APPENDIX F

ADVANCE

Advanced Driver and Vehicle Advisory Navigation Concept

Traffic Related Functions
Evaluation Report (6 of 7)
Documents # 8460.00

CONTAINS:

Base Data and Static Profile Evaluation Report -- Document # 8460-01.01
Data Screening Evaluation Report -- Document # 8460-02.02
Quality of Probe Reports Evaluation Report -- Document # 8460-03.01
Travel Time Prediction and Performance of Probe and Detector Data Evaluation Report -- Document # 8460-04.01
Detector Travel Time Conversion and Fusion of Probe and Detector Data Evaluation Report -- Document # 8460-05.01
- **Frequency of Probe Reports Evaluation Report -- Document # 8460-06.01**

Relationships among Travel Times Evaluation Report -- Document # 8460-07.02

Prepared by
University of Illinois-Chicago
Urban Transportation Center

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ADVANCE Evaluation

Frequency of Probe Reports

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Executive Summary

A key piece of information for probe-based automatic traveler-information systems (ATIS) is the level of deployment required to obtain good quality travel time estimates. Issues relating to this are discussed in this report.

While a complete answer to the question can only be conjectured, better answers can be given to the question: how many probes per time interval are needed to obtain good quality link travel-time estimates.

The question is rendered difficult by the fact that probe reports of travel times are not statistically independent making it impossible to use formulae from introductory statistics which assume independence. A synthetic method is therefore used to estimate the relationship between estimate quality (as measured by variance) and the number of probe reports per time interval.

It is shown that the variance of estimates never goes to zero regardless of the number of probes. Moreover, the quality of estimates essentially stops improving after a fairly small number of probe traversals. This indicates that very high levels of probe deployment might not be necessary for an effective probe-based ATIS.

Because of the fact that one must always tolerate fairly high variances, the architecture used in ADVANCE — specifically not broadcasting dynamic estimates until they differ adequately from static estimates — is indeed most appropriate. However, for future ATIS deployment we recommend that information on signal time changes and the times when probes exit links be used. The quality of estimates and hence of route guidance will be considerably improved.

Based on the study we come to the informal conclusion that the full deployment would have worked quite well. Although slightly higher levels of deployment would have enhanced the quality of guidance, considerably higher levels would not have yielded much additional improvement.

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1 Introduction

The purpose of this report is to provide an assessment of the number of probes that would be required to obtain reasonable estimates of link travel times. Since link travel times may be obtained in many different ways, this report is essentially confined to the way such estimates were constructed for the ADVANCE demonstration project — and only for recurrent congestion. Specifically, the report focuses on means of probe-reported travel times over five-minute time intervals under incident-free conditions, since such estimates constitute the building blocks for dynamic route guidance. Incidents have been evaluated in a separate report.

Assessing what sample size renders the mean of a set of numbers sufficiently reliable for a given purpose might appear to be a trivial exercise in elementary statistics. However, this is not the case for several reasons, the principal one being that probe travel-time reports are not statistically independent, a requirement of the usual formulae found in introductory statistics textbooks. This and other issues are elaborated on in Section 3, in which the various factors that influence the quality of travel-time estimates are described. Unfortunately, the results obtained from this study are link specific. Generalization to links that were not studied does not appear to be trivial.

However, because the links studied included a wide range of congestion levels, some qualitative statements can be made. One of these is that the deployment level originally planned for full deployment would have been adequate to provide fairly good dynamic information. Deployment levels four or five times higher would yield significantly better information. Above that, while information quality would continue to improve with deployment, the improvements would be slight.

2 Study Area and Data Collection

In early 1995 a decision was made to go from a full deployment using 3000-5000 vehicles to be driven by volunteer drivers as they went about their daily lives, to a targeted deployment where fewer vehicles would be driven by paid drivers on strictly-defined routes. These routes and the data-collection exercise for this targeted deployment are described below.

2.1 Study Area and Routes

The entire routes driven on Dundee Road and adjacent arterials were within the municipality of Wheeling, Illinois (north suburban Chicago). Dundee Road was selected because it carries a high volume of traffic and because each signalized intersection is demand actuated by loop detectors (including turning lanes) and there are volume and occupancy detectors in several locations. Although Dundee Road extends for several miles within the ADVANCE study area the number of potential locations along Dundee Road where

the necessary field tests could be performed was very limited. The data-collection route required a convenient location where vehicles could turn around safely and avoid being off the study route for a long period of time. The route also needed a mix of link and intersection characteristics.

Two route configurations were used for the field data collection. The first route is the long route and was used for the majority of tests (Figure 1). The long route consists of twelve links. The section of the route on Schoenbeck Road and Palm Drive (near the west end) was used as a staging and turnaround area and, since it was too short to complete a recognized link, data were not collected for this section of the route. The route was selected to be completed during a fifteen-minute period. This provided the desired frequency of probe reports. Given the limited size of the driving fleet a longer route would not provide the frequency of reports needed for most tests performed because too few vehicles would be available for dispatch at the required headways. During the off-peak period the majority of the drivers completed this twelve-link route in ten to fourteen minutes.

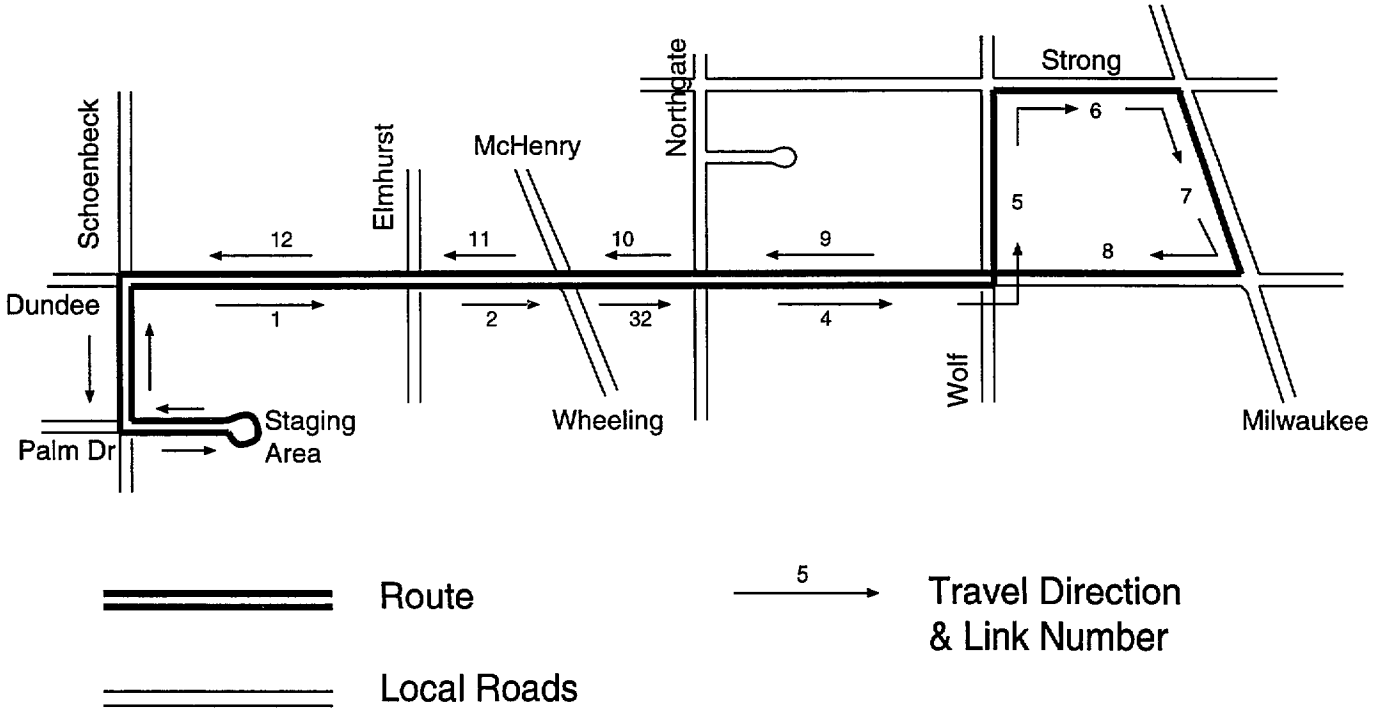

Figure 1: Probe Data Collection: Long Route

During the peak period this route proved to be too long to complete in fifteen minutes and a shorter alternative was used. This is shown in Figure 2. Even the short route could not always be completed in fifteen minutes but this happened infrequently.

