mountain regions. The differences between Types I and II, and I and III result from different degrees of contractions at the constriction and roughness in stream beds and flood plains.

This study is limited to the backwater computation of Type I flow. The application of the Finite Element Model to Types II and III flow conditions require a large additional programming effort which is not incorporated in the model. Supercritical flow conditions are handled by the Finite Element Model by reversing the specifications of the boundary conditions. Depth is specified at the upstream boundary and elevation at the downstream boundary. This is the reverse of the subcritical situation where flow is specified at the upstream location and elevation at the downstream boundary.

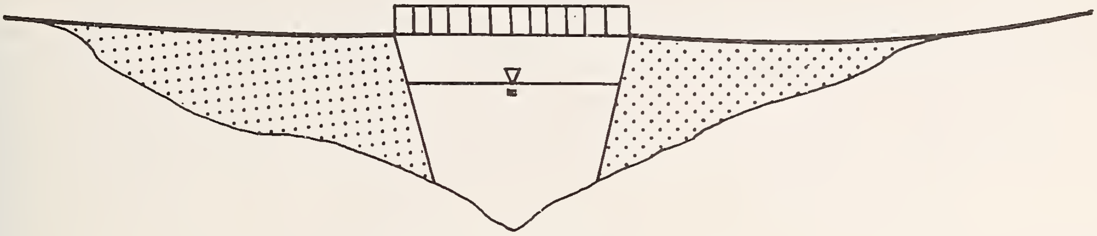
A word of warning is required when modeling systems which, due to flow conditions, can be categorized as Type II or III, respectively. The model does not check for the occurrence of critical or supercritical flow in its present form. The model also does print a warning for the user to do so. This problem is particularly acute at the location of the bridge constriction. The user must check, manually, to ascertain whether the control section (i.e. the critical flow region) is in or below the constriction. If this should occur the model results should be viewed accordingly.

FLOW OVERTOPPING

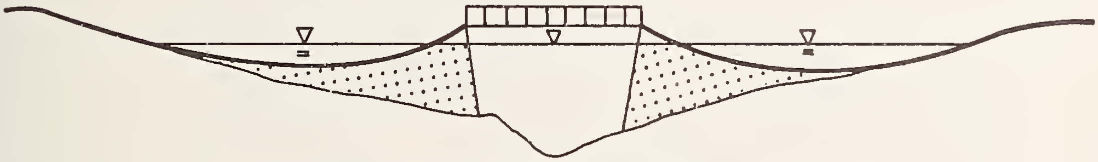
Overtopping is handled in the model as described by equations 100 and 101, respectively. The coefficient gamma (γ) is interpolated from Curve C as shown in Figure 9. This is handled internally by the Finite Element Model. The coefficient C_w is input to the model by the user. Curves A and B are supplied for the convenience of the user. If overtopping should occur, a statement to that effect is printed in the output accompanying execution. Typical situations caused by the embankment and superstructure geometry are shown in Figure 8. When flow overtops the embankments, weir flow prevails. The equation used for computing the overtopping flow is

$$Q = C_w L H^{3/2} \qquad (100)$$

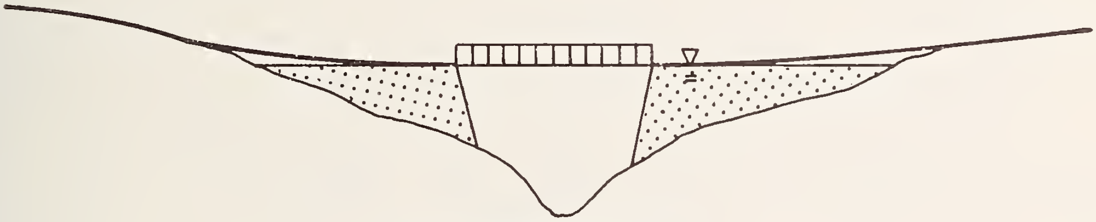
where Q = flow,
C_w = discharge coefficient of broad crest weir,
L = weir length, and
H = head above crest of weir.



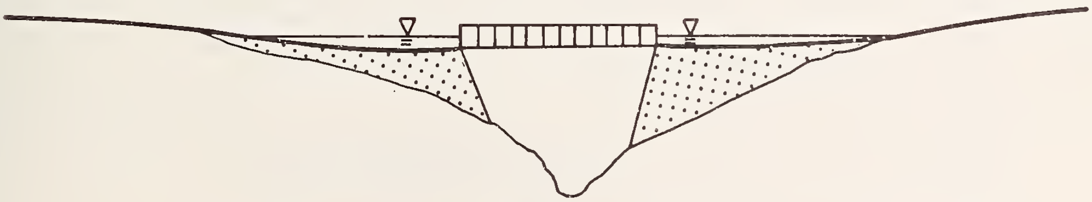
(a) LOW FLOW



(b) COMBINATION of LOW and WEIR FLOWS



(c) ORIFICE FLOW



(d) COMBINATION of ORIFICE and WEIR FLOWS

Figure 8. Water Surface Conditions at a Bridge Section

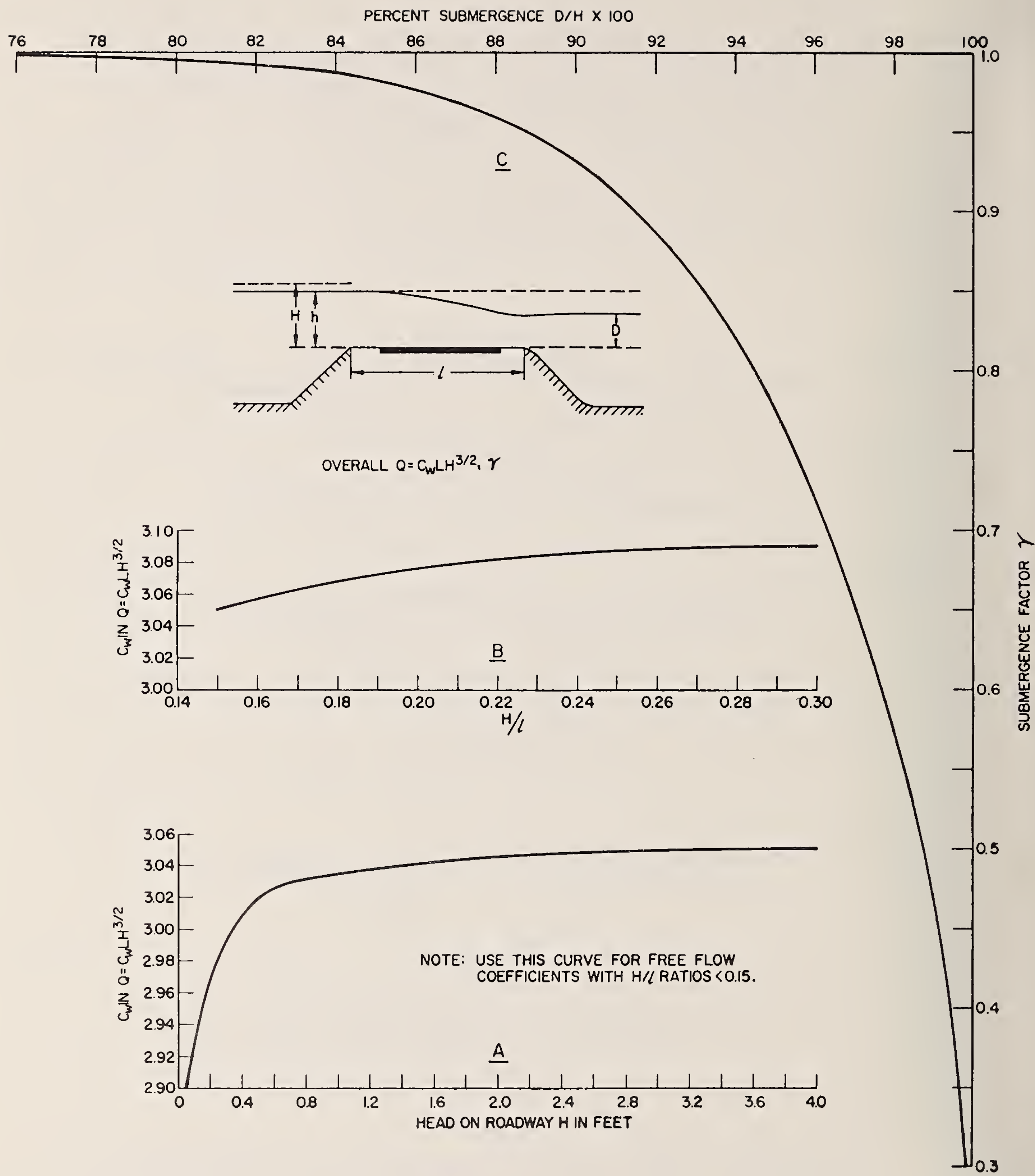


Figure 9. Discharge Coefficients for Flow Over Roadway Embankments (Source: Ref. 9)

Equation 100 is used to compute the discharge under free flow conditions. When the stage at the downstream side of the embankment rises and exceeds the roadway elevation, the flow becomes submerged. The discharge coefficient in Equation 100 is reduced under submerged flow conditions; thus Equation 100 becomes

$$Q = C_s L H^{3/2} \tag{101}$$

where $C_s = \gamma C_w$ and

γ = a correction factor for the discharge coefficient under submerged flow, a function of the ratio of downstream head to upstream head above roadway. The values of γ are obtained from Figure 9 (see Ref. 9).

An iteration procedure has been incorporated in the Finite Element Model to handle the flow overtopping problem. The model initially computes the water surface elevations along the upstream face of the embankment and then compares these with the roadway elevation. If the computed water surface elevations exceed the roadway elevation, an overtopping flow exists. The model will then compute the overtopping flow using the current value of head differential, until two consecutive iterations yield the same flow value. Normally one to two iterations are sufficient to achieve the required accuracy. All of these computations are performed internally once the required inputs for weir characteristics are specified.

There is no provision in the present model to perform orifice flow computations when the bridge deck is unundated. Such a feature may be incorporated into the model during later model development when field data become available for model validation. Presently, the user must manually check for this condition.

EXAMPLE PROBLEMS

EXAMPLE 1. BRIDGE BACKWATER IN A 20.67-FT TEST FLUME

Physical Set Up

The bridge backwater experiments were conducted in a rectangular concrete flume, 20.67 feet wide, 165 feet long and 2.5 feet deep. Bottom slope of the flume was 0.0022. Artificial roughness elements were placed over the entire flume bottom except in the portion under the bridge opening.

An embankment model with a top width of one foot, side slopes of 1 to 0.3 and a height of 20 inches was placed perpendicular to the flow at Station 150, some 60 feet downstream from the flow-establish zone. The size of the bridge opening was three feet. The abutments of the model bridge were wing wall type located equidistant from the center of the flume. Figure 10 shows the general set up of the experiment.

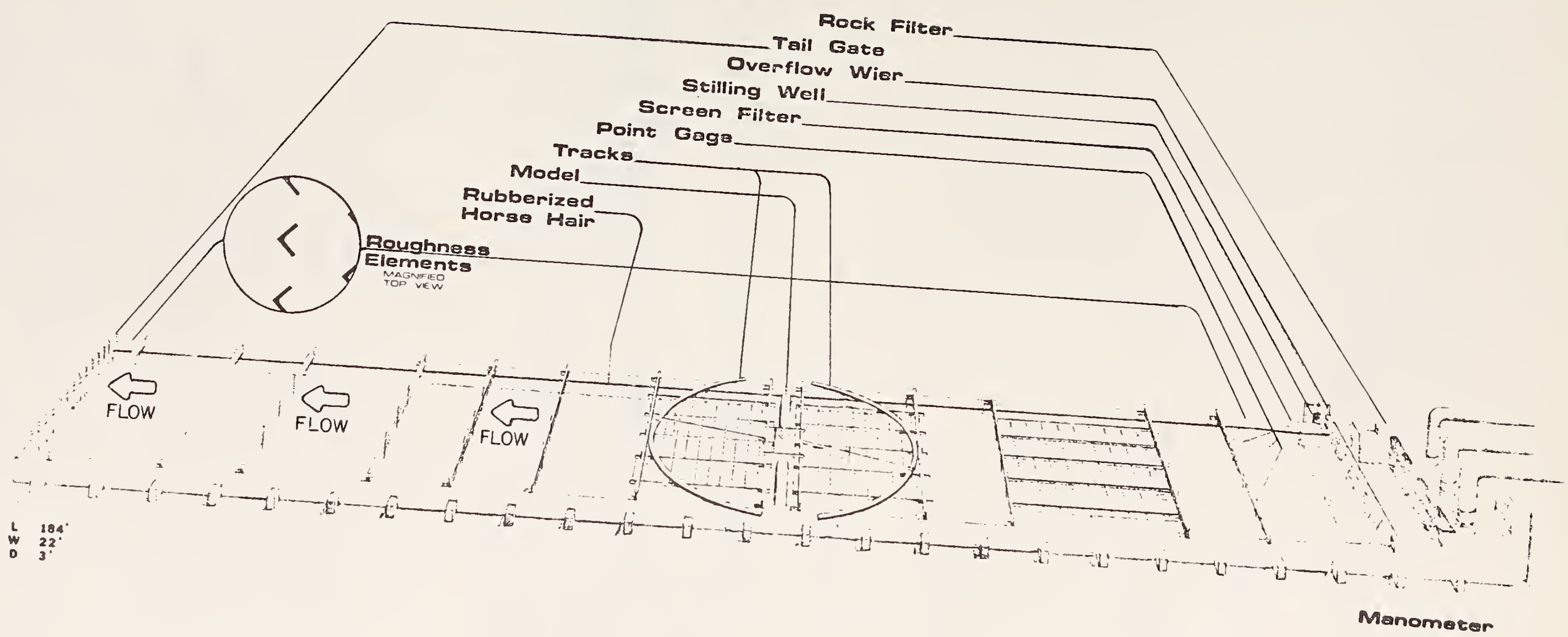


Figure 10. Test Flume - Example 1.

The value of Chezy C was 5.7 in the test flume and 150 across the bridge opening. The test flume and experimental procedures are described in detail in Volume II of this final report series.

Finite Element Representation

In order to simulate the flow conditions in the test flume, a representative network of triangular elements was developed between Stations 100 and 215, as shown in Figure 11. This network defines the entire flow region with a total of 72 elements and 177 nodal points. For each element the roughness coefficient and the eddy viscosities in both X and Y directions are specified. For each node, the X and Y coordinates and the bottom elevation are specified.

The boundary conditions are as follows:

1. Along upstream boundary AG the nodal unit flow r = 0.25 cfs/ft and s = 0, which yields a total flow rate of 5.216 cfs across AG.
2. Along downstream boundary FL, the water surface elevation is 11.26 feet, the elevation under normal flow condition.
3. Along the rigid boundaries AB, BC, CD, DE, EF, GH, HI, IJ, JK and KL, the flow is parallel to the boundary, hence the flow normal to the boundaries is zero.

Model Operation

When the boundary conditions are specified and inputs of system parameters are made, the model computation can be run. To begin computation, the water surface elevation for the entire flume is initialized at a horizontal elevation of 12.00 feet, and then the water is allowed to move to its steady-state position through the iteration process of the

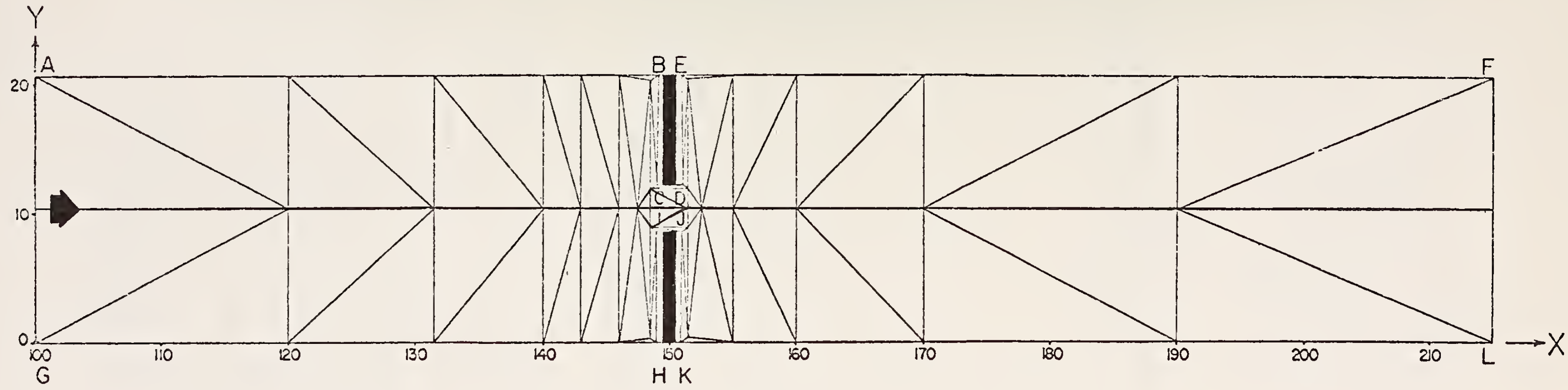


Figure 11. Finite Element Representation.
Example 1 - Test Flume

numerical procedure. Table 1a[2] lists the pertinent input data for the computation. System parameters such as the x and y coordinates, bottom elevation and network geometry data are presented in Tables 1b and 1c. Note that the node and element numbers are omitted in Figure 11 for lack of space. Cross referencing Tables 1b and 1c will give this information.

A total of five iterations are required to obtain the steady-state solution. After five iterations the maximum change of the dependent variables between iteration cycles is:

0.0031 cfs/ft	for x-flow,
0.0019 cfs/ft	for y-flow, and
0.0006 ft	for depth.

The model outputs are listed in Table 2.[2]

Comparison of Computed and Observed Results

Water Surface Elevation. Figure 12 shows the comparison of observed and computed water surface profiles along the centerline of the test flume. The comparison shows excellent agreement.

Velocity Distribution Across the Bridge Opening. A comparison of the observed and computed velocity distribution across the bridge opening is shown in Figure 13. Note that the velocity data in the test flume were taken at the section of the vena contracta. At this location the flow is essentially three-dimensional, with contraction in both the horizontal and vertical planes. The velocity was measured at the center and at both edges of the vena contracta. The measurements showed higher velocities at the edges than at the centerline of the contracted flow.

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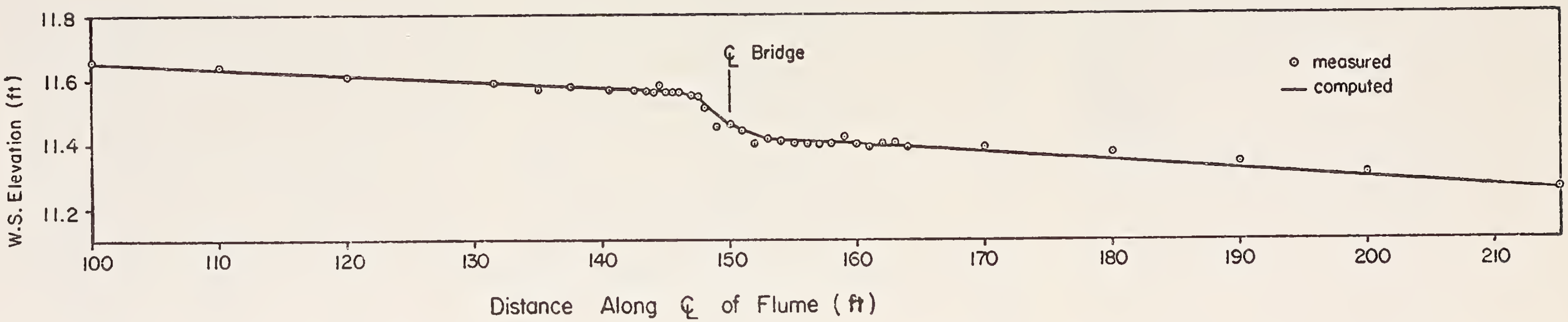
[2]Input and output tables are located at the end of this chapter in order to minimize the disruption of the text discussion.



Q= 5.210 cfs
n = 0.259
S_o= 0.0022

Bridge opening = 3'-0"
Flume width = 20'-8"

Figure 12. Actual Versus Computed Water Surface Profile Along Centerline. Example 1 - Test Flume

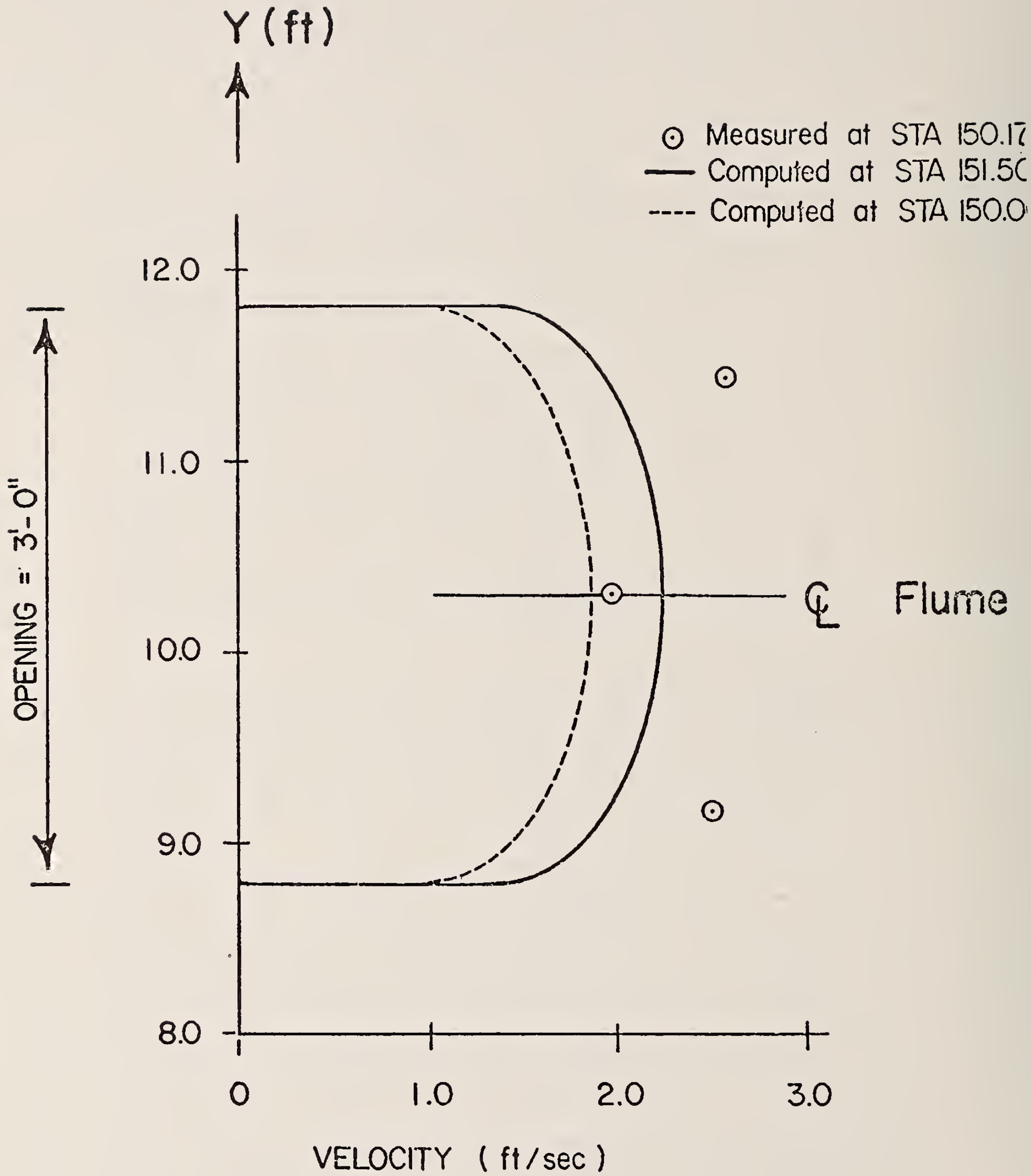


Figure 13. Velocity Profile at Bridge Opening.
Example 1 - Test Flume

The mathematical model cannot reproduce this three-dimensional contracted flow. The computed results are for the mean flow condition which results in a discrepancy in velocity profile between the calculated and the observed velocity data, as shown in Figure 13. Nevertheless, the centerline velocity calculation, which minimizes the above discrepances, is bracketed by model computations. Model velocities on the edge of the vena contracta also demonstrate a reasonable correlation to measured values. These two locations represent points of high turbulence thus making a closer correlation difficult.

EXAMPLE 2. TALLAHALLA CREEK AT STATE HIGHWAY 528, MISSISSIPPI

Site Description

The bridge site selected for the demonstration of the application of the Finite Element Model is located at the Route 528 crossing of Tallahalla Creek near Bay Springs, Mississippi. This bridge site was selected since hydraulic data for major floods have been surveyed and recorded for this site by the USGS. Highwater marks from three major flood events since 1964 are recorded in the vicinity of the bridge site. Data for velocity across the bridge opening are also available for selected floods. A map of the vicinity is shown in Figure 14.

At present there is a 500-foot bridge in place. Figure 15 shows a cross section of the bridge site. From abutment to abutment the bridge begins at Station 363+60 and goes to Station 368+60. The elevation of the bridge is 314.96. The width of the flood plain at the crossing site is approximately 2600 feet, that is, between Stations 257+00 and 383+00. As shown in Figure 14, State Highway 528 crosses Tallahalla Creek at the most narrow section of the stream near Waldrup, Mississippi. The river reach at the vicinity of the crossing is relatively straight, with a slight bend in the left bank. The boundaries of the flood plains are also delineated in Figure 14. A relatively large area of the flood plain is covered by woodlands, and a smaller area is pastureland.

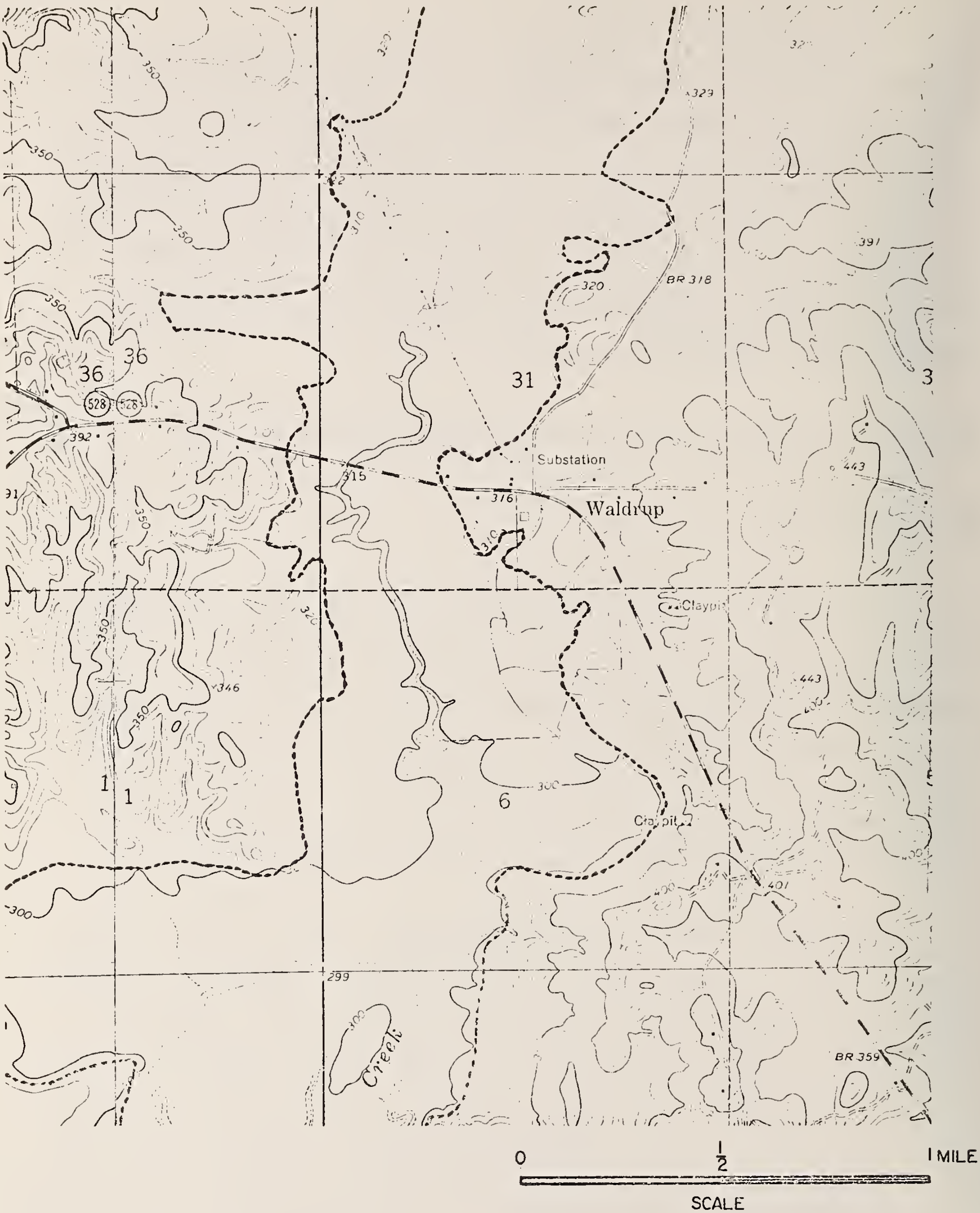


Figure 14. Tallahalla Creek Site Map

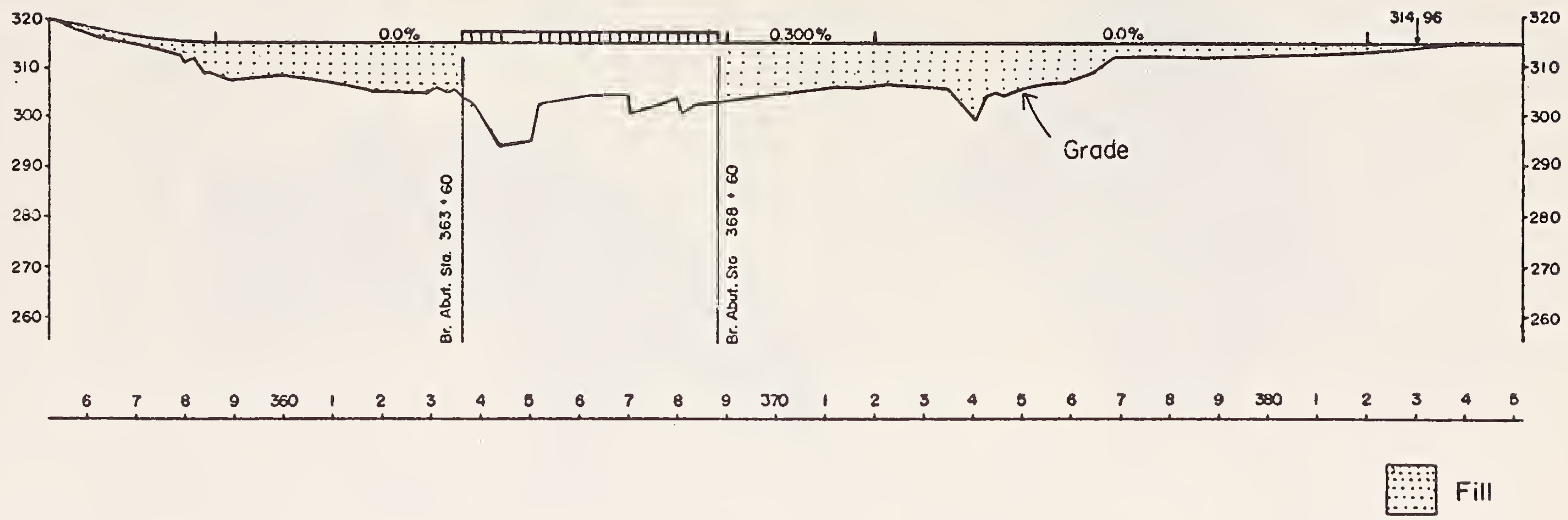


Figure 15. Cross Section of Bridge Site.
Example 2 - Tallahalla Creek, Mississippi.