Links 4 through 9 are on the long route but do not have the same number of link reports for two reasons (Table 1). First, many of the scheduled rest breaks were taken on this portion of the route. The most common break location was on Link 8 and there

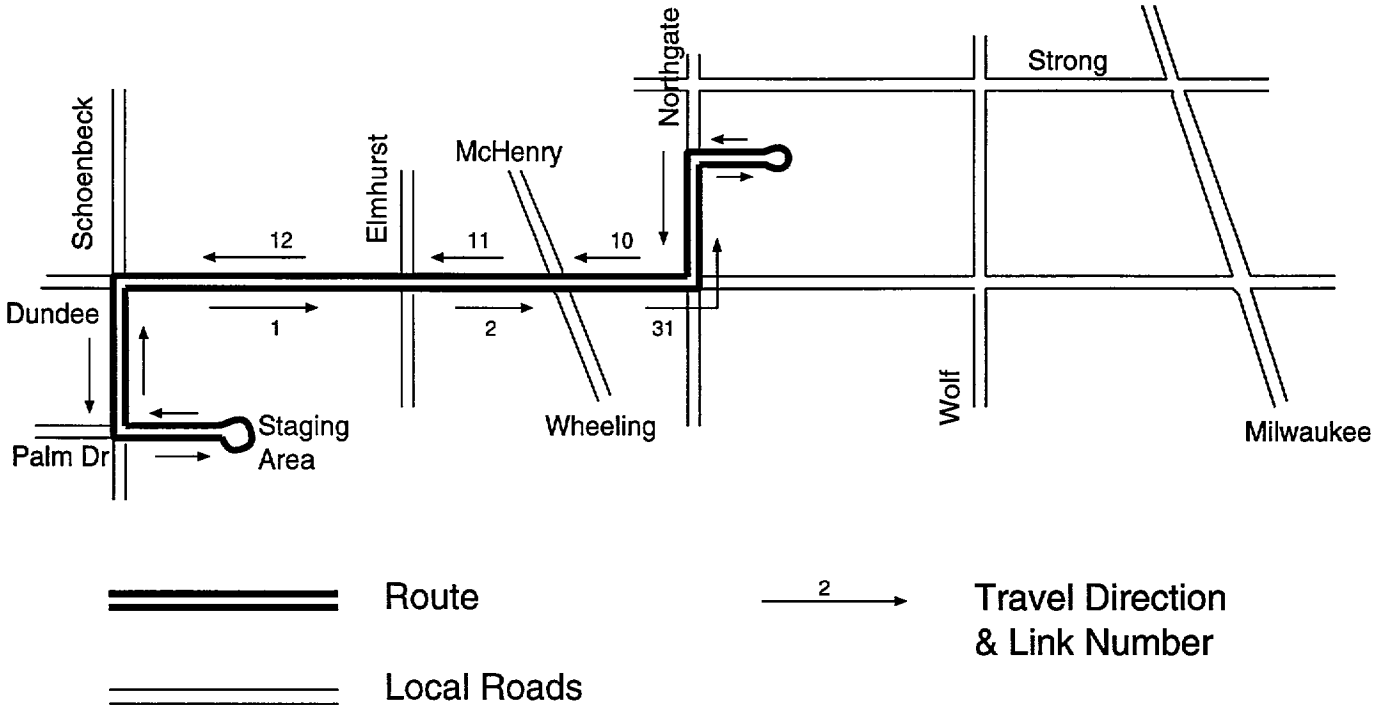

Figure 2: Probe Data Collection: Short Route

are correspondingly fewer reports on this link (the vehicles need to travel the entire link without stopping to create a report). Second, there may have been MNA failures and other reasons for turning off the link and stopping.

During the last three weeks in the Dundee Road study area (July 24 to August 10) the vehicles were being used to test turning relationships. In this case each driver was given a set of randomly drawn routes, to be driven in sequence, which covered the links shown on Figure 3. This consisted of fourteen uni-directional links. On Link H/h the drivers were permitted to stop and study the rest of the routes they had been assigned. Each of these days generally yielded approximately 1000 usable MNA reports, over half of these are on Dundee Road and are used in this study.

2.2 Data-Collection Schedule

Data were collected on the routes described above, with one exception, from Monday to Thursday. Fridays and weekend days were considered, for data-collection purposes, to be different day types. The number of reports received on each day of probe data collection is shown in Table 2. This table includes only the data collected on the main Dundee Road study routes.

At the beginning of each day of data collection, a twelve-noon briefing was held at the ADVANCE office in Schaumburg. At this time the drivers were assigned vehicles and they left the office at approximately 12:30 pm. Each driver used a designated route to drive to the study area. There were several different routes; this report is not concerned

Table 1: MNA Reports by Link

Link	Frequency	Percent	Link	Frequency	Percent
1	5481	10.8	7	2323	4.6
2	6298	12.4	8	2172	4.3
3”	5886	11.6 *	9	2462	4.9
4	2313	4.6	10	6066	12.0
5	2294	4.5	11	7826	15.5
6	2293	4.5	12	5206	10.3
31”	3555	7.0 *			
32*	2331	4.6 *			
			Total	50.620	100.0

* Link 3 consists of two links, 31 and 32. Link 31 is on the short route and includes a left turn at the end of the link. Link 32 is on the long route and has a through movement at the end of the link (no turn).

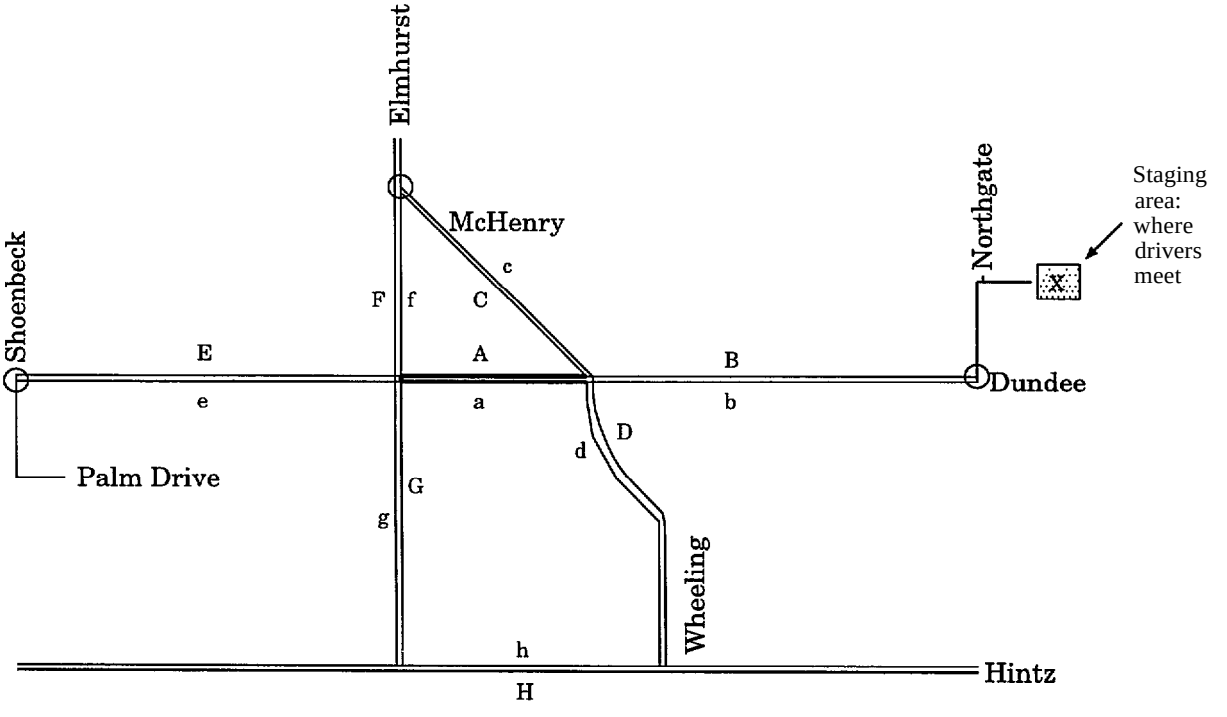

Figure 3: Probe Data Collection: Turning-Relationships Links

Table 2: Probe Reports for each day of Data Collection

Date	No. of Reports	Percent of Total
6/05	660	1.3
6/07	395	0.8
6/08	1140	2.3
6/12	1382	2.7
6/13	1712	3.4
6/14	1014	2.0
6/15	446	0.9
6/19	1178	2.3
6/20	1591	3.1
6/21	1503	3.0
6/22	2372	4.7
6/26	2037	4.0
6/27	1481	2.9
6/28	1744	3.4
6/29	1546	3.1
7/05	1560	3.1
7/06	1996	3.9
7/10	1689	3.3
7/11	1282	2.5
7/12	1507	3.0
7/13	1046	2.1
7/17	2285	4.5
7/18	2252	4.4
7/19	2140	4.2
7/20	1901	3.8
7/24	880	1.7
7/25	907	1.8
7/26	1017	2.0
7/27	899	1.8
7/31	949	1.9
8/01	1069	2.1
8/02	1038	2.1
8/03	1139	2.3
8/04	949	1.9
8/07	1058	2.1
8/08	1050	2.1
8/09	873	1.7
8/10	933	1.8
Total	50,620	100.0

with the routes to and from the study area. Data were collected by probe vehicles driven in the study area between 1:00 pm and 7:00 pm (Table 3), with breaks as described below.

Table 3: Probe Reports for each hour of Data Collection

Hour Beginning	No of Reports	Percent of Total
1:00 pm	8464	16.7
2:00 pm	7980	15.8
3:00 pm	5187	10.2
4:00 pm	8488	16.8
5:00 pm	8433	16.7
6:00 pm	7871	15.5
7:00 pm	4197	8.3
Total	50,620	100.0

On each day of data collection a field manager was present at the staging area. The field manager ensured that vehicles were driving the study route at satisfactory headways and instructed drivers when to take breaks. The field manager also assisted with other problems which routinely occurred.

The drivers were given a ten-minute break at approximately 2:00 pm to 2:l0 pm and another one from approximately 6:00 pm to 6:l0 pm. Each driver took his or her break at a slightly different time, as each was dispatched by the field manager to the break area as they arrived at the staging area. During breaks each probe vehicle was inactive for more than ten minutes as time was lost off-route and also while the vehicle and MNA warmed up. The longest break occurred from 3:30 pm to 4:00 pm. After this break, during the two-hour peak period from 4:00 pm to 6:00 pm, the drivers operated their vehicles without scheduled breaks.

3 Probe-Deployment Considerations

Since in a probe-based ATIS, information on the condition of the network is obtained from probes, a key issue with respect to the future of such systems is the deployment level needed for the system to function well. This section contains a listing of the issues involved in a discussion of deployment levels.

The decision (described above) to proceed with a targeted deployment of probe vehicles saved several million dollars and in many ways the experience gained from the targeted deployment was not significantly less than that which would have been gained from a full deployment. One element that did suffer was an understanding of the level of deployment needed for a well functioning probe-based ATIS. This is not to say that the additional cost of a full deployment could have been justified on this ground; indeed, it possibly could

not have been and perhaps simulation and other methods can be used effectively for this purpose. However, the fact remains that in the evaluation effort which is the subject of the present report, we can offer only a partial answer to the question of what level of probe deployment would be satisfactory.

Since the targeted deployment can give us good information on the links on which probe-reported travel times were recorded we can give answers at a link level. We have attempted to do so in this report. However, a gap remains between link level answers and total system level answers. This section identifies some of the issues involved along with some link level issues.

Several factors need to be considered in order to identify satisfactory probe deployment levels:

- A very important factor is coverage, i.e., the number of links being covered during an appropriate time interval. This issue was addressed in one of the first documents written for ADVANCE design (Hicks, Boyce and Sen, 1992) and the deployment level was chosen in accordance with that study. It is not discussed further here.

  It should however be borne in mind that links not covered are mostly less-traveled links. Such links are less likely to suffer from congestion and, given the nature of the route-choice algorithm used, are less likely to form part of routes that are recommended to drivers.

- The considerations just mentioned apply to links which do have positive numbers of probes, in the sense that more important links would get higher traversal levels per unit time.

- There are enormous variations in probe-reported travel times even within a single traffic-signal cycle. Under conditions of no cycle failure, a vehicle arriving at an intersection at the end of the green time experiences little delay and hence shorter link travel times than one that arrives at the start of the red. Figure 4 illustrates this situation for one of the links on our study route, Link 11, on July 17. The high travel-time observations are for probes arriving during the red phase and the low observations for those arriving late in the green phase. Note that the figure would appear different if progression were to be different.

  While the variance would depend on progression and the proportion of vehicles entering the link via left and right turns, the fact remains that the variance of link travel times would be high.

- Given the above fact, estimating a single number 'link travel time' over a time interval becomes difficult. Such travel time variations are a normal occurence and affect any discussion regarding link travel times.

- However, we would be more interested in route travel times, rather than the travel times on individual links. A route travel time is the sum of travel times on links

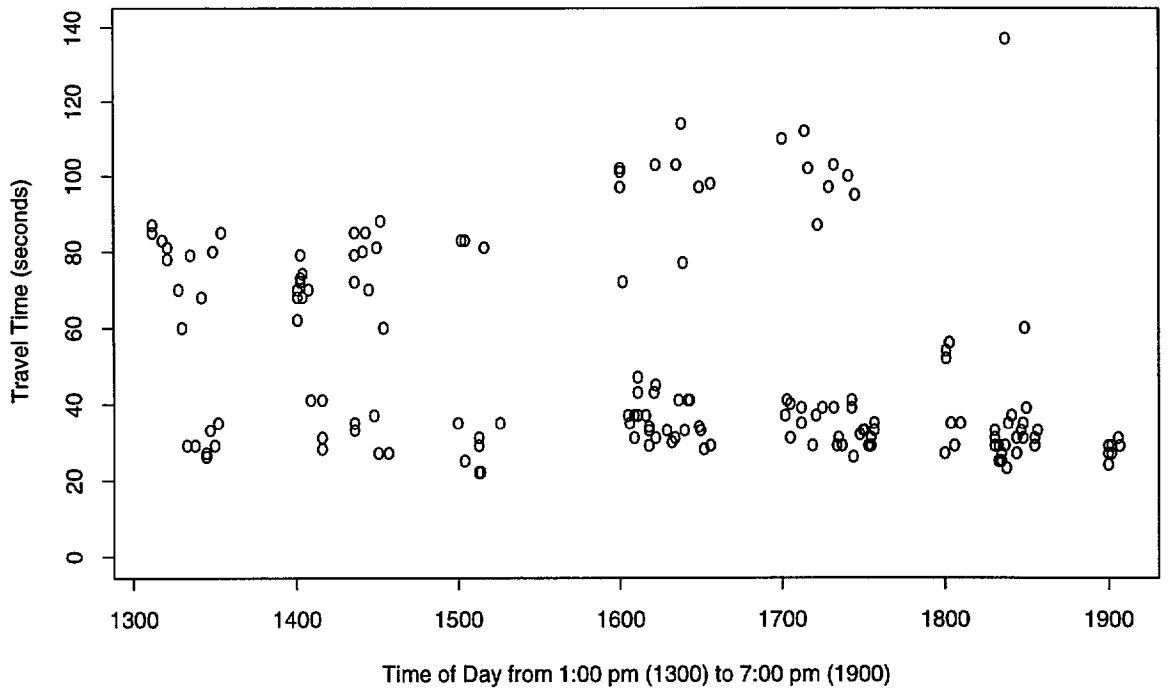

Figure 4: Travel Times on Link 11: July 17, 1995

comprising that route. Thus we would normally conjecture that if an average travel time per link is supplied, the discrepancies between the travel times actually encountered by a vehicle and the average would partially 'cancel out' as we add them over a large number of links comprising the route.

This is not an unusual experience for drivers and would be true if link travel times were independent. However, travel times on contiguous links are not usually independent and depend on progression. Consider the plots of travel times on pairs of links shown in Figure 5. Each point on these plots shows link travel times for the same vehicle on two contiguous links. The plot on the left shows the travel times for each probe vehicle on Links 1 and 2. It may be seen from this plot that when the travel time on Link 1 is high the travel time on Link 2 is low, and vice versa. In this case when a vehicle is stopped at a red light on one link, it nearly always gets a green on the next.

The plot on the right shows the travel times for each probe vehicle on Links 5 and 6. The travel times on these two links are highly correlated and represent a very different situation from the previous example. It may be seen from this plot that when the travel time on Link 5 is high the travel time on Link 6 is also high. In this case each vehicle seems to take its surrounding congestion with it onto the next link.