Site Data

Topographic data in the vicinity of the bridge site including the flood plains were obtained from U.S. Geological Survey quadrangle maps and cross sections of the stream valley taken in the U.S. Geological Survey field surveys. Information on bridge and roadway profiles was also furnished by the staff of the USGS. The spatial distribution of hydraulic roughness in the flow region was estimated based upon aerial photographs and the roughness data were estimated by the USGS field party for each cross section.

Finite Element Network

The finite element network which represents the flow region of the Tallahalla Creek site is shown in Figure 16. The network consists of 199 nodal points and 86 elements. Note that the actual flow boundaries along both sides of the river (dotted lines) have been approximated in the network by straight lines. The dotted lines are the surveyed highwater marks observed by the USGS from the flood of April 14, 1969, which had a peak discharge of 12,500 cfs. These same boundaries have been used for all floods simulated in this study, since field data gathered by the USGS for the larger and smaller floods show approximately the same boundaries as the April 1969 flood.

The main channel is ignored in the network set up except for that portion near the bridge opening. The general rule for laying out the network is to place smaller size triangles in the area where the most significant variation in flow parameters is expected, and larger size triangles where less variation is expected. This approach is clearly illustrated in Figure 16. The spatial distribution of the flood plain vegetative cover is also shown in Figure 16.

Figure 16. Finite Element Representation
Example 2 - Tallahalla Creek, Mississippi

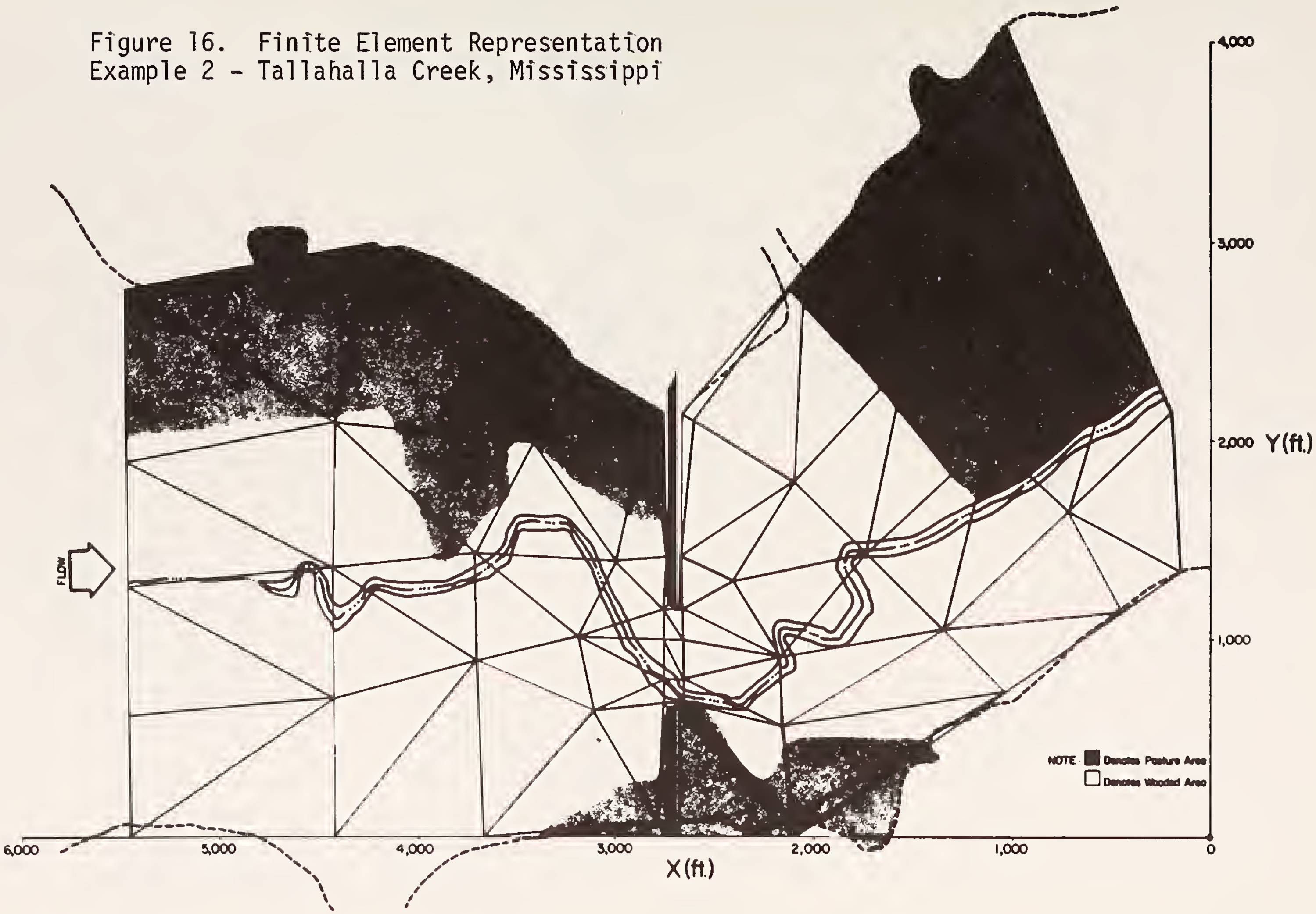
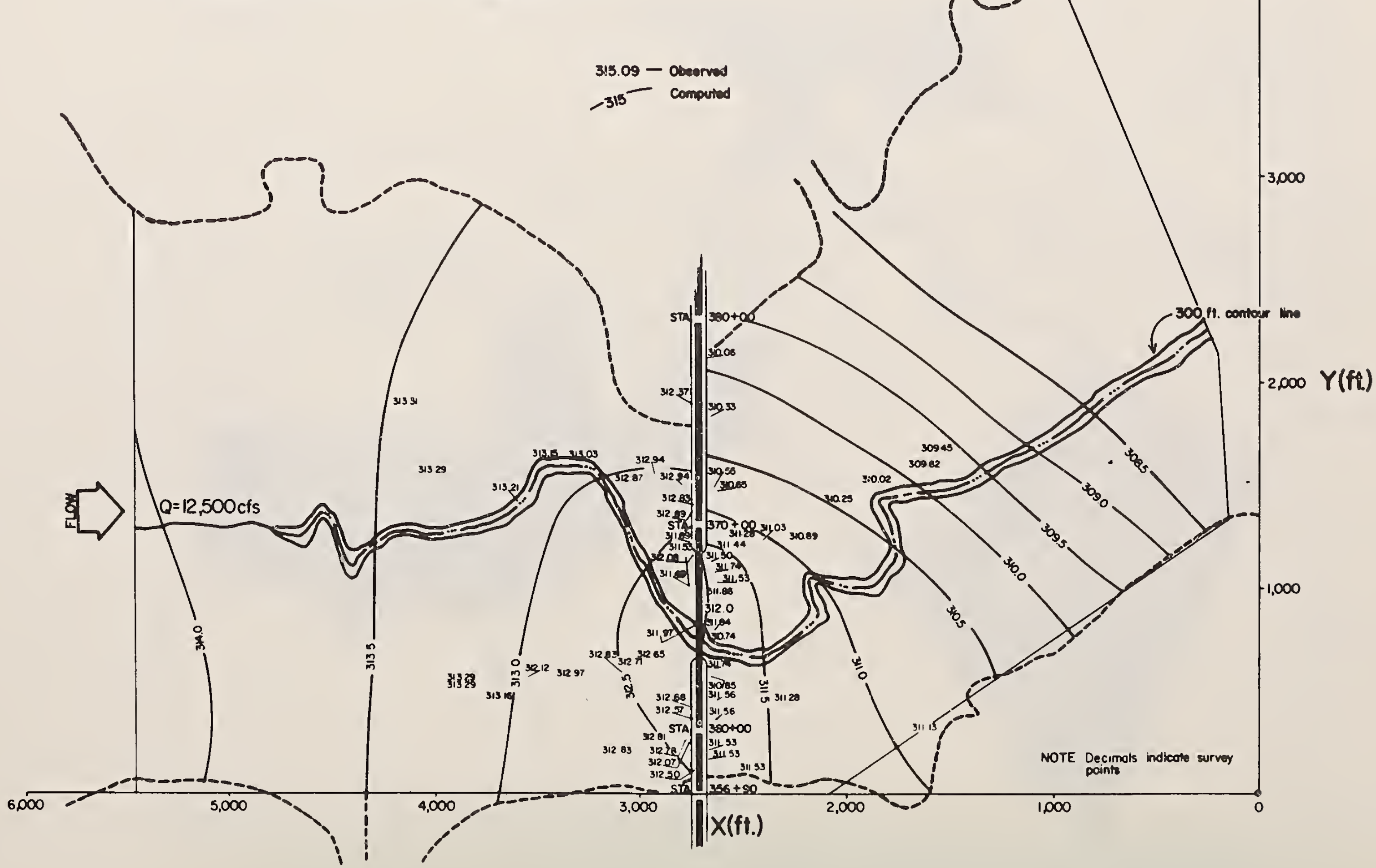


Figure 17. Water Surface Elevations
for 1969 Flood - Example 2



Three separate flood events were computed. Table 3 gives the magnitude of flood peaks and dates of occurrence of these floods.

Table 3. Flood Peaks and Dates of Occurrence

Date	Peak (cfs)	Remarks
April 6, 1964	22,000	Flow overtopping along left bank (Ref. 26)
April 14, 1969	12,500	Data from USGS
February 21, 1971	7,140	Data from USGS

The data from the April 1969 flood were used to calibrate the model. The input data for this flood are given in Table 4, and the outputs in Table 5. Figure 17 shows the results of the computation.

The observed and computed values of velocity under the bridge are compared in Figure 18. Note that the field velocity data were collected at twenty and eighty percent of the depth, respectively. The computed velocities represent the average value at each node. Also note that the computed velocity data are for a flow rate of 9,780 cfs rather than the 12,500 cfs.

After the model had been calibrated for the 1969 flood, it was used to simulate the flow conditions resulting from the two other flood events, in 1964 and 1971. The peak values of these floods are shown in Table 3. Notice that the peak of the 1964 flood is higher than the 1969 flood, and the 1971 peak is lower. Such a selection of flood peaks allows the model to be tested for its predictive capability.

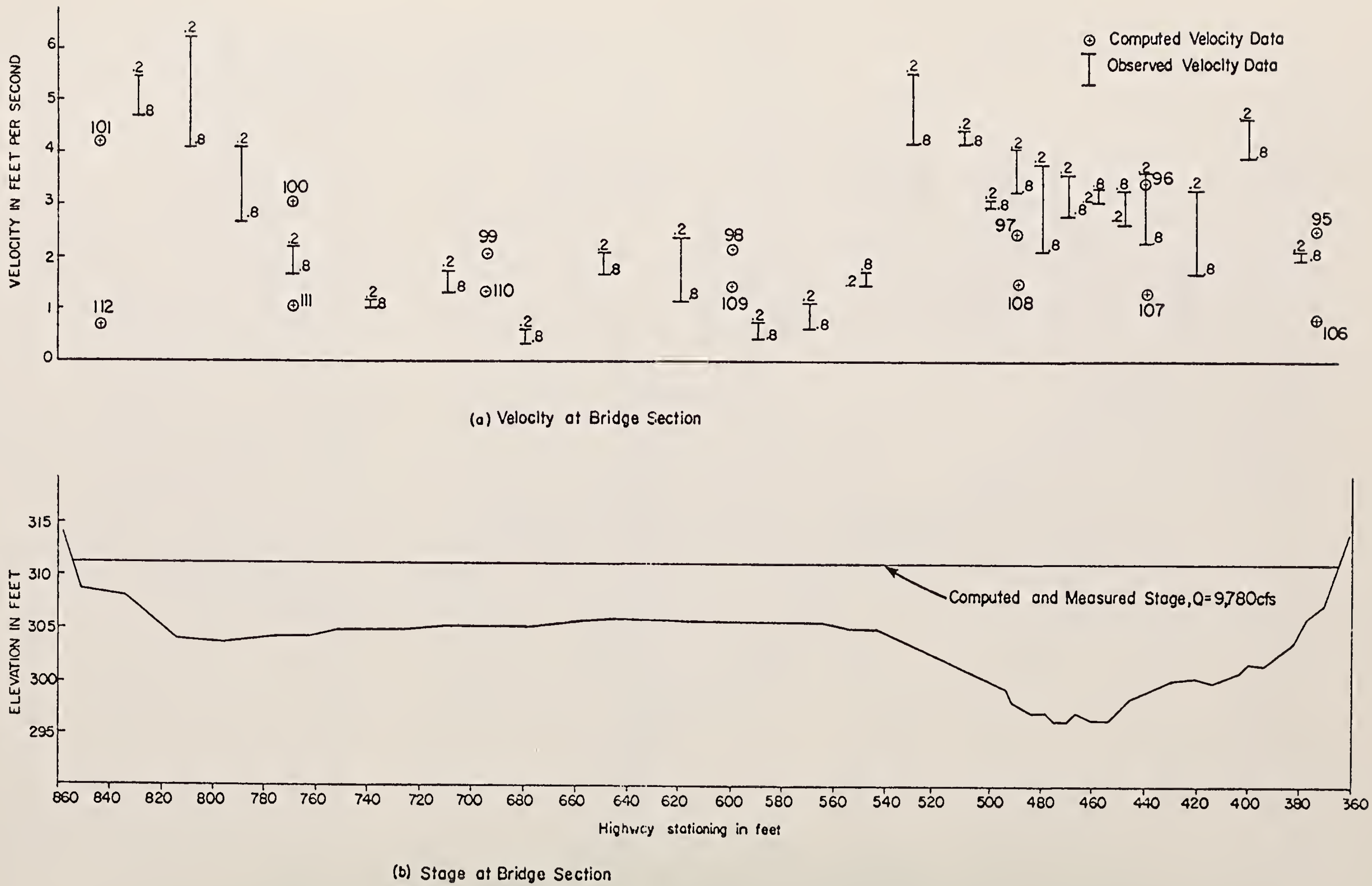


Figure 18. Actual Versus Computed Velocity Profile Across Bridge Opening. Example 2 - Tallahalla Creek, Mississippi

The input data for the flood of 1964 are listed in Table 6, and the output is shown in Table 7. Input and output data for the 1971 flood are shown in Tables 8 and 9, respectively. System parameters for these two flood simulations are the same as those given in Tables 4b and 4c for the 1969 flood simulation. The results of these simulation runs are shown in Figures 19 and 20. As can be seen, field data for the 1971 flood were not as complete as for the 1964 flood. Nevertheless, agreement between the model results and the field data has been satisfactory for both events.

Figure 19. Water Surface Elevations for 1964 Flood - Example 2

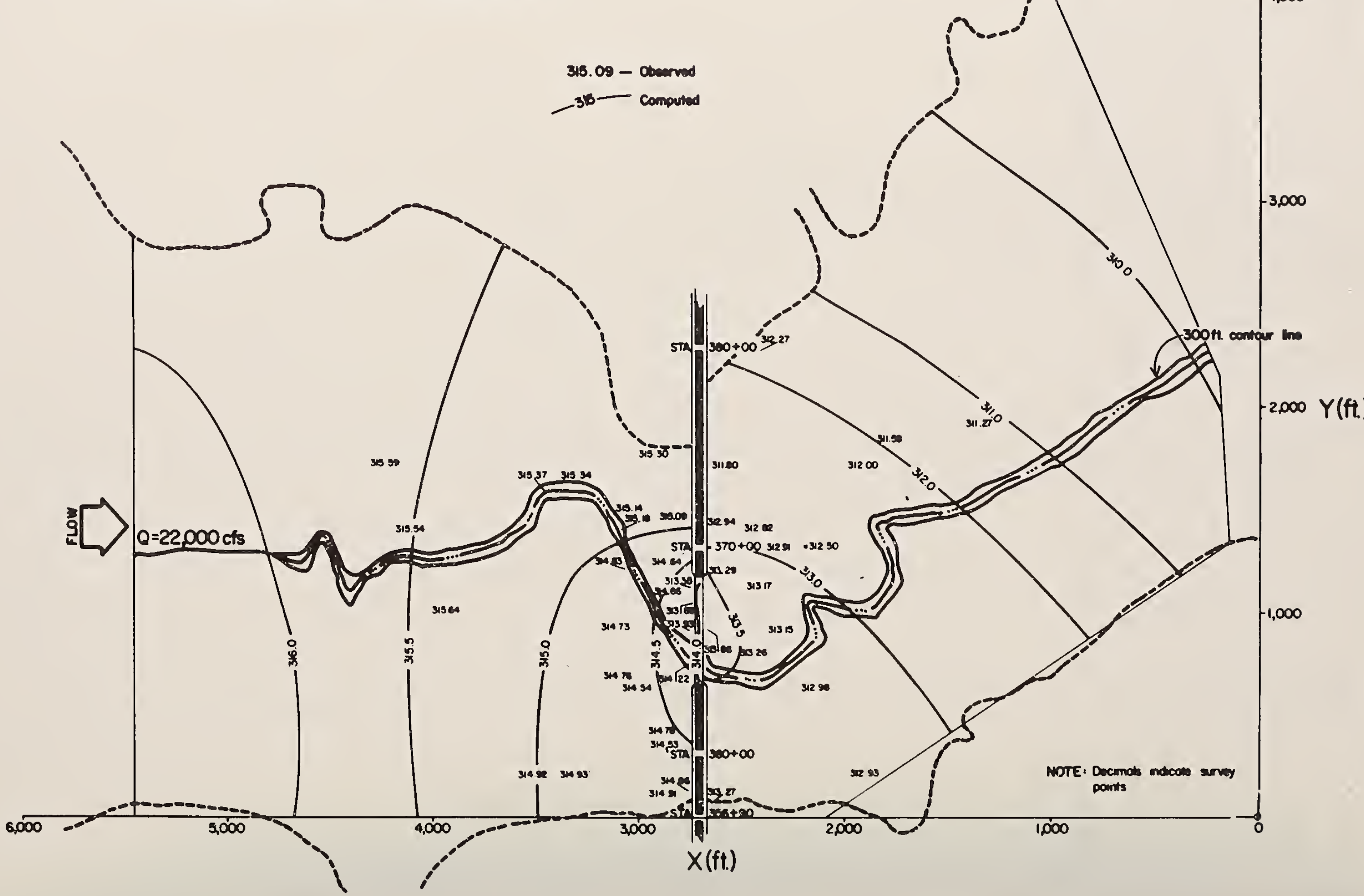


Figure 20. Water Surface Elevations
for 1971 Flood - Example 2

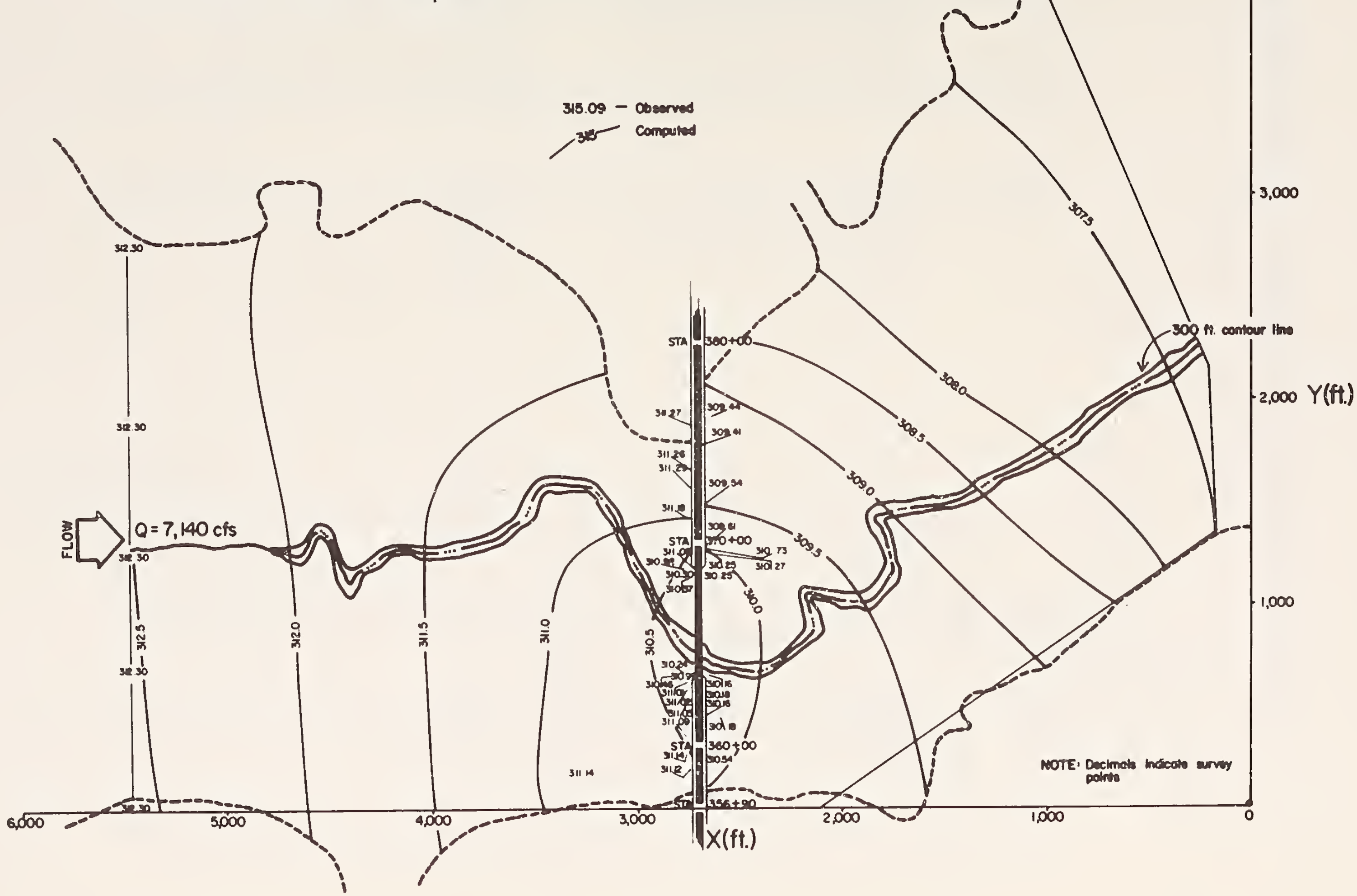


Table 1a. Input Data for Example 1 - Test Flume

1. Run Parameters

Total Number of Nodes	177
Total Number of Elements	72
Element Types	5

2. Element Characteristics

Element Type	ε_x (lb-sec/ft^2)	ε_y (lb-sec/ft^2)	Chezy C
1	1.0	1.0	5.7
2	0.5	0.5	5.7
3	0.5	0.5	150
4	0.5	0.5	150
5	0.5	0.5	150

3. Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
1			11.27
2			11.26
3			11.25
4			11.25
5			11.25
173	0.25		
174	0.25		
175	0.25		
176	0.25		
177	0.25		

Table 1b. Nodal Coordinates and Bottom Elevations

Example 1 - Test Flume

NODE	X-LOC (FEET)	Y-LOC (FEET)	ELEV (FEET)
1	215.00	0.00	10.4
2	215.00	5.15	10.3
3	215.00	10.30	10.3
4	215.00	15.50	10.3
5	215.00	20.70	10.3
6	202.50	0.00	10.4
7	202.50	5.15	10.4
8	202.50	10.30	10.4
9	202.50	15.50	10.4
10	202.50	20.70	10.4
11	190.00	0.00	10.4
12	190.00	5.15	10.4
13	190.00	10.30	10.4
14	190.00	15.50	10.4
15	190.00	20.70	10.4
16	180.00	0.00	10.4
17	180.00	5.15	10.4
18	180.00	10.30	10.4
19	180.00	15.50	10.4
20	180.00	20.70	10.4
21	170.00	0.00	10.4
22	170.00	5.15	10.4
23	170.00	10.30	10.4
24	170.00	15.50	10.4
25	170.00	20.70	10.4
26	165.00	0.00	10.5
27	165.00	5.15	10.5
28	165.00	10.30	10.5
29	165.00	15.50	10.4
30	165.00	20.70	10.6
31	160.00	0.00	10.5
32	160.00	5.15	10.5
33	160.00	10.30	10.5
34	160.00	15.50	10.6
35	160.00	20.70	10.7
36	157.50	0.00	10.5
37	157.50	5.15	10.5
38	157.50	10.30	10.5
39	157.50	15.50	10.6
40	157.50	20.70	10.6
41	155.00	0.00	10.5
42	155.00	5.15	10.5
43	155.00	10.30	10.5
44	155.00	15.50	10.5
45	155.00	20.70	10.5
46	153.00	0.00	10.5
47	153.25	.25	10.5
48	153.75	5.15	10.5
49	153.75	10.30	10.5
50	153.75	15.50	10.5
51	153.25	20.45	10.5
52	153.00	20.70	10.5
53	151.00	0.00	10.5
54	151.25	.25	10.5
55	151.50	.50	10.5
56	152.00	5.40	10.5
57	152.50	10.30	10.5
58	152.00	15.25	10.5
59	151.50	20.20	10.5
60	151.25	20.45	10.5
61	151.00	20.70	10.5
62	151.00	4.30	10.5
63	151.25	4.55	10.5
64	151.50	4.65	10.5
65	152.00	4.55	10.5
66	152.00	10.30	10.5
67	152.00	11.05	10.5
68	151.50	16.00	10.5
69	151.25	16.15	10.5
70	151.00	16.40	10.5
71	151.00	8.60	10.5
72	151.25	8.70	10.5
73	151.50	8.80	10.5
74	151.50	9.55	10.5
75	151.50	10.30	10.5
76	151.50	11.05	10.5
77	151.50	11.80	10.5
78	151.25	11.95	10.5
79	151.00	12.10	10.5
80	150.00	8.60	10.5
81	149.75	8.70	10.5
82	150.00	8.80	10.5
83	150.00	9.55	10.5
84	150.00	10.30	10.5
85	150.00	11.05	10.5
86	150.00	11.80	10.5
87	149.75	11.95	10.5
88	150.00	12.10	10.5
89	149.00	8.60	10.5
90	148.75	8.70	10.5
91	148.50	8.80	10.5
92	148.50	9.55	10.5
93	148.50	10.30	10.5
94	148.50	11.05	10.5
95	148.50	11.80	10.5
96	148.75	11.95	10.5
97	149.00	12.10	10.5
98	149.00	4.30	10.5
99	148.75	4.55	10.5
100	148.50	4.65	10.5

Table 1b. (Continued)

NODE	X-LOC (FEET)	Y-LOC (FEET)	ELEV (FEET)
101	148.00	9.55	10.5
102	148.00	10.30	10.5
103	148.00	11.05	10.5
104	148.50	16.00	10.5
105	148.75	16.15	10.5
106	149.00	16.40	10.5
107	149.00	0.00	10.5
108	148.75	.25	10.5
109	148.50	.50	10.5
110	148.00	5.40	10.5
111	147.50	10.30	10.5
112	148.00	15.25	10.5
113	148.50	20.20	10.5
114	148.75	20.45	10.5
115	149.00	20.70	10.5
116	147.50	0.00	10.5
117	147.25	.25	10.5
118	146.75	5.15	10.5
119	146.75	10.30	10.5
120	146.75	15.50	10.5
121	147.25	20.45	10.5
122	147.50	20.70	10.5
123	146.00	0.00	10.5
124	146.00	5.15	10.5
125	146.00	10.30	10.5
126	146.00	15.50	10.5
127	146.00	20.70	10.5
128	144.50	0.00	10.5
129	144.50	5.15	10.5
130	144.50	10.30	10.5
131	144.50	15.50	10.5
132	144.50	20.70	10.5
133	143.00	0.00	10.5
134	143.00	5.15	10.5
135	143.00	10.30	10.5
136	143.00	15.50	10.5
137	143.00	20.70	10.5
138	141.50	0.00	10.5
139	141.50	5.15	10.5
140	141.50	10.30	10.5
141	141.50	15.50	10.5
142	141.50	20.70	10.5
143	140.00	0.00	10.5
144	140.00	5.15	10.5
145	140.00	10.30	10.5
146	140.00	15.50	10.5
147	140.00	20.70	10.5
148	135.00	0.00	10.6
149	135.00	5.15	10.5
150	135.00	10.30	10.5
151	135.00	15.50	10.5
152	135.00	20.70	10.5
153	130.00	0.00	10.6
154	130.00	5.15	10.6
155	130.00	10.30	10.5
156	130.00	15.50	10.5
157	130.00	20.70	10.5
158	125.00	0.00	10.6
159	125.00	5.15	10.6
160	125.00	10.30	10.6
161	125.00	15.50	10.6
162	125.00	20.70	10.6
163	120.00	0.00	10.6
164	120.00	5.15	10.6
165	120.00	10.30	10.6
166	120.00	15.50	10.6
167	120.00	20.70	10.6
168	110.00	0.00	10.6
169	110.00	5.15	10.6
170	110.00	10.30	10.6
171	110.00	15.50	10.6
172	110.00	20.70	10.6
173	100.00	0.00	10.6
174	100.00	5.15	10.6
175	100.00	10.30	10.6
176	100.00	15.50	10.6
177	100.00	20.70	10.6

Table 1c. Nodal Connections and Material Numbers

Example 1 - Test Flume

ELEMENT	NODES(COUNTERCLOCKWISE)						TYPE	AREA(FT2)
1	163	164	165	169	173	168	1	103,00
2	165	170	175	174	173	169	1	103,00
3	165	171	177	176	175	170	1	104.00
4	165	166	167	172	177	171	1	104.00
5	153	154	155	159	163	158	1	51,50
6	155	160	165	164	163	159	1	51,50
7	155	161	167	166	165	160	1	52,00
8	155	156	157	162	167	161	1	52,00
9	143	144	145	149	153	148	1	51.50
10	145	150	155	154	153	149	1	51,50
11	145	151	157	156	155	150	1	52,00
12	145	146	147	152	157	151	1	52,00
13	133	134	135	139	143	138	1	15,45
14	135	140	145	144	143	139	1	15,45
15	135	141	147	146	145	140	1	15,60
16	135	136	137	142	147	141	1	15,60
17	123	124	125	129	133	128	1	15,45
18	125	130	135	134	133	129	1	15,45
19	125	131	137	136	135	130	1	15,60
20	125	126	127	132	137	131	1	15,60
21	107	108	109	117	123	116	2	,75
22	109	110	111	118	123	117	1	12,50
23	111	119	125	124	123	118	1	7,73
24	111	120	127	126	125	119	1	7,80
25	111	112	113	121	127	120	1	12,63
26	113	114	115	122	127	121	2	,75
27	107	98	89	99	109	108	2	2,15
28	109	99	89	90	91	100	2	2,08
29	109	100	91	101	111	110	2	4,15
30	91	92	93	102	111	101	2	,75
31	93	94	95	103	111	102	2	,75
32	95	104	113	112	111	103	2	4,20
33	95	96	97	105	113	104	2	2,10
34	97	106	115	114	113	105	2	2,15
35	71	81	91	90	89	80	3	,20
36	71	72	73	82	91	81	3	,30