There are also cases where a traffic signal acts as a flow-control valve for congestion; a congested link is followed by an uncongested one. This can be seen on Figure 6 which shows the mean travel time for each time-of-day interval for Links 9, 10 and

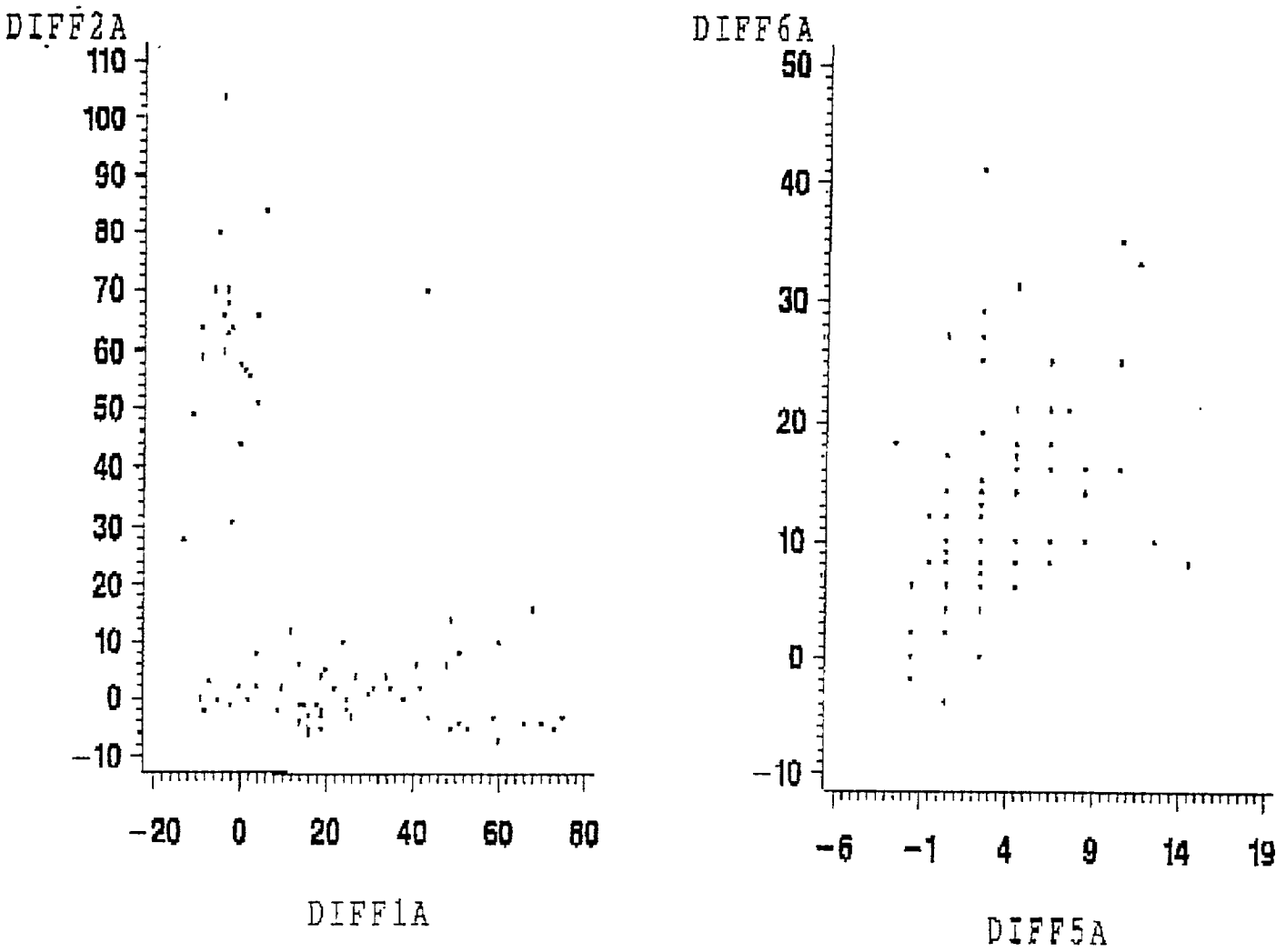

Figure 5: Relationships between Travel-Time Differences on Contiguous Links

Note: DIFF1A on the x-axis of the plot on the left is the travel time for each vehicle on that link less the cruise time on the link. The cruise time is equal to link length/speed limit. Similarly, DIFF2A, DIFF5A and DIFF6A give the travel time less the cruise time on Links 2, 5 and 6. Each point represents a vehicle's travel times on two contiguous links.

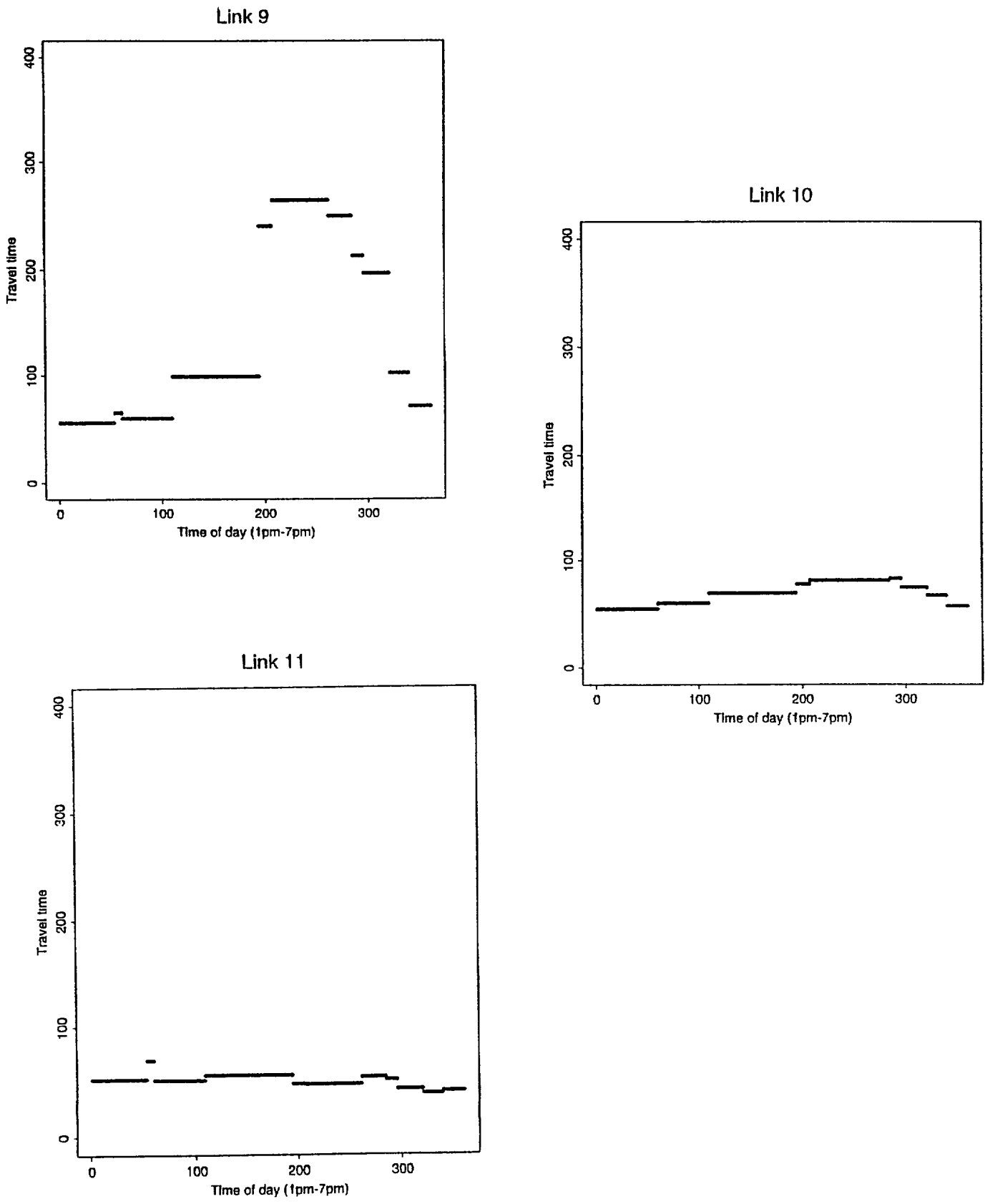

Figure 6: Mean Travel Times on Links, 9, 10 and 11

11. Figure 6 shows high travel times on Link 9 during the peak period, indicating the presence of peak-period congestion on this link. This is not evident for Links 10 and 11.

These examples illustrate that the variation in travel times which exists for one link may well be substantially lower for an entire route. Even when congestion occurs for a few links, it may not exist on a subsequent link.

- Returning to the case of individual links, the formula:

  Travel Time = Cruise Time + Signal Delay

  seems to hold fairly well. The cruise time for a typical fairly long link (of around 800 meters) with a short average queue at the end of it seems to be close to constant.

  The delay encountered by one vehicle in a queue would be very similar to those encountered by neighboring cars in the same queue so long as they depart the link in the same cycle. Thus, very high sample sizes should not be required to estimate link travel times.

- Under moderate to low congestion two vehicles arriving at the beginning of the red signal phase would tend to have similar travel times, regardless of cycle. Similarly, two vehicles arriving during the latter part of the green phase would tend to have similar travel times. In fact, under low and moderate congestion signal phases have an enormous effect on probe-recorded travel times.

  Clearly, if probe arrival times or exit times were precisely recorded and times of phase changes of traffic signals were known, the effect of traffic signals could be corrected for. Indeed, we make the recommendation that in future ATIS design such information be obtained and incorporated into travel-time estimation algorithms.