ELEMENT	NODES(COUNTERCLOCKWISE)						TYPE	AREA(FT2)
37	73	74	75	83	91	82	4	2,25
38	75	84	93	92	91	83	4	2,25
39	75	85	95	94	93	84	4	2,25
40	75	76	77	86	95	85	4	2,25
41	77	78	79	87	95	86	3	,45
42	79	88	97	96	95	87	3	,30
43	53	54	55	63	71	62	2	2,15
44	55	64	73	72	71	63	2	2,08
45	55	56	57	65	73	64	2	4,15
46	57	66	75	74	73	65	5	,75
47	57	67	77	76	75	66	5	,75
48	57	58	59	68	77	67	2	4,20
49	77	68	59	69	79	78	2	2,10
50	59	60	61	70	79	69	2	2,15
51	41	47	55	54	53	46	2	1,00
52	41	48	57	56	55	47	2	17,40
53	41	42	43	49	57	48	2	12,87
54	43	44	45	50	57	49	2	13,00
55	45	51	59	58	57	50	2	17,58
56	45	52	61	60	59	51	2	1,00
57	31	37	43	42	41	36	1	25,75
58	31	32	33	38	43	37	1	25,75
59	33	34	35	39	43	38	1	26,00
60	35	40	45	44	43	39	1	26,00
61	21	27	33	32	31	26	1	51,50
62	21	22	23	28	33	27	1	51,50
63	23	24	25	29	33	28	1	52,00
64	25	30	35	34	33	29	1	52,00
65	11	17	23	22	21	16	1	103,00
66	11	12	13	18	23	17	1	103,00
67	13	14	15	19	23	18	1	104,00
68	15	20	25	24	23	19	1	104,00
69	1	7	13	12	11	6	1	128,75
70	1	2	3	8	13	7	1	128,75
71	3	4	5	9	13	8	1	130,00
72	5	10	15	14	13	9	1	130,00

Table 2. Nodal Velocity and W.S. Elevations

Example 1 - Test Flume

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
1	.204	0.000	.91	11.26
2	.240	.027	.91	11.26
3	.309	.012	.91	11.26
4	.305	.010	.92	11.26
5	.296	0.000	.92	11.26
6	.267	0.000	.91	11.29
7	.284	.011	.92	11.29
8	.272	.016	.92	11.29
9	.273	.004	.92	11.29
10	.280	0.000	.93	11.29
11	.270	0.000	.91	11.32
12	.262	-.004	.92	11.32
13	.272	.003	.92	11.32
14	.278	-.003	.92	11.32
15	.276	0.000	.93	11.32
16	.272	0.000	.92	11.35
17	.275	.004	.92	11.35
18	.274	-.000	.92	11.35
19	.270	.007	.93	11.35
20	.269	0.000	.93	11.35
21	.261	0.000	.93	11.37
22	.277	.006	.93	11.37
23	.274	.004	.93	11.37
24	.276	.014	.93	11.37
25	.282	0.000	.94	11.37
26	.255	0.000	.93	11.38
27	.269	-.008	.93	11.38
28	.265	.006	.93	11.38
29	.275	.010	.93	11.38
30	.252	0.000	.80	11.38
31	.269	0.000	.93	11.39
32	.317	-.040	.93	11.40
33	.313	-.000	.93	11.40
34	.302	.024	.79	11.40
35	.255	0.000	.66	11.40
36	.166	0.000	.92	11.40
37	.267	-.142	.94	11.41
38	.408	.003	.94	11.41
39	.259	.118	.80	11.41
40	.180	0.000	.78	11.40
41	.303	0.000	.51	11.40
42	.269	-.225	.93	11.41
43	.417	.001	.95	11.42
44	.284	.197	.93	11.41
45	.303	0.000	.91	11.40
46	.285	0.000	.91	11.41
47	.290	-.096	.91	11.40
48	.192	-.274	.93	11.42
49	.738	-.002	.95	11.43
50	.188	.259	.93	11.42

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
51	.269	.086	.91	11.40
52	.267	0.000	.91	11.41
53	0.000	0.000	.91	11.41
54	.139	-.024	.91	11.41
55	.194	-.080	.90	11.40
56	-.161	-.256	.93	11.42
57	1.075	-.005	.95	11.44
58	-.125	.260	.93	11.42
59	.200	.068	.91	11.41
60	.147	.016	.91	11.41
61	0.000	0.000	.91	11.41
62	0.000	-.236	.93	11.41
63	-.098	-.238	.92	11.41
64	-.240	-.245	.92	11.41
65	1.047	-.014	.94	11.43
66	2.302	.007	.95	11.43
67	1.159	.003	.95	11.43
68	-.240	.242	.92	11.41
69	-.115	.233	.92	11.41
70	0.000	.229	.92	11.42
71	0.000	0.000	.94	11.42
72	.350	-.074	.94	11.42
73	.603	-.079	.94	11.42
74	2.126	.062	.94	11.42
75	2.199	.030	.94	11.43
76	2.210	-.101	.94	11.42
77	.603	.074	.94	11.42
78	.420	.081	.93	11.42
79	0.000	0.000	.93	11.42
80	1.066	0.000	1.00	11.48
81	1.097	.082	.99	11.47
82	1.263	.034	.99	11.47
83	1.082	.343	.99	11.48
84	1.783	.041	.98	11.47
85	1.732	-.318	.99	11.48
86	1.544	-.047	.99	11.47
87	1.372	-.104	.98	11.47
88	1.342	0.000	.99	11.48
90	.066	.210	1.06	11.54
91	.238	.285	1.05	11.53
92	.814	.403	1.03	11.52
93	1.031	.017	1.02	11.51
94	.844	-.374	1.03	11.52
95	.297	-.288	1.04	11.53
96	.064	-.222	1.05	11.54
97	0.000	0.000	1.06	11.55
98	0.000	.341	1.06	11.56
99	-.021	.345	1.07	11.56
100	.045	.342	1.06	11.55

Table 2. (Continued)

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
101	.696	.172	1.05	11.54
102	.823	.010	1.03	11.53
103	.732	-.165	1.05	11.53
104	.068	-.344	1.05	11.55
105	-.012	-.345	1.07	11.56
106	0.000	-.344	1.07	11.56
107	0.000	0.000	1.06	11.57
108	.089	.029	1.06	11.57
109	.073	.085	1.06	11.58
110	.202	.311	1.06	11.56
111	.614	.004	1.05	11.54
112	.221	-.312	1.06	11.56
113	.065	-.089	1.08	11.58
114	.083	-.032	1.08	11.57
115	0.000	0.000	1.07	11.57
116	-.010	0.000	1.05	11.57
117	-.028	.057	1.05	11.57
118	.210	.234	1.04	11.56
119	.543	.003	1.05	11.55
120	.215	-.235	1.06	11.56
121	-.043	-.058	1.08	11.57
122	-.026	0.000	1.07	11.57
123	.085	0.000	1.04	11.57
124	.243	.187	1.05	11.56
125	.407	.002	1.06	11.56
126	.246	-.188	1.07	11.56
127	.075	0.000	1.07	11.57
128	-.104	0.000	1.04	11.57
129	.235	.119	1.05	11.56
130	.362	.001	1.06	11.56
131	.257	-.120	1.06	11.56
132	.102	0.000	1.07	11.57
133	.195	0.000	1.04	11.57
134	.249	.009	1.05	11.57
135	.258	.001	1.06	11.57
136	.250	-.069	1.06	11.57
137	.196	0.000	1.07	11.57
138	.183	0.000	1.04	11.57
139	.257	.040	1.05	11.57

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
140	.280	.000	1.06	11.57
141	.238	-.040	1.06	11.57
142	.185	0.000	1.06	11.57
143	.229	0.000	1.04	11.57
144	.246	.019	1.04	11.57
145	.250	.000	1.05	11.57
146	.247	-.019	1.06	11.57
147	.231	0.000	1.06	11.57
148	.232	0.000	1.03	11.56
149	.238	.006	1.04	11.58
150	.246	.000	1.05	11.58
151	.239	-.005	1.05	11.58
152	.234	0.000	1.05	11.58
153	.243	0.000	1.03	11.59
154	.244	.000	1.04	11.59
155	.249	.000	1.05	11.59
156	.245	.001	1.05	11.59
157	.245	0.000	1.04	11.59
158	.241	0.000	1.02	11.60
159	.243	.000	1.03	11.60
160	.244	.001	1.04	11.60
161	.243	.001	1.04	11.60
162	.243	0.000	1.03	11.60
163	.244	0.000	1.01	11.61
164	.245	-.000	1.02	11.61
165	.246	.000	1.03	11.61
166	.246	.002	1.03	11.61
167	.245	0.000	1.03	11.61
168	.246	0.000	1.02	11.63
169	.246	.002	1.02	11.63
170	.247	.000	1.03	11.63
171	.246	-.001	1.02	11.63
172	.247	0.000	1.02	11.63
173	.248	0.000	1.02	11.65
174	.245	0.000	1.03	11.65
175	.243	0.000	1.04	11.65
176	.247	0.000	1.02	11.65
177	.250	0.000	1.01	11.65

Table 4a. Input Data for Example 2 -
Flood of 1969, Q = 12,500 cfs

1. Run Parameters

Total Number of Nodes	199
Total Number of Elements	86
Element Types	7

2. Element Characteristics

Element Type	ε_x (lb-sec/ft^2)	ε_y (lb-sec/ft^2)	Chezy C
1	75	50	42
2	50	50	42
3	300	200	12
4	500	250	15
5	500	250	15
6	300	250	42
7	750	750	42

3. Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
1			308
2			308
3			308
4			308
5			308
6			308
7			308
8			308
9			308
191	4.51		
192	4.51		

Table 4a. (Continued)

Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
193	4.51		
194	4.51		
195	4.51		
196	4.51		
197	4.51		
198	4.51		
199	4.44	0.82	

Table 4b. Nodal Coordinates and Bottom Elevations
Example 2 - Tallahalla Creek, Mississippi

NODE	X-LOC (FEET)	Y-LOC (FEET)	ELEV (FEET)
1	-160.00	1350.00	305.0
2	-286.47	1750.00	305.0
3	-412.94	2150.00	305.0
4	-491.99	2400.00	304.7
5	-571.03	2650.00	304.4
6	-673.79	2975.00	304.2
7	-776.54	3300.00	304.0
8	-898.27	3685.00	304.5
9	-1020.00	4070.00	305.0
10	-310.00	1245.62	305.2
11	-440.00	1500.00	305.1
12	-566.47	1900.00	305.1
13	-645.51	2150.00	304.8
14	-840.51	2330.00	304.7
15	-943.27	2655.00	304.5
16	-1153.27	2850.00	304.5
17	-1275.00	3235.00	305.0
18	-1530.00	3466.13	305.5
19	-460.00	1141.24	305.5
20	-590.00	1395.62	305.4
21	-720.00	1650.00	305.3
22	-915.00	1830.00	305.1
23	-1110.00	2010.00	305.0
24	-1320.00	2205.00	305.0
25	-1530.00	2400.00	305.0
26	-1785.00	2631.13	305.5
27	-2040.00	2862.27	306.0
28	-750.00	939.43	304.7
29	-905.00	1095.62	305.2
30	-1035.00	1350.00	305.1
31	-1230.00	1530.00	305.0
32	-1420.00	1725.00	304.2
33	-1630.00	1920.00	304.2
34	-1810.00	2100.00	305.5
35	-2065.00	2331.13	306.0
36	-2315.00	2481.13	306.0
37	-2345.00	2501.13	306.0
38	-1040.00	737.63	304.0
39	-1195.00	893.81	304.5
40	-1350.00	1050.00	305.0
41	-1540.00	1245.00	304.2
42	-1730.00	1440.00	303.5
43	-1910.00	1620.00	304.7
44	-2090.00	1800.00	306.0
45	-2340.00	1950.00	306.0
46	-2590.00	2100.00	306.0
47	-2620.00	2120.00	306.0
48	-2650.00	2140.00	306.0
49	-1570.00	368.81	304.7
50	-1575.00	413.81	304.7
51	-1600.00	648.81	304.7
52	-1755.00	805.00	305.2
53	-1755.00	985.00	304.0
54	-1945.00	1180.00	303.2
55	-2065.00	1370.00	304.2
56	-2245.00	1550.00	305.5
57	-2377.50	1610.00	305.7
58	-2627.50	1760.00	305.7
59	-2657.50	1780.00	305.7
60	-2100.00	0.00	305.5
61	-2105.00	45.00	305.5
62	-2110.00	90.00	305.5
63	-2135.00	325.00	305.5
64	-2160.00	560.00	305.5
65	-2160.00	740.00	304.2
66	-2160.00	920.00	303.0
67	-2280.00	1110.00	304.0
68	-2400.00	1300.00	305.0
69	-2532.50	1360.00	305.2
70	-2665.00	1420.00	305.5
71	-2385.00	330.00	302.7
72	-2375.00	340.00	302.7
73	-2380.00	385.00	302.7
74	-2405.00	620.00	302.7
75	-2405.00	680.00	301.7
76	-2405.00	860.00	300.5
77	-2405.00	960.00	303.5
78	-2405.00	1040.00	303.7
79	-2525.00	1230.00	304.7
80	-2657.50	1290.00	304.9
81	-2667.50	1300.00	304.9
82	-2670.00	660.00	300.0
83	-2660.00	670.00	300.0
84	-2650.00	680.00	300.0
85	-2650.00	740.00	299.0
86	-2650.00	800.00	298.0
87	-2650.00	900.00	301.0
88	-2650.00	1000.00	304.0
89	-2650.00	1080.00	304.2
90	-2650.00	1160.00	304.4
91	-2660.00	1170.00	304.4
92	-2670.00	1180.00	304.4
93	-2700.00	660.00	300.0
94	-2690.00	670.00	300.0
95	-2700.00	680.00	299.0
96	-2700.00	740.00	299.0
97	-2700.00	800.00	298.0
98	-2700.00	900.00	301.0
99	-2700.00	1000.00	304.0
100	-2700.00	1080.00	304.3

Table 4b. (Continued)

NODE	X-LOC (FEET)	Y-LOC (FEET)	ELEV (FEET)
101	-2700.00	1160.00	304.5
102	-2690.00	1170.00	304.5
103	-2700.00	1180.00	304.5
104	-2730.00	660.00	300.0
105	-2740.00	670.00	299.0
106	-2750.00	680.00	298.0
107	-2750.00	740.00	298.0
108	-2750.00	800.00	298.0
109	-2750.00	900.00	301.0
110	-2750.00	1000.00	304.0
111	-2750.00	1080.00	304.3
112	-2750.00	1160.00	304.6
113	-2740.00	1170.00	304.6
114	-2730.00	1180.00	304.6
115	-2730.00	330.00	304.0
116	-2755.00	370.00	304.0
117	-2765.00	380.00	303.0
118	-2930.00	660.00	302.0
119	-2930.00	720.00	302.0
120	-2965.00	910.00	301.0
121	-2965.00	1010.00	304.0
122	-2965.00	1090.00	304.3
123	-2870.00	1280.00	305.3
124	-2740.00	1290.00	305.5
125	-2730.00	1300.00	305.5
126	-2730.00	0.00	308.0
127	-2755.00	40.00	308.0
128	-2780.00	80.00	308.0
129	-2945.00	360.00	307.0
130	-3110.00	640.00	306.0
131	-3145.00	830.00	305.0
132	-3180.00	1020.00	304.0
133	-3085.00	1210.00	305.0
134	-2990.00	1400.00	306.0
135	-2860.00	1410.00	306.3
136	-2730.00	1420.00	306.5
137	-3200.00	0.00	308.1
138	-3225.00	40.00	308.1
139	-3390.00	320.00	307.1
140	-3405.00	770.00	307.1
141	-3440.00	960.00	306.1
142	-3440.00	1235.00	306.1
143	-3345.00	1425.00	307.1
144	-3195.00	1700.00	307.5
145	-2900.00	1770.00	307.5
146	-2770.00	1780.00	307.7
147	-2730.00	1785.00	307.7
148	-3670.00	0.00	308.2
149	-3685.00	450.00	308.2
150	-3700.00	900.00	308.2
151	-3700.00	1175.00	308.2
152	-3700.00	1450.00	308.2
153	-3550.00	1725.00	308.6
154	-3400.00	2000.00	309.0
155	-3105.00	2070.00	309.0
156	-2910.00	2140.00	309.0
157	-2770.00	2145.00	309.0
158	-2730.00	2150.00	309.0
159	-4045.00	0.00	308.6
160	-4060.00	450.00	308.6
161	-4060.00	800.00	308.3
162	-4060.00	1135.00	308.4
163	-4060.00	1410.00	308.4
164	-4050.00	1775.00	308.6
165	-3900.00	2050.00	309.0
166	-3805.00	2475.00	309.1
167	-3510.00	2545.00	309.1
168	-3470.00	2550.00	309.1
169	-3465.00	2575.00	309.1
170	-4420.00	0.00	309.0
171	-4420.00	350.00	308.7
172	-4420.00	700.00	308.4
173	-4420.00	1035.00	308.5
174	-4420.00	1370.00	308.6
175	-4410.00	1735.00	308.8
176	-4400.00	2100.00	309.0
177	-4305.00	2525.00	309.1
178	-4210.00	2950.00	309.2
179	-4205.00	2975.00	309.2
180	-4200.00	3000.00	309.2
181	-4935.00	0.00	309.0
182	-4935.00	350.00	308.7
183	-4935.00	660.00	308.6
184	-4935.00	985.00	308.4
185	-4935.00	1320.00	308.5
186	-4935.00	1635.00	308.9
187	-4925.00	2000.00	309.1
188	-4925.00	2435.00	309.2
189	-4830.00	2860.00	309.4
190	-4825.00	2885.00	309.4
191	-5450.00	0.00	309.1
192	-5450.00	310.00	309.0
193	-5450.00	620.00	308.9
194	-5450.00	945.00	308.6
195	-5450.00	1270.00	308.4
196	-5450.00	1585.00	308.8
197	-5450.00	1930.00	309.2
198	-5450.00	2335.00	309.4
199	-5450.00	2770.00	309.5

Table 4c. Nodal Connections and Material Numbers

Example 2 - Tallahalla Creek, Mississippi

ELEMENT	NODES(COUNTERCLOCKWISE)						TYPE	AREA(FT2)
1	172	182	191	181	170	171	5	360500.00
2	172	183	193	192	191	182	5	319300.00
3	172	184	195	194	193	183	5	334700.00
4	174	185	195	184	172	173	5	345050.00
5	174	186	197	196	195	185	5	324450.00
6	176	187	197	186	174	175	5	381250.00
7	176	188	199	198	197	187	6	456750.00
8	176	177	178	189	199	188	6	509900.00
9	180	190	199	189	178	179	1	30100.00
10	150	160	170	159	148	149	5	337500.00
11	150	161	172	171	170	160	5	252000.00
12	150	162	174	173	172	161	5	241200.00
13	150	151	152	163	174	162	5	198000.00
14	152	164	176	175	174	163	5	262000.00
15	152	153	154	165	176	164	6	290000.00
16	154	166	178	177	176	165	6	474500.00
17	154	155	156	167	179	166	6	376950.00
18	156	157	158	168	178	157	1	79400.00
19	158	169	180	179	178	168	1	41000.00
20	128	138	148	137	126	127	1	77600.00
21	128	129	130	139	148	138	5	282400.00
22	130	140	150	149	148	139	5	281600.00
23	130	131	132	141	150	140	3	193000.00
24	132	142	152	151	150	141	3	143900.00
25	132	133	134	143	152	142	4	139650.00
26	134	144	154	153	152	143	4	202750.00
27	134	145	156	155	154	144	6	205700.00
28	126	115	104	116	128	127	1	16500.00
29	128	116	104	105	106	117	1	6300.00
30	106	118	130	129	128	117	3	107400.00
31	108	119	130	118	106	107	3	21600.00
32	108	120	132	131	130	119	3	74000.00
33	108	109	110	121	132	120	3	43100.00
34	112	122	132	121	110	111	3	74400.00
35	112	123	134	133	132	122	3	68400.00
36	112	124	136	135	134	123	4	73600.00
37	136	146	156	145	134	135	4	94400.00
38	136	147	158	157	156	146	1	29200.00
39	84	94	104	93	82	83	1	600.00
40	84	95	106	105	104	94	1	1030.00
41	84	96	108	107	106	95	2	5000.00
42	84	85	86	97	108	96	2	6000.00
43	86	98	110	109	108	97	2	10000.00
44	86	87	88	99	110	98	2	10000.00
45	88	100	112	111	110	99	2	8000.00
45	88	89	90	101	112	100	2	8000.00
47	90	102	114	113	112	101	1	1000.00
48	90	91	92	103	114	102	1	600.00
49	60	72	84	83	82	71	1	12300.00
50	60	61	62	73	84	72	1	21350.00
51	64	74	84	73	62	63	4	112150.00
52	64	75	86	85	84	74	4	29400.00
53	64	65	66	76	86	75	3	88200.00
54	66	77	88	87	66	76	3	49000.00
55	66	78	90	89	88	77	3	39200.00
56	66	67	68	79	90	78	3	64300.00
57	68	69	70	80	90	79	3	33550.00
58	70	81	92	91	90	80	1	2450.00
59	38	50	62	61	60	49	1	51388.14
60	38	51	64	63	62	50	4	267640.72
61	38	39	40	52	64	51	4	222460.31
62	40	53	66	65	64	52	3	145800.00
63	40	41	42	54	66	53	3	182650.00
64	42	55	68	67	66	54	3	144100.00
65	42	43	44	56	68	55	3	145900.00
66	44	57	70	69	68	56	3	34850.00
67	44	45	46	58	70	57	3	161250.00
68	46	47	48	59	70	58	1	21900.00
69	19	29	40	39	38	28	4	153146.91
70	19	20	21	30	40	29	4	238250.31
71	21	22	23	31	40	30	4	230400.00
72	23	32	42	41	40	31	4	229200.00
73	23	24	25	33	42	32	4	240600.00
74	25	34	44	43	42	33	4	205000.00
75	25	26	27	35	44	34	4	282435.58
76	27	36	46	45	44	35	4	273067.48
77	27	37	48	47	46	36	1	33868.10
78	1	11	21	20	19	10	4	103453.61
79	1	2	3	12	21	11	4	186058.82
80	3	4	5	13	21	12	7	116286.76
81	5	14	23	22	21	13	7	221814.71
82	5	6	7	15	23	14	7	240930.15
83	7	16	25	24	23	15	7	335923.30
84	7	8	9	17	25	16	7	393635.66
85	9	18	27	26	25	17	7	543728.83
86	136	124	112	113	114	125	1	2400.00

Table 5. Nodal Velocity and W.S. Elevation
Example 2 - 1969 Flood Event

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)	NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
1	1.413	.984	3.15	308.15	51	1.014	.412	6.00	310.75
2	1.064	.330	3.08	308.08	52	.861	.529	5.51	310.76
3	1.287	.917	3.00	308.00	53	.785	.647	6.68	310.68
4	1.739	1.289	3.27	307.97	54	.799	.985	7.40	310.65
5	1.370	1.091	3.54	307.94	55	.738	.857	6.32	310.57
6	1.348	.861	3.74	307.94	56	.390	1.010	4.99	310.49
7	.888	.845	3.95	307.95	57	.295	.929	4.66	310.41
8	.657	.535	3.43	307.93	58	.363	1.040	4.61	310.36
9	.310	.367	2.91	307.91	59	.034	1.610	4.61	310.36
10	1.224	.852	3.36	308.61	60	0.000	0.000	5.79	311.29
11	.801	.674	3.24	308.39	61	.927	.016	5.79	311.29
12	1.146	.757	3.16	308.31	62	1.115	.065	5.78	311.28
13	1.540	1.362	3.43	308.28	63	1.120	.229	5.78	311.28
14	1.500	1.300	3.53	308.23	64	1.142	.247	5.70	311.28
15	1.405	1.210	3.74	308.24	65	1.257	.420	6.95	311.20
16	1.191	1.149	3.69	308.19	66	1.222	.626	8.12	311.12
17	1.122	.955	3.17	308.17	67	1.100	.872	7.05	311.05
18	.804	.952	2.66	308.16	68	.737	1.100	5.98	310.98
19	.926	.645	3.56	309.06	69	.373	1.408	5.65	310.90
20	.845	.975	3.45	308.95	70	.026	1.248	5.33	310.83
21	.877	.824	3.33	308.63	71	1.062	-1.230	8.71	311.46
22	.697	1.054	3.43	308.59	72	.929	-1.112	8.75	311.50
23	.723	.911	3.52	308.52	73	1.220	-.483	8.75	311.50
24	.618	1.008	3.47	308.47	74	1.218	-.745	8.75	311.50
25	.703	.881	3.42	308.42	75	1.306	-.508	9.88	311.63
26	.694	.818	2.91	308.41	76	1.596	.259	11.35	311.55
27	.569	.673	2.40	308.40	77	1.384	.535	8.04	311.54
28	1.112	.774	4.90	309.65	78	1.343	.927	7.70	311.40
29	.916	.930	4.40	309.65	79	1.059	1.240	6.63	311.33
30	.821	1.027	4.29	309.44	80	.029	1.749	6.30	311.25
31	.919	1.092	4.38	309.38	81	.039	1.876	6.13	311.39
32	1.009	1.180	5.29	309.34	82	0.000	0.000	11.62	311.62
33	.888	1.226	5.34	309.29	83	.622	-.647	11.67	311.67
34	.805	1.011	3.71	309.21	84	1.334	-.743	11.71	311.71
35	.632	.940	3.20	309.20	85	1.638	-.957	12.85	311.85
36	.029	1.153	3.15	309.15	86	1.884	-.460	13.90	311.98
37	1.378	1.632	3.15	309.15	87	1.879	-.154	10.97	311.97
38	1.029	.716	6.23	310.23	88	1.915	.072	7.97	311.97
39	[illegible]	[illegible]	5.74	[illegible]	89	2.119	.537	7.62	311.82
40	.946	.875	5.24	310.24	90	1.513	.884	7.27	311.67
41	.915	1.022	5.95	310.20	91	.700	.873	7.11	311.51
42	.907	1.096	6.67	310.17	92	0.000	0.000	6.94	311.34
43	.755	1.121	5.33	310.08	93	2.259	0.000	11.94	311.94
44	.593	.998	3.99	309.98	94	2.448	.538	11.98	311.98
45	.418	.985	3.95	309.95	95	2.522	1.378	12.93	311.93
46	.489	1.096	3.90	309.90	96	3.452	1.277	12.87	311.87
47	.508	1.304	3.90	309.90	97	2.379	-.014	14.00	312.00
48	0.000	0.000	3.90	309.90	98	2.109	.047	11.01	312.01
49	1.360	.947	6.01	310.76	99	1.930	-.792	6.00	312.00
50	1.224	.254	6.01	310.76	100	3.948	-1.239	7.77	312.07

Table 5. (Continued)

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
101	4.241	-.673	7.42	311.92
102	3.796	.277	7.41	311.91
103	3.510	0.000	7.24	311.74
104	0.000	0.000	12.26	312.26
105	.311	.600	13.20	312.20
106	.811	.804	14.14	312.14
107	1.346	.677	14.08	312.08
108	1.504	.131	14.02	312.02
109	1.442	-.165	11.03	312.03
110	1.237	-.609	8.04	312.04
111	.998	-.925	7.81	312.11
112	.628	-1.223	7.57	312.17
113	.539	-.912	7.56	312.16
114	0.000	0.000	7.54	312.14
115	0.000	1.362	8.38	312.38
116	.034	1.181	8.38	312.38
117	.212	.906	9.32	312.32
118	1.215	.516	10.32	312.32
119	1.281	.253	10.26	312.26
120	1.470	-.338	11.36	312.36
121	1.333	-.473	8.37	312.37
122	1.260	-.846	8.14	312.44
123	.651	-1.542	7.25	312.55
124	.236	-2.352	7.01	312.56
125	0.000	-2.500	7.00	312.55
126	0.000	0.000	4.51	312.51
127	.919	.234	4.51	312.51
128	1.178	.152	4.51	312.51
129	.839	.415	5.50	312.50
130	.902	-.002	6.50	312.50
131	1.381	-.462	7.60	312.60
132	.979	-.629	8.71	312.71
133	.873	-1.071	7.82	312.82
134	.575	-1.025	6.93	312.93
135	.203	-1.217	6.69	312.94
136	0.000	-1.279	6.45	312.95
137	.940	0.000	4.63	312.73
138	.663	.349	4.63	312.73
139	.649	.681	5.6[illegible]	31[illegible]
140	.964	-.295	5.69	312.79
141	.811	-.279	6.80	312.90
142	.666	-.700	6.88	312.93
143	.552	-.697	6.00	313.10
144	.723	-.933	5.63	313.13
145	.405	-1.163	5.60	313.10
146	.183	-.712	5.36	313.11
147	0.000	-.943	5.35	313.10
148	.892	0.000	4.75	312.95
149	1.009	-.179	4.82	313.02
150	.837	-.160	4.89	313.09

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
151	.786	-.328	4.97	313.17
152	.772	-.455	5.06	313.26
153	.748	-.555	4.70	313.30
154	.835	-.718	4.34	313.34
155	1.051	-1.430	4.30	313.30
156	.599	-1.002	4.26	313.26
157	.328	-.611	4.26	313.26
158	0.000	0.000	4.26	313.26
159	.887	0.000	4.65	313.25
160	.878	-.100	4.72	313.32
161	.882	.025	5.03	313.33
162	.795	-.232	4.93	313.33
163	.700	-.140	5.02	313.42
164	.829	-.298	4.81	313.41
165	1.238	-.743	4.45	313.45
166	1.087	-.279	4.36	313.46
167	1.049	-.496	4.32	313.42
168	1.065	-.542	4.32	313.42
169	1.055	-.610	4.31	313.41
170	.855	0.000	4.56	313.56
171	.844	-.037	4.86	313.56
172	.843	.005	5.17	313.57
173	.824	-.070	5.08	313.58
174	.808	.005	4.98	313.58
175	.812	-.035	4.77	313.57
176	.828	.075	4.56	313.56
177	1.073	-.214	4.47	313.57
178	.750	-.232	4.38	313.58
179	.407	-.057	4.37	313.57
180	0.000	0.000	4.37	313.57
181	.834	0.000	4.83	313.88
182	.837	.005	5.14	313.99
183	.836	.133	5.23	313.98
184	.827	.004	5.45	313.85
185	.764	.236	5.35	313.85
186	.634	.255	4.87	313.77
187	.883	.343	4.65	313.75
188	1.401	.606	4.40	313.65
189	1.022	.036	4.32	313.67
190	1.016	.187	4.31	313.66
191	.884	0.000	5.11	314.21
192	.868	0.000	5.20	314.20
193	.853	0.000	5.29	314.19
194	.820	0.000	5.50	314.15
195	.789	0.000	5.72	314.12
196	.862	0.000	5.24	314.04
197	.950	0.000	4.75	313.95
198	1.002	0.000	4.50	313.85
199	1.043	.192	4.25	313.75

Table 6. Input Data for Example 2 -
Flood of 1964, Q = 22,000 cfs

1. Run Parameters

Total Number of Nodes	199
Total Number of Elements	86
Element Types	7

2. Element Characteristics

Element Type	ε_x (lb-sec/ft^2)	ε_y (lb-sec/ft^2)	Chezy C
1	75	50	42
2	50	50	42
3	300	250	12
4	500	250	15
5	500	250	15
6	300	250	42
7	750	750	42

3. Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
1			309.9
2			309.9
3			309.9
4			309.9
5			309.9
6			309.9
7			309.9
8			309.9
9			309.9
191	7.94		
192	7.94		

Table 6. (Continued)

Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
193	7.94		
194	7.94		
195	7.94		
196	7.94		
197	7.94		
198	7.94		
199	7.81	1.44	

Table 7. Nodal Velocity and W.S. Elevation
Example 2 - 1964 Flood Event

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)	NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
1	1.569	1.092	5.07	310.07	51	1.741	1.261	8.28	313.03
2	1.056	.329	4.97	309.97	52	1.172	.616	7.76	313.01
3	1.280	1.057	4.88	309.86	53	.567	.875	6.96	312.96
4	1.867	1.581	5.15	309.85	54	.624	1.101	9.61	312.86
5	1.491	1.350	5.43	309.83	55	.643	1.101	6.54	312.79
6	1.529	1.038	5.64	309.84	56	.276	1.332	7.18	312.68
7	1.070	1.021	5.85	309.85	57	.211	1.205	6.65	312.60
8	.839	.739	5.34	309.84	58	.294	1.278	6.61	312.56
9	.514	.609	4.82	309.82	59	.025	1.649	6.61	312.56
10	1.338	.931	5.35	310.50	60	0.000	0.000	7.84	313.34
11	.792	.778	5.15	310.30	61	.910	-.112	7.83	313.33
12	1.208	.586	5.05	310.20	62	1.615	-.219	7.82	313.32
13	1.660	1.661	5.33	310.18	63	2.014	-.146	7.77	313.27
14	1.604	1.593	5.39	310.09	64	1.697	-.391	7.73	313.23
15	1.564	1.407	5.61	310.11	65	1.563	.333	8.93	313.18
16	1.315	1.330	5.56	310.06	66	1.591	.703	10.13	313.13
17	1.280	1.137	5.04	310.04	67	1.522	1.042	9.06	313.06
18	.989	1.171	4.54	310.04	68	.643	1.389	8.00	313.00
19	1.205	.838	5.63	311.13	69	.435	1.817	7.67	312.92
20	.964	1.184	5.43	310.63	70	.042	1.968	7.33	312.83
21	.962	.994	5.23	310.53	71	1.513	-1.752	10.59	313.34
22	.777	1.298	5.29	310.44	72	1.294	-1.795	10.66	313.41
23	.623	1.143	5.36	310.36	73	1.932	-1.310	10.66	313.41
24	.761	1.173	5.31	310.31	74	2.063	-2.067	10.61	313.36
25	.613	1.086	5.27	310.27	75	2.037	-1.555	11.82	313.57
26	.668	.961	4.75	310.26	76	2.149	.134	13.02	313.52
27	.748	.860	4.26	310.26	77	1.925	.763	10.03	313.53
28	1.199	.634	7.23	311.98	78	1.836	1.311	9.61	313.31
29	1.046	1.155	6.72	311.97	79	1.416	1.385	8.54	313.24
30	.973	1.208	6.51	311.66	80	-.113	1.996	8.21	313.16
31	.993	1.281	6.58	311.58	81	.041	1.987	7.96	312.91
32	1.099	1.317	7.22	311.47	82	0.000	0.000	13.35	313.35
33	1.013	1.312	7.16	311.43	83	1.056	-.910	13.42	313.42
34	.947	1.148	5.81	311.31	84	2.320	-1.057	13.49	313.49
35	.759	1.088	5.31	311.31	85	2.949	-1.371	14.70	313.70
36	.806	1.352	5.27	311.27	86	3.110	-.684	15.92	313.92
37	1.799	2.130	5.27	311.27	87	2.930	-.597	12.92	313.92
38	1.553	1.081	8.33	312.83	88	3.112	-.730	9.93	313.93
39	1.358	1.402	8.31	312.81	89	3.600	.058	9.51	313.71
40	1.110	1.237	7.80	312.80	90	2.553	.656	9.08	313.48
41	.866	1.383	8.44	312.69	91	1.079	.682	8.83	313.23
42	.920	1.396	9.09	312.59	92	0.000	0.000	8.58	312.98
43	.794	1.390	7.72	312.47	93	3.308	0.000	13.85	313.85
44	.713	1.261	6.36	312.36	94	3.574	.638	13.92	313.92
45	.513	1.119	6.33	312.33	95	3.554	1.678	14.86	313.86
46	.640	1.333	6.29	312.29	96	4.941	2.080	14.77	313.77
47	.596	1.433	6.29	312.29	97	3.630	.169	15.99	313.99
48	.027	.031	6.28	312.28	98	3.295	-.053	13.00	314.00
49	1.852	1.289	8.33	313.08	99	2.698	-2.034	10.01	314.01
50	1.905	1.130	8.32	313.07	100	4.171	-3.022	9.82	314.12

Table 7. (Continued)

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
101	6.023	-1.557	9.40	313.90
102	5.434	.236	9.33	313.63
103	5.046	0.000	9.08	313.58
104	0.000	0.000	14.36	314.36
105	.428	.736	15.29	314.29
106	1.086	.983	16.22	314.22
107	1.776	.718	16.14	314.14
108	1.997	-.155	16.06	314.06
109	1.951	-.647	13.07	314.07
110	1.637	-1.360	10.09	314.09
111	1.294	-1.843	9.90	314.20
112	.794	-2.195	9.71	314.31
113	.774	-1.515	9.65	314.25
114	0.000	0.000	9.58	314.18
115	0.000	1.417	10.46	314.46
116	.026	1.267	10.46	314.46
117	.266	1.041	11.39	314.39
118	1.641	.666	12.40	314.40
119	1.791	.296	12.32	314.32
120	2.075	-.603	13.46	314.46
121	1.880	-.900	10.47	314.47
122	1.691	-1.257	10.28	314.58
123	.884	-2.554	9.35	314.65
124	.460	-4.506	9.12	314.67
125	0.000	-4.750	9.05	314.60
126	0.000	0.000	6.56	314.56
127	.823	.304	6.56	314.56
128	1.092	.291	6.55	314.55
129	.955	.414	7.57	314.57
130	1.038	-.148	8.58	314.58
131	1.405	-.972	9.72	314.72
132	1.267	-1.019	10.86	314.86
133	1.353	-1.554	9.93	314.93
134	1.012	-1.758	9.00	315.00
135	.504	-2.334	8.76	315.01
136	.001	-2.145	8.52	315.02
137	1.020	0.000	6.69	314.79
138	.706	.229	6.69	314.79
139	.830	.147	7.70	314.80
140	.962	-.392	7.74	314.84
141	.863	-.393	8.88	314.98
142	.991	-.907	8.90	315.00
143	1.190	-1.654	7.97	315.07
144	1.294	-1.376	7.61	315.11
145	.715	-1.357	7.58	315.08
146	.489	-1.176	7.34	315.09
147	.010	-1.152	7.33	315.08
148	.957	0.000	6.83	315.03
149	.977	-.031	6.86	315.06
150	.947	-.069	6.90	315.10

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
151	.884	-.046	6.92	315.12
152	.996	-.291	6.94	315.14
153	1.457	-1.153	6.58	315.18
154	1.571	-1.144	6.22	315.22
155	1.166	-1.451	6.19	315.19
156	.825	-1.200	6.16	315.16
157	.500	-.628	6.15	315.15
158	.027	-.016	6.15	315.15
159	.958	0.000	6.78	315.38
160	.929	-.015	6.81	315.41
161	.917	.103	7.10	315.40
162	.888	-.015	6.97	315.37
163	.845	.112	7.00	315.40
164	1.006	-.157	6.74	315.34
165	1.685	-.785	6.38	315.38
166	1.514	-.766	6.31	315.41
167	1.467	-.992	6.28	315.38
168	1.536	-.892	6.27	315.37
169	1.552	-.897	6.26	315.36
170	.953	0.000	6.73	315.73
171	.962	.037	7.02	315.72
172	.976	.075	7.31	315.71
173	.934	-.030	7.18	315.68
174	.949	.133	7.05	315.65
175	.689	.040	6.80	315.60
176	1.009	.198	6.54	315.54
177	1.483	-.289	6.47	315.57
178	.991	-.230	6.39	315.59
179	.540	-.019	6.39	315.59
180	0.000	0.000	6.38	315.58
181	.996	0.000	7.02	316.22
182	.997	.023	7.31	316.21
183	.993	.219	7.40	316.20
184	.989	.197	7.71	316.11
185	.899	.471	7.59	316.09
186	.691	.508	6.99	315.89
187	1.046	.539	6.74	315.84
188	1.926	1.044	6.42	315.67
189	1.351	.055	6.35	315.70
190	1.344	.247	6.34	315.69
191	1.086	0.000	7.31	316.71
192	1.073	0.000	7.40	316.70
193	1.060	0.000	7.49	316.69
194	1.017	0.000	7.81	316.61
195	.978	0.000	8.12	316.52
196	1.055	0.000	7.52	316.32
197	1.146	0.000	6.93	316.13
198	1.200	0.000	6.62	315.97
199	1.239	.228	6.30	315.80

Table 8. Input Data for Example 2 -
Flood of 1971, Q = 7,140 cfs

1. Run Parameters

Total Number of Nodes	199
Total Number of Elements	86
Element Types	7

2. Element Characteristics

Element Type	ε_x (lb-sec/ft^2)	ε_y (lb-sec/ft^2)	Chezy C
1	75	50	42
2	50	50	42
3	300	250	12
4	500	250	15
5	500	250	15
6	300	250	42
7	750	750	42

3. Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
1			307.40
2			307.40
3			307.40
4			307.40
5			307.40
6			307.40
7			307.40
8			307.40
9			307.40
191	2.58		
192	2.58		

Table 8. (Continued)

Boundary Conditions

Node	X-Flow (cfs/ft)	Y-Flow (cfs/ft)	W.S. Elevation (ft)
193	2.58		
194	2.58		
195	2.58		
196	2.58		
197	2.58		
198	2.58		
199	2.44	0.82	

Table 9. Nodal Velocity and W.S. Elevation
Example 2 - 1971 Flood Event

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)	NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
1	1.057	.735	2.50	307.50	51	.779	.239	4.69	309.44
2	.782	.230	2.45	307.45	52	.657	.385	4.19	309.44
3	.964	.693	2.40	307.40	53	.613	.501	5.39	309.39
4	1.269	.963	2.68	307.38	54	.625	.684	6.11	309.36
5	.998	.826	2.95	307.35	55	.565	.656	5.06	309.31
6	.959	.649	3.16	307.36	56	.301	.748	3.75	309.25
7	.610	.618	3.36	307.36	57	.239	.633	3.45	309.20
8	.449	.363	2.85	307.35	58	.271	.770	3.42	309.17
9	.188	.222	2.34	307.34	59	.025	1.135	3.41	309.16
10	.934	.650	2.59	307.84	60	0.000	0.000	4.33	309.83
11	.607	.514	2.52	307.67	61	.673	.011	4.33	309.83
12	.370	.572	2.47	307.62	62	.793	.056	4.32	309.82
13	1.164	1.021	2.75	307.60	63	.807	.169	4.32	309.82
14	1.143	.985	2.86	307.56	64	.829	.208	4.33	309.83
15	1.047	.922	3.06	307.56	65	.916	.329	5.52	309.77
16	.865	.868	3.02	307.52	66	.898	.484	6.71	309.71
17	.806	.695	2.51	307.51	67	.818	.662	5.66	309.66
18	.561	.664	2.00	307.50	68	.543	.810	4.61	309.61
19	.637	.478	2.69	308.19	69	.279	1.016	4.31	309.56
20	.625	.754	2.62	308.02	70	.019	.909	4.01	309.51
21	.655	.611	2.55	307.85	71	.504	-.931	7.19	309.94
22	.525	.834	2.66	307.81	72	.694	-.814	7.22	309.97
23	.533	.689	2.77	307.77	73	.904	-.371	7.22	309.97
24	.439	.775	2.72	307.72	74	.923	-.460	7.22	309.97
25	.517	.667	2.68	307.68	75	.975	-.354	8.30	310.05
26	.465	.619	2.17	307.67	76	1.168	.217	9.49	309.99
27	.393	.466	1.65	307.65	77	1.011	.376	6.49	309.99
28	.843	.590	3.87	308.62	78	.972	.662	6.20	309.90
29	.691	.744	3.38	308.63	79	.737	.912	5.15	309.85
30	.617	.792	3.30	308.45	80	.022	1.282	4.84	309.79
31	.715	.836	3.41	308.41	81	.023	1.354	4.74	309.69
32	.791	.931	4.14	308.39	82	0.000	0.000	10.06	310.06
33	.699	.967	4.10	308.35	83	.431	-.462	10.09	310.09
34	.618	.766	2.78	308.28	84	.920	-.509	10.12	310.12
35	.473	.698	2.27	308.27	85	1.175	-.696	11.19	310.19
36	.476	.860	2.24	308.24	86	1.329	-.369	12.27	310.27
37	1.059	1.254	2.23	308.23	87	1.344	-.120	9.26	310.26
38	.794	.553	5.05	309.05	88	1.341	.079	6.26	310.26
39	.745	.725	4.56	309.06	89	1.445	.429	5.97	310.17
40	.714	.664	4.06	309.06	90	1.014	.634	5.68	310.08
41	.705	.784	4.79	309.04	91	.480	.632	5.57	309.97
42	.713	.850	5.51	309.01	92	0.000	0.000	5.46	309.86
43	.585	.861	4.20	308.95	93	1.490	0.000	10.25	310.25
44	.444	.724	2.88	308.88	94	1.643	.348	10.28	310.28
45	.314	.654	2.85	308.85	95	1.753	.912	11.24	310.24
46	.364	.812	2.82	308.82	96	2.325	.630	11.20	310.20
47	.377	.985	2.82	308.82	97	1.629	-.172	12.28	310.28
48	0.000	0.000	2.82	308.82	98	1.492	-.069	9.28	310.28
49	1.019	.709	4.69	309.44	99	1.453	-.639	6.28	310.28
50	.937	.186	4.69	309.44	100	2.328	-.888	6.03	310.33

Table 9. (Continued)

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
101	3.003	-.440	5.74	310.24
102	2.654	.184	5.74	310.24
103	2.440	0.000	5.64	310.14
104	0.000	0.000	10.44	310.44
105	.272	.409	11.40	310.40
106	.635	.554	12.37	310.37
107	1.058	.474	12.33	310.33
108	1.202	.075	12.29	310.29
109	1.160	-.120	9.29	310.29
110	.961	-.488	6.30	310.30
111	.777	-.757	6.05	310.35
112	.503	-.984	5.81	310.41
113	.396	-.719	5.81	310.41
114	0.000	0.000	5.81	310.41
115	0.000	.982	6.51	310.51
116	-.002	.845	6.51	310.51
117	.093	.629	7.47	310.47
118	.892	.333	8.48	310.48
119	.941	.094	8.44	310.44
120	1.114	-.341	9.54	310.54
121	1.005	-.396	6.54	310.54
122	.937	-.697	6.30	310.60
123	.478	-1.197	5.41	310.71
124	.159	-1.816	5.17	310.72
125	0.000	-1.934	5.17	310.72
126	0.000	0.000	2.59	310.59
127	.875	.146	2.58	310.58
128	1.238	-.016	2.58	310.58
129	.758	.288	3.59	310.59
130	.728	-.022	4.59	310.59
131	.685	-.420	5.69	310.69
132	.837	-.525	6.79	310.79
133	.733	-.923	5.90	310.90
134	.508	-.872	5.01	311.01
135	.160	-.996	4.77	311.02
136	0.000	-1.059	4.53	311.03
137	.769	0.000	2.73	310.83
138	.479	.332	2.73	310.83
139	.818	.072	3.73	310.83
140	.831	-.215	3.61	310.91
141	.761	-.201	4.91	311.01
142	.649	-.624	5.00	311.10
143	.526	-.613	4.11	311.21
144	.676	-.847	3.76	311.26
145	.336	-1.082	3.71	311.21
146	.151	-.596	3.46	311.21
147	0.000	-.857	3.46	311.21
148	.738	0.000	2.87	311.07
149	.855	-.147	2.95	311.15
150	.709	-.154	3.02	311.22

NODE	X VEL (FPS)	Y VEL (FPS)	DEPTH (FT)	ELEV (FT)
151	.689	-.244	3.12	311.32
152	.688	-.362	3.21	311.41
153	.648	-.495	2.57	311.47
154	.700	-.554	2.52	311.52
155	1.039	-1.418	2.46	311.46
156	.539	-.019	2.40	311.40
157	.319	-.579	2.40	311.40
158	0.000	0.000	2.39	311.39
159	.804	0.000	2.87	311.47
160	.798	-.063	2.94	311.54
161	.824	.031	3.24	311.54
162	.774	-.194	3.16	311.56
163	.690	-.106	3.25	311.65
164	.827	-.322	3.04	311.64
165	1.284	-.784	2.69	311.69
166	1.063	-.215	2.60	311.70
167	.967	-.469	2.54	311.64
168	.996	-.508	2.54	311.64
169	1.002	-.579	2.53	311.63
170	.777	0.000	2.86	311.86
171	.820	.000	3.16	311.86
172	.865	.075	3.47	311.87
173	.752	-.113	3.38	311.88
174	.738	-.014	3.29	311.89
175	.552	-.005	3.03	311.88
176	.781	.004	2.87	311.67
177	1.105	-.211	2.78	311.88
178	.863	-.241	2.68	311.88
179	.478	-.047	2.68	311.88
180	0.000	0.000	2.67	311.67
181	.659	0.000	3.23	312.26
182	.653	.018	3.53	312.28
183	.660	.104	3.61	312.26
184	.738	.098	3.78	312.18
185	.674	.143	3.70	312.20
186	.559	.188	3.22	312.12
187	.812	.219	3.01	312.11
188	1.386	.391	2.77	312.02
189	1.005	-.021	2.68	312.03
190	.994	.183	2.67	312.02
191	.715	0.000	3.60	312.70
192	.700	0.000	3.68	312.68
193	.686	0.000	3.76	312.65
194	.656	0.000	3.93	312.58
195	.628	0.000	4.10	312.50
196	.711	0.000	3.63	312.43
197	.819	0.000	3.15	312.35
198	.886	0.000	2.91	312.26
199	1.661	.306	2.67	312.17

VII. COMMENTS ON MODEL APPLICATION

In Chapter VI it is shown that the Finite Element Model is capable of reproducing both laboratory and field hydraulic conditions with reasonable accuracy. As is the case with most existing mathematical models, however, the Finite Element Model is subject to certain practical limitations. These limitations may eventually be overcome in the course of applying the model. In particular, better approximations will be made for some of the coefficients (e.g., eddy viscosities) in the model. For field applications the Finite Element Model has four limitations:

1. *Fixed Flow Boundary*. In the construction of the finite element network system the delineation of flow boundaries along both channel banks is preassumed. While such an assumption for flows in rectangular channels is essentially correct, it is an approximation for natural channels. Flow in natural open channels seldom follows common flow pathways along the banks for all discharges; the greater the flow, the wider the cross-sectional surface width over flood plains. This is particularly true for streams having mild bank slopes.

   One way of delineating the flow boundaries is to take the average boundary of high and low flows. Unless the flood plain is unusually flat, the blow boundary thus defined would be sufficiently accurate for network construction. It would be most helpful if data on high water marks were available for an intermediate flood.

2. *Effect of Element Size on Solution Accuracy*. Mathematically the finite element approximation converges to an exact solution as element size approaches zero. In practical application it is unrealistic to specify a finite element network in such great detail to give a near closed-form solution. The user is often under the constraint of computational cost and limited computer storage. The user

is interested in a reasonably acceptable level of accuracy within a reasonable range of computer costs. Consequently, from the user's viewpoint a network with the least amount of detail, with an acceptably accurate solution, will be the desirable selection. Unfortunately there has not yet been developed a procedure for determining the most cost-effective FEM network. We may speculate that, as more experience is gained in finite difference numerical methods, it will be shown that the level of network detail is governed by the degree of variation in system variables that the model is to solve. Therefore we suggest that networks should be constructed with more detail in areas where the gradients of the system variables are expected to be great and with less detail in areas where gradients are expected to be small or where accuracy is not required. In the case of two-dimensional problems, this means that different levels of detail can be used in regions where gradients are expected to be small. This is illustrated in the example problems in the preceding chapter. For both laboratory and field examples, larger size elements were specified for upstream and downstream flow regions, and detailed or smaller size elements were specified for the areas near the bridge openings.

3. *Estimate of Turbulent Exchange Coefficients*. An input required to operate the FEM is the coefficients representing eddy viscosities. Although there has been much research on these coefficients over the years, equations to develop a reasonable estimate of these values are still lacking. It has been recognized that the magnitude of eddy viscosities is determined by the factors of scale of motion, depth, velocity, velocity gradient, and relative time scale. Thus the value of eddy viscosities for the prototype would be different from that of the scale model.

   The significance of the turbulent exchange coefficients is determined by the relative magnitude of the terms containing these coefficients and the inertial terms in the equations of motion. In cases where the flow behavior is dominated by the inertial terms the viscous forces become insignificant; thus the solutions of flow equations will not be very sensitive to the exchange coefficients. In practical applications however, we frequently encounter a situation where the inertial forces and viscous forces are approximately equal in magnitude. In this case proper specification of exchange coefficients becomes essential to accurate model results.

The hydraulic behavior of bridge waterways is a case in point where the inertial forces and viscous forces may have equal importance in the vicinity of the bridge. When flood water passes through the bridge opening, its flow is contracted upstream from the bridge and under the bridge, and expanded downstream from the bridge. During the process of flow contraction and expansion, the flow separates around the bridge abutments and thus generates eddies and intensive turbulence exchange. On the other hand, in the same region flow contraction and expansion result in rapid variation in water surface and flow velocity which are phenomena of inertial forces. In this region both the forces of viscosity and the inertial forces have a significant effect on the flow hydraulics. Away from the bridge opening the flow tends to be gradual and less turbulent. In such flow regions the effect of the viscosity forces of turbulence exchange becomes less significant and the inertial forces dominate.

From the experience of model calibration at the Tallahalla field site and the laboratory data, the values of turbulence exchange coefficients (eddy viscosities) listed in Table 10 give the most satisfactory results. While the magnitude of the coefficients may vary from site to site, these values may be considered typical, and can be used for hydraulic simulations.

4. *Computer Requirements.* The execution of the Finite Element Model requires a high speed computer with a very large storage capacity (on the order of 200K octal words). A large memory is needed for the solution of the large system of simultaneous equations and the in-core storage of the resulting coefficient matrix.

   In terms of computer speed, it is advisable to use the fastest machine available because FEM has a very high computational requirement. The model development was undertaken on a Control Data 6700 computer. Other versions of the model have been run on a UNIVAC 1108 computer. Computers such as the CDC 6000 and 7600 series, and UNIVAC 1110 are the types best suited for the Finite Element Model. The large IBM 360 configurations (350 K bytes) are also suited for model execution.

Table 10. Values of Turbulence Exchange Coefficients Used in Example Problems.

Type of Simulation	Turbulence Exchange Coefficients (Eddy Viscosity) ($lb\text{-}sec/ft^2$) Gradually Varied Zone		Flow Contracting Zone		Flow Expansion Zone	
	Ex	Ey	Ex	Ey	Ex	Ey
Field Site (Tallahalla Creek at Rt. 528, Miss.)	500* 750**	250* 750**	50	50	300	250
Wing Wall Model	1.0	1.0	0.5	0.5	0.5	0.5

* Wooded Flood Plain
** Pasture Area

VIII. SUMMARY

A mathematical model describing the steady, two-dimensional subcritical flow in bridge waterways has been developed using the *Finite Element Method* of numerical analysis. The basic fluid equations comprising the model consist of the phenomenologic motion equations (i.e., the Navier-Stokes Equations with Reynold's stress terms) and the continuity equation. These partial differential equations are solved simultaneously by numerical methods to yield the spatial distribution of water surface elevations and velocities within the flow region for prescribing boundary conditions.

Galerkin's variation of the method of weighted residuals has been applied to the basic differential equations to form the finite element representation. The basic element shape selected for the model is the triangle, with quadratic shape functions defined for unit flows and linear shape functions defined for the depth. Such an approximation results in six degrees of freedom for each unit flow component and three for depth on any triangular element.

The model is capable of simulating flow characteristics of arbitrary geometry and of accepting boundary specifications for any dependent variable at any location. Hydraulic computations for various types of highway stream crossings, such as normal, skewed, eccentric, or a combination of these can easily be performed by the model. The model is capable of simulating flow overtopping over roadway embankments and can perform hydraulic computation for a series of bridges across a stream valley without requiring prior assumption of the flow distribution for each bridge opening. In fact the model may be used to determine the flow passing through each bridge opening under this type of crossing condition. It may be possible to use the Finite Element Model to compute scour. The model is coded in FORTRAN language for solution by a digital computer.

The model has been tested for two example problems: one for a field site near Tallahalla, Mississippi, and one for hydraulic flume data. In both examples good agreement between the model and the observed data was demonstrated.

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GLOSSARY OF MATHEMATICAL TERMS

A = Area of triangular element

B = Function dependent on ψ

C = Chezy coefficient

C_f = Depth-dependent resistance coefficient

C_s = γC_w = Weir coefficient for submerged condition

C_w = Discharge coefficient of broad crest weir

c_x, c_y = Direction cosines of outward normal in x and y directions

d = Depth

F = Body force

F_{b_x} = Unit force due to bottom friction in x direction

F_{b_y} = Unit force due to bottom friction in y direction

f = Error function

g = Gravitational constant

H = Head above weir

h = Local water depth

L = Weir length

ℓ = Distance along a stream line

N_i = Shape function of element node

p = Instantaneous value of pressure

$\bar{p}$ = Mean pressure over Δt

p' = Fluctuation in pressure

Q = Flow

q = Number of integration points

r = Unit flow in the x direction

S = Contour

s = Unit flow in the y direction

t = Time

u = Instantaneous velocity component in the x direction

$\bar{u}$ = Mean velocity in the x direction

u' = Velocity fluctuations with respect to mean value in the x direction

V = Element area

v = Instantaneous velocity component in the y direction

$\bar{v}$ = Mean velocity in the y direction

v' = Velocity fluctuations with respect to mean value in the y direction

w = Instantaneous velocity component in the z direction

$\bar{w}$ = Mean velocity in the z direction

w' = Velocity fluctuations with respect to mean value in the z direction

w_g = Numerical integration weighting factor at location g

x = Distance along x-coordinate

$\mathbf{x}$ = Body force acting on x direction

y = Distance along y-coordinate

$\mathbf{y}$ = Body force acting on y direction

z = Water surface elevation

$\mathbf{z}$ = Body force acting on z direction

z_o = Elevation of bottom surface

α = Coefficient

$\beta = \left(\frac{2}{\Delta t} r + \frac{\partial r}{\partial t}\right)^t$

γ = Correction factor for the discharge coefficient under submerged flow, a function of the ratio of downstream head to upstream head above roadway

ε = Turbulence exchange coefficient (eddy viscosity)

ε_x, ε_y, ε_z = Turbulence exchange coefficients in the x, y, and z directions

ξ = Scalar quantity or vorticity

μ = Dynamic viscosity of fluid

ρ = Fluid density

τ = Fluid shear stress = $(\mu + \varepsilon) \frac{\partial \bar{u}}{\partial y}$

ϕ_a = Location function

ψ = Stream function

APPENDIX A

USERS' GUIDE

Finite Element Model for Bridge Backwater Computation

INTRODUCTION

This section of the report presents a brief description of the information needed to apply the Finite Element Model to a backwater problem. Included is a description of input requirements and important model parameters. This section is intended to be used to assist in the application of the model to a physical problem. If a more in-depth understanding of the program formulation or the method of solution is needed the reader is referred to earlier sections of this report.

Input data for the Finite Element Model is surprisingly concise. Among the most important items required is a precise estimate of upstream and downstream boundary conditions along with an accurate description of the physical geometry of the area under study. These topics are covered in detail sufficient for implementation by model users. Important system parameters such as bottom elevations, abutment geometry and overtopping parameters, along with typical roughness and eddy viscosity parameters are discussed in detail. An important section on model calibration procedures is also included. This section attempts to formalize the procedures necessary to provide an accurate and realistic application of the model.

The Appendix concludes with a description of input format requirements, followed by a description of important variables in COMMON and a complete listing of the program.

GUIDELINES FOR GRID NETWORK LAYOUT

NETWORK LAYOUT

The first requirement for constructing a grid network system for Finite Element Modeling is a map showing details of the physical system. A U.S. Geological Survey (USGS) quadrangle map with a scale of 1:24,000 is useful, but a larger scale map (1:500) is more desirable. The map should show all the major characteristics of the physical system, including the main channel configuration, flood plain boundaries, bottom elevations, spatial distribution of vegetation cover within the flood plain, embankment and abutment geometries, and horizontal and vertical clearances of the bridge. If a sufficient amount of flood plain cross-sectional data are available, these data may be used to substitute for the topographic maps. As a minimum, cross-sectional data extending one to two miles in both directions from the crossing site are generally required.

Construction of the network system begins with the delineation of flow boundaries along the banks and along the upstream and downstream boundaries of the overall system. The upstream and downstream boundaries should be selected at locations where boundary conditions can be specified with a degree of certainty. Delineation of the boundaries of the flood plain requires a judgmental decision since natural streams have sloping sides rather than rectangular sides which are used by the FEM model. Since water surface elevation is a dependent variable, the flow boundaries along the banks cannot be determined *a priori*. Highwater marks of historical flood events may be helpful in delineating these flow boundaries.

As a general rule, the system should be described with as few corners as possible. The interior elements of the flow region can be described after the overall flow boundaries have been set. Experience has shown that the best network system, in terms of computational efficiency, follows the flow net of potential flow in a very general way. It is not necessary that the network be orthogonal as is the case of a potential flow network. Basically, the finite element network consists of a system of interlocking triangles.(Figure A1 The first step of constructing such a system of network is to draw a series of lines from bank to bank and another from the upstream boundary to a downstream boundary forming a system of quadrangles in the flow region. Diagonals can then be drawn in the quadrangles to form the basic grid network of the computational system. As in most numerical methods the grid size is essentially governed by the expected degree of variation in water surface elevation, flow velocity, channel roughness and geometry, and bottom elevation.