  Since in the ADVANCE system time intervals over which estimates are made are of fixed length and different intervals include different proportions of red and green phases, averages of probe-reported travel times would tend to vary from interval to interval for this reason and increasing the number of probes will not substantially reduce this variation. Therefore, in the ADVANCE system some variance in estimates will have to be tolerated and increasing the number of probes will not necessarily reduce this.

  This fact and the one described in the last item are demonstrated mathematically in Section 4.

  It may be mentioned that these problems do not substantially affect static forecasts, which, although they do not reflect current conditions, are inherently more reliable than dynamic estimates. This fact has been noted in several different elements of ADVANCE evaluation. The ADVANCE design takes account of this very well by broadcasting dynamic updates only when dynamic estimates substantially differ from static updates.

After this discussion of the issues, in the rest of this report we focus on the relationship between frequency of probe reports and the variance of mean travel-time estimates. This relationship is complicated by the lack of independence of probe reports as described above. However, empirical relationships can be obtained and are presented.

ADVANCE used a 5-minute time interval and therefore the discussion in this report focuses only on this interval. A longer interval would trap more probes and by including more cycles could improve estimates. On the other hand longer intervals would not be able to handle small variations in travel times and would therefore make estimates supplied to vehicles less current.

4 Results

As mentioned earlier, for each link and for each (5-minute) time interval, means of the link travel times from different vehicles are computed and act as building blocks for most of the route-guidance information supplied to in-vehicle MNAs. If a link is detectorized, then data from detectors and from probe vehicles are used to estimate link travel-time estimates. For links without detectors (most links are not detectorized) mean travel times from probe data only are supplied to vehicles as 'current travel times.' Forecasts of link travel times made for 5, 10 and 15 minutes into the future are also computed. Five-minute means are used as inputs in the formulæ used for constructing these forecasts. Therefore, the key statistic is mean link travel time over each 5-minute interval and the rest of this discussion is focused on this estimate.

4.1 Variance of Mean Link Travel Times

The variance of the mean $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ of (5-minute mean) probe-reported travel times, $x_1, x_2, \ldots x_n$, for the same link over some time interval is

$$\begin{aligned}
\mathrm{var}\,[\bar{x}] &= \mathrm{E}[\bar{x} - \mathrm{E}(\bar{x})]^2 = \mathrm{E}[n^{-1}\sum_{i=1}^{n}(x_i - \mathrm{E}[x_i])]^2 \\
&= n^{-2}\,\mathrm{E}[\sum_{i=1}^{n}(x_i - \mathrm{E}[x_i])^2 + \sum_{\substack{i,j \\ i\neq j}}(x_i - \mathrm{E}[x_i])(x_j - \mathrm{E}[x_j])] \\
&= n^{-2}\sum_{i=1}^{n}\mathrm{var}\,[x_i] + \sum_{\substack{i,j \\ i\neq j}}\mathrm{Cov}\,[x_i, x_j]
\end{aligned} \tag{1}$$

where $\mathrm{Cov}\,[\cdot]$ stands for covariance. Let $\eta = n^{-1}\sum_{i=1}^{n}\mathrm{var}\,[x_i]$ be the average variance of x_i's and let

$$\nu = [n(n-1)]^{-1}\sum_{\substack{i,j \\ i\neq j}}\mathrm{Cov}\,[x_i, x_j]$$

be the average covariance of all pairs (x_i, x_j). Then, (1) may be written as

$$\begin{aligned} \text{var}\,[\bar{x}] &= n^{-2}[n\eta + n(n-1)\nu] = \eta n^{-1} + [1 - n^{-1}]\nu \\ &= \nu + n^{-1}[\eta - \nu]. \end{aligned} \tag{2}$$

Of course, if the covariances were zeros, then $\nu = 0$, and we would get $\text{var}\,[\bar{x}] = n^{-1}\eta$, and if all variances of x_i were equal to σ^2, then we would get the familiar $\text{var}\,[\bar{x}] = n^{-1}\sigma^2$. In this case as $n \to \infty$, $\text{var}\,[\bar{x}] \to 0$, as is very well known. On the other hand, if the correlation between every pair of x_i's were one and all variances were equal, then for all i and j, $\text{Cov}\,[x_i, x_j] = \text{var}\,[x_i]$ and $\nu = \eta$. Then $\text{var}\,[\bar{x}] = \nu$ and then $\text{var}\,[\bar{x}] = \nu$ stays the same no matter what n is. This is to be expected, because then all the observations x_i would be exactly the same. In practice we would expect something in between, where $\eta > \nu > 0$. Even then as $n \to \infty$, $\text{var}\,[\bar{x}] \to 0$. While our discussion has been in terms of the mean, a similar situation would hold for most reasonable estimators.

The implication of these facts is that if $\nu > 0$, then no matter how many link travel times of distinct vehicles we measure over a short time interval, the variance of the mean would be large. Clearly, how large is a critical question and can be answered by examining values of ν's. In fact, even if we computed the mean of all vehicles during a given time interval, the variance of the mean would remain above the value of ν for that link.

This might appear counter-intuitive, since when a sample becomes the same as the population one might expect the variance of the sample mean to go to zero. This does not happen here because each x_i is assumed to be one out of a continuum of (and hence an infinite number of) random variables. This is appropriate for forecasting applications, because it is not nearly as important for us to know what exactly the mean travel time for a given set of vehicles was during some interval in the past as to know what travel times will be in the future if similar conditions persist. For this purpose, sets of travel times need to be considered as samples from an infinite population.

4.2 Test of No-Correlation Hypothesis

While in Section 3 we have given the underlying causes of a lack of independence, it is perhaps appropriate to formally test the hypothesis of no correlation against the presence of correlation. If the x_i's were uncorrelated, the average covariance ν would be zero. Therefore, all we need to do is test if $\nu = 0$. One method of testing the hypothesis $H : \nu = 0$ against the alternative $A : \nu \neq 0$ is afforded by (2) itself. If we have reasonable estimates of $\text{var}\,[\bar{x}]$, we could regress this against the corresponding values of n^{-1}, where n is the number of observations used to compute $\bar{x}$. The intercept term would then be an estimate of ν and could be used to test H against A.

One estimate of $\text{var}\,[\bar{x}]$ is $[\bar{x} - \widehat{\text{E}[\bar{x}]}]^2$, where $\widehat{\text{E}[\bar{x}]}$ is an estimate of $\text{E}[\bar{x}]$. In order to estimate $\widehat{\text{E}[\bar{x}]}$, we used the model

$$\text{E}[x_{d,t}] = \gamma + \alpha_d + \beta_t, \tag{3}$$

where α_d and β_t are respectively day effects and time-of-day effects. The model (3) is implied by

$$\mathrm{E}[x_{d,t,i}] = \gamma + \alpha_d + \beta_t \tag{4}$$

where $x_{d,t,i}$ is the ith observation during day d and time-period t. Model (4) can be estimated by least squares, after coding the independent variables corresponding to α_d's and β_t's as indicator (or dummy) variables (one indicator variable for each time interval t and one for each day d) with the restrictions $\sum_d \alpha_d = 0$ and $\sum_t \beta_t = 0$, in order to avoid multicollinearity. The residuals $e_{d,t,i}$ from (4) are

$$e_{d,t,i} = x_{d,t,i} - \hat{\gamma} - \hat{\alpha}_d - \hat{\beta}_t \tag{5}$$

where $\hat{\alpha}_d$ denotes an estimate of the parameter α_d. For every day and time period, the mean over all i of these residuals is therefore,

$$\bar{e}_{d,t} = \bar{x}_{d,t} - \hat{\gamma} - \hat{\alpha}_d - \hat{\beta}_t. \tag{6}$$

Thus, if (3) holds, a reasonable estimate of $[\bar{x} - \mathrm{E}[\bar{x}]]$ is $\bar{e}_{d,t}$ and $[\bar{e}_{d,t}]^2$ estimates $\mathrm{var}\,[\bar{x}]$. Therefore, the model estimated is

$$[\bar{e}_{d,t}]^2 = \nu + \gamma_1[1/n_{d,t}] + \epsilon_{d,t} \tag{7}$$

where the $\eta = \nu + \gamma_1$ and $\epsilon_{d,t}$ is the error term relating to the tth time interval on day d. These parameters may be estimated by some regression procedure. Thus a test of the hypothesis H against A could be conducted by using least squares to carry out the regression and then test $\nu = 0$ in the usual way using the t-statistic.

However, before conducting the estimation, we need to make sure that the underlying assumptions of least squares are at least approximately met. Figures 7 and 8 show plots of the dependent variable $y_{d,t} = [\bar{e}_{d,t}]^2$ against the independent variable n^{-1} for links 1 and 11 during the peak period. In these plots n is the number of probes per 5-minute interval; indeed all empirical work was done for 5-minute interval data in order to be consistent with ADVANCE design. It would appear that we have a wedge or funnel-shaped pattern of points indicating the presence of heteroscedasticity or unequal variance violating the assumption that $\mathrm{E}[\epsilon_{d,t}^2] = \sigma^2$.