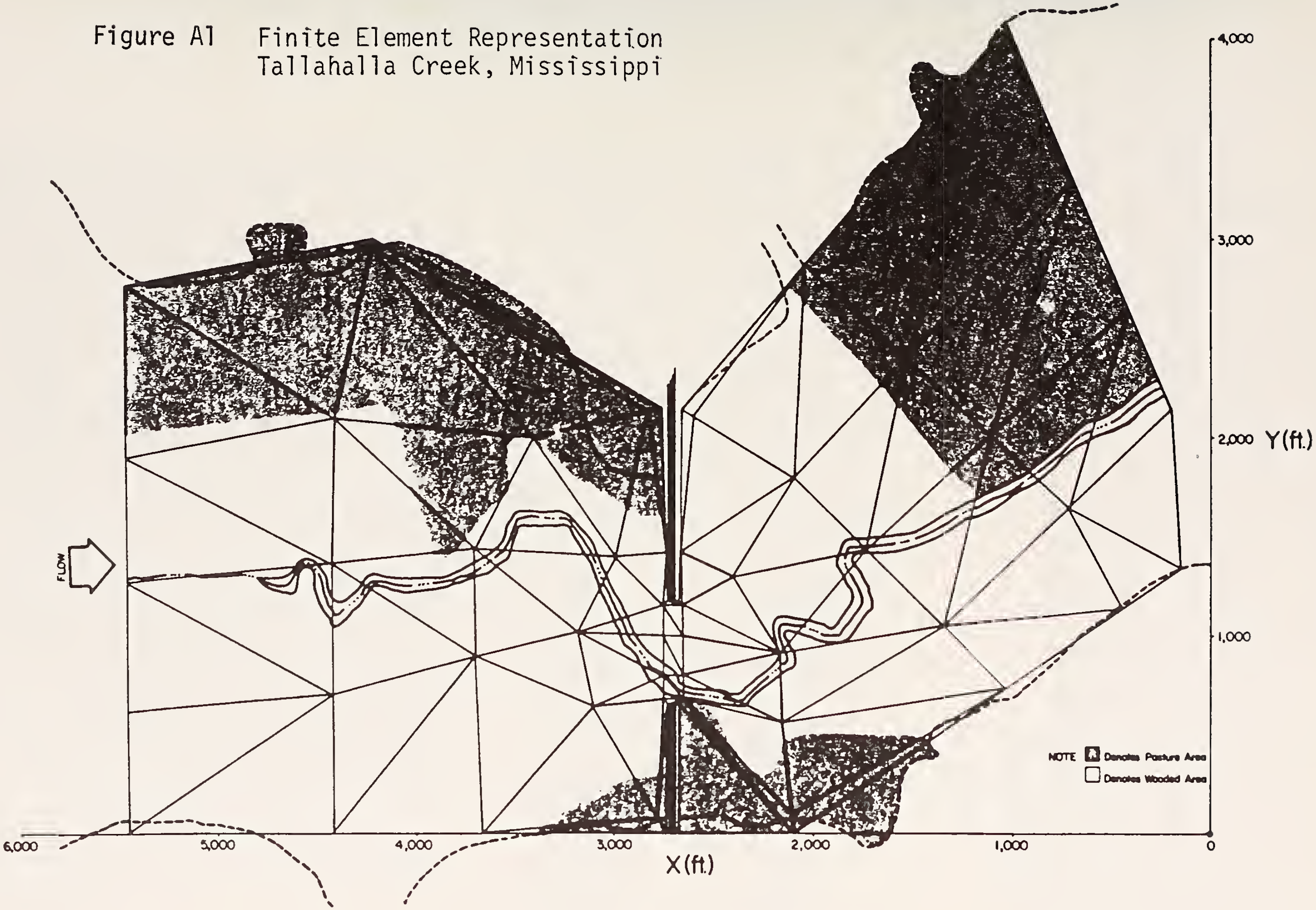
With the network complete, the nodes and elements are numbered and checked against the capacity of the program. If program capacity has been exceeded, one can either consider enlarging the program dimensions (assuming a sufficiently large computer) or restructuring the network to a lesser number of nodes and elements. The best possible network is the smallest one which will adequately and accurately describe the prototype system.

It should be noted that FEM treats the corner nodes along the exterior boundary as stagnation points. Such an assumption may not be completely valid for some of the physical systems but is necessary for present versions of FEM. Special consideration is thus required to make these boundary elements near corners as small as possible.

NETWORK NUMBERING

Each nodal point in the network system must be assigned a unique number, both for interpretation of input and output, and for internal computation. The general rule of nodal numbering is to keep the maximum difference in node numbers across any element a minimum. This will usually result if the nodes are numbered sequentially along transverse lines. Node numbers must be assigned to the mid-side nodes as well as corner nodes.

Figure A1 Finite Element Representation
Tallahalla Creek, Mississippi



If one errs in network numbering, discovers an order which will reduce the maximum node number difference across an element, or wishes to add elements to an existing network which do not fit in with the existing numbering system, there is no need to renumber and repunch all the data associated with the network. An option in the computer program allows the internal renumbering of the node system, while at the same time allowing the old, external numbering system to be used for input and output. This option is detailed on Card Sequence Number 19 in the section entitled "Input Card Formats".

BOUNDARY CONDITIONS

Proper boundary conditions must be described prior to the execution of FEM. The system contains three dependent variables: flow (or velocity) in the longitudinal direction, flow (or velocity) in the transverse direction, and the head (or water surface elevation). The program permits one or more of these variables, or, in certain instances, a combination of them, to be specified by the user for each nodal point. There are six types of conditions possible at each nodal point:

1. *No Boundary Condition Specified*

   When no boundary condition is specified at a node for a particular variable, this variable simply enters the calculation scheme as one of the basic unknowns. Normally there will be no specified values at any of the interior nodes of the network.

2. *Exact Boundary Condition*

   The exact boundary condition is the most familiar type of boundary condition, and simply means that a specified value is assigned to one or more variables at a particular node point. Typically this is the type of specification that is used to set upstream flows.

3. *Parallel Flow Boundary Condition*

   In order to make the model as realistic as possible, a feature has been programmed into the computer codes which allows flow to move parallel to a fixed boundary. This feature is particularly useful in reducing the required level of element detail when straight-sided systems are being simulated. When using the parallel flow option, it is important to insure the condition that the fixed boundary along which flow is allowed to move is really straight. In general this cannot be expected to occur if node points are taken from an ordinary graphical layout. For this reason an option has been built into the program which allows the exact calculation of selected node point coordinates to insure that they lie along a straight line. This option has been used in the demonstration data deck. Input information found on card types 19, 24 and 25 are necessary for implementation. A description of the card types can be found in the section entitled "Input Card Formats".

4. *Corner Boundary Conditions*

The flow or velocity specifications at a corner node are zero-zero because corners must be considered as stagnation points in the model. This may cause certain problems in the overall solution if the corner elements are too large, but it is necessary to preserve overall system continuity.

5. *Exit Boundary Conditions*

The mathematical formulation of FEM has resulted in a situation whereby the specification of exit boundary conditions for head is handled in a slightly different fashion from the exact boundary condition. From a user's point of view this will simply mean that the head at the downstream boundary will have to be identified as such, and that in some cases the model will not reproduce these specifications with the same numerical precision as an exact boundary condition.

6. *Sink-Source Boundary Condition*

Special boundary conditions are required at locations where the flows leave and re-enter the system. This type of flow condition occurs at channel confluences, and in bridge waterway, at the top of embankment where flow overtopping takes place.

Typically, the user specifies the exact unit flows in both space dimensions at each nodal point on the upstream boundary, and the exact water surface elevation at corner points on the downstream boundary. Water surface elevation along the upstream boundary and the flows along the downstream boundary are left as unknow variables to be solved by FEM. No exact boundary conditions along the boundaries of channel banks are required other than those at corner nodes where zero flows are specified. Generally a parallel flow boundary condition is specified along the channel banks and approach embankments.

PREPARATION OF INPUT DATA

The *Input Data* required for the Finite Element Model consist of (a) network geometric parameters, (b) system coefficients, (c) initial conditions, (d) boundary conditions, and (e) run control parameters. Brief descriptions of these input data are given in the following paragraphs. Input card formats are found in the section under that title.

The *Network Geometric Parameters* are the basic framework of the model. The data in this category include the X and Y coordinates and bottom elevation of nodal points. Specification of these data are required only at corner nodes of each element. The data at mid-nodes are obtained through interpolation of the data from adjacent corner nodes. Connectivity information of nodes around each element must also be specified.

The *System Coefficients* include bottom friction coefficients, namely the Manning's roughness coefficients and eddy viscosities. Data describing the spatial distribution of channel roughness in the flow region are very important. Roughness data must be developed for both the main channel and the flood plain. Vegetation maps or aerial photographs at the bridge site are helpful in estimating channel roughness. The locations of high water marks from past flood events are a good way to determine the channel roughness under flood stages when such data are available. Information on eddy viscosities is generally sparse and they must be obtained from past studies (See Chapter VIII and Table 10). Both of the data types are specified for each element of the network system. For cases where overtopping of approach embankments and bridge decks occurs, discharge coefficients for Weir flow must be specified and input to the model. Wier coefficients are input on Card Type 10 of the input data deck. Typical coefficients are given in Figure 10.

Initial Conditions must be specified before the computation begins. In dynamic computations the initial conditions are specified to depict the state of the system at the beginning of the next time step. In steady state computations the specification of initial conditions generally result in increased computational efficiency. The input data describing these initial conditions are estimates of unit flows (cfs/ft) both the X and Y directions and the water surface elevation at each node of the network system. As a general guideline, values of flow parameters under uniform flow condition usually are good initial conditions.

Boundary Conditions must be described as input data in accordance with the proper types of boundary outlined in the section entitled Boundary Conditions. Generally it is a relatively easy task to prepare this group of input data, since most of the needed information, such as flow data, is readily available.

Run Control Parameters input to the model consist of: total number of nodes, elements, element types, corner nodes and boundary nodes; print option, continuity checks across given sections, degrees of freedom, and average water surface elevation at beginning, latitude and scale factors. In addition, time and iteration control such as total number of iteration cycles, time steps, time intervals and total run time are also input to the model.

MODEL OUTPUT

Depending upon the specification of print option in the input data, model output can either be displayed for every iteration cycle or for selected cycles. The model output consists of the values of velocity components in the X and Y directions, water depth and water surface elevation at each nodal point. These data can either be output to a line printer or stored on magnetic tapes. Information regarding convergence and continuity at each iteration are also output. When embankment overtopping occurs, the location and flow rate printed is also output.

Sample output for the test sites are given in Chapter VI of this report.

MODEL CALIBRATION

Before the FEM can be applied to study the hydraulic characteristics at a bridge site the model must be calibrated. Model calibration is necessary because of the assumptions and simplifications which are made to approximate the real world condition by a comparatively idealized model, e.g., the irregular boundary of a physical system has been approximated by simple geometric representations. Furthermore, a number of complicated flow phenomena in the real system are represented by empirical relationships in the model, using coefficients developed from past studies or from field observations. Some coefficients are dependent on the characteristics of the physical system and thus vary from place to place. The essence of model calibration, therefore, entails a systematic adjustment of network properties and system parameters or coefficients in the model to improve model output until satisfactory agreement is obtained between model results and field observations. The calibrated model is then ready to be used to simulate the effects of other conditions at the site.

The major parameters that can be adjusted to calibrate FEM are: (1) channel roughness, (2) turbulence exchange coefficients (or eddy viscosities), (3) network properties, and (4) discharge coefficients for weir flow. The first estimates of channel roughness data are generally obtained from the literature or from historical flood measurements. As these data are applied to FEM, adjustments may be needed to give good calibration results. Care must be taken in adjusting parameters such as roughness. There is only a finite range of channel roughness that is permissable. It would be unreasonable to obtain a good calibration while using a roughness which has no physical meaning. As stated previously, information regarding eddy viscosities is generally lacking in the literature. The estimate of eddy viscosities is a rather difficult task since they tend to vary with scale of fluid motion, location and time.

Model calibration generally begins with the selection of historical flood events for which most of the pertinent field data have been collected. After an initial network layout has been constructed, a set of estimated roughness and turbulence exchange coefficients is assigned. Normally solutions will converge after three cycles of iteration. If the solution fails to converge rapidly or computed results do not conform with measured data favorably, then adjustment of network layout and/or roughness and eddy viscosities may be needed. The model outputs of continuity checks across the selected river sections and convergence parameters provide reliable information for determining model performance. The ultimate calibration results, however, must be judged based on comparison of the computed water surface elevations and velocities to observed field data. If the computed values are generally higher than the observed data, then either the estimated values of roughness are too high or the outer flow boundary has been set too close. In such case, the location of the flow boundary should be examined and the roughness adjusted until satisfactory model output is obtained.

Stage and velocity near the bridge opening are sensitive to the turbulence exchange coefficients, but this sensitivity diminishes with distance away from the bridge opening. Adjustment of these coefficients is significant in calibrating the model to reproduce flow conditions near the bridge opening. Experience with the FEM has shown that major adjustment of roughness is generally not needed when high water marks were used for roughness computation.

Discharge coefficients for both free and submerged flow over highway embankments are well documented, but local roadway configuration and construction practice may affect these values slightly. Minor tuning may be required to determine discharge coefficients accurately.

For model calibration, data describing the velocity distribution across a channel cross section are useful. The velocity data used for model calibration on natural conditions should be taken at the section of the proposed bridge site. In the case of existing bridge sites the data can be taken either at the bridge or at other sections upstream or downstream from the bridge. Velocity data taken in the flood plain and in the main channel, where distinct channel roughness exists, are extremely useful to model calibration but they are usually difficult to obtain.

Information on embankment overtopping, including the location and quantity of overflow, is also important for model calibration and verification. If the bridge deck is inundated by flood water, the water surface elevations both upstream and downstream, and the velocity data across the bridge section must be measured. Such data are important for calibrating the flow conditions when the bridge deck is submerged and orifice flow prevails.

Historical flood data at the proposed bridge site are a good source of information. High water marks observed for various flood peaks provide invaluable information for model calibration and validation. The water surface elevation measured at various locations throughout the flow region is most desirable, but reliable stage data only along the banks is also useful.

At existing bridge sites water surface elevations under the bridge and along the approach embankments may have been measured in addition to the data collected along both banks. Stage data at specific locations such as the upstream and downstream stagnation points and around the abutments should be sought.

INPUT CARD FORMATS

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN[1] Type	Description of Input Value
1:1	1-80	TIT	A	CYCLE - Multiple discharges NOCYCLE - Single discharge
2:1	1-5	INORM	I	INORM=0 Overtopping considered INORM=1 No overtopping
(Omit card groups 3-16 if TIT = NOCYCLE)				
3:1	1-20	TIT5	A	Any comment
3:1	21-25	NLEN	I	·No. of configurations considered
3:1	26-50	IIN(5)	I	Input tape no's. for configuration data
4:1	1-20	TIT5	A	Any comment
4:1	21-70	FLEN(5)	R	Bridge length considered
5:1	1-20	TIT5	A	Any comment
5:1	21-30	NFLFR	I	No. of discharges
6:NFLFR*	1-10	FLFR(I,1)	R	Flow magnitude (cfs)
6:NFLFR*	11-20	FLFR(I,2)	R	Return interval
7:1	1-20	TIT5	A	Any comment
7:1	21-25	NEMB	I	No. of embankments to be considered for each bridge configuration
8:1	1-10	EMB(10	R	1 Embankment elevation (ft)
8:1	11-20	EMB(2)	R	2 Embankment elevation (ft)
8:1	21-30	EMB(3)	R	3 Embankment elevation (ft)
8:1	31-40	EMB(4)	R	4 Embankment elevation (ft)

* Repeat for each discharge considered.

[1] A = Alphanumeric, R = Real, I = Integer.

INPUT CARD FORMATS (Cont.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
(Omit card groups 9 and 10 if INORM = 1)				
9:(NLEN-1)*	1-20	TIT5	A	Any comment
	21-25	NNDS(I)	I	No. of overtopping nodes for each bridge configuration
10:(NLEN-1)*	1-10	NUP(I,J)	I	Upstream overtopping at Node No. J
	11-20	NDWN(I,J)	I	Downstream overtopping at Node No. J
	21-30	WELEV(I,J)	R	Water surface elevation (ft) at Node No. J
	31-40	WCF(I,J)	R	Weir coefficient at Section J
	41-50	NVPP(I,J)	I	Upstream corner node at Node No. J
	51-60	NDWNN(I,J)	I	Downstream corner node at Node No. J
11:1	1-5	NNDSUP	I	Number of upstream boundary condition nodes
	6-10	NNDSD	I	Number of downstream boundary condition nodes
12:NLEN**	1-5	KNDSUP(**,1)	I	Upstream BC Node 1
	6-10	KNDSUP(**,2)	I	Upstream BC Node 2
	11-15	KNDSUP(**,3)	I	Upstream BC Node 3
	16-20	KNDSUP(**,4)	I	Upstream BC Node 4
	21-25	KNDSUP(**,5)	I	Upstream BC Node 5
	26-30	KNDSUP(**,6)	I	Upstream BC Node 6
	31-35	KNDSUP(**,7)	I	Upstream BC Node 7
	36-40	KNDSUP(**,8)	I	Upstream BC Node 8
	41-45	KNDSUP(**,9)	I	Upstream BC Node 9
	46-50	KNDSUP(**,10)	I	Upstream BC Node 10

*Repeat card groups 9 and 10 for each bridge configuration.

**Repeat card group 12 for each bridge configuration and for the normal configuration (no bridge).

INPUT CARD FORMATS (Cont.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
13:(NLEN-1)*	1-5	KNNDS(*,1)	I	Downstream BC Node 1
	6-10	KNNDS(*,2)	I	Downstream BC Node 2
	11-15	KNNDS(*,3)	I	Downstream BC Node 3
	16-20	KNNDS(*,4)	I	Downstream BC Node 4
	21-25	KNNDS(*,5)	I	Downstream BC Node 5
	26-30	KNNDS(*,6)	I	Downstream BC Node 6
	31-35	KNNDS(*,7)	I	Downstream BC Node 7
	36-40	KNNDS(*,8)	I	Downstream BC Node 8
	41-45	KNNDS(*,9)	I	Downstream BC Node 9
	46-50	KNNDS(*,10)	I	Downstream BC Node 10
14:NFLFR**	1-80 (8 @ 10)	BXV(I,J)	R	X velocity at upstream boundary node J (ft/sec)
15:NFLFR**	1-80 (8 @ 10)	BXZ(I,J)	R	Z velocity at upstream boundary node J (ft/sec)
16:NFLFR**	1-80 (8 @ 10)	HTI(I,J)	R	Downstream head (ft) at node J
17:1	1-10	DEMBK	R	Embankment height (ft)
18:1	1-80	TITLE	A	Any heading comment
19:1	1-5	NP	I	The maximum node number in the network
	6-10	NE	I	The maximum element number in the network
	11-15	NMAT	I	Number of element types
	16-20	NPX	I	Number of corner nodes
	21-25	NBX	I	Number of nodes with boundary conditions specified
	26-30	IPRT	I	Control for output printing: IPRT=0, all input data printed; IPRT=1, node and element input data suppressed

* Repeat card group 13 for each bridge configuration.
**Repeat this card group for the number of discharges considered.

INPUT CARD FORMATS (Cont.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
19:1	31-35	NXZL	I	Number of line segments along which the Z coordinates are to be computed internally (X from card input)
	36-40	NZXL	I	Number of line segments along which the X coordinates are to be computed internally (Z from card input)
	41-45	NCL	I	The number of line segments along which flow continuity is to be checked
	46-50	IWIND	I	Control for wind field input: IWIND=0, no wind field input IWIND=1, wind field input
	51-55	NDF	I	Number of degrees of freedom (set NDF=3) NDF=3; flow, head NDF=4; flow, head, density NDF=5; flow, head, density, temperature
	56-60	IRO	I	Control for internal node reordering: IRO=0; no reordering IRO=1; reordering
20:1	1-10	OMEGA	R	Local average latitude (deg.)
	11-20	ELEV	R	Average initial water surface elevation (ft)
	21-30	TEMP	R	Average initial water temperature (°C)
	31-40	XSCALE	R	Scale factor for X coordinate card inputs
	41-50	ZSCALE	R	Scale factor for Z coordinate card inputs
21:1	1-5	NITI	I	Number of iterations for initial solution

(If NITI ≤ 0, a set of initial conditions will be read at sequence 32 below. If NITI < 0, NITI will be set to the absolute value of NITI after input of the initial conditions.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
21:1	6-10	NITN	I	Number of iterations per dynamic solution step (set to zero)
	11-15	NCBC	I	Number of time steps between updates of boundary conditions (set to zero)
	16-25	DELT	R	Length of time step (hr)
	26-35	TMAX	R	Total simulation time (hr) (set to zero)
22:NMAT	1-10	J	I	Element type (9 or less)
	11-20	ORT(J,1)	R	X-turbulent exchange coefficient ($lb\text{-}sec/ft^2$)
	21-30	ORT(J,2)	R	Z-turbulent exchange coefficient ($lb\text{-}sec/ft^2$)
	31-40	ORT(J,3)	R	X-turbulent diffusion coefficient (ft^2/sec)
	41-50	ORT(J,4)	R	Z-turbulent diffusion coefficient (ft^2/sec)
	51-60	ORT(J,5)	R	Chezy coefficient for this element type
23:NPX	1-10	J	I	Node number--corner
	11-20	CORD(J,1)	R	X-coordinate
	21-30	CORD(J,2)	R	Z-coordinate
	31-40	AO(J)	R	Bottom elevation at node J (ft)

(The coordinate values read from Card 23 are multiplied by the appropriate scale factors, XSCALE and ZSCALE, and should result in the proper X and Z coordinates (ft) after transformation.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
24:NXZL**	1-5	NA	I	A corner node at one end of a straight line segment
	6-10	NB	I	The corner node at the other end of a straight line segment

**Input optional: controlled by values specified on Card 19.

INPUT CARD FORMATS (Cont.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
24:NXZL	11-80 (14@5)	NIP	I	Up to 14 corner nodes between nodes NA and NB for which the Z coordinate is to be interpolated using the input values of the X coordinate
25:NZXL**	1-5	NA	I	A corner node at one end of a straight line segment
	6-10	NB	I	A corner node at the other end of a straight line segment
	11-80	NIP	I	Up to 14 corner nodes between nodes NA and NB for which the X coordinate is to be interpolated using the input values of the Z coordinate
26:NP/16**	1-80	NFIXH	I	A complete list of NP node numbers which will be used to reorder the internal formation of the system equations. This feature can be used to achieve more efficient core storage allocation without repunching other existing system data
27:NE	1-5	J	I	Element number
	6-35 (6@5)	NOP(J,K)	I	The six node numbers for element J, listed counterclockwise around the element starting from any corner
	36-40	IMAT(J)	I	Element type (the number entered here corresponds to the parameters specified in the ORT array, defined above)

(A card must be entered for all element numbers even if they are missing from the network. Enter the missing element number in columns 1-5 and a zero in column 40.)

**Input optional: controlled by values specified on Card 19.

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
28:NCL**	1-80 (16@5)	LINE(J,K)	I	Lists of node numbers which define line segments across which total flow is to be computed for continuity checking. Please note that the continuity checker assumes the sequence if the input list is positive, and thus the sign of computed flow will depend on the sequence of input values. Up to 16 values may be specified.
29:NBX	1-10	J	I	Node number
	16	NFIX(J)	I	Enter 1 if the X-direction flow is to have an exact boundary condition; otherwise leave blank
	17	NFIX(J)	I	Enter 1 if the Z-direction flow is to have an exact boundary condition; otherwise leave blank
	18	NFIX(J)	I	Enter 1 if the water surface elevation (head) is to have an exact boundary condition; enter 2 if the head is to have an exit boundary condition; otherwise leave blank
	19	NFIX(J)	I	Enter 1 if the density is to have an exact boundary condition; otherwise leave blank
	20	NFIX(J)	I	Enter 1 if the temperature is to have an exact boundary condition; otherwise leave blank
				NOTE: If zero is entered in column 16 and 1 in column 17, flow will be allowed to move parallel to the boundary at this node

**Input optional: controlled by values specified on Card 19.

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
29:NBX	21-30	SPEC(J,1)	R	The specified X-direction flow (ft^3/sec/ft)
	31-40	SPEC(J,2)	R	The specified Z-direction flow (ft^3/sec/ft)
	41-50	SPEC(J,3)	R	The specified water surface elevation (ft)
	51-60	SPEC(J,4)	R	The specified density (slugs/ ft^3)
	61-70	SPEC(J,5)	R	The specified temperature (°C)
(If a node number is missing from the network system, a card with the missing node number in columns 1-10 and 1 in columns 16-20 should be entered.)				
30:1**	1-10	ATMOS(1)	R	Net shortwave solar radiation (Kcal/m^2/sec)
	11-20	ATMOS(2)	R	Net longwave solar radiation (Kcal/m^2/sec)
	21-30	ATMOS(3)	R	Dry bulb air temperature (°F)
	31-40	ATMOS(4)	R	Relative humidity (%)
	41-50	ATMOS(5)	R	Average wind speed (mi/hr)
(The above data are to be input only if NDF = 5.)				
31:NP**	1-10	J	I	Node number
	11-20	SIGMA(J,1)	R	Wind velocity at node J (mi/hr)
	21-30	SIGMA(J,2)	R	Angle between the wind velocity and the X-axis (deg.)
(The wind values read here are used only for the calculation of the wind effect on fluid motion and are input only if IWIND = 1.)				
32:NP**	1-10	J	I	Node number
	11-20	VEL(1,J)	R	Initial value of X-velocity (ft/sec)

**Input optional: controlled by values specified on Card 19.

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
32:NP	21-30	VEL(2,J)	R	Initial value of Z-velocity (ft/sec)
	31-40	VEL(3,J)	R	Initial value of water surface elevation (ft)

(Input Card 32 will be read only if NITI ≤ 0.)

(Omit Card 33 if TIT = CYCLE.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
33:1	1-10	NOVTOP	I	Number of overtopping nodes for single discharge option

(Omit Card 34 if NOVTOP = 0.)

Card Sequence: No. of Cards	Card Columns	FORTRAN Name	FORTRAN Type	Description of Input Value
34:NOVTOP	1-10	NVP(1,K)	I	Upstream embankment node K
	11-20	NDWN(1,K)	I	Downstream embankment node K
	21-30	WELEV(1,K)	R	Water surface elevation K
	31-40	WCF(1,K)	R	Weir coefficient K
	41-50	QOV1(K)	R	Initial estimate of overtopping flow (cfs) at section K
	51-60	NVPP(1,K)	I	Upstream corner node K
	61-70	NDWNN(1,K)	I	Downstream corner node K

DEFINITION OF VARIABLES

FORTRAN Variable Name	COMMON Block	Definition
AEVAP	BLKF	An evaporation coefficient
AFACT(J)	BLKC	Coefficients of the coordinates for numerical integration along lines
ALFA(J)	BLKB	Slope of boundary at node J
ALTM	BLKA	Multiplier for time stepping scheme
AO(J)	BLKB	Bottom elevation at node J
APRESS	BLKF	The average atmospheric pressure
ATEMP(J,K)	BLKC	Coefficients of the triangular coordinates for numerical integration of areas
ATMOS(J)	BLKF	The current values of the meteorological inputs
BEVAP	BLKF	An evaporation coefficient
CHI	BLKA	Empirical coefficient in wind stress term
CORD(J,K)	BLKB	X and Z coordinates for node J
DELT	BLKA	Time step for dynamic problems
EAVG(J)*	blank	A temporary array of the average values of solution corrections for variable J
EMAX(J)*	blank	A temporary array of the maximum values of solution corrections for variable J
ESTIFM(J,K)	BLKE	The element coefficient matrix
F(J)	BLKE	The element RHS vector
FACT	BLKA	Relaxation coefficient in Newton-Raphson scheme
GRAV	BLKA	The gravitational constant
HEAT(J,K)	BLKF	Coefficients used in the linearization of the heat exchange at the air-water interface

*Indicates variable which is printed in output.

DEFINITION OF VARIABLES (Cont.)

FORTRAN Variable Name	COMMON Block	Definition
HFACT(J)	BLKC	Weighting factor for numerical integration along lines
ICYC*	BLKA	Current time step for dynamic problems
IMAT(J)	BLKB	The type identifier for element J
LI	BLKA	The maximum half bandwidth for the global coefficient matrix
LINE(J,K)	BLKD	A list of K nodes in line J along which continuity is to be checked
LMT(K)	BLKD	The number of nodes in continuity line J
MBAND	BLKA	Semi-bandwidth of global coefficient matrix
MFIX(J)	blank	A temporary array of the fixed conditions for input boundary conditions
NB	BLKA	Not used
NBC(J,K)	BLKB	Equation number for the K^{th} variable at node J
NCBC	BLKA	Number of time steps between inputs of new boundary conditions for dynamic problems
NCL	BLKA	Number of lines for continuity checking
NCN	BLKA	Number of corner nodes
NDF	BLKA	Number of Degrees of Freedom
NE	BLKA	Maximum element number
NFIXH(J)	BLKB	On input the numbers for node reordering; later the fix values for dynamic boundary conditions
NIP(J)	blank	A temporary array of nodes defining a line along which coordinate interpolation is to be made
NITI	BLKA	Iteration control for first solution
NLL	BLKA	Not used
NMAT	BLKA	Number of different types of elements
NOP(J,K)	BLKB	The nodes around element J
NOPT	BLKA	Size of element coefficient matrix

*Indicates variable which is printed in output.

DEFINITION OF VARIABLES (Cont.)

FORTRAN Variable Name	COMMON Block	Definition
NP	BLKA	Number of system nodes
NPR*	BLKA	Controlling index on iteration loop
NSZF	BLKA	Total number of system equations
OMEGA	BLKA	Constant part of Coriolis terms
ORT(J,K)	BLKB	The characteristics of elements of type J
R1(J)	blank	Before solution the RHS vector; after solution the vector of solution corrections
ROAVG	BLKA	Local fluid density
RPERM(J)	BLKB	The RHS vector containing the influence of external surface integrals
SIGMA(J,K)	BLKB	X and Z wind stress values at node J
SK(J)	blank	The global coefficient matrix
SLOAD(J)	BLKC	A sign correction factor for the numerical integration of the head surface integrals
SPEC(J,K)	BLKB	Boundary condition K at node J
SPECH(J)	blank	A temporary array of exit boundary conditions
TET*	BLKA	Current value of time in dynamic problems
TITLE(J)*	BLKA	Input title comment
TMAX	BLKA	Maximum time for dynamic problems
URF	BLKA	Relaxation coefficient in Newton-Raphson scheme
VEL(K,J)*	BLKB	Current value of the K^{th} variable at node J
VDOT(K,J)	BLKB	Value of the K^{th} time derivative at node J
VOLD(K,J)	BLKB	Value of the K^{th} variable at node J at the beginning of the time step
WAIT(J)	BLKC	Weighting factor for numerical integration of areas
XAREA(J)	blank	A temporary array of element areas
XVEL(J,K)*	blank	A temporary array of the state variables in output format

*Indicates variable which is printed in output.

DEFINITION OF VARIABLES (Cont.)

FORTRAN Variable Name	COMMON Block	Definition
IFLG	BLKTK	Flag to indicate CYCLE or NOCYCLE option
IIN(I)	BLKTK	Input tape file for bridge configuration data
FLEN(I)	BLKTK	Bridge lengths of designs considered
NLEN	BLKTK	Number of bridge lengths plus one
FLFR(I,1)	BLKTK	Discharge (cfs) I
FLFR(I,2)	BLKTK	Return frequency of discharge I
NFLFR	BLKTK	Number of discharges considered
EMB(JJ)	BLKTK	Embankment elevation (ft)
NEMB	BLKTK	Number of embankments
NNDS(J)	BLKTK	Number of overtopping nodes for each bridge configuration
NVPP(I,J)	BLKTK	Upstream corner nodes for bridge configuration I
NDWN(I,J)	BLKTK	Downstream embankment nodes for bridge configuration I
NNDSUP	BLKTK	Number of upstream boundary nodes
NNDSD	BLKTK	Number of downstream boundary nodes
INORM	BLKTK	Indicator variable to flag overtopping option
WELEV(I,J)	BLKTK	Bottom embankment elevation (ft) for bridge configuration I
WELEV1(K)	BLKTK	Bottom embankment elevation (ft) for one configuration case
WCF(I,J)	BLKTK	Weir coefficients for section J for bridge configuration I
NVP(I,J)	BLKTK	Upstream embankment nodes for bridge configuration I
ICK*	BLKTK	Overtopping variable to indicate overtopping has occurred
NDWNN(I,J)	BLKTK	Downstream corner nodes for bridge configuration I

*Indicates variable which is printed in output.

DEFINITION OF VARIABLES (Cont.)

FORTRAN Variable Name	COMMON Block	Definition
NI	BLKTK	Card input read tape number
NO	BLKTK	Output tape number for system output
QOV1(K)	BLKTK	Overtopping flow in embankment section K
BXV(I,J)	BLKTK	X velocity (ft/sec) at upstream node J for bridge configuration I
BZV(I,J)	BLKTK	Z velocity (ft/sec) at upstream node J for bridge configuration I
HTI(I,J)	BLKTK	Downstream stage (ft) for node J for bridge configuration I
KNDSUP(I,J)	BLKTK	Upstream boundary condition node for bridge configuration I
KNNDS(I,J)	BLKTK	Downstream boundary condition node for bridge configuration I
NOVTOP*	BLKTK	Number of overtopping nodes for single discharge case
JMN	BLKTK	Number of the bridge configuration being considered
IND	BLKTK	Set to 1 if overtopping option is exercised; otherwise set to 0
DEMBK	BLKTK	Embankment height (ft) for single discharge case

*Indicates variable which is printed in output.

FINITE ELEMENT MODEL

PROGRAM LISTING

```
C      PROGRAM FEM1(INPUT=128,OUTPUT=128,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=1
C     228)
C  CONTROL MAIN PROGRAM
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKF/ ATMOS(10),HEAT(6,2),AEVAP,BEVAP,APRESS,A1,A2,A3
      COMMON/BLKTK/IFLG,IIN(5),FLEN(5),NLEN,FLFR(6,2),NFLFR,EMB(5),NEMB,
     1NNDS(4),NUPP(4,40),NDWN(4,40),NNDSUP,NNDSD,INORM,
     2WELEV(4,40),WELV1(40),WCF(4,40),NUP(4,40),ICK,NDWNN(4,40),
     3NI,NO,QOV1(40),BXV(6,10),BZV(6,10),HTI(6,10),KNDSUP(5,10),
     4KNNDS(5,10),NOVTOP,JMN,IND,DEMBK
      COMMON R1(500),SK(60000)
      DATA MAXROW/500/,MAXSTO/60000/,IERR/0/
      ICK=0
C-
C-.....INPUT DATA FOR INITIAL SOLUTION.....
C-
      CALL INPUT1
      IND=0
      IF (IFLG) 10,500,10                                               00024000
10    CALL LOAD
      CALL ERRCHK(NSZF,MBAND)
      NTEQ=NSZF
      WRITE(7) IIN,FLEN,NLEN,FLFR,NFLFR,EMB,NEMB,NNDS,NDSUP,NNDSUP,NUPP
     1,NDWN,WELEV,WCF,NUP,NDWNN
      DO 180 J=1,NFLFR
      IF(J.EQ.1) GO TO 160
      DO 30 K=1,NNDSUP
      L=KNDSUP(1,K)
      SPEC(L,1)=BXV(J,K)
30    SPEC(L,2)=BZV(J,K)
      DO 35 K=1,NNDSD
      L=KNNDS(1.K)
35    SPEC(L,3)=HTI(J,K)
      CALL INPUT4
      CALL LOAD2
160   IDP=2
      DO 170 NPR=1,NITI
      CALL FANDS(R1,SK,NTEQ)
      CALL OUTPUT(IDP)
      CALL CHECK
170   CONTINUE
180   CONTINUE
      IF(INORM.EQ.1) STOP
      IND=1
      NOVTOP=0
      DO 200 J=2,NLEN
      JMN=J-1
      IN=IIN(J)
      REWIND IN
```

```
      CALL INPUT3
      CALL LOAD
      JNOV=NNDS(JMN)
      DO 195 JJ=1,NEMB
      DO 197 J1=1,NFLFR
      DO 31 K=1,NNDSUP
      L=KNDSUP(J,K)
      SPEC(L,1)=BXV(J1,K)
31    SPEC(L,2)=BZV(J1,K)
      DO 32 K=1,NNDSD
      L=KNNDS(J,K)
32    SPEC(L,3)=HTI(J1,K)
      CALL ERRCHK(NSZF,MBAND)
      NTEQ=NSZF
      IF(NITI.LT.0) STOP 40
      IDP=2
      DO 42 K=1,JNOV
42    WELV1(K)=WELEV(JMN,K)+EMB(JJ)
      DO 40 I=1,40
40    QOV1(I)=0.
      WRITE(NO,5020) (NUP(JMN,K),NDWN(JMN,K),WELV1(K),WCF(JMN,K),
     1QOV1(K),NUPP(JMN,K),NDWNN(JMN,K),K=1,JNOV)
      DO 188 K=1,NITI
      NPR=K
      DO 190 K1=1,JNOV
      N=NUP(JMN,K1)
      M=NDWN(JMN,K1)
      VEL(1,N)=0.
      VEL(1,M)=0.
      IF(NBC(N,2).EQ.0) VEL(2,N)=0.
      IF(NBC(M,2).EQ.0) VEL(2,M)=0.
      N1=NUPP(JMN,K1)
      N2=NDWNN(JMN,K1)
      Q=TOPPER(VEL(3,N),WELV1(K),VEL(3,M),WCF(JMN,K1),AO(N),
     1AO(M),QOV1(K1))
      IF(Q.LT.0.) Q=0.
      VEL(1,N)=Q
      VEL(1,M)=Q
      IF(N1.NE.0)VEL(2,N)=(CORD(N,2)-CORD(N1,2))/
     1 (CORD(N,1)-CORD(N1,1))*VEL(1,N)
      IF(N2.NE.0)VEL(2,M)=(CORD(N2,2)-CORD(M,2))/
     1 (CORD(N2,1)-CORD(M,1))*VEL(1,M)
      QC=QOV1(K1)-Q
      WRITE(NO,6070) N,VEL(1,N),QC
      QOV1(K1)=VEL(1,N)
190   CONTINUE
      CALL FANDS(R1,SK,NTEQ)
      CALL OUTPUT(IDP)
      CALL CHECK
188   CONTINUE
197   CONTINUE
195   CONTINUE
200   CONTINUE
C-
```

```
C-.....SETUP EQUATIONS AND CHECK SIZE ALLOCATIONS.....
C-
500   CALL LOAD
      CALL ERRCHK(NSZF,MBAND)
C-
C-.....PERFORM INITIAL STEP SOLUTION.....
C-
      NTEQ = NSZF
      IF(NITI.GT.0) GO TO 161
      IDP = 0
      CALL OUTPUT( IDP )
      GO TO 181
161   IDP=2
      READ(NI,5010) NOVTOP
      IF(NOVTOP.EQ.0) GO TO 163
      IND=1
      DO 45 I=1,40
45    QOV1(I)=0.
      READ(NI,5020) (NUP(1,K),NDWN(1,K),WELEV(1,K),WCF(1,K),
     1QOV1(K),NUPP(1,K),NDWNN(1,K),K=1,NOVTOP)
      DO 999 K=1,NOVTOP
999   WELEV(1,K)=WELEV(1,K)+DEMBK
      WRITE(NO,6020) (K,NUP(1,K),NDWN(1,K),WELEV(1,K),WCF(1,K),QOV1(K),
     1NUPP(1,K),NDWNN(1,K),K=1,NOVTOP)
163   DO 171  NPR=1,NITI
      IF(NOVTOP.EQ.0) GO TO 166
      WRITE(NO,6070) NPR
      DO 165 K=1,NOVTOP
      N=NUP(1,K)
      M=NDWN(1,K)
      VEL(1,N)=0.
      VEL(1,M)=0.
      IF(NBC(N,2).EQ.0) VEL(2,N)=0.
      IF(NBC(M,2).EQ.0) VEL(2,M)=0.
      N1=NUPP(1,K)
      N2=NDWNN(1,K)
      Q=TOPPER(VEL(3,N),WELEV(1,K),VEL(3,M),WCF(1,K),AO(N),
     1 AO(M),QOV1(K))
      IF(Q.LT. 0.) Q=0.
      VEL(1,N)=Q
      VEL(1,M)=Q
      IF(N1.NE.0) VEL(2,N)=(CORD(N,2)-CORD(N1,2))/(CORD(N,1)-CORD(N1,1))
     1      *VEL(1,N)
      IF(N2.NE.0) VEL(2,M)=(CORD(N2,2)-CORD(M,2))/(CORD(N2,1)-CORD(M,1))
     1      *VEL(1,M)
      QC=Q-QOV1(K)
      WRITE(NO,6080) N,VEL(1,N),QC
      QOV1(K)=VEL(1,N)
  165 CONTINUE
166   CALL FANDS(R1,SK,NTEQ)
      CALL OUTPUT( IDP )
      CALL CHECK
171   CONTINUE
C-
```

```
C-.....SETUP FOR TIME SOLUTION.....
C-
181    IF(TMAX.LT.1.0E-4) STOP 140
C-
C-.....ERROR OUTPUT FORMATS.....
C-
 5010 FORMAT(I10)
6020  FORMAT(//,4X,'EMBANKMENT',7X,'UPSTREAM',11X,'DOWNSTREAM',8X,'EMBAN
     1KMENT',6X,'WEIR',9X,'INITIAL',7X,'CORNER NODES',/,
     35X,'SECTION',6X,'OVERTOPPING NODES',3X,'OVERTOPPING NODES',
     35X,'HEIGHT',6X,'COEFFICIENT',5X,'FLOW',6X,'UPSTREAM DOWNSTREAM',
     5/,I9,9X,I10,10X,I10,10X,F10.3,2X,F10.3,2X,F10.3,8X,I5,I9)
 5020 FORMAT(2I10,3F10.3,2I10)
6070  FORMAT(//,5X,'OVERTOPPING INFORMATION FOR CYCLE',I5,/)
6080  FORMAT(5X,'OVERFLOW AT NODE',I5,/,5X,'FLOW',F8.4,/,5X,
     1'FLOW CHANGE THIS CYCLE',F8.4)
      END
```

```
      SUBROUTINE ERRCHK(L1,L2)
      DATA MAXROW/500/,MAXSTO/60000/,IERR/0/
      NO=6
      ISTO=L1*L2
      WRITE(NO,6060) L1,L2,ISTO
 6060 FORMAT( // 10X, 'PROBLEM SIZE IS',I5, ' EQUATIONS WITH A BAND WID
     1H OF', I5, ' COLUMNS AND TOTAL STORAGE OF',I7 )
      IF(L1.LE.MAXROW) GO TO 100
      IERR=1
      WRITE(NO,6050) MAXROW,L1
 6050 FORMAT( // 10X, '-----ERROR.....TOO MANY EQUATIONS..RAISE NFIX ANI
     1 RPERM IN ABLK FROM',I6, ' TO AT LEAST',I6 )
100   IF(ISTO.LE.MAXSTO) GO TO 110
      IERR=1
      WRITE(NO,6055) MAXSTO,ISTO
 6055 FORMAT( // 10X, '-----ERROR.....INSUFFICIENT BLANK COMMON..RAISE S
     2K FROM',I10, ' TO AT LEAST',I10 )
110   IF(IERR.EQ.1) STOP 111
      RETURN
      END
      FUNCTION TOPPER(HU,HR,HD,CF,B,B1,QOLD)
      DIMENSION X(7),Y(7)
      DATA X/0.,82.,89.,94.,97.,99.,100./
      DATA Y/1.,.99,.94,.82,.62,.45,0./
      DWN=HD+B1
      UP=HU+B
      IF(UP-HR) 300,300,10
10    IF(DWN-HR) 290,290,20
20    RATIO=(DWN-HR)/(UP-HR)
      IF(RATIO-.75) 290,290,30
30    DELH=RATIO*100.
      DO 40 J=1,6
      I=7-J
40    IF(DELH.LE. X(I)) K=I
      FK=Y(K)+(X(K)-DELH)/(X(K)-X(K-1))*(Y(K-1)-Y(K))
      TOPPER=(FK*CF*((UP-HR)**1.5)-QOLD)*.6+QOLD
      GO TO 310
290   TOPPER=(CF*((UP-HR)**1.5)-QOLD)*.6+QOLD
      GO TO 310
300   TOPPER=0.
310   RETURN
      END
```

```
      SUBROUTINE INPUT1
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKD/ LINE(10,16),LMT(10)
      COMMON NIP(14),XAREA(500)
      COMMON/BLKTK/IFLG,IIN(5),FLEN(5),NLEN,FLFR(6,2),NFLFR,EMB(5),NEMB,
     1NNDS(4),NUPP(4,40),NDWN(4,40),NNDSUP,NNDSD,INORM,
     2WELEV(4,40),WELV1(40),WCF(4,40),NUP(4,40),ICK,NDWNN(4,40),
     3NI,NO,QOV1(40),BXV(6,10),BZV(6,10),HTI(6,10),KNDSUP(5,10),
     4KNNDS(5,10),NOVTOP,JMN,IND,DEMBK
      DIMENSION TIT(20),TIT5(5)
      DATA CYCL/4HCYCL/
      DATA NORM/4HNORM/
      IFLG=0
      NI=5
      NO=6
      READ(NI,7000) TIT
 7000 FORMAT(20A4)
      IF(TIT(1).EQ.CYCL) IFLG=1
      IF (IFLG) 300,15,300                                              00245000
  300 READ(NI,1) INORM
    1 FORMAT(10I5)
      READ(NI,7010) TIT5,NLEN,(IIN(I),I=1,NLEN)
 7010 FORMAT(5A4,6I5)
      READ(NI,7030) TIT5,(FLEN(I),I=1,NLEN)
 7030 FORMAT(5A4,5E10.0)
      READ(NI,7010) TIT5,NFLFR
      DO 310 I=1,NFLFR
  310 READ(NI,7020) FLFR(I,1),FLFR(I,2)
 7020 FORMAT(2F10.0)
      READ(NI,7040) TIT5,NEMB
 7040 FORMAT(5A4,I5)
      READ(NI,7025) (EMB(I),I=1,NEMB)
 7025 FORMAT(8F10.0)
      IF(INORM.EQ.1) GO TO 10
      IK=NLEN-1
      DO 320 I=1,IK
      READ(NI,7027) TIT5,NNDS(I)
      L1=NNDS(I)
      DO 315 J=1,L1
 7027 FORMAT(5A4,I5)
  315 READ(NI,7029) NUP(I,J),NDWN(I,J),WELEV(I,J),WCF(I,J),
     1 NUPP(I,J),NDWNN(I,J)
  320 CONTINUE
 7029 FORMAT(2I10,2F10.0,2I10)
   10 READ(NI,1) NNDSUP,NNDSD
      DO 1000 I=1,NLEN
      READ(NI,1) (KNDSUP(I,J),J=1,NNDSUP)
      READ(NI,1) (KNNDS(I,J),J=1,NNDSD)
 1000 CONTINUE
```

```
      DO 1001 I=1,NFLFR
      READ(NI,3) (BXV(I,J),J=1,NNDSUP)
3     FORMAT((8F10.4))
      READ(NI,3) (BZV(I,J),J=1,NNDSUP)
      READ(NI,3) (HTI(I,J),J=1,NNDSD)
1001  CONTINUE
C******DATA ECHO **********
      WRITE(NO,6000)
      WRITE(NO,7070)
7070  FORMAT(1H0,40X,'    CYCLE OPTION ')
      WRITE(NO,7080) (IIN(I),FLEN(I),I=1,NLEN)
7080  FORMAT(//,30X,'  INPUT TAPES   ',20X,
     1' BRIDGE LENGTH   ',//,5(/,35X,I5,20X,F15.4))
      WRITE(NO,7090) (FLFR(I,1),FLFR(I,2),I=1,NFLFR)
7090  FORMAT(1H0,//,30X, '  FLOW   ',15X,'RETURN INTERVAL',///,
     16(/,20X,F20.4,10X,F15.4))
      WRITE(NO,8000) (EMB(I),I=1,NEMB)
8000  FORMAT(////,40X,'  EMBANKMENT HEIGHTS ',//,
     15(/,33X,5F10.4))
C     WRITES RUN ORDER
      WRITE(NO,6000)
      WRITE(NO,7095)
7095  FORMAT(///,60X,'RUN IDENTIFICATION',
     1  /,10X,'RUN NUMBER',T30,'OPENING WIDTH',T50,'FLOW (CFS/FT)',
     2  T70,'DELT EMBNKMNT HT')
      DO 115 J = 1, NP
      IF( NFIX(J)/1000 .GT. 1 ) ALFA(J) = 0.0
  115 CONTINUE
      IF( IPRT .GT. 0 ) GO TO 500
      ENTRY INPUT4
C
C                                           PRINT INPUT DATA
C
      WRITE(NO,6000)
      WRITE(NO,6005) TITLE
      WRITE(NO,6040)
      WRITE(NO,6041)
      INT = (NP+1)/2
      DO 485 J = 1, INT
      WRITE(NO,6050) (N,CORD(N,1),CORD(N,2),AO(N),NFIXH(N),N=J,NP,INT)
  485 CONTINUE
      WRITE(NO,6000)
      WRITE(NO,6005) TITLE
      WRITE(NO,6060)
      WRITE(NO,6061)
      INT = ( NE + 1 ) / 2
      DO 490 J = 1, INT
      WRITE(NO,6070) (N,(NOP(N,M),M=1,6),IMAT(N),XAREA(N),N=J,NE,INT)
  490 CONTINUE
      WRITE(NO,6000)
      WRITE(NO,6005) TITLE
      WRITE(NO,6080)
      WRITE(NO,6081)
      INT = ( NP + 1 ) / 2
      DO 495 J = 1, INT
      WRITE(NO,6090) (N,NFIX(N),(SPEC(N,M),M=1,3),N=J,NP,INT)
  495 CONTINUE
  500 CONTINUE
```

```
      DO 515 J = 1, NCL
      NA = LMT(J)
      WRITE(NO,6125) J,(LINE(J,K),K=1,NA)
  515 CONTINUE
  520 CONTINUE
C-
C-..... INITIALIZE FOR BOUNDARY CONDITIONS.....
C-
      DO 550 J = 1, NP
      IF( SPEC(J,3) .GT. 0.0 ) SPEC(J,3) = SPEC(J,3) - AO(J)
      DO 545 K = 1, NDF
      IF( SPEC(J,K) .NE. 0.0 ) SPEC(J,K) = SPEC(J,K) - VEL(K,J)
  545 CONTINUE
  550 CONTINUE
      RETURN
C-
C-.....ENTRY FOR TINE DEPENDENT INPUT DATA.....
C-
C-
C-.....INPUT DATA CARD FORMATS.....
C-
 5005 FORMAT( 20A4 )
 5010 FORMAT( 16I5 )
 5020 FORMAT( 8E10.0 )
 5025 FORMAT( 3I5, 2E10.0 )
 5030 FORMAT( I10, 5E10.0 )
 5035 FORMAT( I10, 3E10.0 )
 5040 FORMAT( 8I5 )
 5050 FORMAT(2I10,3E10.0)
 5060 FORMAT(4F10.1)
6000  FORMAT(1H1 / 10X,'TWO-DIMENSIONAL FINITE ELEMENT BACKWATER MODEL',
     1/,10X,'DEVELOPED FOR THE FEDERAL HIGHWAY ADMINISTRATION',/,10X,
     2'BY WATER RESOURCES ENGINEERS, SPRINGFIELD,VA.',/,10X,
     3'AUGUST 1974........')
 6005 FORMAT( / 5X, 20A4 )
 6010 FORMAT( // 2X, 'RUN CONTROL PARAMETERS' //
     1  5X, 'TOTAL NODES',I10 / 5X, 'TOTAL ELTS',I11 / 5X, 'ELEMENT TYPE
     1S',I8 / 5X, 'CORNER NODES',I9 /
     1        5X, 'BOUNDARY NODES',I7 / 5X, 'PRINT OPTION',I9 / 5X, 'X-Z
     2INTERP',I11 / 5X, 'Z-X INTERP',I11 / 5X, 'CONT CHECKS',I10 /
     3  5X, 'WIND FIELD',I11 / 5X, 'DEG OF FREEDOM',I7 /
     4  5X, 'NODE REORDER',I9 )
 6020 FORMAT( / 5X, 'AVG LAT(DEG)', F13.2 / 5X, 'AVG WS ELEV(FT)',F10.2/
     1  5X, 'AVG TEMP(DEG C)', F10.2 / 5X, 'X-SCALE FACTOR', F11.2 /
     2  5X, 'Z-SCALE FACTOR', F11.2 )
 6025 FORMAT( // 2X, 'TIME AND ITERATION CONTROL' //
     1  5X, 'CYCLES-FIRST ITERATION',I7 / 5X, 'CYCLES-NEXT ITERATIONS',
     2  I7 / 5X, 'TIME STEPS/DATA UPDATE',I7, / 5X, 'TIME INTERVAL(HOURS
     3)',F9.2/ 5X, 'TOTAL RUN TIME(HOURS)',F8.2 )
 6030 FORMAT( // 5X, 'ELEMENT CHACTERISTICS' //
     1  9X, 'NUMBER      X EDDY VIS      Z EDDY VIS          X DIFF
     2 Z DIFF            CHEZY' / 18X, '(LB-SEC/FT2)    (LB-SEC/FT2)       (
     3FT2/SEC)       (FT2/SEC)'  / ( I15, 1P5E15.3 ) )
 6040 FORMAT(   / 10X, 'FIXED NODAL COORDINATES AND ELEVATIONS.....' )
```

```
 6041 FORMAT( / 8X,'NODE      X-LOC      Z-LOC       ELEV      ORDER
     1NODE      X-LOC      Z-LOC       ELEV      ORDER' / 16X, '(FEET)     (F
     2EET)     (FEET)', 26X, '(FEET)     (FEET)     (FEET)' )
 6050 FORMAT( 2( I12, 2F10.2, F10.1, I10 ) )
 6060 FORMAT(  /  10X, 'NODAL CONNECTIONS AND MATERIAL NUMBERS....' )
 6061 FORMAT( / 3X, 'ELEMENT    NODES(COUNTERCLOCKWISE)      TYPE  AREA(F
     1T2)     ELEMENT    NODES(COUNTERCLOCKWISE)      TYPE  AREA(FT2)' )
 6070 FORMAT( 2( I10,7I5,F12.2,2X  ) )
 6080 FORMAT(  // 10X, 'BOUNDARY CONDITIONS.....'    )
6081  FORMAT(/3X,'NODE       FIX     X-FLOW     Z-FLOW       ELEV
     1                NODE     FIX     X-FLOW     Z-FLOW       ELEV',
     2/16X,'(CFS/FT)  (CFS/FT)     (FEET)
     3          (CFS/FT)  (CFS/FT)     (FEET)')
 6090 FORMAT(2(2I7,3F10.2,20X))
 6115 FORMAT( /// 10X, 'NO CONTINUNITY CHECKES REQUESTED.....' )
 6120 FORMAT(   //// 10X, 'CONTINUNITY CHECKS TO BE MADE ALONG THE FOLLO
     1WING LINES'  // 6X, 'LINE       NODES' )
 6125 FORMAT( I10, 4X, 16I6 )
      END
```

```
      SUBROUTINE OUTPUT( IDP )
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKTK/IFLG,IIN(5),FLEN(5),NLEN,FLFR(6,2),NFLFR,EMB(5),NEMB,
     1NNDS(4),NUPP(4,40),NDWN(4,40),NNDSUP,NNDSD,INORM,
     2WELEV(4,40),WELV1(40),WCF(4,40),NUP(4,40),
     3ICK,NDWNN(4,40),NI,NO,QOV1(40),BXV(6,10),BZV(6,10),HTI(6,10),
     4KNDSUP(5,10),KNNDS(5,10),NOVTOP,JMN,IND,DEMBK
      COMMON R1(500),XVEL(5,250),EMAX(5),EAVG(5)
      DIMENSION IVAR(5,2)
      DATA IVAR/4H(X-F,4H(Z-F,4H(DEP,4H(DEN,4H(TEM,
     1  4HLOW),4HLOW),4HTH) ,4HSTY),4HP)
C-
C-.....SETUP FOR SOLUTION CORRECTIONS.....
C-
      DO 100 J = 1, NDF
      EAVG(J) = 0.0
      EMAX(J) = 0.0
  100 CONTINUE
      IF( IDP .EQ. 0 ) GO TO 180
      FACT = 1.00
      IF( NPR .GT. 1 ) GO TO 115
      DO 110 J = 1, NP
      DO 105 K = 1, NDF
      VEL(K,J) = SPEC(J,K) + VEL(K,J)
  105 CONTINUE
  110 CONTINUE
  115 IF( NPR .GT. 1 ) FACT = URF
C-
C-.....COMPUTE SOLUTION CORRECTIONS....
C-
      DO 160 K = 1, NDF
      EMAX(K) = 0.0
      EAVG(K) = 0.0
      COUNT = 0.0
      DO 150 J = 1, NP
      IF( NBC(J,K) ) 150, 150, 125
  125 I = NBC(J,K)
      COUNT = COUNT + 1.0
      EX = FACT * R1(I)
      AEX = ABS( EX )
      EAVG(K) = EAVG(K) + AEX
      IF( AEX .GT. EMAX(K) ) EMAX(K) = AEX
      IF( K .GT. 2 ) GO TO 140
      IF( ALFA(J)  ) 130, 140,130
  130 VEL(K,J) = VEL(K,J) + EX*COS( ALFA(J) )
      VEL(K+1,J) = VEL(K+1,J) + EX*SIN( ALFA(J) )
      GO TO 150
  140 VEL(K,J) = VEL(K,J) + EX
  150 CONTINUE
```

```
      EAVG(K) = EAVG(K) / COUNT
  160 CONTINUE
      IF( IDP .EQ. 1 ) RETURN
C-
C-.....OUTPUT RESULTS.....
C-
  180 WRITE(6,6000)
      WRITE(6,6001) TITLE
      WRITE(6,6003) ICYC, TET, NPR
      WRITE(6,6005)
      DO 200 J = 1, NDF
      WRITE(6,6010) J,EAVG(J),EMAX(J),IVAR(J,1),IVAR(J,2)
  200 CONTINUE
      WRITE(6,6015)
      INT = (NP+1)/2
C-
C-.....COMPUTE VALUES FOR SECONDARY OUTPUT.....
C-
      DO 233 J = 1, NP
      DO 230 K = 1, 5
      XVEL(K,J) = 0.0
  230 CONTINUE
  233 CONTINUE
      DO 240 J = 1, NE
      IF( IMAT(J) .EQ. 0 ) GO TO 240
      DO 235 K = 2, 6, 2
      N1 = NOP(J,K-1)
      N2 = NOP(J,K)
      N3 = MOD(K+1,6)
      N3 = NOP(J,N3)
      VEL(3,N2) = 0.5*( VEL(3,N1) + VEL(3,N3) )
      XVEL(3,N1) = VEL(3,N1)
      XVEL(3,N2) = 0.5*( VEL(3,N1) + VEL(3,N3) )
      XVEL(3,N3) = VEL(3,N3)
  235 CONTINUE
  240 CONTINUE
      DO 245 J = 1, NP
      IF( XVEL(3,J) .LE. 0.0 ) GO TO 245
      XVEL(1,J) = VEL(1,J) / XVEL(3,J)
      XVEL(2,J) = VEL(2,J) / XVEL(3,J)
      XVEL(3,J) = XVEL(3,J) + AO(J)
  245 CONTINUE
  255 WRITE(6,6018)
      DO 260 I = 1, INT
      WRITE(6,6020) (J,(XVEL(K,J),K=1,2),VEL(3,J),XVEL(3,J),J=I,NP,INT)
  260 CONTINUE
      IF(NITI-NPR) 265,265,280
 265  IF(IND) 270,280,270                                                00721000
270   DO 271 J=1,40
271   IF(QOV1(J).NE.0.) ICK=1
      IF(NOVTOP.EQ.0) GO TO 10
      WRITE(7,1) NOVTOP,ICK,(XVEL(1,NUP(1,N)),XVEL(2,NUP(1,N)),VEL(3,NUP
```

```
     1(1,N)),XVEL(3,NUP(1,N)),N=1,NOVTOP)
1     FORMAT(2I5/(4E12.5))
      GO TO 280
10    KN=NNDS(JMN)
      WRITE(7)KN,ICK,(XVEL(1,NUP(JMN,N)),XVEL(2,NUP(JMN,N)),VEL(3,NUP(JM
     1N,N)),XVEL(3,NUP(JMN,N)),N=1,KN)
      ICK=0
280   RETURN
6000  FORMAT(1H1 / 10X,'TWO-DIMENSIONAL FINITE ELEMENT BACKWATER MODEL',
     1/,10X,'DEVELOPED FOR THE FEDERAL HIGHWAY ADMINISTRATION',/,10X,
     2'BY WATER RESOURCES ENGINEERS, SPRINGFIELD,VA.',/,10X,
     3'AUGUST 1974........')
 6001 FORMAT( / 5X, 20A4 )
 6003 FORMAT( / 5X, 'RESULTS AT THE END OF',I4, ' TIME STEPS...TOTAL TIM
     1E =',F8.2 , ' HOURS....ITERATION CYCLE IS' , I5 )
 6005 FORMAT( // 10X,   'CONVERGENCE PARAMETERS'  // 8X, 'DF         AVG
     1CHG         MAX CHG' )
 6010 FORMAT( I10, 2F15.4, 1X, 2A4 )
6015  FORMAT(//10X,'NODAL VELOCITIES AND HEAD'/)
6018  FORMAT(3X,'NODE      X VEL      Z VEL     DEPTH     ELEV
     1            NODE      X VEL      Z VEL     DEPTH     ELEV',
     2/,12X,'(FPS)      (FPS)      (FT)      (FT)',
     3  32X,'(FPS)      (FPS)      (FT)      (FT)')
6020  FORMAT(2(I7,2F10.3,2F9.2,20X))
      END
```

```
      SUBROUTINE LOAD
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKC/ ATEMP(7,3),WAIT(7),AFACT(4),HFACT(4),SLOAD(2)
      COMMON R1(500),NLEFT(1000),NRIGHT(1000),DL(2),R(3),HTEMP(25)
      COMMON SPECH(500),MFIX(500)
C-
C-.....COPY HEAD SPECS AND FIX BOUNDARY CONDITIONS.....
C-
      NA = 10**(5-NDF)
      DO 195 J = 1, NP
      MFIX(J) = NFIX(J)
      IF( MOD( NFIX(J)/100,10 ) .EQ. 2 ) NFIX(J) = NFIX(J) - 200
  190 NFIX(J) = NFIX(J) / NA
  195 CONTINUE
      DO 196 J = 1, 1000
      NLEFT(J) = 0
      NRIGHT(J) = 0
  196 CONTINUE
C
C                                       FORM DEGREE OF FREEDOM ARRAY
C
      DO 220 N=1,NP
      DO 200 M=1,NDF
      NBC(N,M)=1
      IF(NFIX(N) / (10**(NDF-M)).EQ.0) GO TO 200
      NFIX(N)=NFIX(N)-10**(NDF-M)
      NBC(N,M)=0
  200 CONTINUE
  220 CONTINUE
      DO 250 M=1,NE
      IF( IMAT(M) .EQ. 0 ) GO TO 250
      DO 240 NN = 2, NCN,2
      N=NOP(M,NN)
      NBC(N,3) = 0
  240 CONTINUE
  250 CONTINUE
C
C                                       REARRANGE ARRAY
C
      NSZF=0
      N = 0
      DO 300 NN = 1, NP
      IF( NFIXH(NN) .NE. 0 ) N = NFIXH(NN) - 1
      N = N + 1
      DO 300 M=1,NDF
      IF(NBC(N,M).EQ.0) GO TO 300
      NSZF=NSZF+1
      NBC(N,M)=NSZF
  300 CONTINUE
```

```
      MBAND=0
      LI=0
      DO 400 M=1,NE
      DO 380 I=1,NCN
      III=NOP(M,I)
      DO 380 II=1,NDF
      IROW=NBC(III,II)
      IF(IROW.EQ.0) GO TO 380
      DO 370 J=1,NCN
      JJJ=NOP(M,J)
      DO 370 JJ=1,NDF
      ICOL=NBC(JJJ,JJ)
      IF(ICOL.EQ.0) GO TO 370
      NDIF=ICOL-IROW
      IF(NDIF.GT.NRIGHT(IROW)) NRIGHT(IROW)=NDIF
      IF(NDIF.LT.NLEFT(IROW)) NLEFT(IROW)=NDIF
      IF( IABS(NDIF) - LI ) 370,370,360
  360 LI = IABS(NDIF)
  370 CONTINUE
  380 CONTINUE
  400 CONTINUE
      NFIX(1)=1
      MBAND=NRIGHT(1)+1
      DO 450 N=2,NSZF
      NFIX(N)=1-NLEFT(N)
      IF(NRIGHT(N).LT.NRIGHT(N-1)) NRIGHT(N)=NRIGHT(N-1)-1
      IF(MBAND.LT.NFIX(N)+NRIGHT(N)) MBAND=NFIX(N)+NRIGHT(N)
  450 CONTINUE
      DO 130 J = 1, NP
      NFIXH(J) = MFIX(J)
  130 CONTINUE
      GO TO 145
      ENTRY LOAD2
  145 DO 150 J = 1, NSZF
      R1(J) = 0.0
      RPERM(J) = 0.0
  150 CONTINUE
      DO 160 J = 1, NP
      MFIX(J) = NFIXH(J)
      IF( MOD( NFIXH(J)/100,10 ) .EQ. 2 ) GO TO 155
      NFIXH(J) = 0
      GO TO 160
  155 NFIXH(J) = 1
      SPECH(J) = SPEC(J,3) + VEL(3,J)
      SPEC(J,3) = 0.0
  160 CONTINUE
C-
C-.....COMPUTE EXTERNAL LOAD FORCES.....
C-
      DO 173 LL = 1, NE
      IF( IMAT(LL) .EQ. 0 ) GO TO 173
      N1 = NOP(LL,1)
      N2 = NOP(LL,3)
      N3 = NOP(LL,5)
```

```
      NA = NFIXH(N1) + NFIXH(N2) + NFIXH(N3)
      IF( NA .LE. 1 ) GO TO 173
      DO 171 NN = 1, 5 , 2
      N1 = NOP(LL,NN)
      N2 = NOP(LL,NN+1)
      N3 = MOD(NN+2,6)
      N3 = NOP(LL,N3)
      NA = NFIXH(N1) + NFIXH(N3)
      IF( NA .LE. 1 ) GO TO 171
      H1 = SPECH(N1)
      H3 = SPECH(N3)
      DL(1) = CORD(N3,2) - CORD(N1,2)
      DL(2) = CORD(N3,1) - CORD(N1,1)
      DO 170 M = 1, 2
      DO 166 J = 1, 3
  166 R(J) = 0.0
      TEMP = DL(M)*GRAV/8.0
      DO 168 N = 1, 4
      XL = AFACT(N)
      H = H1 + AFACT(N)*( H3 - H1 )
      HP = TEMP*HFACT(N)*H**4
      R(1) = R(1) + HP*( 2.0*XL**2 - 3.0*XL + 1.0 )
      R(2) = R(2) + HP*4.0*( XL - XL**2 )
      R(3) = R(3) + HP * ( 2.0*XL**2 - XL )
  168 CONTINUE
      NQ = N1
      DO 169 J = 1, 3
      IF( J .EQ. 2 ) NQ = N2
      IF( J .EQ. 3 ) NQ = N3
      IC=NBC(NQ,M)
      IF( ALFA(NQ) .NE. 0.0 ) GO TO 175
      IF(IC.EQ.0) GO TO 169
      R1(IC)=R1(IC)+SLOAD(M)*R(J)
      GO TO 179
  175 N=NBC(NQ,1)
      IF(M.EQ.1) R1(N)=R1(N)+R(J)*COS(ALFA(NQ))*SLOAD(1)
      IF(M.EQ.2) R1(N)=R1(N)+R(J)*SIN(ALFA(NQ))*SLOAD(2)
  179 CONTINUE
  169 CONTINUE
  170 CONTINUE
  171 CONTINUE
  173 CONTINUE
      DO 180 J = 1, NSZF
  180 RPERM(J) = R1(J)
      DO 185 J = 1, NP
      NFIXH(J) = MFIX(J)
  185 CONTINUE
      RETURN                                                          43
      END
```

```