One solution to this problem is to weight the regression. In order to find appropriate weights, ignoring subscripts for the moment, we write the dependent variable as $y = u^2$ where $u = \bar{e}$, with $\bar{e}$ as in (7). For any differentiable function $f(u)$, the variance $\mathrm{var}\,[f(u)]$ of $f(u)$, may be approximately written as $\mathrm{var}\,[f(u)] \approx [f'(z)]^2\,\mathrm{var}\,[u]$, where a prime denotes a derivative and z is the mean of u. Since $f(u) = u^2$ here, and an estimate of $\mathrm{var}\,[u]$ is y, we get $\mathrm{var}\,[f(u)] \approx 4\,\mathrm{E}[y]^2 \propto \mathrm{E}[y]^2$. Thus a proper weight would appear to be the reciprocal of the square of an estimate of $E[y]$. In our examination of diagnostics from the different regressions weighted as above, no heteroscedasticity was noticeable.

Notice that under the hypothesis of no correlation, the dependent variable values are only slightly correlated — the only correlation would be that due to the parameters γ,

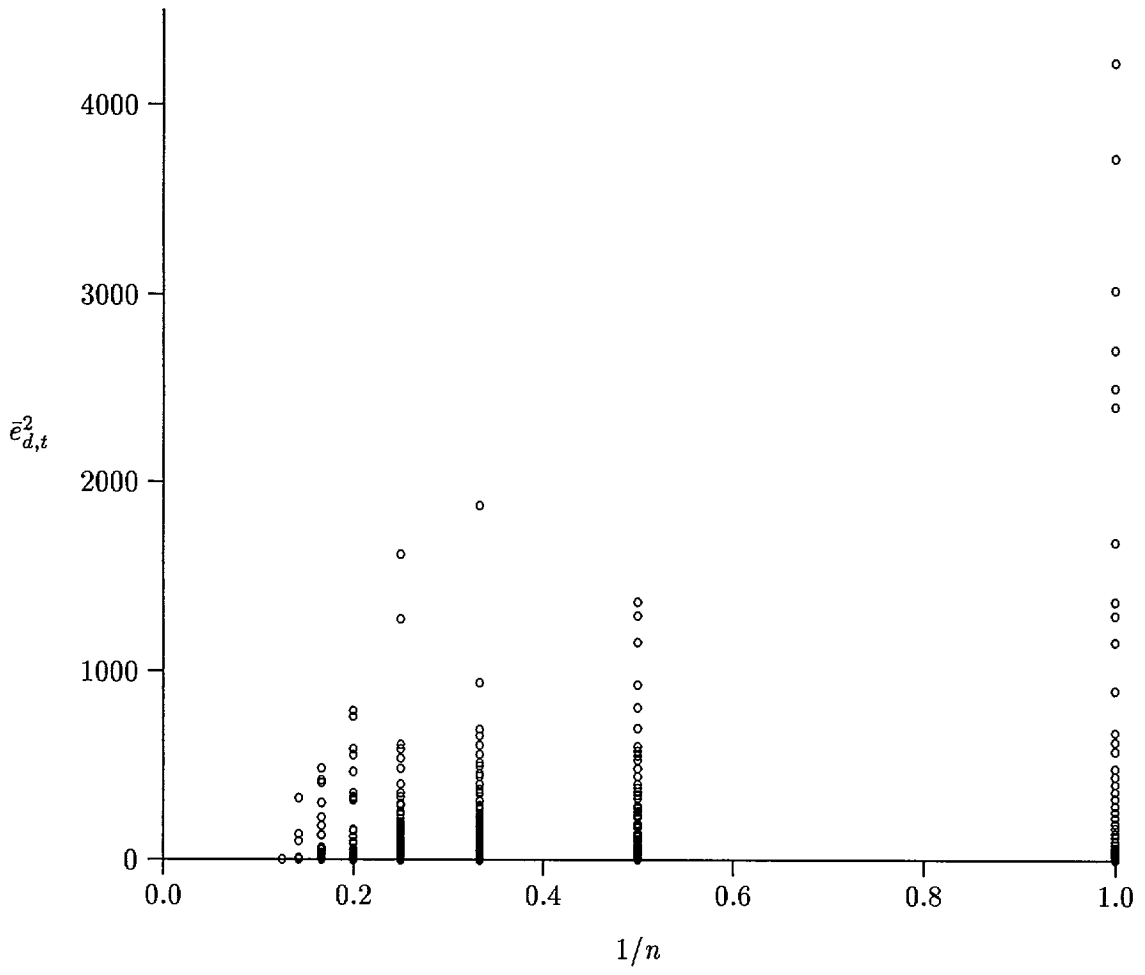

Figure 7: Plot of $[\bar{e}_{d,t}]^2$ against $1/n$ for Link 1 during the Peak Period

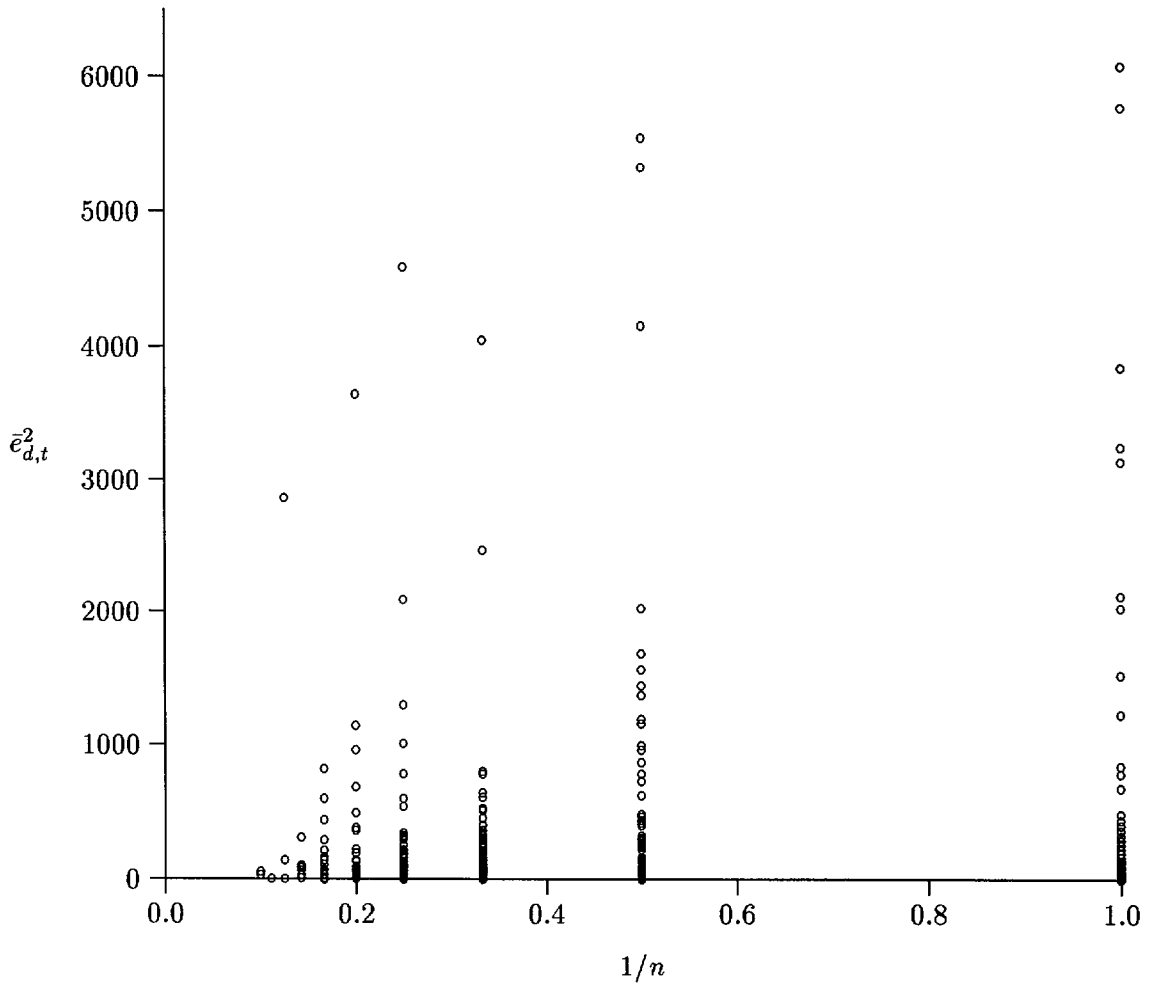

Figure 8: Plot of $[\bar{e}_{d,t}]^2$ against $1/n$ for Link 11 during the Peak Period

α_d and β_t being common to different $\bar{e}_{d,t}$'s. The model, (2), is based on a mathematical identity. Therefore, there would appear to be no significant violations to the Gauss-Markov Conditions *under the hypothesis*, save for the presence of outliers. The presence of outliers would only tend to increase rather than decrease standard errors. If there is any slight heteroscedasticity left that too, by violating the minimum variance property of least squares estimates, would only tend to raise standard errors. Thus the test we conducted (using t-values to examine the size of the intercept) is an appropriate test, albeit perhaps lacking somewhat in power; that is, we would err on the side of accepting rather than rejecting H. Notice further that, given the large sample sizes, it is appropriate to assume that the estimate of the intercept is approximately normally distributed (Sen and Srivastava, 1990, Ch. 5), although the dependent variable values might not be — they are in fact highly skewed.