      SUBROUTINE CHECK
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKD/ LINE(10,16),LMT(10)
      IF( NCL .LE. 0 ) RETURN
      WRITE(6,6030)
      DO 180 J = 1, NCL
      SUMX = 0.0
      SUMY = 0.0
      DO 150 K = 1, 13, 2
      IF( LINE(J,K+1) .LE. 0 ) GO TO 160
      NA = LINE(J,K)
      NB = LINE(J,K+1)
      NC = LINE(J,K+2)
      DX = ( CORD(NC,1) - CORD(NA,1) ) / 6.0
      DY = ( CORD(NC,2) - CORD(NA,2) ) / 6.0
      IF(DY.LT.0.) DY=-DY
      IF(DX.LT.0.) DX=-DX
      SUMX = SUMX + DY*( VEL(1,NA) + 4.0*VEL(1,NB) + VEL(1,NC) )
      SUMY = SUMY + DX*( VEL(2,NA) + 4.0*VEL(2,NB) + VEL(2,NC) )
  150 CONTINUE
160   TOTAL=ABS(SUMX)+ABS(SUMY)
      IF( J .EQ. 1 ) REF = TOTAL
      PCT = 100.0*TOTAL/REF
      MX = LMT(J)
      WRITE(6,6035) J,TOTAL,SUMX,SUMY,PCT
  180 CONTINUE
      RETURN
 6030 FORMAT( // 10X, 'CONTINUNITY CHECKS' //  10X, 'LINE          TOTAL
     1         X FLOW          Z FLOW   PERCENT' )
 6035 FORMAT( 10X, I4, 1P3E15.3, 0PF10.1 )
      END
      BLOCK DATA
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKC/ ATEMP(7,3),WAIT(7),AFACT(4),HFACT(4),SLOAD(2)
      DATA ATEMP
     1 /0.33333333,0.05971587,2*0.47014206,0.79742699,2*0.10128651,
     2 0.33333333,0.47014206,0.05971587,0.47014206,0.10128651,
     3 0.79742699,0.10128651,0.33333333,2*0.47014206,0.05971587,
     4 2*0.10128651,0.79742699/
      DATA WAIT/0.225,3*0.13239415,3*0.12593918/
      DATA AFACT/0.0694319,0.3300095,0.6699905,0.9305682/
      DATA HFACT/0.3478548,2*0.6521451,0.3478548/
      DATA SLOAD/-1.0,1.0/,NDF,NCN/3,6/,GRAV/32.2/,ROAVG/1.935/
      DATA CHI/1.4E-6/,FACT/1.00/,URF/1.00/
      DATA ALTM/0.00/,ICYC/0/,TET/0.00/
      DATA A1/1.939938/,A2/5.588599E-5/,A3/-1.108539E-5/
C                                                                       00960000
      END                                                               00961000
```

```
      SUBROUTINE COEFS(NN)
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKC/ ATEMP(7,3),WAIT(7),AFACT(4),HFACT(4),SLOAD(2)
      COMMON/BLKE/ ESTIFM(30,30),F(30)
      COMMON R1(500)
      DIMENSION ACOF(3,3),FEEA(6),FEEB(6),FEEC(6),FEED(6),
     1  XN(6),DNX(6),DNY(6),XM(3),DMX(3),DMY(3)
C-
C-.....SETUP PARAMETERS FOR THIS ELEMENT.....
C-
      NP1 = NOP(NN,1)
      NP2 = NOP(NN,2)
      NP3 = NOP(NN,3)
      NP4 = NOP(NN,4)
      NP5 = NOP(NN,5)
      NP6 = NOP(NN,6)
C-
C-.....INITALIZE NECESSARY LOCATIONS.....
C-
      DO 50 M = 1, NOPT
      F(M) = 0.0
      DO 45 N = 1, NOPT
      ESTIFM(M,N) = 0.0
   45 CONTINUE
   50 CONTINUE
C-
C- COMPUTE CENTROID AND ELEMENT AREA
C-
      XBAR = (CORD(NP1,1) + CORD(NP3,1) + CORD(NP5,1))/3.0
      YBAR = (CORD(NP1,2) + CORD(NP3,2) + CORD(NP5,2))/3.0
      AI = CORD(NP1,1) - XBAR
      AJ = CORD(NP3,1) - XBAR
      AK = CORD(NP5,1) - XBAR
      BI = CORD(NP1,2) - YBAR
      BJ = CORD(NP3,2) - YBAR
      BK = CORD(NP5,2) - YBAR
      AM = (AJ*BK - AK*BJ - AI*BK + AK*BI + AI*BJ - AJ*BI)/2.0
C-
C- COMPUTE SHAPE FUNCTION COEFS.
C-
      ACOF(1,1) = 2.0*AM/3.0
      ACOF(1,2) = ACOF(1,1)
      ACOF(1,3) = ACOF(1,1)
      ACOF(2,1) = BJ - BK
      ACOF(2,2) = BK - BI
      ACOF(2,3) = BI - BJ
      ACOF(3,1) = AK - AJ
      ACOF(3,2) = AI - AK
      ACOF(3,3) = AJ - AI
```

```
C-
C-.....FIND AVERAGE ELEMENT DENSITY.....
C-
      MR = IMAT(NN)
      DCX = ORT(MR,3)
      DCZ = ORT(MR,4)
   75 EPSX = ORT(MR,1)/ROAVG
      EPSZ = ORT(MR,2)/ROAVG
      FFACT = GRAV/ORT(MR,5)**2
      TWOAM = 2.0 * AM
C-
C-.....ENTER NUMERICAL INTEGRATION LOOP..........
C-
   80 DO 500 I = 1, 7
      AMW = AM*WAIT(I)
      XTEMP = ATEMP(I,1)*(CORD(NP1,1)-XBAR)+ATEMP(I,2)*(CORD(NP3,1)
     1 -XBAR)+ATEMP(I,3)*(CORD(NP5,1)-XBAR)
      YTEMP = ATEMP(I,1)*(CORD(NP1,2)-YBAR)+ATEMP(I,2)*(CORD(NP3,2)
     1 - YBAR)+ATEMP(I,3)*(CORD(NP5,2)-YBAR)
C
C.....COMPUTE LINEAR SHAPE FUNCTIONS AND DERIVATIVES.....
C
      DO 90 J = 1, 3
      XM(J) = ( ACOF(1,J) + XTEMP*ACOF(2,J) + YTEMP*ACOF(3,J) )/(2.0*AM)
      DMX(J) = ACOF(2,J) / ( 2.0*AM )
      DMY(J) = ACOF(3,J) / ( 2.0*AM )
   90 CONTINUE
C
C.....COMPUTE THE QUADRATIC SHAPE FUNCTIONS AND DERIVATIVES.....
C
      DO 120 K = 2, 6, 2
      J = K / 2
      L = J + 1
      IF( L .EQ. 4 ) L = 1
      XN(K-1) = XM(J)*( 2.0*XM(J) - 1.0 )
      DNX(K-1) = DMX(J)*( 4.0*XM(J) - 1.0 )
      DNY(K-1) = DMY(J)*( 4.0*XM(J) - 1.0 )
      XN(K) = 4.0*XM(J)*XM(L)
      DNX(K) = 4.0*( XM(J)*DMX(L) + XM(L)*DMX(J) )
      DNY(K) = 4.0*( XM(J)*DMY(L) + XM(L)*DMY(J) )
  120 CONTINUE
C.....COMPUTE R, S, H AND THEIR DERIVATIVES.....
      R = 0.0
      S = 0.0
      DRDX = 0.0
      DRDZ = 0.0
      DSDX = 0.0
      DSDZ = 0.0
      DO 270 M = 1, 6
      MR = NOP(NN,M)
      R = R + XN(M)*VEL(1,MR)
      DRDX = DRDX + DNX(M)*VEL(1,MR)
      DRDZ = DRDZ + DNY(M)*VEL(1,MR)
      S = S + XN(M)*VEL(2,MR)
```

```
      DSDX = DSDX + DNX(M)*VEL(2,MR)
      DSDZ = DSDZ + DNY(M)*VEL(2,MR)
  270 CONTINUE
      H = 0.0
      DHDX = 0.0
      DHDZ = 0.0
      DAODX = 0.0
      DAODZ = 0.0
      GAMMA = 0.0
      DO 275 M = 1, 3
      MC = 2*M - 1
      MR = NOP(NN,MC)
      H = H + XM(M)*VEL(3,MR)
      DHDX = DHDX + DMX(M)*VEL(3,MR)
      DHDZ = DHDZ + DMY(M)*VEL(3,MR)
      DAODX = DAODX + DMX(M)*AO(MR)
      DAODZ = DAODZ + DMY(M)*AO(MR)
  275 CONTINUE
      GHC = GRAV*H**3
      VECQ = SQRT( R**2 + S**2 )
      TFRIC = 0.0
      IF( VECQ .GT. 1.0E-6 ) TFRIC = FFACT / VECQ
      FRN = 0.0
      FSN = 0.0
C.....EVALUATE THE BASIC EQUATIONS WITH PRESENT VALUES.....
C
C.....MOMENTUM TERMS.....
C
  279 FRN = FRN + H*(R*DRDX + S*DRDZ) - R*(R*DHDX + S*DHDZ)
      FSN = FSN + H*(S*DSDZ + R*DSDX) - S*(R*DHDX + S*DHDZ)
C
C.....VISCOUS TERMS.....
C
      FRN = FRN - EPSX*(2.0*R*(DHDX**2 + DHDZ**2) -
     1  2.0*H*(DHDX*DRDX + DHDZ*DRDZ) )
      FRNX = EPSX*(DRDX*H**2 - H*R*DHDX)
      FRNZ = EPSX*(DRDZ*H**2 - H*R*DHDZ)
      FSN = FSN - EPSZ*(2.0*S*(DHDX**2 + DHDZ**2) -
     1  2.0*H*(DHDX*DSDX + DHDZ*DSDZ) )
      FSNX = EPSZ*(DSDX*H**2 - H*S*DHDX)
      FSNZ = EPSZ*(DSDZ*H**2 - H*S*DHDZ)
C
C.....SURFACE AND BOTTOM SLOPE (PRESSURE) TERMS.....
C
      FRN = FRN + GHC*DAODX
      FSN = FSN + GHC*DAODZ
      FRNX = FRNX - 0.25*H*GHC
      FSNZ = FSNZ - 0.25*H*GHC
C
C.....BOTTOM FRICTION TERMS.....
C
      FRN = FRN + FFACT*VECQ*R
      FSN = FSN + FFACT*VECQ*S
C
```

```
C.....CORIOLIS TERMS.....
C
      FRN = FRN - OMEGA*S*H**2
      FSN = FSN + OMEGA*R*H**2
C-
C-.....EVALUATE TERMS FOR VARIABLE DENSITY.....
C-
C
C.....MOTION EQUATIONS.....
C
      DO 285 M = 1, NCN
      IA = 1 + NDF*(M-1)
      F(IA) = F(IA) - AMW*(XN(M)*FRN + DNX(M)*FRNX + DNY(M)*FRNZ)
      IA = IA + 1
      F(IA) = F(IA) - AMW*(XN(M)*FSN + DNX(M)*FSNX + DNY(M)*FSNZ)
  285 CONTINUE
C
C.....CONTINUNITY EQUATION.....
C
      FRN = DRDX + DSDZ
      DO 290 M = 1, 3
      IA = 3 + 2*NDF*(M-1)
      F(IA) = F(IA) - AMW*XM(M)*FRN
  290 CONTINUE
C-
C-.....CONVECTION-DIFFUSION EQUATION FOR DENSITY.....
C-
C
C.....FORM THE X MOTION EQUATIONS.....
C
C.....FLOW TERMS.....
C
      DO 300 M = 1, 6
      WRITE(NO,7096) (NORM,I,FLFR(I,1),I=1,NFLFR)
7096  FORMAT(13X,A4,I5,T55,F10.3)
      I1=0
      DO 321 I=2,NLEN
      DO 322 J=1,NFLFR
      DO 323 K=1,NEMB
      I1=I1+1
      WRITE(NO,7097) I1,FLEN(I),FLFR(J,1),EMB(K)
7097  FORMAT(17X,I5,T35,F10.3,T55,F10.3,T66,F10.3)
323   CONTINUE
322   CONTINUE
321   CONTINUE
      IF(INORM.EQ.1) GO TO 11
      DO 330 I=1,IK
      L=I+1
      L=IIN(L)
      L1=NNDS(I)
      WRITE(NO,6000)
      WRITE(NO,8010) L,(NUP(I,J),NDWN(I,J),WELEV(I,J),WCF(I,J),
     1NUPP(I,J),NDWNN(I,J),J=1,L1)
8010  FORMAT(1H0,//////,30X,' OVERTOPPING NODES FOR TAPE ',I5,//,
     1  17X,'IN',9X,'OUT',4X,'EMBANK',10X,'WCF',//,
     2  40(/,10X,2I10,2F15.4,2I10))
330   CONTINUE
11    DO 2000 I=1,NLEN
```

```
      WRITE(NO,10001) (KNNDS(I,J),J=1,NNDSD)
10001 FORMAT(1H0,//,10X,'DOWNSTREAM NODES',10(/,10X,I12))
2000  CONTINUE
      DO 2001 I=1,NFLFR
      WRITE(NO,10002) I
10002 FORMAT(1H0,///,20X,'FLOOD NO.',I5)
      WRITE(NO,10003) (BXV(I,J),BZV(I,J),J=1,NNDSUP)
10003 FORMAT(1H0,/,10X,'X VEL.',10X,'Z VEL.',10(/,2(10X,F6.2)))
      WRITE(NO,10004) (HTI(I,J),J=1,NNDSD)
10004 FORMAT(1H0,//,15X,'STAGE',10(/,15X,F7.2))
2001  CONTINUE
      IF(IFLG.EQ.1) NI=IIN(1)
      GO TO 15
      ENTRY INPUT3
15    CONTINUE
      READ(NI,3) DEMBK
C
C                                       READ AND PRINT TITLE AND CONTROL
C
      WRITE(NO,6000)
      READ(NI,5005) TITLE
      WRITE(NO,6005) TITLE
      READ(NI,5010) NP,NE,NMAT,NPX,NBX,IPRT,NXZL,NZXL,NCL,IWIND,NDFX,IRO
      WRITE(NO,6010)NP,NE,NMAT,NPX,NBX,IPRT,NXZL,NZXL,NCL,IWIND,NDFX,IRO
      READ(NI,5020)  OMEGA,ELEV,TEMP,XSCALE,ZSCALE
      WRITE(NO,6020) OMEGA,ELEV,TEMP,XSCALE,ZSCALE
      READ(NI,5025)  NITI,NITN,NCBC,DELT,TMAX
      WRITE(NO,6025) NITI,NITN,NCBC,DELT,TMAX
      NDF = NDFX
C-
C-.....INITALIZE ARRAYS NOT COVERED BY INPUT.....
C-
      DO 25 J = 1, NP
      AO(J) = 0.0
      CORD(J,1) = 0.0
      CORD(J,2) = 0.0
      ALFA(J) = 0.0
      NFIX(J) = 0
      NFIXH(J) = 0
      DO 20 K=1,3
      SPEC(J,K) = 0.0
      NBC(J,K) = 0
   20 CONTINUE
   25 CONTINUE
C-
C-.....READ ELEMENT CHARACTERISTICS.....
C-
      READ(NI,5030)  (J,(ORT(J,K),K=1,5),I=1,NMAT)
      WRITE(NO,6030) (J,(ORT(J,K),K=1,5),J=1,NMAT)
C-
C-.....READ FIXED NODAL PARAMETERS.....
C-
      READ(NI,5035) (J,CORD(J,1),CORD(J,2),AO(J),K=1,NPX)
C
```

```
C.....COMPUTE INTERMEDIATE POINTS.....
C
      IF( NXZL .LE. 0 ) GO TO 95
      DO 94 K = 1, NXZL
      READ(NI,5010) NA,NB,NIP
      RATIO = ( CORD(NB,2) - CORD(NA,2) ) / ( CORD(NB,1) - CORD(NA,1 ) )
      DO 93 J = 1, 14
      IF( NIP(J) .LE. 0 ) GO TO 94
      NC = NIP(J)
      CORD(NC,2) = CORD(NA,2) + RATIO*( CORD(NC,1) - CORD(NA,1) )
   93 CONTINUE
   94 CONTINUE
   95 IF( NZXL .LE. 0 ) GO TO 100
      DO 97 K = 1, NZXL
      READ(NI,5010) NA,NB,NIP
      RATIO = ( CORD(NB,1) - CORD(NA,1) ) / ( CORD(NB,2) - CORD(NA,2) )
      DO 96 J = 1, 14
      IF( NIP(J) .LE. 0 ) GO TO 97
      NC = NIP(J)
      CORD(NC,1) = CORD(NA,1) + RATIO*( CORD(NC,2) - CORD(NA,2) )
   96 CONTINUE
   97 CONTINUE
  100 IF( IRO .LE. 0 ) GO TO 119
      READ(NI,5010) (NFIXH(J),J=1,NP)
  119 CONTINUE
C-
C-.....READ FIXED ELEMENT PARAMETERS.....
C-
      READ(NI,5040) (J,(NOP(J,K),K=1,6),IMAT(J),I=1,NE)

C-
C-..... READ LINES FOR CONTINUNITY CHECKS.....
C-
      IF( NCL .LT. 1 ) GO TO 129
  121 DO 123 J = 1, NCL
      READ(NI,5010) (LINE(J,K),K=1,16)
      DO 122 K = 1, 16
      IF( LINE(J,K) .LE. 0 ) GO TO 122
      LMT(J) = K
  122 CONTINUE
  123 CONTINUE
  129 CONTINUE
C
C ..... READ VARIABLE NODAL PARAMETERS .....
C
      READ(NI,5050) (N,NFIX(N),(SPEC(N,M),M=1,3),L=1,NBX)
C-
C-.....INPUT INITIAL CONDITIONS, IF SPECIFIED.....
C-
  150 CONTINUE
      DO 102 J = 1, NE
      IF( IMAT(J) .EQ. 0 ) GO TO 102
      DO 101 K = 2, 6, 2
      N1 = NOP(J,K-1)
      N2 = NOP(J,K)
```

```
      N3 = MOD(K+1,6)
      N3 = NOP(J,N3)
      CORD(N2,1) = 0.5*( CORD(N1,1) + CORD(N3,1) )
      CORD(N2,2) = 0.5*(CORD(N1,2) + CORD(N3,2) )
      AO(N2) = 0.5*( AO(N1) + AO(N3) )
  101 CONTINUE
      N1 = NOP(J,1)
      N2 = NOP(J,3)
      N3 = NOP(J,5)
  102 CONTINUE
      IF(NITI.GT.0) GO TO 170
      READ(NI,5070) ((VEL(M,N),M=1,3),N=1,NP)
 5070 FORMAT(10X,3F10.2)
      NITI = IABS( NITI )
  560 DO 563 J = 1, NP
      VEL(3,J) = VEL(3,J) - AO(J)
      VEL(1,J) = VEL(1,J) * VEL(3,J)
      VEL(2,J) = VEL(2,J) * VEL(3,J)
  563 CONTINUE
      ROAVG=1.93994
      GO TO 175
C-
C-.....INITIALIZE PRIMARY STATE VARIABLE ARRAY.....
C-
  170 ROAVG=1.93994
      DO 103 J = 1, NP
      VEL(1,J) = 0.0
      VEL(2,J) = 0.0
      VEL(3,J) = ELEV - AO(J)
  103 CONTINUE
C-
C-.....SETUP ONE TIME RUN PARAMETERS.....
C-
  175 NOPT = NDF * NCN
      OMEGA = 1.458E-4*SIN( OMEGA/57.3 )
C-
C-.....COMPUTE MIDSIDE VALUES.....
C-
      DO 99 J = 1, NP
      CORD(J,1) = CORD(J,1) * XSCALE
      CORD(J,2) = CORD(J,2) * ZSCALE
   99 CONTINUE
C-
C-.....COMPUTE AREAS AND SIDE SLOPES FOR NODES WITH BC'S.....
C-
      DO 110 J = 1, NE
      IF( IMAT(J) .EQ. 0 ) GO TO 110
      N1 = NOP(J,1)
      N2 = NOP(J,3)
      N3 = NOP(J,5)
      XAREA(J) = 0.5*( CORD(N1,1)*CORD(N2,2) + CORD(N2,1)*CORD(N3,2) +
     1  CORD(N3,1)*CORD(N1,2) - CORD(N3,1)*CORD(N2,2) -
     2  CORD(N2,1)*CORD(N1,2) - CORD(N1,1)*CORD(N3,2) )
      DO 105 K = 2, 6, 2
```

```
      N = NOP(J,K)
      IF( NFIX(N)/1000 .EQ. 0 ) GO TO 105
      N1 = K - 1
      N1 = NOP(J,N1)
      N2 = MOD(K+1,6)
      N2 = NOP(J,N2)
      DY = CORD(N2,2) - CORD(N1,2)
      DX = CORD(N2,1) - CORD(N1,1)
      IF( DX ) 104,105,104
  104 DEL = DY / DX
      ALFA(N) = ATAN( DEL )
      ALFA(N1) = ALFA(N)
      ALFA(N2) = ALFA(N)
  105 CONTINUE
  110 CONTINUE
C
C.....INERTIAL COMPONENTS.....
C
      FEEA(M) = AMW*( XN(M)*( H*DRDX - 2.0*R*DHDX - S*DHDZ )
     1  + DNX(M)*H*R + DNY(M)*H*S )
      FEED(M) = AMW*XN(M)*( H*DRDZ - R*DHDZ )
C
C.....VISCOUS COMPONENTS.....
C
      FEEA(M) = FEEA(M) - AMW*EPSX*( XN(M)*( 2.0*DHDX**2 + 2.0*DHDZ**2 )
     1  - DNX(M)*2.0*H*DHDX - DNY(M)*2.0*H*DHDZ )
      FEEB(M) = - AMW*EPSX*( XN(M)*H*DHDX - DNX(M)*H**2 )
      FEEC(M) = - AMW*EPSX*( XN(M)*H*DHDZ - DNY(M)*H**2 )
C
C.....BOTTOM FRICTION COMPONENTS.....
C
      FEEA(M) = FEEA(M) + AMW*XN(M)*TFRIC*( 2.0*R**2 + S**2 )
      FEED(M) = FEED(M) + AMW*XN(M)*TFRIC*R*S
C-
C-.....CORIOLIS COMPONENTS.....
C-
```

```
      FEED(M) = FEED(M) - AMW*XN(M)*OMEGA*H**2
  300 CONTINUE
C-
C-.....FORM THE TIME TERMS.....
C-
      DO 310 M = 1, NCN
      IA = 1 + NDF*(M-1)
      DO 305 N = 1, NCN
      IB = 1 + NDF*(N-1)
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + XN(M)*FEEA(N) + DNX(M)*FEEB(N)
     1  + DNY(M)*FEEC(N)
      IB = IB + 1
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + XN(M)*FEED(N)
  305 CONTINUE
  310 CONTINUE
C
C.....FORM THE HEAD TERMS.....
C
      DO 315 M = 1, 3
C-
C-.....INERTIAL COMPONENTS.....
C-
      FEEA(M) = AMW*( XM(M)*( R*DRDX + S*DRDZ ) - DMX(M)*R**2
     1  - DMY(M)*R*S )
C
C.....VISCOUS COMPONENTS.....
C
      FEEA(M) = FEEA(M) - AMW*EPSX*( XM(M)*( -2.0*DHDX*DRDX -
     1  2.0*DHDZ*DRDZ ) + DMX(M)*( 4.0*R*DHDX - 2.0*H*DRDX )
     1  + DMY(M)*( 4.0*R*DHDZ - 2.0*H*DRDZ ) )
      FEEB(M) = -AMW*EPSX*( XM(M)*( R*DHDX - 2.0*H*DRDX ) + DMX(M)*R*H )
      FEEC(M) = -AMW*EPSX*( XM(M)*( R*DHDZ - 2.0*H*DRDZ ) + DMY(M)*R*H )
C
C.....PRESSURE COMPONENTS.....
C
      FEEA(M) = FEEA(M) + AMW*XM(M)*3.0*GRAV*DAODX*H**2
      FEEB(M) = FEEB(M) - AMW*XM(M)*GHC
C
C.....CORIOLIS COMPONENTS.....
C
      FEEA(M) = FEEA(M) - AMW*XM(M)*2.0*OMEGA*S*H
C
C.....WIND STRESS COMPONENTS.....
C-
  315 CONTINUE
C-
C-.....FORM THE TIME TERMS.....
C-
      DO 325 M = 1, NCN
      IA = 1 + NDF*(M-1)
      DO 320 N = 1, 3
      IB = 3 + 2*NDF*(N-1)
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + XN(M)*FEEA(N) + DNX(M)*FEEB(N)
     1  + DNY(M)*FEEC(N)
```

```
  320 CONTINUE
  325 CONTINUE
C-
C-.....FORM THE DENSITY TERMS.....
C-
C
C.....FORM THE Y MOTION EQUATIONS.....
C
C.....FLOW TERMS.....
C
      DO 330 M = 1, 6
C
C.....INERTIAL COMPONENTS.....
C
      FEEA(M) = AMW*( XN(M)*( H*DSDZ - 2.0*S*DHDZ - R*DHDX )
     1  + DNX(M)*H*R + DNY(M)*S*H )
      FEED(M) = AMW*XN(M)*( H*DSDX - S*DHDX )
C
C.....VISCOUS COMPONENTS.....
C
      FEEA(M) = FEEA(M) - AMW*EPSZ*( XN(M)*( 2.0*DHDX**2 + 2.0*DHDZ**2 )
     1  - DNX(M)*2.0*H*DHDX - DNY(M)*2.0*H*DHDZ )
      FEEB(M) = -AMW*EPSZ*( XN(M)*H*DHDX - DNX(M)*H**2 )
      FEEC(M) = -AMW*EPSZ*( XN(M)*H*DHDZ - DNY(M)*H**2 )
C
C.....BOTTOM FRICTION COMPONENTS.....
C
      FEEA(M) = FEEA(M) + AMW*XN(M)*TFRIC*( 2.0*S**2 +R**2 )
      FEED(M) = FEED(M) + AMW*XN(M)*TFRIC*R*S
C
C.....CORIOLIS COMPONENTS.....
C
      FEED(M) = FEED(M) + AMW*XN(M)*OMEGA*H**2
  330 CONTINUE
C-
C-.....FORM THE TIME TERMS.....
C-
      DO 340 M = 1, NCN
      IA = 2 + NDF*(M-1)
      DO 335 N = 1, NCN
      IB = 1 + NDF*(N-1)
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + XN(M)*FEED(N)
      IB = IB + 1
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + XN(M)*FEEA(N) + DNX(M)*FEEB(N)
     1  + DNY(M)*FEEC(N)
  335 CONTINUE
  340 CONTINUE
C
C.....HEAD TERMS.....
C
      DO 345 M = 1, 3
C-
C-.....INERTIAL COMPONENTS.....
C-
```

```
      FEEA(M) = AMW*( XM(M)*( S*DSDZ + R*DSDX ) - DMX(M)*R*S
     1  - DMY(M)*S**2 )
C-
C-.....VISCOUS COMPONENTS.....
C-
      FEEA(M) = FEEA(M) - AMW*EPSZ*( XM(M)*( -2.0*DHDX*DSDX
     1  - 2.0*DHDZ*DSDZ ) + DMX(M)*( 4.0*S*DHDX - 2.0*H*DSDX )
     2  + DMY(M)*( 4.0*S*DHDZ - 2.0*H*DSDZ ) )
      FEEB(M) = -AMW*EPSZ*( XM(M)*( S*DHDX - 2.0*H*DSDX ) + DMX(M)*S*H )
      FEEC(M) = -AMW*EPSZ*( XM(M)*( S*DHDZ - 2.0*H*DSDZ ) + DMY(M)*S*H )
C
C.....PRESSURE COMPONETS.....
C
      FEEA(M) = FEEA(M) + AMW*XM(M)*3.0*GRAV*DAODZ*H**2
      FEEC(M) = FEEC(M) - AMW*XM(M)*GHC
C
C.....CORIOLIS COMPONENTS.....
C
      FEEA(M) = FEEA(M) + AMW*XM(M)*2.0*OMEGA*R*H
C-
C-.....WIND STRESS COMPONENTS.....
C-
  345 CONTINUE
C-
C-.....FORM THE TIME TERMS.....
C-
      DO 355 M = 1, NCN
      IA = 2 + NDF*(M-1)
      DO 350 N = 1, 3
      IB = 3 + 2*NDF*(N-1)
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + XN(M)*FEEA(N) + DNX(M)*FEEB(N)
     1  + DNY(M)*FEEC(N)
  350 CONTINUE
  355 CONTINUE
C-
C-.....FORM THE DENSITY TERMS.....
C-
C
C.....FORM THE CONTINUNITY EQUATIONS.....
C
      DO 365 M = 1, 3
      IA = 3 + 2*NDF*(M-1)
      DO 360 N = 1, NCN
      IB = 1 + NDF*(N-1)
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + AMW*DNX(N)*XM(M)
      IB = IB + 1
      ESTIFM(IA,IB) = ESTIFM(IA,IB) + AMW*DNY(N)*XM(M)
  360 CONTINUE
  365 CONTINUE
C-
C-
C-.....CONVECTION-DIFFUSION EQUATION FOR DENSITY.....
C-
C-
```

```
  500 CONTINUE
C-
C- APPLY TRANSFORMATIONS TO STIFFNESS AND FORCE MATRICES FOR SLOPING B. C.
C-
      DO 1000 N=1,NCN
      N1=NOP(NN,N)
      IF(ALFA(N1)) 820,1000,820
  820 CA=COS(ALFA(N1))
      SA=SIN(ALFA(N1))
      IB=NDF*(N-1)+1
      DO 900 M=1,NCN
      DO 900 MM=1,NDF
      IA=NDF*(M-1)+MM
      TEMP=ESTIFM(IA,IB)*CA + ESTIFM(IA,IB+1)*SA
      ESTIFM(IA,IB+1)=-ESTIFM(IA,IB)*SA + ESTIFM(IA,IB+1)*CA
      ESTIFM(IA,IB)=TEMP
  900 CONTINUE
      DO 990 M=1,NCN
      DO 990 MM=1,NDF
      IA=NDF*(M-1)+MM
      TEMP=ESTIFM(IB,IA)*CA + ESTIFM(IB+1,IA)*SA
      ESTIFM(IB+1,IA)=-ESTIFM(IB,IA)*SA + ESTIFM(IB+1,IA)*CA
      ESTIFM(IB,IA)=TEMP
  990 CONTINUE
      TEMP=CA*F(IB) + SA*F(IB+1)
      F(IB+1)=-F(IB)*SA + F(IB+1)*CA
      F(IB)=TEMP
 1000 CONTINUE
C
C                                             INSERT BOUNDARY CONDITIONS
C
      IF(NPR.GT.1) GO TO 2410
      DO 2400 MM=1,NCN
      N=NOP(NN,MM)
      DO 2350 M=1,NDF
      IF(SPEC(N,M)) 2275,2350,2275
 2275 IB = NDF*(MM-1)+M
      IA=0
      DO 2300 NC=1,NCN
      DO 2300 K=1,NDF
      IA=IA+1
 2300 F(IA)=F(IA)-ESTIFM(IA,IB)*SPEC(N,M)
 2350 CONTINUE
 2400 CONTINUE
 2410 CONTINUE
      DO 1050 I=1,NCN
      J = NOP(NN,I)
      IA=NDF*(I-1)
      DO 1050 K=1,NDF
      IA=IA+1
      JA=NBC(J,K)
      IF(JA.EQ.0) GO TO 1050
      R1(JA)=R1(JA)+F(IA)
 1050 CONTINUE
 1100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FANDS(R1,SK,NTEQ)
      COMMON/BLKA/ TITLE(20),NP,NE,NB,NDF,NCN,NMAT,NSZF,LI,MAXN,NPR,
     1             MBAND,NLL,NCL,GRAV,ROAVG,OMEGA,NOPT,NITI,NITN,NCBC,
     2             ICYC,DELT,TMAX,ALTM,CHI,FACT,URF,TET
      COMMON/BLKB/ CORD(200,2),NBC(200,3),VEL(3,200),SPEC(200,3) ,
     1             ALFA(200),AO(200),NOP(90,6),IMAT(90),ORT(10,5),
     2             NFIXH(200),NFIX(500),RPERM(500)
      COMMON/BLKE/ ESTIFM(30,30),F(30)
      DIMENSION R1(1),SK(NTEQ,1)
C
C                                       SET BANDMAX AND NO OF EQUATIONS
C
      NBAND=MBAND
      DO 290 J = 1, NSZF
  290 R1(J) = RPERM(J)
      DO 300 N=1,NSZF
      DO 300 M=1,NBAND
  300 SK(N,M)=0.
C
C                                       SCAN ELEMENTS
C
      DO 400 N=1,NE
      IF( IMAT(N) .EQ. 0 ) GO TO 400
      CALL COEFS(N)
C
C                                       RETURNS ESTIFM AS STIFFNESS MATRIX
C
C                                       STORE ESTIFM IN SK
C
C                                       FIRST ROWS
C
      DO 350 JJ=1,NCN
C
      NROW=NOP(N,JJ)
      DO 350 J=1,NDF
      NROWB=NBC(NROW,J)
      IF(NROWB.EQ.0) GO TO 350
      I=(JJ-1)*NDF+J
C
C                                       THEN COLUMNS
C
      DO 330 KK=1,NCN
      NCOB=NOP(N,KK)
      DO 320 K=1,NDF
      NCOLB=NBC(NCOB,K)
      IF(NCOLB.EQ.0) GO TO 320
      L=(KK-1)*NDF+K
      NCOL=NCOLB-NROWB+NFIX(NROWB)
  310 SK(NROWB,NCOL)=SK(NROWB,NCOL)+ESTIFM(I,L)
  320 CONTINUE
  330 CONTINUE
  350 CONTINUE
  400 CONTINUE
C
```

```
C**** FORWARD ELIMINATION
C
   30 FORMAT(1H0,6(E15.6,1X))
      NEQS=NSZF
C
C**   MAIN LOOP ON EQUATIONS
C
      NED=NEQS-1
      DO 280 I=1,NED
      JMIN=NFIX(I)
      JMAX=LI+JMIN
      IF(JMAX.GT.MBAND) JMAX=MBAND
C
C**   NORMALIZE COEFFICIENTS
C
      AATEMP = SK(I,JMIN)
      IF(ABS(AATEMP) .LT. 1.0E-04) WRITE(6,10) I,AATEMP
   10 FORMAT(//, 1H ,I10,F20.4)
      DO 200 J=JMIN,JMAX
      SK(I,J) = SK(I,J) / AATEMP
  200 CONTINUE
      R1(I) = R1(I) / AATEMP
C
C**   SET-UP ROWS FOR ELIMINATION
C
      KMIN = I + 1
      KMAX=I+LI
      IF (KMAX .GT. NEQS) KMAX=NEQS
      DO 260 K=KMIN,KMAX
      JK=NFIX(K)+I-K
      IF(JK) 260,260,205
  205 CONTINUE
      IF( SK(K,JK) ) 210,260,210
  210 CONTINUE
C
C**   ELIMINATE VARIABLE I FROM EQUATION K
C
      JJ=JK
      JJMIN=JMIN+1
      JJMAX= JMAX
      J=JJ+JMAX-JMIN
      IF(J.GT.MBAND) JJMAX=JJMAX+MBAND-J
      DO 240 J=JJMIN,JJMAX
      JJ=JJ+1
      SK(K,JJ) = SK(K,JJ) - SK(I,J)*SK(K,JK)
  240 CONTINUE
      R1(K) = R1(K) - R1(I)*SK(K,JK)
C
  260 CONTINUE
  280 CONTINUE
C
C**** BACK SUBSTITUTION
C
      JNEQ=NFIX(NEQS)
      R1(NEQS) = R1(NEQS) / SK(NEQS,JNEQ)
      DO 480 II=2,NEQS
      I = NEQS + 1 - II
      JJ=NFIX(I)+1
      JMAX=NFIX(I)+LI
      IF(JMAX.GT.MBAND) JMAX=MBAND
      DO 460 J=JJ,JMAX
      K=J-JJ+1+I
      R1(I) = R1(I) - SK(I,J)*R1(K)
  460 CONTINUE
  480 CONTINUE
      RETURN
      END
```