Column 6 in Tables 4 and 5 shows the t-values for $\hat{\nu}$ for various links for the peak and off-peak periods respectively. The $\hat{}$ symbol denotes an estimate ($\hat{\eta}$ is an estimate of η), s.e. and t refer to standard errors and t-values respectively. The two tables also give $\hat{\nu}$, $\widehat{\eta - \nu}$, and their standard errors and $\hat{\eta}$ along with the total sample size (obs.) and $\bar{x}_\ell$, or the mean travel times for all links ℓ that were analyzed. A description of the data has been given in Section 3 and the time interval over which means were computed was 5 minutes. It is easily seen that all estimates of ν are positive and are significant at the 5% level or better, with the exception of Links 5, 6 and 8 in the peak period and Links 2, 5 and 6 in the off-peak period.

Link	mean ($\bar{x}_\ell$)	obs.	$\hat{\nu}$	s.e.	t	$\widehat{\eta - \nu}$	s.e.	t	$\hat{\eta}$
1	69.73	1233	102.77	26.04	3.95	246.88	73.17	3.37	349.65
2	59.29	1242	93.36	40.06	2.33	518.30	119.67	4.33	611.66
31	64.90	886	228.16	71.25	3.20	553.98	171.28	3.23	782.14
32	39.14	308	113.69	44.56	2.55	55.95	79.93	0.70	169.64
4	114.47	320	440.99	162.43	2.72	298.12	236.11	1.26	739.11
5	38.20	327	36.55	61.15	0.60	44.97	87.53	0.51	81.70
6	53.63	321	100.13	58.03	1.73	124.56	121.29	1.03	224.69
7	196.3	309	966.11	248.69	3.89	24.26	332.95	0.07	990.37
8	85.36	297	30.50	94.02	0.32	804.41	209.61	3.89	834.91
9	167.98	355	1082.86	228.85	4.73	172.97	325.73	0.53	1255.83
10	81.38	1253	102.98	50.18	2.05	479.67	91.35	5.26	582.66
11	47.55	1275	122.25	45.96	2.67	279.56	82.75	3.38	401.91
12	88.83	1260	250.41	46.20	5.42	249.33	82.94	3.01	499.74

Table 4: Estimates from Day/Time-of-Day Model for the Peak Period

The situation in Link 5 can be easily explained; it is a very lightly traveled link with no traffic control. On this link, a vehicle rarely affects the travel time of another vehicle. Link 6 is also very lightly traveled but has a stop sign. Of course, for neither was there

Link	mean ($\bar{x}_\ell$)	obs.	$\hat{\nu}$	s.e.	t	$\widehat{\eta - \nu}$	s.e.	t	$\hat{\eta}$
1	69.86	1746	56.98	20.98	2.72	287.83	63.74	4.52	344.81
2	47.40	1728	31.23	28.56	1.09	615.80	88.62	6.95	647.03
31	60.43	627	205.93	64.70	3.18	412.19	113.45	3.63	618.12
32	31.56	1028	39.61	19.15	2.07	140.42	53.38	2.63	180.03
4	93.08	1048	346.53	74.06	4.68	258.09	126.69	2.04	604.62
5	36.29	1063	20.81	47.22	0.44	56.24	94.46	0.60	77.05
6	44.78	1059	11.15	6.24	1.79	57.62	17.18	3.35	68.77
7	96.22	1068	516.91	91.08	5.68	494.69	177.27	2.79	1011.60
8	58.59	961	77.73	29.42	2.64	163.45	48.51	3.37	241.17
9	65.72	1042	277.22	58.40	4.75	282.80	114.66	2.47	561.00
10	58.86	1680	203.63	53.27	3.82	623.81	89.47	6.97	827.44
11	50.98	1712	148.54	43.27	3.43	546.22	75.20	7.26	694.76
12	75.18	1588	159.53	31.35	5.09	170.94	55.32	3.09	330.47

Table 5: Estimates from Day/Time-of-Day Model for the Off-Peak Period

any effect from traffic signals. Link 8 is more difficult to explain, particularly because the lack of significance occurs only for the peak. However, a partial explanation is that all the probe vehicles that were the source of the data entered the link via a right turn. Because traffic during the peak period on the major westbound route is heavy, most vehicles would execute the turn on green, and would, therefore, often encounter sparse traffic until they reached the end of the link. This is particularly the case during the peak when progression was excellent.

With the exception of the links and time periods mentioned above, the t-values were always large indicating that the hypothesis of no correlation and of $\nu = 0$ may be rejected for them.

4.3 Estimation of Parameters

The least squares procedure we used above to test for lack of correlations is also a reasonable method for estimating η and ν. One advantage of treating average covariances and variances as parameters in a regression model is that we do not have to directly measure them. Direct measurement would be complicated because, owing to the presence of traffic signals, the mean link travel time at any instant would be very difficult to obtain without using additional information, such as on signal timing.

4.4 Implications of Dependence

Although, we have examined only a small number of links, we would expect (2) with $\nu > 0$ to be true for well-traveled, signalized links in general. In this section, we draw

some implications of this result for probe-based ATIS. Figures 9 and 10 show the relationship between the standard error of mean link travel times *computed using model (2) and parameter values from Table 4*, for Links 32 and 11, against n, the number of probes during a five-minute interval. Thus, while figures comparable to Figures 9 and 10 would vary from link to link, we would expect these figures to illustrate curves of generally the same shape provided the links were well-traveled and signalized.