FINITE ELEMENT MODEL PROGRAM
SAMPLE INPUT DATA

```
CARD TYPE
  1  -   NOCYCLE
 17  -       350.0
 18  -   TWO-DIMENSIONAL FINITE ELEMENT BACKWATER MODEL, TALLAHALA CREEK FLOOD
 19  -     199    86       7   57     52     0      2     2      4     0     3     0
 20  -          0.00    312.00        0.00    -200.00     200.00
 21  -       -5
                   1      75.0        50.0                                        42
                   2      50.0        50.0                                        42
                   3     300.0       250.0                                        12.
 22  -             4     500.0       250.0                                        15
                   5     500.0       250.0                                        15.
                   6     300.0       250.0                                        42
                   7     750.0       750.0                                        42
                   1      0.80        6.75      305.00
                   3      1.00       10.75      305.00
                   5      2.05       13.25      304.40
                   7      3.50       16.50      304.00
                   9      5.10       20.35      305.00
                  19      2.30        5.70      305.50
                  21      3.60       8.250      305.3
                  23      5.55       10.05      305.00
                  25      7.65       12.00      305.00
                  27     10.20       14.35      306.00
                  38      5.20        3.70      304.00
                  40      6.75        5.25      305.00
                  42      8.65        7.20      303.50
                  44     10.45        9.00      306.00
                  46     12.95       10.50      306.00
                  48     13.25       10.70      306.00
 23  -            60     10.50        0.00      305.50
                  62     10.55        0.45      305.50
                  64     10.80        2.80      305.50
                  66     10.80        4.60      303.00
                  68     12.00        6.50      305.00
                  70     13.30        7.10      305.50
                  82     13.35        3.30      300.00
                  84     13.25        3.40      300.00
                  86     13.25        4.00      298.00
                  88     13.25        5.00      304.00
                  90     13.25        5.80      304.40
                  92     13.35        5.90      304.40
                 104     13.65        3.30      300.00
                 106     13.75        3.40      298.00
                 108     13.75        4.00      298.00
                 110     13.75        5.00      304.00
                 112     13.75        5.80      304.60
```

		114		13.65		5.90		304.60
		126		13.65		0.00		308.00
		128		13.90		0.40		308.00
		130		15.55		3.20		306.00
		132		15.90		5.10		304.00
		134		14.95		7.00		306.00
		136		13.65		7.10		306.50
		148		18.35		0.00		308.20
		150		18.50		4.50		308.20
		152		18.50		7.25		308.20
		154		17.00		10.00		309.00
		156		14.05		10.70		309.00
23 –		158		13.65		10.75		309.00
		170		22.10		0.00		309.00
		172		22.10		3.50		308.40
		174		22.10		6.85		308.60
		176		22.00		10.50		309.00
		178		21.05		14.75		309.20
		180		21.00		15.00		309.20
		191		27.25		0.00		309.10
		193		27.25		3.10		308.90
		195		27.25		6.35		308.40
		197		27.25		9.50		309.20
		199		27.25		13.85		309.50
24 –	60	1	38	19				
25 –	48	9	27					
	1	9	3	5	7			
	92	48	70					
	1	172	182	191	181	170	171	5
	2	172	183	193	192	191	182	5
	3	172	184	195	194	193	183	5
	4	174	185	195	184	172	173	5
	5	174	186	197	196	195	185	5
	6	176	187	197	186	174	175	5
	7	176	188	199	198	197	187	6
	8	176	177	178	189	199	188	6
	9	180	190	199	189	178	179	1
	10	150	160	170	159	148	149	5
27 –	11	150	161	172	171	170	160	5
	12	150	162	174	173	172	161	5
	13	150	151	152	163	174	162	5
	14	152	164	176	175	174	163	5
	15	152	153	154	165	176	164	6
	16	154	166	178	177	176	165	6
	17	154	155	156	167	178	166	6
	18	156	157	158	168	178	167	1
	19	158	169	180	179	178	168	1
	20	128	138	148	137	126	127	1
	21	128	129	130	139	148	138	5
	22	130	140	150	149	148	139	5
	23	130	131	132	141	150	140	3
	24	132	142	152	151	150	141	3
	25	132	133	134	143	152	142	4
	26	134	144	154	153	152	143	4

27 —

27	134	145	156	155	154	144	6
28	126	115	104	116	128	127	1
29	128	116	104	105	106	117	1
30	106	118	130	129	128	117	3
31	108	119	130	118	106	107	3
32	108	120	132	131	130	119	3
33	108	109	110	121	132	120	3
34	112	122	132	121	110	111	3
35	112	123	134	133	132	122	3
36	112	124	136	135	134	123	4
37	136	146	156	145	134	135	4
38	136	147	158	157	156	146	1
39	84	94	104	93	82	83	1
40	84	95	106	105	104	94	1
41	84	96	108	107	106	95	2
42	84	85	86	97	108	96	2
43	86	98	110	109	108	97	2
44	86	87	88	99	110	98	2
45	88	100	112	111	110	99	2
46	88	89	90	101	112	100	2
47	90	102	114	113	112	101	1
48	90	91	92	103	114	102	1
49	60	72	84	83	82	71	1
50	60	61	62	73	84	72	1
51	64	74	84	73	62	63	4
52	64	75	86	85	84	74	4
53	64	65	66	76	86	75	3
54	66	77	88	87	86	76	3
55	66	78	90	89	88	77	3
56	66	67	68	79	90	78	3
57	68	69	70	80	90	79	3
58	70	81	92	91	90	80	1
59	38	50	62	61	60	49	1
60	38	51	64	63	62	50	4
61	38	39	40	52	64	51	4
62	40	53	66	65	64	52	3
63	40	41	42	54	66	53	3
64	42	55	68	67	66	54	3
65	42	43	44	56	68	55	3
66	44	57	70	69	68	56	3
67	44	45	46	58	70	57	3
68	46	47	48	59	70	58	1
69	19	29	40	39	38	28	4
70	19	20	21	30	40	29	4
71	21	22	23	31	40	30	4
72	23	32	42	41	40	31	4
73	23	24	25	33	42	32	4
74	25	34	44	43	42	33	4
75	25	26	27	35	44	34	4
76	27	36	46	45	44	35	4
77	27	37	48	47	46	36	1
78	1	11	21	20	19	10	4
79	1	2	3	12	21	11	4
80	3	4	5	13	21	12	7

	81	5	14	23	22	21	13	7			
	82	5	6	7	15	23	14	7			
27 —	83	7	16	25	24	23	15	7			
	84	7	8	9	17	25	16	7			
	85	9	18	27	26	25	17	7			
	86	136	124	112	113	114	125	1			
	191	192	193	194	195	196	197	198	199		
28 —	126	127	128	129	130	131	132	133	134	135	136
	60	61	62	63	64	65	66	67	68	69	70
	1	2	3	4	5	6	7	8	9		
		1		01200						307.68	
		2		00200						307.68	
		3		00200						307.68	
		4		00200						307.68	
		5		00200						307.68	
		6		00200						307.68	
		7		00200						307.68	
		8		00200						307.68	
		9		01200						307.68	
		191		11000		3.53					
		192		11000		3.53					
		193		11000		3.53					
		194		11000		3.53					
		195		11000		3.53					
		196		11000		3.53					
		197		11000		3.53					
		198		11000		3.53					
		199		11000		3.47		0.64			
29 —		190		1000							
		180		11000							
		169		1000							
		158		11000							
		147		10000							
		136		10000							
		125		10000							
		114		11000							
		103		1000							
		92		11000							
		81		1000							
		70		1000							
		59		1000							
		48		11000							
		37		1000							
		27		1000							
		18		1000							
		10		1000							
		19		1000							
		28		1000							
		38		1000							
		49		1000							
		60		11000							
		71		1000							
		82		11000							
		93		1000							

	104	11000				
	115	10000				
	126	11000				
	137	1000				
29 —	148	1000				
	159	1000				
	170	1000				
	181	1000				
	1	1.24685	.86765	308.32418	1.93994	0.00000
	2	.89172	.22178	308.15760	1.93994	0.00000
	3	.99457	.88068	307.99102	1.93994	0.00000
	4	1.89678	1.62259	307.94911	1.93994	0.00000
	5	1.31762	1.28387	307.90720	1.93994	0.00000
	6	1.46127	.88750	307.92502	1.93994	0.00000
	7	.91617	.89092	307.94283	1.93994	0.00000
	8	.71001	.51706	307.92271	1.93994	0.00000
	9	.26828	.31766	307.90258	1.93994	0.00000
	10	1.07094	.74524	309.06053	1.93994	0.00000
	11	.54090	.61336	308.61804	1.93994	0.00000
	12	.90647	.66074	308.45146	1.93994	0.00000
	13	1.51444	1.71071	308.40955	1.93994	0.00000
	14	1.43846	1.58908	308.26423	1.93994	0.00000
	15	1.45672	1.36624	308.28204	1.93994	0.00000
	16	1.15845	1.26413	308.20479	1.93994	0.00000
	17	1.20537	1.00995	308.18466	1.93994	0.00000
	18	.86004	1.01833	308.17851	1.93994	0.00000
	19	.90584	.63035	309.79689	1.93994	0.00000
	20	.65025	.94388	309.35440	1.93994	0.00000
	21	.71439	.68953	308.91191	1.93994	0.00000
	22	.45280	.95200	308.76658	1.93994	0.00000
32 —	23	.45331	.73183	308.62125	1.93994	0.00000
	24	.42301	.78476	308.54399	1.93994	0.00000
	25	.48969	.71673	308.46674	1.93994	0.00000
	26	.59480	.63744	308.46058	1.93994	0.00000
	27	.39429	.46686	308.45443	1.93994	0.00000
	28	.86645	.60294	310.71676	1.93994	0.00000
	29	.72810	.91719	310.69395	1.93994	0.00000
	30	.65474	.97722	310.25147	1.93994	0.00000
	31	.73418	1.01726	310.10614	1.93994	0.00000
	32	.83884	1.04759	309.94650	1.93994	0.00000
	33	.76347	1.02583	309.86924	1.93994	0.00000
	34	.69913	.84693	309.71915	1.93994	0.00000
	35	.53750	.79372	309.71300	1.93994	0.00000
	36	.56821	1.01544	309.64136	1.93994	0.00000
	37	1.80347	2.13540	309.63799	1.93994	0.00000
	38	1.16433	.81023	311.63663	1.93994	0.00000
	39	.98490	1.10552	311.61383	1.93994	0.00000
	40	.77676	.92778	311.59102	1.93994	0.00000
	41	.56977	1.04225	311.43139	1.93994	0.00000
	42	.67294	1.01662	311.27175	1.93994	0.00000
	43	.56182	1.02840	311.12166	1.93994	0.00000
	44	.52164	.87930	310.97157	1.93994	0.00000
	45	.32585	.79465	310.89993	1.93994	0.00000
	46	.45789	.97558	310.82828	1.93994	0.00000

	47	.56529	1.41737	310.82491	1.93994	0.00000
	48	0.00000	0.00000	310.82154	1.93994	0.00000
	49	1.47699	1.02780	311.81928	1.93994	0.00000
	50	1.54178	.85305	311.81787	1.93994	0.00000
	51	1.32034	1.09532	311.78774	1.93994	0.00000
	52	.70501	.54480	311.76494	1.93994	0.00000
	53	.16441	.56420	311.72954	1.93994	0.00000
	54	.28528	.70852	311.56990	1.93994	0.00000
	55	.40735	.64916	311.52159	1.93994	0.00000
	56	.14428	.83667	311.37150	1.93994	0.00000
	57	.16152	.72632	311.29865	1.93994	0.00000
	58	.23234	.83041	311.22700	1.93994	0.00000
	59	.02852	1.36902	311.22363	1.93994	0.00000
	60	0.00000	0.00000	312.00192	1.93994	0.00000
	61	.90417	-.25288	312.00051	1.93994	0.00000
	62	1.57164	-.32019	311.99910	1.93994	0.00000
	63	1.88405	-.14004	311.96897	1.93994	0.00000
	64	1.19734	-.12319	311.93885	1.93994	0.00000
	65	.78766	.24086	311.90345	1.93994	0.00000
	66	.93629	.42102	311.86805	1.93994	0.00000
	67	.85667	.58661	311.81974	1.93994	0.00000
	68	.51153	.88466	311.77143	1.93994	0.00000
	69	.28222	1.07969	311.69858	1.93994	0.00000
	70	.02055	.98641	311.62573	1.93994	0.00000
	71	1.48031	-1.71404	312.08997	1.93994	0.00000
	72	1.39152	-1.60229	312.13391	1.93994	0.00000
	73	1.84798	-1.13047	312.13249	1.93994	0.00000
32 —	74	1.54880	-1.38059	312.10237	1.93994	0.00000
	75	1.29640	-.91163	312.23789	1.93994	0.00000
	76	1.12641	.08211	312.20249	1.93994	0.00000
	77	1.00870	.30507	312.22936	1.93994	0.00000
	78	1.00001	.66460	312.12027	1.93994	0.00000
	79	.83291	.95973	312.07196	1.93994	0.00000
	80	.00523	1.42287	311.99911	1.93994	0.00000
	81	.03148	1.51117	311.85733	1.93994	0.00000
	82	0.00000	0.00000	312.17803	1.93994	0.00000
	83	.66399	-.76650	312.22196	1.93994	0.00000
	84	1.42141	-1.07144	312.26589	1.93994	0.00000
	85	1.64888	-1.39029	312.40141	1.93994	0.00000
	86	1.58937	-.80327	312.53694	1.93994	0.00000
	87	1.44435	-.60300	312.56380	1.93994	0.00000
	88	1.58051	-.48711	312.59067	1.93994	0.00000
	89	1.86211	.02876	312.48158	1.93994	0.00000
	90	1.32287	.52926	312.37249	1.93994	0.00000
	91	.66215	.64019	312.23071	1.93994	0.00000
	92	0.00000	0.00000	312.08893	1.93994	0.00000
	93	2.24002	0.00000	312.48649	1.93994	0.00000
	94	2.44267	.50977	312.53042	1.93994	0.00000
	95	2.52280	1.30286	312.47637	1.93994	0.00000
	96	3.32705	1.09837	312.42072	1.93994	0.00000
	97	2.17255	-.36241	312.55624	1.93994	0.00000
	98	1.82726	-.53093	312.58901	1.93994	0.00000
	99	1.75310	-2.01031	312.61587	1.93994	0.00000
	100	3.29211	-2.32488	312.74861	1.93994	0.00000

101	4.60670	-1.04277	312.63952	1.93994	0.00000
102	4.01874	.07741	312.61288	1.93994	0.00000
103	3.69711	0.00000	312.47110	1.93994	0.00000
104	0.00000	0.00000	312.79494	1.93994	0.00000
105	.28300	.48889	312.74089	1.93994	0.00000
106	.74772	.60665	312.68684	1.93994	0.00000
107	1.21581	.43652	312.63119	1.93994	0.00000
108	1.32064	-.13002	312.57554	1.93994	0.00000
109	1.24548	-.34612	312.60831	1.93994	0.00000
110	1.00834	-.61676	312.64108	1.93994	0.00000
111	.79266	-1.24483	312.77382	1.93994	0.00000
112	.48631	-1.43706	312.90656	1.93994	0.00000
113	.49643	-1.04975	312.87992	1.93994	0.00000
114	0.00000	0.00000	312.85328	1.93994	0.00000
115	0.00000	1.14761	312.91643	1.93994	0.00000
116	.03075	.97464	312.91436	1.93994	0.00000
117	.22449	.73350	312.86030	1.93994	0.00000
118	1.06382	.34417	312.88562	1.93994	0.00000
119	1.12155	.05308	312.82997	1.93994	0.00000
120	1.26128	-.44627	312.99186	1.93994	0.00000
121	1.11070	-.56344	313.02463	1.93994	0.00000
122	.99835	-.66416	313.15737	1.93994	0.00000
123	.55520	-1.60756	313.22575	1.93994	0.00000
124	.31419	-3.94423	313.22987	1.93994	0.00000
125	0.00000	-4.16435	313.20323	1.93994	0.00000
126	0.00000	0.00000	313.03792	1.93994	0.00000
127	.68371	.27703	313.03585	1.93994	0.00000
128	.81116	.12780	313.03377	1.93994	0.00000
129	.67603	.24770	313.05909	1.93994	0.00000
130	.66878	-.07094	313.08440	1.93994	0.00000
131	.82092	-.65052	313.24629	1.93994	0.00000
132	.76593	-.67154	313.40818	1.93994	0.00000
133	.84042	-.99916	313.47656	1.93994	0.00000
134	.71369	-1.32918	313.54494	1.93994	0.00000
135	.41212	-2.03504	313.54906	1.93994	0.00000
136	0.00000	-1.86247	313.55318	1.93994	0.00000
137	.73650	0.00000	313.25487	1.93994	0.00000
138	.43682	.14344	313.25280	1.93994	0.00000
139	.58113	.08694	313.27812	1.93994	0.00000
140	.55367	-.34399	313.31793	1.93994	0.00000
141	.32479	-.36219	313.47982	1.93994	0.00000
142	.52871	-.74379	313.53156	1.93994	0.00000
143	.87358	-1.63715	313.59994	1.93994	0.00000
144	1.01653	-1.15407	313.63879	1.93994	0.00000
145	.70414	-.99613	313.59946	1.93994	0.00000
146	.48196	-.79205	313.60358	1.93994	0.00000
147	0.00000	-.77451	313.59936	1.93994	0.00000
148	.65270	0.00000	313.47183	1.93994	0.00000
149	.77614	-.02138	313.51164	1.93994	0.00000
150	1.17293	-.13851	313.55145	1.93994	0.00000
151	.70239	-.01112	313.60320	1.93994	0.00000
152	.90961	-.00799	313.65494	1.93994	0.00000
153	.97295	-.91202	313.69379	1.93994	0.00000
154	.96060	-1.10044	313.73265	1.93994	0.00000

	155	1.03871	-1.14111	313.69331	1.93994	0.00000
	156	.67424	-1.02613	313.65398	1.93994	0.00000
	157	.37624	-.56538	313.64976	1.93994	0.00000
	158	0.00000	0.00000	313.64554	1.93994	0.00000
	159	.93249	0.00000	313.69447	1.93994	0.00000
	160	.19689	-.06986	313.73428	1.93994	0.00000
	161	-.50266	-.13775	313.68935	1.93994	0.00000
	162	.49075	-.30251	313.90407	1.93994	0.00000
	163	.59507	.25596	313.95581	1.93994	0.00000
	164	.63601	.34940	313.78543	1.93994	0.00000
	165	1.08675	-.80569	313.82428	1.93994	0.00000
	166	1.12593	-.87021	313.89788	1.93994	0.00000
	167	1.24302	-.86703	313.85855	1.93994	0.00000
	168	1.30679	-.75550	313.85433	1.93994	0.00000
	169	1.32625	-.76688	313.84295	1.93994	0.00000
	170	3.99858	0.00000	313.91711	1.93994	0.00000
	171	2.47803	.32003	313.87218	1.93994	0.00000
	172	2.87096	.39267	313.82725	1.93994	0.00000
	173	1.02605	-.95452	314.04197	1.93994	0.00000
	174	1.30902	.56203	314.25668	1.93994	0.00000
32 —	175	.76403	-.34833	314.08629	1.93994	0.00000
	176	1.65642	.86933	313.91591	1.93994	0.00000
	177	2.02579	-.75905	313.98952	1.93994	0.00000
	178	1.46242	-.38926	314.06312	1.93994	0.00000
	179	.81454	-.04796	314.05175	1.93994	0.00000
	180	0.00000	0.00000	314.04037	1.93994	0.00000
	181	-.12297	0.00000	318.52673	1.93994	0.00000
	182	-.42878	-.35146	318.48180	1.93994	0.00000
	183	.07468	.69244	318.13413	1.93994	0.00000
	184	.93666	.64297	316.74674	1.93994	0.00000
	185	.75654	.29419	316.96145	1.93994	0.00000
	186	.37283	.84229	315.27069	1.93994	0.00000
	187	.57059	.10579	315.10031	1.93994	0.00000
	188	1.20400	1.78466	314.46771	1.93994	0.00000
	189	1.29818	-.03478	314.54132	1.93994	0.00000
	190	1.26781	.23328	314.52995	1.93994	0.00000
	191	.32852	0.00000	323.13635	1.93994	0.00000
	192	.33455	0.00000	322.78868	1.93994	0.00000
	193	.34080	0.00000	322.44101	1.93994	0.00000
	194	.36827	0.00000	321.05362	1.93994	0.00000
	195	.40054	0.00000	319.66622	1.93994	0.00000
	196	.49181	0.00000	317.97547	1.93994	0.00000
	197	.63695	0.00000	316.28471	1.93994	0.00000
	198	.71605	0.00000	315.65212	1.93994	0.00000
	199	.80406	.14802	315.01952	1.93994	0.00000
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