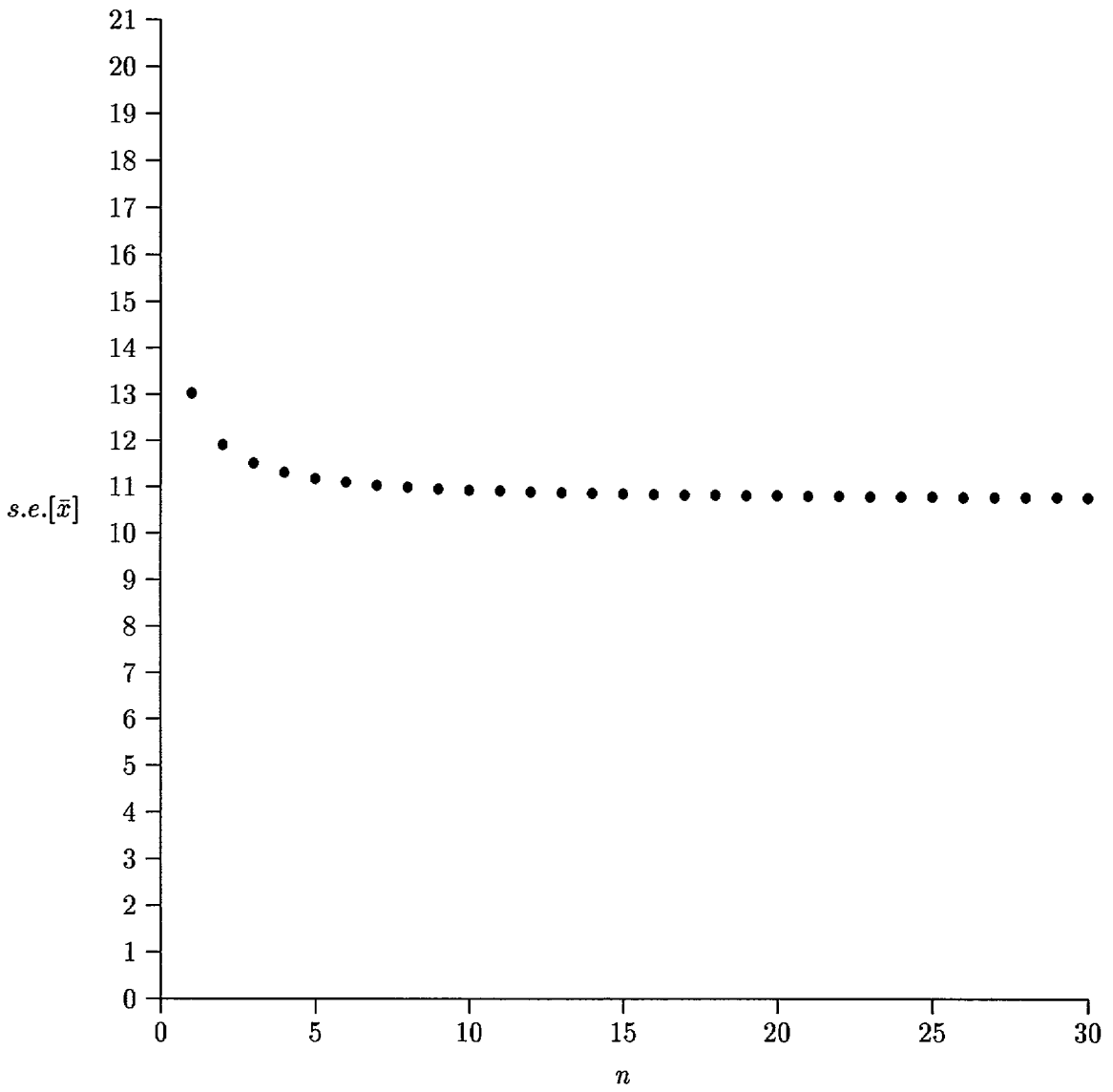

Figure 9: Relationship between Standard Error $s.e.[\bar{x}]$ and Frequency n of Probes on Link 32 during the Peak Period Estimated from the Model $[\bar{e}_{d,t}]^2 = \nu + \gamma_1[1/n_{d,t}] + \epsilon_{d,t}$.

Notice from Figure 9, the plot for Link 32 during the peak period, that the points flatten out rapidly with increasing n. For $n \to \infty$, the standard errors would approach $\hat{\nu} = 10.66$. Therefore, if we are willing to accept a roughly 10% increase in standard

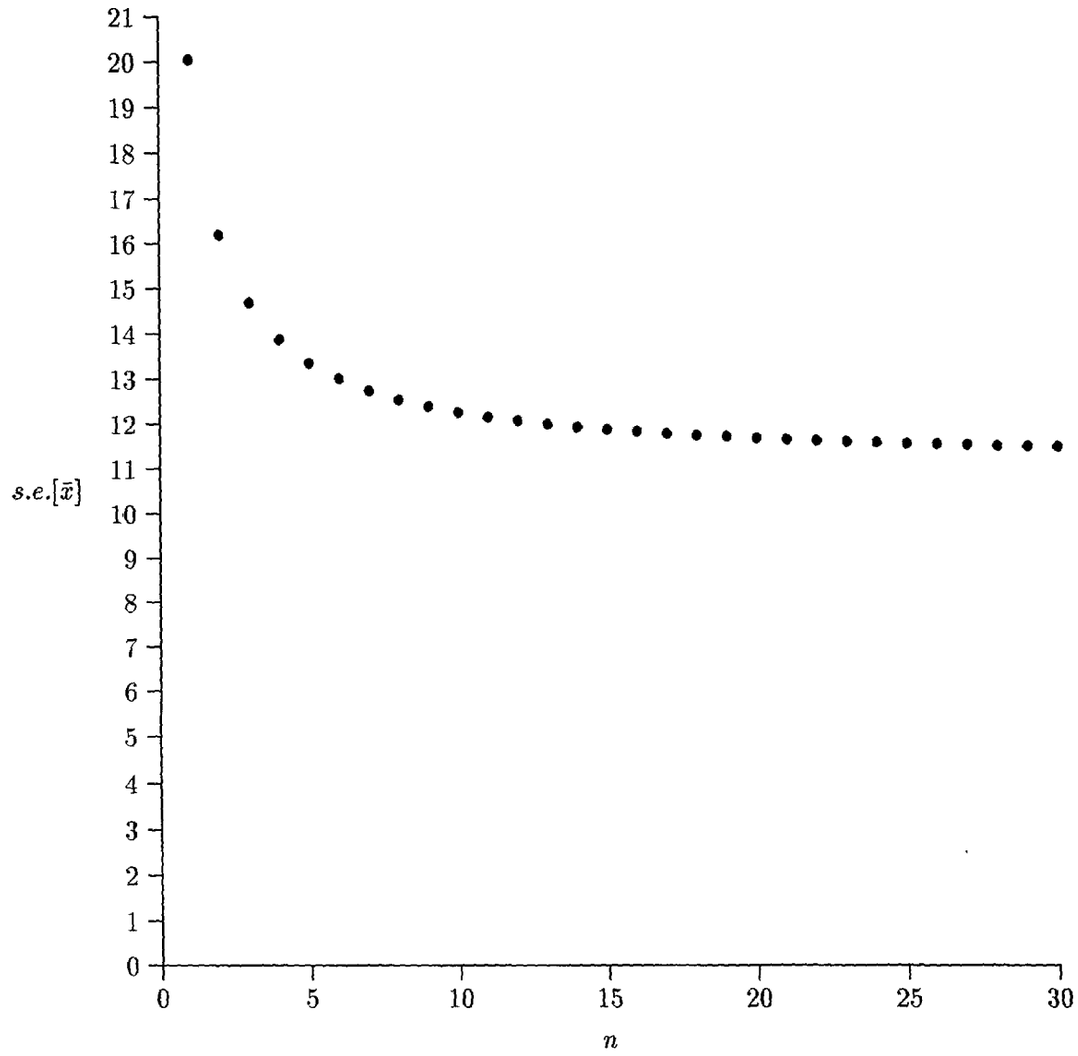

Figure 10: Relationship between Standard Error $s.e.[\bar{x}]$ and Frequency n of Probes on Link 11 during the Peak Period Estimated from the Model $[\bar{e}_{d,t}]^2 = \nu + \gamma_1[1/n_{d,t}] + \epsilon_{d,t}$.

errors, we could get by with two observations and if we accept a 20% increase in standard error, even a single observation could suffice.

The rate of flattening depends on the relative sizes of $\hat{\nu}$ and $\hat{\eta} - \hat{\nu}$ and we chose Link 32 to illustrate a case where $\hat{\eta}$ was somewhat close to $\hat{\nu}$. Consider now the case of Link 11 for the peak period (Figure 10). Here the ratio of $\widehat{\eta - \nu}$ to $\hat{\nu}$ is considerably larger than for Link 32. For Link 11, then, a 10% increase in standard errors is achieved with about 10 observations, and a 20% increase with 5 observations.

Usually highly-traveled links have high correlations between travel times of vehicles. Therefore, under situations of high congestion the sample sizes required would be smaller. Thus, ATIS would be of major benefit in recurrent congestion situations because, when congestion is low, travel times would tend to be closer to historic averages (suitably conditioned on day-of-week and time-of-day considerations) and, when congestion is high, even low deployment levels would usually achieve acceptable probe frequencies because of higher overall traffic volume. For example, suppose we have a moderately-high congestion level represented by 1500 vehicles per hour for a two-lane arterial or 125 vehicles per five-minute period. Then a 1% deployment level would give us 1 vehicle per five minutes on average and a 5% deployment would give us 6 vehicles.

These observations coupled with the discussion in Section 3 lead us to the statements we made at the end of the Introduction.

- the deployment level originally planned for full deployment would have been adequate to provide fairly good dynamic information.
- deployment levels four or five times higher than originally planned would yield significantly better informtaion, above this level information would continue to improve with deployment level but the improvements would be slight.

Two additional conclusions also emerge:

- The variance of mean link travel times remains quite far from zero no matter how many probes traverse the link.
- After deployment of a certain number of probes per unit time, additional probes do not decrease the variance very much.

High levels of deployment would of course be necessary to cover and monitor a wider area of the network. However, the second conclusion above suggests that very high levels of market penetration by probe-based ATIS may be unnecessary in order to improve estimates of link travel times. Methods must therefore be devised which take the variance of link travel times explicitly into account in any sample size computation for probe-based systems.

5 Recommendations

From the discussion above and that of Section 3, we make the following judgment: the deployment level originally planned for full deployment would have been adequate to provide fairly good dynamic travel-time information. Deployment levels four or five times higher would yield significantly better information. Above that, while information quality would continue to improve with deployment, the improvements would be slight.

In future efforts involving probe-based ATIS, we strongly recommend that

- Exit times of probes from each link traversed be accurately noted
- Times at which phase changes occur at each traffic signal be made available to travel time prediction algorithm.

Although, since no such data were available, numerical estimates cannot be provided, we expect that with these additions extremely high quality travel-time estimates can be produced, even with relatively low deployment levels.

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